Chapter 12

Kinetic Theory of Gases

Solutions (Set-1)

Very Short Answer Type Questions :

1. Can we increase the temperature of a gas keeping its pressure and volume constant?

Sol. No, temperature cannot be changed without changing pressure and volume.

2. The absolute temperature of a gas is increased 4 times. What will be the change in the rms velocity of the gas molecules?

Sol. $v_{rms} \propto \sqrt{T}$

New rms velocity is $2v_{rms}$

So change = $2v_{rms} - v_{rms} = v_{rms}$

3. What will be the ratio of translational kinetic energy of hydrogen and nitrogen molecules at the same temperature?

Sol. One, because the kinetic energy per molecule of the gas depends only upon the temperature.

4. What will be the ratio of the rms velocities of the molecules of two gases *A* and *B* if their vapour densities are in the ratio 9 : 16?

Sol. $\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{9}{16}} = 3:4$

5. Calculate the root mean square velocity of a given sample having three molecules having velocity 1, 2 and 3 m/s.

Sol. $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}} = \sqrt{\frac{1+4+9}{3}} = \sqrt{\frac{14}{3}}$ m/s

6. Define degree of freedom.

Sol. The term degree of freedom of a system refers to the possible independent motions a system can have or number of possible independent ways in which system can have energy.

7. Give the dependence of mean free path of molecules of a gas on number density and size of molecule.

Sol. Mean free path (l) is given by

$$l = \frac{1}{\sqrt{2}n\pi d^2}$$

where n is no. density and d is the diameter of the gas molecule.

8. If the temperature of a gas increases, what will happen to its mean square speed?

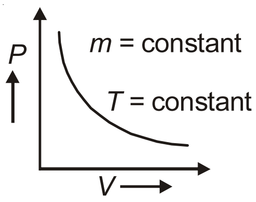
Sol. If the temperature increases, the mean square speed of gas molecules will increase as $\bar{v}^2 \propto T$.

9. How many degrees of freedom a diatomic gas molecule can have?

Sol. A diatomic gas have 5 degree of freedom, 3 translational and 2 rotational. If diatomic molecule is not rigid then due to vibratory motion 2 degree of freedom will be added. *i.e.*, 7 degree of freedom.

10. If temperature and mass of a gas are kept constant, draw variation of P(pressure) with V(volume).

Sol.



Short Answer Type Questions :

11. On the basis of kinetic theory of gases, explain how the temperature of a gas rises on heating.

Sol. When heat is given to a gas, the rms velocity of the gas molecules increases. As $v_{rms} \propto \sqrt{T}$, so the temperature of the gas increases.

12. On increasing the temperature of a gas filled in a cylinder, the pressure of the gas increases. Explain.

Sol. On raising the temperature, the average velocity of the gas molecules increases. As a result of which more and more molecules collide with the wall of the cylinder per second. Hence, greater momentum is transferred to the wall per second and the pressure increases.

13. Explain, on the basis of kinetic theory, how the pressure of a gas changes if its volume is reduced at constant temperature.

Sol. On reducing the volume, the space for the given number of molecules of the gas decreases *i.e.*, number of molecules per unit volume increases. As a result of which more molecules collide with the walls of the container per second transferred more momentum to the walls. Hence, the pressure increases.

14. Distinguish between average speed and rms speed.

Sol. Average speed is the arithmetic mean of the speeds of the molecules where as rms speed is the root mean square speed *i.e.*, the square root of the mean of the squares of different speeds of the individual molecules.

15. State Boyle's law.

Sol. According to Boyle's law, when the temperature of a certain mass of a gas is kept constant, the volume V occupied by the gas is inversely proportional to the pressure P exerted by the gas *i.e.*,

when T is constant, $V \propto \frac{1}{P}$ or $V = \frac{\text{constant}}{P}$

$$PV = \text{constant}$$

16. State Charles's law.

Sol. According to this law, when pressure P of a certain mass of a gas is kept constant, the volume V occupied by the gas is directly proportional to the temperature T of the gas, *i.e.*, when P is constant

$V \propto T$

$\boxed{\frac{V}{T} = \text{constant}}$

17. Two vessels A and B are filled with the same gas, where volume, temperature and pressure in A is twice the volume, temperature and pressure in B. Calculate the ratio of the number of molecules of gas in A and B.

Sol. $PV = nRT$

$$\Rightarrow \quad n = \frac{PV}{RT}\text{, number of molecules} = n_A N$$

$$\frac{P_A V_A}{T_A} = n_A R, \quad \frac{P_B V_B}{T_B} = n_B R$$

$$\frac{n_A}{n_B} = \frac{P_A V_A}{T_A} \times \frac{T_B}{P_B V_B} = \frac{2P \times 2V}{2T} \times \frac{T}{PV} = \frac{2}{1} = 2:1$$

18. On what factor the average kinetic energy of gas molecules depend? What will the kinetic energy of the molecules at the absolute zero?

Sol. The average kinetic energy of gas molecules depends upon the temperature of the gas. At absolute zero, rms velocity of the gas molecules becomes zero so mean K.E. per molecules of the gas becomes zero at absolute zero.

19. A gas in a cylinder is at pressure P. If the masses of all the molecules be made one-third and their speeds be made doubled, then find the resultant pressure.

Sol. $P = \frac{1}{3}\frac{mnv_{\text{rms}}^2}{V}$

$$m' = \frac{m}{3}, \quad v'_{\text{rms}} = 2v_{\text{rms}}$$

$$P' = \frac{1}{3} \times \frac{\left(\frac{m}{3}\right) n \times (2v_{\text{rms}})^2}{V} = \frac{4P}{3}$$

$$\boxed{P' = \frac{4}{3}P}$$

20. A vessel contains a gas at pressure P_0. Show that the pressure becomes double, if the masses of all the molecules are halved and their speeds doubled.

Sol. $P = \frac{1}{3}\frac{mn}{V}v_{\text{rms}}^2$

m is halved and v_{rms} is doubled. Hence, P will become two times.

$$P' = \frac{1}{3} \times \frac{\left(\frac{1}{2}m\right) n \times (2v_{\text{rms}})^2}{V}$$

$$\boxed{P' = 2P_0}$$

Hence proved.

21. The root mean square velocity of the gas molecules is 400 m/s. If the atomic weight is doubled and absolute temperature is halved then, calculate the root mean square speed of the gas molecules.

Sol. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

$v'_{rms} = \sqrt{\dfrac{3RT'}{M'}}$

$T' = \dfrac{T}{2}$, $M' = 2M$

$\therefore \quad v'_{rms} = \sqrt{\dfrac{3RT}{4M}} = \dfrac{1}{2}v_{rms} = \dfrac{1}{2}\times 400 = 200$ m/s

22. A vessel contains N molecules of a gas at absolute temperature T. If the number of molecules is halved, what should be the absolute temperature so as the kinetic energy of the molecules remains the same.

Sol. $\text{K.E.} = \dfrac{3}{2}nkT$

when the number of molecules is halved. K.E. becomes double so, to keep the K.E. same temperature should be doubled.

23. Show that the r.m.s. velocity of oxygen is $\sqrt{2}$ times that of sulphur dioxide.

Sol. $\dfrac{(v_{r.m.s.})_{O_2}}{(v_{r.m.s.})_{SO_2}} = \sqrt{\dfrac{M_{SO_2}}{M_{O_2}}}$ Atomic weight of sulphur is 32 and that of oxygen is 16.

$= \sqrt{\dfrac{32+32}{32}}$

$= \sqrt{2}$

or $(v_{r.m.s.})_{O_2} = \sqrt{2}(v_{r.m.s.})_{SO_2}$

24. Calculate the average kinetic energy of a oxygen molecule (in ergs), if it occupies a volume of 2 × 10^4 cc at a pressure of 10^5 dyne/cm^2. (use N_0 = 6 × 10^{23})

Sol. Average KE $= \dfrac{1}{2}mv_{rms}^2$

$= \dfrac{1}{2}m\cdot\dfrac{3PV}{M}$

$= \dfrac{1}{2}m\cdot\dfrac{3PV}{mN_A} = \dfrac{1}{2}\dfrac{3PV}{N_A}$

$= \dfrac{1}{2}\times\dfrac{3\times 10^5\times 2\times 10^4}{6.023\times 10^{23}}$

$= 0.5 \times 10^{-14}$ erg.

25. The average kinetic energy of a gas molecule at 27°C is 3.3×10^{-21} J. Calculate its average kinetic energy at 227°C.

Sol. $\text{K.E.} = \frac{3}{2}k_B T$

at 27°C $\Rightarrow T = 300$ K

at 227°C $\Rightarrow T = 500$ K

$$\frac{\text{KE}_1}{\text{KE}_2} = \frac{T_1}{T_2}$$

$$\text{KE}_2 = \frac{\text{KE}_1 \times T_2}{T_1} = \frac{3.3 \times 10^{-21} \times 500}{300} = 5.5 \times 10^{-21} \text{ J}$$

26. Mention the condition when a real gas obeys ideal gas equation and why?

Sol. At low pressure and high temperature, the molecules are farther apart so that molecular size is negligible as compared to the size of the container.

27. Two perfect gases at temperature T_1 and T_2 are mixed. If there is no loss of energy and the masses and number of molecules of the gases are m_1, m_2 and n_1, n_2 respectively. Calculate the temperature of the mixture.

Sol. $\text{K.E.}_1 = n_1 \frac{3}{2}kT_1$

$$\text{K.E.}_2 = n_2 \frac{3}{2}kT_2$$

Total K.E. $= (n_1 + n_2)\frac{3}{2}kT$

Total K.E. = K.E.$_1$ + K.E.$_2$

$$(n_1 + n_2)\frac{3}{2}kT = n_1 \frac{3}{2}kT_1 + n_2 \frac{3}{2}kT_2$$

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

28. A gas is filled in a cylinder fitted with a piston at a definite temperature and pressure. Explain on the basis of kinetic theory why on pulling the piston out; the pressure of gas decreases.

Sol. On pulling the piston out, the volume of gas increases. Due to this, less number of molecules collide with the walls of cylinder per second and hence less momentum is transferred to the walls per second. Also, area of the walls on which these collisions take place increases. Due, to this pressure decreases.

29. Two vessels *A* and *B* are identical. *A* has 1 gm of hydrogen at 0°C and *B* has 1 gm oxygen at 0°C.

(i) Which vessel does contain more molecules and how much?

(ii) In which vessel the pressure of the gas is higher and how much?

Sol. (i) Vessel *A* contains 16 times more (hydrogen) molecules contained in *B*, because the number of molecules in 1 gm hydrogen is 16 times the number of molecules in 1 gm oxygen.

(ii) At a given temperature, the average kinetic energy per molecule $\left(\frac{3}{2}kT\right)$ is independent of the nature of the gas. Now, from the gas equation $PV = nKT$ for n molecules.

Thus, for a given volume, the gas pressure is proportional to the number of molecules. Hence, pressure in *A* will be 16 times higher than that in *B*.

30. State law of equipartition of energy and hence calculate molar specific heats of mono, di and triatomic gases at volume and constant pressure.

Sol. According to law of equipartition of energy $\frac{1}{2}k_BT$ energy is associated with every degree of freedom of a gas molecule, where k_B is Boltzmann constant and T is temperature of the system.

(a) Specific heat capacity of monoatomic gases

In case of mono-atomic gases, a molecule has 3 degrees of freedom.

Average energy associated with three degrees of freedom = $\frac{3}{2}k_BT$.

The total energy of one gram mole of the monoatomic gas $U = \frac{3}{2}k_BT \times N_A = \frac{3}{2}RT$

but $C_v = \frac{dU}{dT}$. Hence $C_v = \frac{d}{dT}\left(\frac{3}{2}RT\right) = \frac{3}{2}R$.

We know that $C_p - C_v = R$

Hence $C_p = R + C_v = \frac{3}{2}R + R = \frac{5}{2}R$

(b) In case of diatomic gases, a molecule is treated like a rigid rotator. It has 5 degrees of freedom, 3 translational and 2 rotational.

Similarly as in case of mono-atomic gases

$$U = 5\times\left(\frac{1}{2}k_BT\right)\times N_A = \frac{5}{2}RT$$

$$C_v = \frac{dU}{dT} = \frac{5}{2}R$$

$$C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R$$

If diatomic gas is not rigid rotator, it also has one vibrational mode, hence 7 degrees of freedom. With vibrational mode, 2 degrees of freedom are associated.

Hence $U = \left(\frac{5}{2}k_BT + k_BT\right)\times N_A = \frac{7}{2}RT$

$$C_v = \frac{dU}{dT} = \frac{7}{2}R$$

$$C_p = C_v + R = \frac{9}{2}R.$$

(c) For triatomic gases

(i) In case of linear triatomic gases, there are seven degrees of freedom

$$U = 7\times\frac{1}{2}k_BT \times N_A = \frac{7}{2}RT$$

$$C_v = \frac{dU}{dT} = \frac{7}{2}R$$

$$C_p = C_v + R = \frac{7}{2}R + R = \frac{9}{2}R$$

(ii) In case of non-linear triatomic gas molecules, there are six degrees of freedom,

Hence $U = 6 \times \frac{1}{2} k_B T \times N_A = 3RT$

$$C_v = \frac{dU}{dT} = \frac{d}{dT}(3RT) = 3R$$

$$C_p = C_v + R = 3R + R = 4R$$

Long Answer Type Questions :

31. What is an ideal gas? Under what conditions of pressure and temperature can a gas be assumed as an ideal gas?

Sol. Ideal gas is a gas which strictly obey gas laws. For such a gas, the size of the molecules of a gas is zero and there is no force of attraction or repulsion amongst its molecules. The ideal gas equation connecting pressure (P), volume (V) and absolute temperature (T) is given by

$$PV = \mu RT = k_B NT$$

where μ is the number of moles and N is the number of molecules. R and k_B are constants

$R = 8.314$ J mol^{-1} K^{-1}, $k_B = \frac{R}{N_A} = 1.38 \times 10^{-23}$ JK^{-1}

Real gases satisfy the ideal gas equation only at low pressures and high temperatures only approximately.

32. Give the postulates of kinetic theory of gases and on its basis, find the expression for the pressure of an ideal gas.

Sol. Assumptions of kinetic theory of gases

(i) A gas consists of a large number of identical, tiny, spherical, neutral and elastic particles called molecules.

(ii) In a gas molecules are moving in all possible directions with all possible speeds in accordance with Maxwell's distribution law.

(iii) The space occupied by the molecules is much smaller than the volume of the gas.

(iv) There is no force of attraction among the molecules.

(v) The pressure of gas is due to elastic collision of gas molecules with the walls of the container.

(vi) The time of contact of a moving molecules with the walls of the container is negligible as compared to the time interval between two successive collisions on the same wall of the container.

Pressure P exerted by an ideal gas is given by

$$P = \frac{1}{3}\frac{mN}{V}\overline{v^2}$$

where $\overline{v^2}$ is the mean square velocity and m is the mass of each molecule.

$$\overline{v^2} = \left[\frac{v_1^2 + v_2^2 + \ldots}{N}\right]$$

N is the total number of molecules in the vessel having volume V.

33. Assuming the relation $P = \frac{1}{3}\rho\bar{v}^2$ of kinetic theory, prove that the average kinetic energy of a molecule of an ideal gas is directly proportional to the absolute temperature of the gas.

Sol. The average kinetic energy of 1 gm molecule of the gas

$$= \frac{1}{2}M\overline{v}^2 = \frac{1}{2}M\left(\frac{3P}{\rho}\right) \qquad \left[\text{Using, } P = \frac{1}{3}\rho\overline{v}^2\right]$$

$$\Rightarrow \quad \text{K.E.} = \frac{3}{2}M\left(\frac{RT}{\rho V}\right) = \frac{3}{2}RT \qquad \left[\rho = \frac{M}{V}\right]$$

There are N molecules in 1 gram molecule of the gas. Hence the average kinetic energy of 1 molecule

$$= \frac{\frac{3}{2}RT}{N} = \frac{3}{2}\left(\frac{R}{N}\right)T = \frac{3}{2}kT$$

where $k = \frac{R}{N}$ is Boltzmann constant. Thus energy $\propto T$.

34. Write the formula for the pressure of an ideal gas in terms of molecular mass, number of molecules and their velocity on the basis of kinetic theory and with the help of it, establish the relation between kinetic energy of the molecules and temperature of the gas.

Sol. The pressure exerted by an ideal gas is given by

$$P = \frac{1}{3}nm\overline{v^2}$$

Multiplying both sides by V

$$PV = \frac{1}{3}nmV\overline{v^2}$$

$$PV = \frac{1}{3}Nm\overline{v^2} \qquad \ldots\text{(i)}$$

where $N = (nV)$ is the number of molecules in the sample.

The internal energy E of an ideal gas is purely kinetic

$$E = N\frac{1}{2}mv^2 \qquad \ldots\text{(ii)}$$

Using equation (ii) in (i), we get

$$PV = \left(\frac{2}{3}\right)E \qquad \ldots\text{(iii)}$$

Comparing equation (iii) with the ideal gas equation, we get

$$\frac{2}{3}E = Nk_BT$$

$$E = \frac{3}{2}Nk_BT$$

or $$\frac{E}{N} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT \qquad \ldots\text{(iv)}$$

35. Two vessels A and B are identical. A has 1 gm hydrogen at 0°C and B has 1 gm nitrogen at 0°C.

(i) Which vessel does contain more molecules and how much?

(ii) In which vessel is the pressure of the gas higher and how much?

(iii) In which vessel is the average speed of molecules larger and how much?

Sol. (i) Vessel A contains 14 times more (hydrogen) molecules than the nitrogen molecules contained in vessel B, because the number of molecules in 1 gm of hydrogen is 14 times the number of molecules in 1 gm oxygen.

(ii) At a given temperature, the average kinetic energy per molecule $\left(\frac{3}{2}k_BT\right)$ is independent of the nature of the gas. Now, from the ideal gas equation $PV = nk_BT$ for n molecules, the pressure of the gas is

$$P = \frac{nk_BT}{V}$$

Thus, for a given volume, the gas pressure is proportional to the number of molecules. Hence, pressure in A will be 14 times higher than that in B.

(iii) At a given temperature $v_{rms} \propto \frac{1}{\sqrt{M}}$, where M is molecular mass of the gas. Since, the molecular mass of hydrogen is $\left(\frac{1}{14}\right)$th the molecular mass of oxygen, the rms speed of hydrogen in vessel A is $\sqrt{14}$ times larger than that of nitrogen in vessel B.

36. A vessel contains two non-reacting gases, neon (monatomic) and oxygen (diatomic). The ratio of their partial pressures is 3 : 2. Find the ratio of (a) number of molecules, and (b) mass density of Ne and O_2 in the vessel. The atomic mass of Ne is 20.2 and the molecular mass of O_2 is 32.0.

Sol. The partial pressure of a gas in a mixture of gases filled in a vessel is that pressure which the gas would have if it alone occupied the whole vessel at the same temperature. (The total pressure of a mixture of non-reacting gases is the sum of partial pressures of its constituent gases).

Since, V and T are common for the gases neon and oxygen (assumed ideal), we can write

$$P_1V = \mu_1RT \text{ and } P_2V = \mu_2RT$$

where P_1 and P_2 are the partial pressure, μ_1 and μ_2 the number of moles of neon and oxygen respectively in the vessel. Thus,

$$\frac{P_1}{P_2} = \frac{\mu_1}{\mu_2} \text{ as } \frac{P_1}{P_2} = \frac{3}{2} \text{ (given)}$$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

(a) If n_1 and n_2 be the number of molecules of neon and oxygen respectively, and N be the Avogadro number, then by definition, we have

$$\mu_1 = \frac{n_1}{N} \text{ and } \mu_2 = \frac{n_2}{N}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

(b) If m_1 and m_2 be the masses, and M_1 and M_2 the molecular masses of neon and oxygen respectively, then we have

$$\mu_1 = \frac{m_1}{M_1} \text{ and } \mu_2 = \frac{m_2}{M_2}$$

$$\frac{\rho_1}{\rho_2} = \frac{m_1/v}{m_2/v} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{M_1}{M_2}\right) = \frac{3}{2} \times \frac{20.2}{32.0} = 0.947$$

37. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder.

(R = 8.3 J mol^{-1} K^{-1}; molecular mass M of O_2 = 32)

Sol. Let P_1 and T_1 be the initial pressure and absolute temperature of oxygen in the cylinder, and P_2 and T_2 their values after some oxygen is withdrawn. Let μ_1 the number of moles of oxygen initially present in the cylinder and μ_2 the number of moles left over. The volume V (say) remains unchanged. Now, writing gas equation, we have

$$P_1V = \mu_1RT_1$$

and $P_2V = \mu_2RT_2$

Here, P_1 = 15 atm = 15 × 1.01 × 10^5 Nm^{-2}, P_2 = 11 × 1.01 × 10^5 Nm^{-2}, T_1 = 27 + 273 = 300 K,

T_2 = 17 + 273 = 290 K, V = 30 litre = 30 × 10^{-3} m^3 and R = 8.3 J mol^{-1} K^{-1}. Therefore,

$$\mu_1 = \frac{P_1V}{RT_1} = \frac{(15\times1.01\times10^5)\times(30\times10^{-3})}{8.3\times300} = 18.25$$

and $$\mu_2 = \frac{P_2V}{RT_2} = \frac{(11\times1.01\times10^5)\times(30\times10^{-3})}{8.3\times290} = 13.84$$

∴ Number of moles of oxygen taken out of the cylinder is

$$\mu_1 - \mu_2 = 18.25 - 13.84 = 4.41$$

The molecular mass of O_2 is 32 g.

Hence, the mass of oxygen taken out of the cylinder is

$$m = (\mu_1 - \mu_2)M = 4.41 \times 32 \text{ g} = 141.12 \text{ g}$$

38. A vessel A contains hydrogen and another vessel B whose volume is twice of A contains same mass of oxygen at the same temperature. Compare

(i) Average translational kinetic energies of hydrogen and oxygen molecules,

(ii) Root-mean square speeds of the molecules,

(iii) Pressures of gases in A and B.

(Molecular weights of hydrogen and oxygen are 2 and 32 respectively).

Sol. (i) For all gases at the same temperature, the kinetic energy per molecule is the same $\left(\frac{3}{2}k_BT\right)$. Since, gases in both vessels are at the same temperature, the average kinetic energy per molecule is the same (1 : 1).

(ii) We know that $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\therefore \quad \frac{(v_{\text{rms}})_H}{(v_{\text{rms}})_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{32}{2}} = 4:1$$

(iii) According to the kinetic theory, we have

$$P = \frac{1}{3}\frac{mn}{V}\overline{v^2} = \frac{1}{3}\frac{M}{V}\overline{v^2}$$

Masses of both the gases hydrogen and oxygen are equal.

Hence, the ratio of pressures is

$$\frac{P_H}{P_O} = \frac{\overline{v_H^2}}{\overline{v_O^2}} \times \frac{V_O}{V_H} = \frac{(v_{rms}^2)_H}{(v_{rms}^2)_O} \times \frac{V_O}{V_H} = \frac{16}{1} \times \frac{2}{1} = 32:1$$

39. What do you mean by the degree of freedom and law of equipartition of energy?

Sol. Degree of freedom of a system refers to the possible independent motions a system can have or number of possible independent ways in which a system can have energy.

(i) Monoatomic molecule has 3 degrees of freedom all translational.

(ii) A diatomic molecule has 5 degrees of freedom 3 translational and 2 rotational.

(iii) A diatomic molecule that is free to vibrate will have 7 degrees of freedom (3 + 2 + 2).

(iv) A nonlinear polyatomic molecule has 6 degrees of freedom 3 translational and 3 rotational.

The law of equipartition of energy states that if a system is in equilibrium at absolute temperature T, the total energy is distributed equally in different energy modes of absorption. The energy in each mode being equal to $\frac{1}{2}k_BT$.

Each translation and rotational degree of freedom corresponds to one energy mode of absorption and has energy $\frac{1}{2}k_BT$. Each vibrational frequency has two modes of energy with corresponding energy equal to $2 \times \frac{1}{2}k_BT = k_BT$.

40. Explain the concept of mean free path.

Sol. The size of gas molecules is very small and they are undergoing collisions, due to which their motion is not straight or we can say that the path of molecules keep on getting deflected due to collisions. Suppose the molecules of a gas are spheres of diameter d.

Let us focus on a single molecule having average speed $<v>$. It will suffer collision with any molecule that comes with in a distance d, (between the centres).

In Δt time the volume sweeps by the molecule is given by

Volume = Area × distance travelled by the molecule in Δt time

$$V = \pi d^2 <v> \Delta t \qquad \ldots(i)$$

Equation (i) gives the volume in which any other molecule can collide with it. If n is number of molecules per unit volume, then the molecule will suffer $n\pi d^2 <v> \Delta t$ collisions in Δt time. The rate of collision is

$$\frac{n\pi d^2 <v> \Delta t}{\Delta t} = n\pi d^2 <v>$$

and the time (τ) between two successive collision is given by

$$\tau = \frac{1}{(n\pi <v> d^2)} \qquad \ldots(ii)$$

The average distance between two successive collisions, called the mean free path (l) which is given by

$$l = \langle v \rangle \tau = \frac{1}{(n\pi d^2)} \quad \text{...(iii)}$$

While doing the above derivation we have not considered the motion of other molecules. We assumed them to be at rest. But actually all are moving. Hence $\langle v \rangle$ is replaced by $\langle v_r \rangle$ *i.e.*, average relative velocity of the molecules. Hence more exact treatment gives

$$l = \frac{1}{\sqrt{2}n\pi d^2} \quad \text{...(iv)}$$

The mean free path of equation (iv) gives its dependence on the number density and size of molecules.

41. On the basis of law of equipartition of energy, calculate the molar specific heats of monoatomic and diatomic gases at constant volume and pressure separately.

Sol. A monoatomic gas has only three translational degrees of freedom therefore, its average energy at temperature T is given by

$$E = \frac{3}{2}k_B T$$

Total internal energy of one mode of such gas is

$$U = \frac{3}{2}k_B T \times N_A \quad \text{...(i)}$$

where N_A is Avogadro's number.

As we know, $k_B = \frac{R}{N_A}$, substituting in equation (i), we get

$$U = \frac{3}{2}RT$$

$$\frac{dU}{dT} = \frac{3}{2}R$$

The molar specific heat at constant volume, C_v is

$$\frac{dU}{dT} = C_v \text{ (monoatomic gas)} = \frac{3}{2}R$$

For an ideal gas

$$C_p - C_v = R$$

where C_p is the molar specific heat at constant pressure. Thus,

$$C_p = R + C_v = R + \frac{3}{2}R = \frac{5}{2}R$$

Ratio of specific heats $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$

A diatomic molecule is treated as a rigid rotator with 5 degrees of freedom, 3 translational and 2 rotational. So the average energy of a molecule of diatomic gas at temperature T is given by

$$E = \frac{5}{2}k_B T$$

and the total internal energy of a mole of diatomic gas is

$$U = \frac{5}{2}k_B T \times N_A = \frac{5}{2}RT$$

and hence, molar specific heat C_v (at constant volume) and molar specific heat C_p (at constant pressure) is given by

$$C_v = \frac{dU}{dT} = \frac{5}{2}R$$

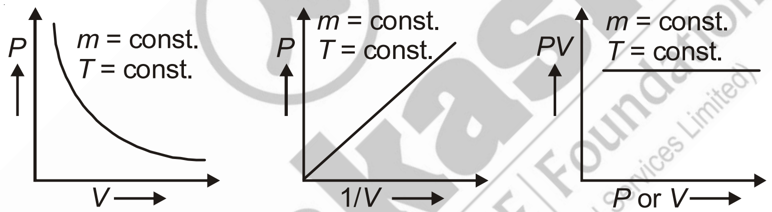
$$C_v = \frac{5}{2}R$$

$$C_p = R + C_v = R + \frac{5}{2}R = \frac{7}{2}R$$

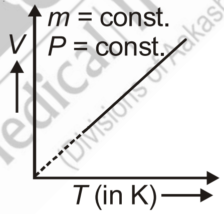
$$\gamma = \frac{C_p}{C_v} = \frac{7}{5}$$

42. Explain Boyle's law and Charles's law and draw the corresponding graphs related to P, V and T.

Sol. Boyle's law : According to it, for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, *i.e.*, $V \propto \frac{1}{P}$ if mass of gas and T = constant



Charles's law : According to it, for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature *i.e.*, $V \propto T$, if m and P are constant.



43. On the basis of kinetic theory of gases, explain the kinetic interpretation of temperature.

Sol. The pressure exerted by an ideal gas is given by

$$P = \frac{1}{3}nm\overline{v^2}$$

Multiplying both sides by V

$$PV = \frac{1}{3}nmV\overline{v^2}$$

$$PV = \frac{1}{3}Nm\overline{v^2} \qquad \ldots(i)$$

where N = (nV) is the number of molecules in the sample.

The internal energy E of an ideal gas is purely kinetic

$$E = N\frac{1}{2}mv^2 \qquad \text{...(ii)}$$

Using equation (ii) in (i), we get

$$PV = \left(\frac{2}{3}\right)E \qquad \text{...(iii)}$$

Comparing equation (iii) with the ideal gas equation, we get

$$\frac{2}{3}E = Nk_BT$$

$$E = \frac{3}{2}Nk_BT$$

or $$\frac{E}{N} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT \qquad \text{...(iv)}$$

From equation (iv) we can see that the average kinetic energy of a molecule is proportional to the absolute temperature of the gas. It is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating the temperature to the internal energy of a molecule. This is kinetic interpretation of temperature.

The *rms* velocity (root mean square velocity) is defined as the square root of the mean of the squares of the random velocities of the individual molecules of a gas

$$v_{\text{rms}} = \sqrt{\overline{v^2}}$$

As from the above equation we can see that $\overline{v^2} \propto T$

$\therefore \quad v_{\text{rms}} \propto \sqrt{T}$

44. Establish a relation between kinetic energy of the molecules of a gas and temperature and show that the root mean square velocity of the molecules of a gas is directly proportional to the square root of the absolute temperature of the gas.

Sol. The pressure exerted by an ideal gas is given by

$$P = \frac{1}{3}nm\overline{v^2}$$

Multiplying both sides by V

$$PV = \frac{1}{3}nmV\overline{v^2}$$

$$PV = \frac{1}{3}Nm\overline{v^2} \qquad \text{...(i)}$$

where $N = (nV)$ is the number of molecules in the sample.

The internal energy E of an ideal gas is purely kinetic

$$E = N\frac{1}{2}mv^2 \qquad \text{...(ii)}$$

Using equation (ii) in (i), we get

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$$v_{\text{rms}} = \sqrt{\overline{v^2}}$$

As from the above equation we can see that $\overline{v^2} \propto T$

$\therefore \quad v_{\text{rms}} \propto \sqrt{T}$

❑ ❑ ❑

Chapter 12

Kinetic Theory of Gases

Solutions (Set-2)

[Behaviour of Gases]

1. If Boyle's law is written in the form $PV = C$; and the temperature remains constant then, in the above relation, the magnitude of C depends upon

(1) The nature of the gas used in the experiment
(2) The molecular mass of gas in the laboratory
(3) The atmospheric pressure
(4) The quantity of gas enclosed

Sol. Answer (4)

$PV = \mu RT = CT$

$C = \mu R$

μ depends on the mass of the gas.

2. A closed vessel A having volume V contains N_2 at pressure P and temperature T. Another closed vessel B having the same volume V contains He at the same pressure P. But temperature $2T$. The ratio of masses of N_2 and He in the vessels A and B is

(1) 1 : 2
(2) 3 : 2
(3) 5 : 2
(4) 14 : 1

Sol. Answer (4)

$PV = nRT$

n = number of moles, $n = \dfrac{m}{M}$

For N_2 : $P = \dfrac{m_{N_2}}{M_{N_2}} \times \dfrac{RT}{V}$

For He : $P = \dfrac{m_{He}}{M_{He}} \times \dfrac{R(2T)}{V}$

$$\frac{m_{N_2}}{m_{He}} = \frac{M_{N_2}}{M_{He}} \times \frac{R(2T)}{V} \times \frac{V}{RT}$$

$$= \frac{2 \times 28}{4} = \frac{14}{1} = 14:1$$

3. By what percentage should the pressure of a given mass of a gas be increased, so as to decrease its volume by 10% at a constant temperature?

(1) 8.1% (2) 9.1% (3) 10.1% (4) 11.1%

Sol. Answer (4)

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{PV}{T} = \frac{P' \times \frac{90}{100} V}{T}$$

$$\frac{P'}{P} = \frac{100}{90} = \frac{10}{9}$$

$$\frac{P' - P}{P} = \frac{1}{9} \times 100 = 11.1\%$$

4. For Boyle's law to hold the gas should be

(1) Perfect and at constant temperature and mass

(2) Real and at constant temperature and mass

(3) Perfect and at constant temperature but variable mass

(4) Real and at constant temperature but variable mass

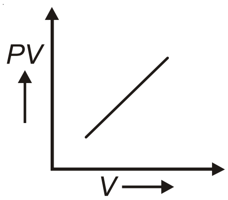
Sol. Answer (1)

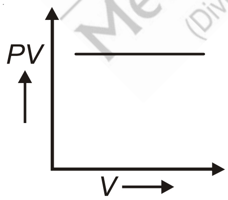
5. Boyle's law is applicable in

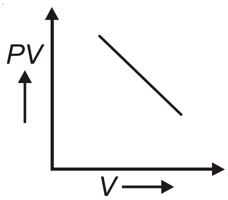
(1) Isobaric process (2) Isochoric process (3) Isothermal process (4) Adiabatic process

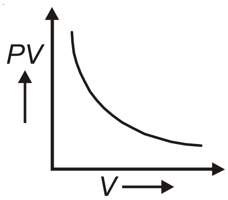
Sol. Answer (3)

6. The variation of PV with V of fixed mass of an ideal gas at constant temperature is graphically represented by the curve

(1) 

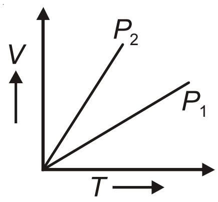
(2) 

(3) 

(4) 

Sol. Answer (2)

7. For V versus T curves at constant pressure P_1 and P_2 for an ideal gas are shown in the figure given below



(1) $P_1 > P_2$ (2) $P_1 < P_2$ (3) $P_1 = P_2$ (4) $P_1 \geq P_2$

Sol. Answer (1)

Slope of graph $= \frac{1}{P}$

Slope is greater for P_2, hence $P_1 > P_2$

8. Two identical cylinders contain helium at 1.5 atm and argon at 1 atm respectively. If both the gases are filled in one of the cylinders, the pressure would be

(1) 1 atm (2) 1.75 atm (3) 2.5 atm (4) 0.5 atm

Sol. Answer (3)

$P = P_1 + P_2$

$= 1.5 + 1$

$= 2.5$ atm

9. Two gases *A* and *B* having the same temperature *T*, same pressure *P* and same volume *V* are mixed. If the mixture is at the same temperature *T* and occupies a volume *V*, the pressure of the mixture is

(1) $2P$ (2) P (3) $P/2$ (4) $4P$

Sol. Answer (1)

$P' = P_1 + P_2,\ P'$

$= P + P = 2P$

10. A perfect gas at 27°C is heated at constant pressure so as to double its volume. The temperature of the gas will be

(1) 300°C (2) 200°C (3) 327°C (4) 600°C

Sol. Answer (3)

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \frac{V}{300} = \frac{2V}{T}$$

$T = 600$ K $= 600 - 273 = 327°$C

11. Which of the following methods will enable the volume of ideal gas to be increased four times?

(1) Double the temperature and reduce the pressure to half

(2) Double the temperature and also double the pressure

(3) Reduce the temperature to half and double the pressure

(4) Reduce the temperature to half and reduce the pressure to half

Sol. Answer (1)

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{PV(2T)}{T\left(\frac{P}{2}\right)} = 4V$$

12. We have a jar A filled with gas characterised by parameters P, V and T and another jar B filled with gas with parameters $2P$, $\frac{V}{4}$ and $2T$. The ratio of the number of molecules of jar A to those of jar B is

(1) 1 : 1 (2) 1 : 2 (3) 2 : 1 (4) 4 : 1

Sol. Answer (4)

$$PV = \frac{m_0 N}{M} RT$$

For jar A : $PV = \frac{m_0 N_1}{M} RT$

For jar B : $(2P)\left(\frac{V}{4}\right) = \frac{m_0 N_2 R(2T)}{M}$

$\therefore \quad \frac{N_1}{N_2} = \frac{4}{1}$

13. During an experiment an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at temperature T and volume V, when it expands to volume $2V$, the resulting temperature is

(1) $\frac{T}{2}$ (2) $2T$ (3) $\sqrt{2}T$ (4) $\frac{T}{\sqrt{2}}$

Sol. Answer (3)

$PV = RT$ = constant

$$P = \frac{RT}{V}$$

VP^2 = constant

$\Rightarrow \quad \frac{V \times R^2 T^2}{V^2} = \text{constant}$

$\frac{R^2 T^2}{V} = \text{constant}$

$\Rightarrow \quad \frac{T^2}{V} = \text{constant}$

$\frac{T^2}{V} = \frac{T'^2}{2V}$

or $T'^2 = 2T^2$

$T' = \sqrt{2}T$

14. Two gases A and B having the same pressure P, volume V and temperature T are mixed. If mixture has volume and temperature as V and T respectively, then the pressure of the mixture will be

(1) $4P$ (2) $3P$ (3) $2P$ (4) P

Sol. Answer (3)

15. A gas is heated through 1°C in a closed vessel. Its pressure is increased by 0.4%. The initial temperature of the gas is

(1) 250°C (2) 100°C (3) –75°C (4) –23°C

Sol. Answer (4)

$T_1 = T$, $P_1 = P$

P_2 = 0.4% more than P_1 $= P + \frac{0.4P}{100} = \frac{100.4P}{100}$

So, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{PV}{T} = \frac{100.4}{100}\frac{PV}{(T+1)}$$

On solving,

T = 250 K = –23°C

16. A balloon contains 1500 m^3 of helium at 27°C and 4 atmospheric pressure. The volume of helium at –3°C temperature and 2 atmospheric pressure will be

(1) 1500 m^3 (2) 1700 m^3 (3) 1800 m^3 (4) 2700 m^3

Sol. Answer (4)

V_1 = 1500 m^3, T_1 = 27°C = 300 K

P = 4 atm, T_2 = –3°C = 270 K, P_2 = 2 atm

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{P_2T_1} = \frac{4\times1500\times270}{2\times300}$$

V_2 = 2700 m^3

17. If the pressure and temperature of an ideal gas is doubled and volume is halved, the number of molecules of the gas

(1) Remains constant (2) Becomes half (3) Becomes two times (4) Becomes four times

Sol. Answer (2)

$PV = nRT$

$$n \propto \frac{PV}{T}$$

Number of molecules are directly proportional to the number of moles of the gas (number of molecules = nN)

Here P and T are doubled while volume is halved. Therefore, the number of moles and hence the number of molecules will become half.

18. A gas at pressure P_0 is contained in a vessel. If the masses of all the molecules are halved and their speeds doubled, the resulting pressure would be

(1) $4P_0$ (2) $2P_0$ (3) P_0 (4) $\frac{P_0}{2}$

Sol. Answer (2)

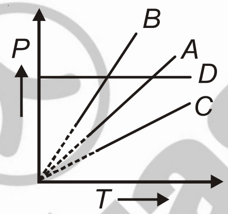
$$P = \frac{1}{3}\frac{mn}{V}v_{rms}^2$$

$$P \propto m$$

$$P \propto v_{rms}^2$$

m is halved and v_{rms} is doubled. Hence, P will become two times. [In the above equation m is the mass of one gas molecule and n is the total number of gas molecules].

19. Pressure-temperature graph of the ideal gas at constant volume V is shown by a straight line A. Now, pressure P of the gas is doubled and the volume is halved; then the corresponding pressure-temperature graph will be shown by the line



(1) A (2) B (3) C (4) D

Sol. Answer (2)

$$P = \frac{nRT}{V} = \frac{mRT}{MV}$$ (where $n = \frac{m}{M}$ = number of moles)

So, at constant volume pressure-temperature graph is a straight line passing through origin with slope $\frac{mR}{MV}$. As the mass is doubled and volume is halved, slope becomes four times. Therefore, pressure versus temperature graph will be shown by the line B.

20. Two balloons are filled, one with pure He gas and other by air respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is

(1) More in the He filled balloon

(2) Same in both balloons

(3) More in air filled balloons

(4) In the ratio of 1 : 4

Sol. Answer (2)

Assuming the balloons have the same volume, as $PV = nRT$, if P, V and T are the same, n the number of moles present will be the same, whether it is He or air. Hence, number of molecules per unit volume will be same in both the balloons.

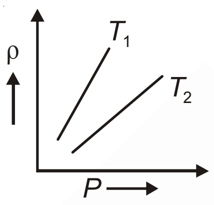
21. The temperature of a gas contained in a closed vessel increases by 2°C, when the pressure is increased by 2%. The initial temperature of the gas is

(1) 200 K (2) 100 K (3) 200°C (4) 100°C

Sol. Answer (2)

Using, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

22. The density (ρ) versus pressure (P) graphs of a given mass of an ideal gas is shown at two temperatures T_1 and T_2. Then relation between T_1 and T_2 may be



(1) $T_1 > T_2$ (2) $T_2 > T_1$ (3) $T_1 = T_2$ (4) None of these

Sol. Answer (2)

According to ideal gas equation $PV = nRT$

$$PV = \frac{m}{M}RT$$

$$P = \frac{\rho}{M}RT$$

$$\frac{\rho}{P} = \frac{M}{RT}$$

$$\Rightarrow \frac{\rho}{P} \propto \frac{1}{T}$$

Hence, $T_2 > T_1$

23. If both the temperature and the volume of an ideal gas are doubled, the pressure

(1) Increases by a factor of 4 (2) Is also doubled

(3) Remains unchanged (4) Is diminished by a factor of 4

Sol. Answer (3)

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

24. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(1) $T_1 + T_2$ (2) $\frac{(T_1 + T_2)}{2}$ (3) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$ (4) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$

Sol. Answer (3)

The number of moles of the system remains same.

$$\frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2}=\frac{P(V_1+V_2)}{RT}$$

or $$P=\frac{T(P_1V_1T_2+P_2V_2T_1)}{T_1T_2(V_1+V_2)}$$

According to Boyle's law,

$P_1V_1 + P_2V_2 = P(V_1 + V_2)$

Put value of P, then

$$T=\frac{(P_1V_1+P_2V_2)T_1T_2}{(P_1V_1T_2+P_2V_2T_1)}$$

25. In a process the pressure of a gas remains constant. If the temperature is doubled, then the change in the volume will be

(1) 100% (2) 200% (3) 50% (4) 25%

Sol. Answer (1)

P = constant

$T_1 = T$

$T_2 = 2T$, Let $V_1 = V$

$\Delta V = ?$

$PV = nRT$

$V \propto T$

$$\frac{V_1}{V_2}=\frac{T_1}{T_2}$$

$$\frac{V_2}{V_1}=\frac{T_2}{T_1}$$

$$V_2=\frac{2T}{T}.V$$

$V_2 = 2V$

$\Delta V = 2V - V = V$

100%

26. A gas is found to obey the law P^2V = constant. The initial temperature and volume are T_0 and V_0. If the gas expands to a volume $3V_0$, then the final temperature becomes

(1) $\sqrt{3}T_0$ (2) $\sqrt{2}T_0$ (3) $\frac{T_0}{\sqrt{3}}$ (4) $\frac{T_0}{\sqrt{2}}$

Sol. Answer (1)

$\boxed{P^2V = \text{Constant}}$

$V_1 = V_0$, $T_1 = T_0$

$V_2 = 3V_0$, T_2 = ? $\quad \because PV = nRT$

P^2V = constant

$$\frac{T^2}{V^2}V = \text{constant}$$

$$\frac{T^2}{V} = \text{constant}$$

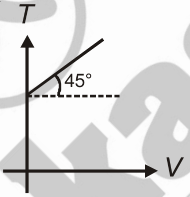
$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{V_1}{V_2}\right)$$

$$\left(\frac{T_0}{T_2}\right)^2 = \left(\frac{V_0}{3V_0}\right)$$

$$T_0\sqrt{3} = T_2$$

$$\boxed{T_2 = \sqrt{3}.\,T_0}$$

27. The given curve represent the variation of temperature as a function of volume for one mole of an ideal gas. Which of the following curves best represents the variation of pressure as a function of volume?



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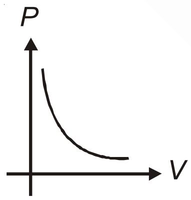

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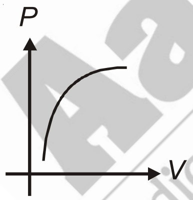

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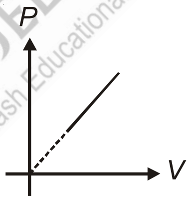

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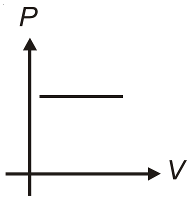
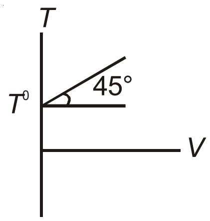


Sol. Answer (1)

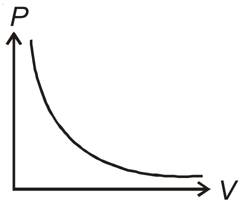
Equation will be $\boxed{T = V + T_0}$



$$\frac{PV}{nR} = V + T_0$$

$$P = nR + nR.\ T_0\frac{1}{V}$$

So, *PV* graph will be



[Kinetic Theory of Gases]

28. The root mean square speed of molecules of two ideal gases *A* and *B*, at the same temperature are

(1) The same

(2) Inversely proportional to the square root of the molecular weight

(3) Directly proportional to the molecular weight

(4) Inversely proportional to the molecular weight

Sol. Answer (2)

For all gases at the same temperature,

$$v_{\text{rms}} = \sqrt{\frac{1}{M}}$$

29. Which of the following gases possess maximum rms velocity, all being at the same temperature?

(1) Oxygen (2) Nitrogen (3) Hydrogen (4) Carbondioxide

Sol. Answer (3)

At same temperature

$$v_{\text{rms}} = \sqrt{\frac{1}{M}}$$

M is smallest for Hydrogen.

30. If the masses of all molecules of a gas are halved and their speeds doubled, then the ratio of initial and final pressures would be

(1) 2 : 1 (2) 1 : 2 (3) 4 : 1 (4) 1 : 4

Sol. Answer (2)

$$P = \frac{1}{3}\frac{mN\overline{v^2}}{V}, \quad P' = \frac{1}{3}\frac{\left(\frac{m}{2}\right)N}{V}(2\overline{v})^2$$

$$\frac{P'}{P} = \frac{2}{1} \text{ or } \frac{P}{P'} = 1 : 2$$

31. Two vessels having equal volume contain molecular hydrogen at one atmosphere and helium at two atmospheres respectively. If both samples are at the same temperature, the rms velocity of hydrogen molecule is

(1) Equal to that of helium (2) Twice that of helium

(3) Half that of helium (4) $\sqrt{2}$ times that of helium

Sol. Answer (4)

$$v_{\text{rms}} = \sqrt{\frac{3PV}{M}}$$

$$(v_{\text{rms}})_{\text{H}} = \sqrt{\frac{3V}{1}}$$

$$(v_{rms})_{He} = \sqrt{\frac{6V}{4}}$$

$$\frac{(v_{rms})_H}{(v_{rms})_{He}} = \sqrt{\frac{3V}{1} \times \frac{4}{6V}} = \sqrt{2}$$

$$(v_{rms})_H = \sqrt{2}(v_{rms})_{He}$$

32. At a given temperature, the pressure of a gas

(1) Varies inversely as its mass

(2) Varies inversely as the square of its mass

(3) Varies linearly as its mass

(4) Is independent of its mass

Sol. Answer (3)

$$P = \frac{1}{3}\left(\frac{mN}{V}\right)\overline{v^2}$$

$$P \propto mN$$

33. A sample of gas is at 0°C, to what temperature must it be raised in order to double the rms speed of its molecules

(1) 100°C (2) 273°C (3) 819°C (4) 919°C

Sol. Answer (3)

$$v_{rms} \propto \sqrt{T}$$

$$v_{rms} \propto \sqrt{273} \text{ or, } 2v_{rms} \propto \sqrt{T}$$

$$2 = \sqrt{\frac{T}{273}} \text{ or } T = 4 \times 273 = 1092 \text{ K}$$

$$T = 1092 - 273 = 819°\text{C}$$

34. The root mean square speed of the gas molecules is 300 m/s. What will be the root mean square speed of the molecules if the atomic weight is doubled and absolute temperature is halved?

(1) 300 m/s (2) 200 m/s (3) 150 m/s (4) 100 m/s

Sol. Answer (3)

$$v_{rms} = \sqrt{\frac{3RT}{M}} \text{ or } 300 = \sqrt{\frac{3RT}{M}}$$

$$\text{and } v'_{rms} = \sqrt{\frac{3R\left(\frac{T}{2}\right)}{2M}} = \frac{1}{2} \times 300 = 150 \text{ m/s}$$

35. RMS speed of a particle is v_{rms} at pressure P. If pressure is increased to two times, then at constant temperature rms speed becomes

(1) $2\,v_{rms}$ (2) $3\,v_{rms}$ (3) v_{rms} (4) Zero

Sol. Answer (3)

v_{rms} depends on temperature and is independent of pressure.

36. Four particles have speeds 2, 4, 6, 8 ms^{-1} respectively. Then rms speed is

(1) $\sqrt{30}$ m/s (2) 120 m/s (3) $2\sqrt{30}$ m/s (4) 60 m/s

Sol. Answer (1)

$$V_{rms} = \sqrt{\frac{v^2}{n}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2}{4}}$$

$$= \sqrt{\frac{4+16+36+64}{4}}$$

$$= \sqrt{\frac{120}{4}} = \sqrt{30}\ \text{ms}^{-1}$$

37. The rms speed of a gas molecule is

(1) $\sqrt{\frac{M}{3RT}}$ (2) $\frac{M}{3RT}$ (3) $\sqrt{\frac{3RT}{M}}$ (4) $\left(\frac{3RT}{M}\right)^2$

Sol. Answer (3)

38. The temperature of a gas in a rigid container is raised the pressure exerted by the gas on the walls of the container increases because

(1) The molecules have higher average speed, so strike the walls more often

(2) The molecules lose more energy each time they strike the walls

(3) The molecules are now in contact with the walls for small intervals

(4) The molecules collide with each other more often

Sol. Answer (1)

39. A container has N molecules at absolute temperature T. If the number of molecules is doubled but kinetic energy in the box remain the same as before, the absolute temperature of the gas is

(1) T (2) $\frac{T}{2}$ (3) $3T$ (4) $4T$

Sol. Answer (2)

$$\overline{\text{KE}} = \frac{3}{2}kT$$

On doubling the number of molecules and keeping total K.E. same, average KE $(\overline{\text{KE}})$ becomes half, resulting in half of temperature.

40. 2000 small balls, each weighing 1 g, strike one square cm of area per second with a velocity of 100 m/s normal to the surface and rebounds with the same velocity. The pressure on the surface is

(1) 2×10^3 N/m^2 (2) 2×10^5 N/m^2 (3) 4×10^3 N/m^2 (4) 4×10^6 N/m^2

Sol. Answer (4)

Change in the momentum = force

$2mv = F$, $N = 2000$

$$P = \frac{NF}{A} = \frac{2\times1\times10^{-3}\times100\times2000}{1\times10^{-4}}$$

$= 2 \times 10^3 \times 2 \times 10^3$

$= 4 \times 10^6$ N/m^2

41. Some gas at 100 K is enclosed in a container. Now the container is placed on a fast moving train. While the train is in motion, the temperature of the gas

(1) Rises above 100 K (2) Falls below 100 K (3) Remains the same (4) Becomes unsteady

Sol. Answer (3)

Random motion of molecules changes the temperature not the ordered motion.

42. The root mean square speed of hydrogen molecules at a certain temperature is 300 m/s. If the temperature is doubled and hydrogen gas dissociates into atomic hydrogen, the rms speed will become

(1) 424.26 m/s (2) 300 m/s (3) 600 m/s (4) 150 m/s

Sol. Answer (3)

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

T is double and M is halved. Therefore v_{rms} is doubled *i.e.*, 2 × 300 = 600 m/s

43. If gas molecules undergo inelastic collision with the wall of the container

(1) The temperature of the gas will decrease (2) The pressure of the gas will increase

(3) Neither the temperature nor the pressure will change (4) The temperature of the gas will increase

Sol. Answer (3)

When the temperature of the gas is same as that of the wall, there is no exchange of energy. Hence, molecules return with the same average speed whether the collision is elastic or inelastic. Hence, the temperature and pressure would not change.

44. If the rms velocity of a gas is v, then

(1) v^2T = constant (2) $\frac{v^2}{T}$ = constant

(3) vT^2 = constant (4) v and T are independent

Sol. Answer (2)

$$v = \sqrt{\frac{3KT}{M}} \text{ or } v^2 = \frac{3KT}{M}$$

$$\frac{v^2}{T} = \frac{3K}{M} = \text{constant (as } K \text{ and } M \text{ are constant)}$$

45. When the temperature of a gas is increased

(1) Its molecular kinetic energy increases

(2) Molecular kinetic energy and potential energy decreases, total energy remaining constant

(3) Molecular potential energy increases and molecular kinetic energy decreases; total energy remaining constant

(4) Its molecular potential energy increases

Sol. Answer (1)

The internal energy of a gas is totally kinetic and this kinetic energy for a molecule of a gas is given by $\frac{3}{2}kT$.

$$\text{K.E.} = \frac{3}{2}kT$$

$$\text{K.E.} \propto T$$

As the temperature of the gas is increased, its molecular kinetic energy increases.

46. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square speed of gas molecules is v, then at 480 K it will be

(1) $4v$ (2) $2v$ (3) $\frac{v}{2}$ (4) $\frac{v}{4}$

Sol. Answer (2)

Root mean square speed $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

Since, M remains the same $v_{\text{rms}} \propto \sqrt{T}$

$$\frac{(v_{\text{rms}})_1}{(v_{\text{rms}})_2} = \sqrt{\frac{T_1}{T_2}} = \frac{1}{2}$$

$(v_{\text{rms}})_2 = 2v$

47. If average speed becomes 4 times then what will be the effect on rms speed at that temperature?

(1) 1.4 times (2) 4 times (3) 2 times (4) $\frac{1}{4}$ times

Sol. Answer (2)

$$v'_{rms} = \sqrt{v'^2} = \sqrt{(4v)^2} = \sqrt{16v^2} = 4\sqrt{v^2} = 4v_{rms}$$

48. At what temperature, pressure remaining constant, will the r.m.s. speed of a gas molecules increase by 10% of the r.m.s. speed at NTP?

(1) 57.3 K (2) 57.3ºC (3) 557.3 K (4) –57.3ºC

Sol. Answer (2)

As $\frac{v_{rms}}{(v_{rms})_0} = \sqrt{\frac{T}{T_0}}$

and rms speed increases by 10%

therefore $\frac{v_{rms}}{(v_{rms})_0} = \frac{110}{100} = \sqrt{\frac{273+t}{273+0}}$

$$1.1 = \sqrt{\frac{273+t}{273}}$$

or $\frac{273+t}{273} = (1.1)^2 = 1.21$

t = 273 (1.21 – 1)

= 57.3ºC

49. The temperature of a gas is due to

(1) The potential energy of its molecules (2) The kinetic energy of its molecules

(3) The attractive force between its molecules (4) The repulsive force between its molecules

Sol. Answer (2)

50. The temperature of an ideal gas is increased from 27°C to 927°C. The rms speed of its molecules become

(1) Twice (2) Half (3) Four times (4) One fourth

Sol. Answer (1)

T_1 = 27°C

T_2 = 927°C

V_{rms} = ?

$V_{rms} \propto \sqrt{T}$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{927+273}{27+273}}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{1200}{300}} = 2$$

$\boxed{V_2 = 2V_1}$ Twice.

51. Two closed vessels *A*, *B* are at the same temperature *T* and contain gases which obey Maxwellian distribution of velocities. Vessel *A* contains O_2, and *B* contain mixture of H_2 and O_2. If the average speed of the O_2 molecule in vessel *A* is v_1, then average speed of H_2 in container *B* is

(1) Zero (2) $\frac{v_1}{4}$ (3) $\frac{v_1}{2}$ (4) $4\,v_1$

Sol. Answer (4)

At constant temperature, kinetic energy of molecules of O_2 as well as H_2 are same. Also r.m.s speed is proportional to average speed. So $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$

$$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{32}{2}} \Rightarrow v_2 = 4v_1$$

52. A vessel contains 28 gm of N_2 and 32 gm of O_2, at temperature T = 1800 K and pressure 2 atm. Find the pressure if N_2 dissociate 30% and O_2 dissociate 50% if temperature remains constant.

(1) 2 atm (2) 1 atm (3) 2.8 atm (4) 1.4 atm

Sol. Answer (3)

No. of moles before dissociation = $\frac{28}{28} + \frac{32}{32} = 1 + 1 = 2$

No. of moles after dissociation = $(0.7 + 2 \times 0.3) + (0.5 + 2 \times 0.5) = 2.8$

If all other conditions remain unchanged, then

$P \propto n$

$$\Rightarrow \frac{P}{2\,atm} = \frac{2.8}{2}$$

$\Rightarrow P = 2.8$ atm.

53. A container of volume 1 m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300 K and the other part is vacuum. The whole system is isolated from surroundings. When the partition is removed, the gas expands to occupy the whole volume. The temperature will now be

(1) 300 K (2) 150 K (3) 100 K (4) 200 K

Sol. Answer (1)

$Q = \Delta U + W$

$Q = 0$, because system is isolated.

$W = 0$, because the external pressure for the expanding gas is zero.

$\therefore \Delta U = 0$

$\Rightarrow$ Change in temperature is zero.

54. The total momentum of the molecules of one gram-mole of hydrogen gas in a container (at rest) at a temperature 300 K is

(1) $2\times\sqrt{900\,R}$ g cm s^{-1} (2) 1800 R g cm s^{-1} (3) $\sqrt{900\,R}$ g cm s^{-1} (4) Zero

Sol. Answer (4)

Momentum is a vector quantity. Due to random motion of molecules, the sum of momenta of all molecules is zero. (Since container is at rest)

[Law of Equipartition of Energy]

55. On the basis of kinetic theory of gases. The mean kinetic energy of one mole of gas per degree of freedom is

(1) $\frac{1}{2}kT$ (2) $\frac{3}{2}kT$ (3) $\frac{3}{2}RT$ (4) $\frac{1}{2}RT$

Sol. Answer (4)

56. Kinetic energy of an ideal gas is proportional to

(1) T (2) $\frac{1}{T}$ (3) T^0 (4) T^2

Sol. Answer (1)

57. Pressure of an ideal gas is increased by keeping temperature constant. What is the effect on kinetic energy of molecules?

(1) Increases (2) Decreases (3) No change (4) Cannot be determined

Sol. Answer (3)

[Specific Heat Capacity]

58. An ideal gas, contained in a cylinder by a frictionless piston is allowed to expand from volume v_1 and pressure p_1 to volume v_2 and pressure p_2, its temperature is kept constant throughout. The work done by gas

(1) Zero (2) Negative (3) Positive (4) May be negative or positive

Sol. Answer (3)

Work done in isothermal expansion, $W = nRT \ln\frac{V_2}{V_1}$

$\because$ $V_2 > V_1$ (ln $a > 0$ if $a > 1$)

$\Rightarrow$ W = positive

59. If 2 moles of an ideal monoatomic gas at temperature T_0 is mixed with 4 moles of another ideal monoatomic gas at temperature $2T_0$, then the temperature of mixture is

(1) $\frac{5}{3}T_0$ (2) $\frac{3}{2}T_0$ (3) $\frac{4}{3}T_0$ (4) $\frac{5}{4}T_0$

Sol. Answer (1)

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