CHAPTER 04

# Arithmetic Progression

## Number patterns

Look at the following numbers:-

2, 4, 6, 8, 10, ..........................................................................

Do you see some order or pattern in these numbers?

Salma - Here, every number is 2 less than the next number.

Mohan - Here, every number is a successive multiple of 2. On multiplying 2 by1, we get 2, on multiplying 2 by 2 we get 4, on multiplying 2 by 3 we get 6 and so on.....

John - Here, the second number 4 is two times of the first number 2, third number 6 is one and a half times of second number 4 and so on.

You can see that we will need a different rule for every number in John's pattern while in the patterns of Salma and Mohan all numbers will be formed by a single rule.

Look at the numbers given below:-

6, 11, 16, 21, ---------------------------------------------------

You can say that except for the first number, each number is formed by adding 5 to the previous number.

Can you see any other pattern in this (Discuss)?

Following are some more examples of numbers:

1. -5, -7, -9, -11, -13, ..........................
2. 4, 9, 14, 19, ..........................
3. 3, 7, 11, 15, ..........................

## What is a series?

You can see that numbers in each of the series given above increase or decrease by a certain amount from the previous number, for example, numbers decrease by 2 in the first series, increase by 5 in the second series and increase by 4 in the third series. These types of number series in which there is a certain relation between successive numbers are known as progressions.

### Try These

Find out the pattern in each of the given progressions:-

(1) 4, 10, 16, 22, ..........................................

(2) 0, 3, 6, 9, ...............................................

(3) -1, -3, -5, -7, ..........................................

## Arithmetic progressions

You saw that in the series given above except the first term, each term is formed by adding a certain number to the previous term. These types of series of numbers are called Arithmetic Progressions or A.P. and the certain number added to each term is called common difference of the arithmetic progression. Common difference can be positive, negative or zero.

Look at the number series given below:

8, 13, 18, 23, ...................................................................

The first term of this series is 8, the second term is 13, the third term is 18 and the fourth term is 23. Here, we get the next term by adding 5 to the previous term. Therefore, the common difference of this progression is 5.

**Example-1.** Find the first term, the fourth term and the common difference for the given arithmetic progression.

-7, -11, -15, -19.....

**Solution:** First term = -7, Fourth term = -19

Common difference = Second term - First term

= -11 - (-7)

= -11 + 7

= -4

### Try These

1. Find out which of the following sequences are arithmetic progressions:-

(i) 9, 16, 23, 30, ..........................

(ii) 11, 15, 18, 20, ..........................

(iii) 4, 13, 19, 28, ..........................

(iv) 0, -3, -6, -9, ..........................

(v) 2, 2, 2, 2, ..........................

(vi) $9\frac{1}{7}, \frac{7}{7}, \frac{9}{7}, \frac{13}{7}$, .......................

2. Write the first term and the common difference for the given arithmetic progressions:-

(i) 9, 12, 15, 18, ...............................

(ii) 2, 8, 14, 20, ................................

(iii) 3, -2, -7, -12, ...............................

(iv) -5, 2, 9, 16, ..................................

(v) 0.4, 0.9, 1.4, 1.9, ..........................

(vi) 5, 5, 5, 5, ......................................

(vii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$, .......... .......................

## Finding the next term:

An arithmetic progression is given below:-

3, 10, 17, ................................................

Can we extend the progression? How do we find the next term, i.e. the fourth term of this arithmetic progression?

Ajita - The fourth term is 24 which is obtained by adding the common difference 7 to the third term which is 17.

Now, write the next four terms i.e. The fifth, the sixth, the seventh and the eighth terms of this progression.

| | | | |
|---|---|---|---|
| Fifth term | | Seventh term | |
| Sixth term | | Eighth term | |

**Try These**

1. Find the next three terms of the arithmetic progressions given below:

(i) 5, 11, 17, 23, ........................

(ii) -11, -8, -5, -2, .......................

(iii) $\frac{4}{9}, \frac{7}{9}, \frac{10}{9}, \frac{13}{9}$, ......................

(iv) 0, 9, 18, 27, .......................

## Expressing an arithmetic progression in a general form

So far, we have seen many examples of arithmetic progressions. Each progression has a first term and a common difference. If we denote the first term by a and the common difference by d then we can find each term of an arithmetic progression using a and d. The Second term of the arithmetic progression can be obtained by adding the common difference d to the first term, so that second term is a + d. In this way, by adding d to the second term (a + d), we can get the third term which will be a + d + d. We can write any arithmetic progression in the following way:

$a, a + d, a + d + d, a + d + d + d, \ldots\ldots\ldots\ldots\ldots$

Or

$a, a + d, a + 2d, a + 3d, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

This is called the generalized form of an arithmetic progression. When number of terms of the AP is finite it is said to be a finite arithmetic progression and when number of terms is infinite it is said to be an infinite arithmetic progression.

You can see that the number of terms in the arithmetic progression -7, -11, -15, -19, ............... given in example (1) is infinite, therefore this is an infinite arithmetic progression.

### Try These

1. Form an infinite arithmetic progression where the first term is 5 and common difference is 3.
2. Form two finite arithmetic progressions each having 5 terms.
3. If a = 11 and d = 6 in a finite arithmetic progression having 10 terms, then find the largest member of the series.

**Note:-** It is not necessary that the common difference should always be a natural number, it can be any real number.

**Example-2.** Write the first three terms of the arithmetic progression where the first term a = 10 and the common difference, d = -3.

**Solution:**

| | |
|---|---|
| First term | a=10 |
| Common difference | d= -3 |
| Second term | = a+d |
| | = 10 + (-3) |
| | = 7 |
| Third term | = a+2d |

$= 10+2(-3)$
$= 10-6$
$= 4$

Hence, the first three terms of the arithmetic progression are 10, 7, 4.

If we denote the common difference by d and the starting term or the first term of an AP by $a_1$, the second term by $a_2$, the third term by $a_3$ ..............., the $n^{th}$ term by $a_n$ then the arithmetic progression can be expressed as $a_1, a_2, a_3, .....a_n$.

Here, the common difference can be expressed by $d = a_2 - a_1 = a_3 - a_2 = .....a_n - a_{n-1} = a_{n+1} - a_n$

**Example-3.** Sort the arithmetic progression from the following number series. Write the next two terms of the arithmetic progressions among the following.

(i) 9, 27, 81, ...........................

(ii) $4, 4+\sqrt{3}, 4+2\sqrt{3}, 4+3\sqrt{3}, ................$

(iii) 1, -1, -3, -5, ......................................

(iv) 0.2, 0.22, 0.222, 0.2222, ........................

**Solution:** (i) $a_1=9,\ a_2=27,\ a_3=81$

$a_2-a_1 = 27 - 9 = 18$

$a_3-a_2 = 81-27 = 54$

Since, $a_3 - a_2 \neq a_2 - a_1$, therefore 9, 27, 81, .............. is not an arithmetic progression.

(ii) $a_1 = 4, a_2 = 4+\sqrt{3},\ a_3 = 4+2\sqrt{3},\ a_4 = 4+3\sqrt{3},$

$$a_2 - a_1 = 4+\sqrt{3}-4 = \sqrt{3}$$

$$a_3 - a_2 = 4+2\sqrt{3}-(4+\sqrt{3}) = \sqrt{3}$$

$$a_4 - a_3 = 4+3\sqrt{3}-(4+2\sqrt{3}) = \sqrt{3}$$

Since $(a_{k+1} - a_k)$, where k = 1, 2, 3 ......is same each time, therefore the given list of number is an arithmetic progression whose common difference is $d = \sqrt{3}$. The next two terms of progression are:

$$(4+3\sqrt{3})+(\sqrt{3}) = 4+4\sqrt{3} \text{ and}$$

$$(4+4\sqrt{3})+(\sqrt{3}) = 4+5\sqrt{3}$$

(iii) $a_1 = 1, a_2 = -1, a_3 = -3, a_4 = -5$

$$a_2 - a_1 = -1-1 = -2$$

$$a_3 - a_2 = -3-(-1) = -2$$

$$a_4 - a_3 = -5-(-3) = -2$$

Since $(a_{k+1} - a_k)$, where k=1,2,3......is same each time, therefore given list of number is an arithmetic progression whose common difference d =-2. Next two terms of progression are:-

$$-5+(-2)=-7 \text{ and } -7+(-2)=-9$$

(iv) $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$

$$a_2 - a_1 = 0.22 - 0.2 = 0.02$$

$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$

Since $a_3 - a_2 \neq a_2 - a_1$ therefore the given list of numbers is not an arithmetic progression.

## $n^{th}$ term of Arithmetic Progression

Suppose that $a_1, a_2, a_3,$ .........$a_n$ is an arithmetic progression, whose first term is a and the common difference is d, then

First term $a_1 = a$

Second term $a_2 = a + d = a + (2-1)d$

Third term $a_3 = a + 2d = a + (3-1)d$

Fourth term $a_4 = a + 3d = a + (4-1)d$

Fifth term $a_5 = a + 4d = a + (5-1)d$

By looking at the pattern we can say that

$n^{th}$ term $a_n = a + (n-1)d$

If there are m terms in an arithmetic progression, then $a_m$ represents the last term. The last term can also be denoted by l.

Let us try to understands by some examples:-

**Example-4.** Find the $10^{th}$ term of the arithmetic progression 4, 7, 10, 13, .....

**Solution:** Here $a = 4$, $d = 7 - 4 = 3$ and $n = 10$

$\therefore a_{10} = a + (10-1)d$ [$\because$ $n^{th}$ term, $a_n = a + (n-1)d$ ]

$= 4 + 9 \times 3$

$= 4 + 27$

$= 31$

**Example-5.** Arithmetic progression 2, 6, 10, ..... contains m terms. Find the last term.

**Solution:** Here the first term a = 2, the common difference d = 6 - 2 = 4 and the number of terms are m. Therefore the last term will be m. So n = m.

$m^{th}$ term $a_m = a + (m-1)d$ $\quad [\because$ nth term, $a_n = a + (n-1)d$ ]

$a_m = 2 + (m-1)4$

$= 2 + 4m-4$

$= 4m-2$

### Try These

1. Arithmetic progression 3, 5, 7, ............ contains 15 terms. Find the last term.
2. If the last term of the arithmetic progression -9, -5, -1, ............is 67, then how many terms are there in the progression?
3. Find the $m^{th}$ and $p^{th}$ terms of arithmetic progression 10, 15, 20, ..........

Let us see some more examples:-

**Example-6.** Check whether 301 is a term in the AP 5, 11, 17, 23, .............Give reasons.

**Solution:**

Here, $a = 5$, $d = 11-5 = 6$

Let $n^{th}$ term be 301 i.e. $a_n = 301$

We have to find the value of n

$\therefore a_n = a + (n-1)d$

$301 = 5 + (n-1)6$

$301 = 5 + 6n - 6$

$301 = 6n - 1$

$6n = 302$

$$n = \frac{302}{6}$$

$$n = \frac{151}{3}$$

Since number of terms is n, so the $n^{th}$ term should be a positive whole number, but n is a fraction here. Therefore 301 is not a term of the given arithmetic progression.

**Example-7.** There are 50 terms in an arithmetic progression whose third term is 12 and the last term is 106. Find the $29^{th}$ term of this arithmetic progression.

**Solution:**

Let the first term be a and the common difference be d in the given arithmetic progression

Third term $= 12$

$a_3 = 12$

$a + (3-1) d = 12$

$a + 2d = 12$ ........... (1)

and the last term = 106

$50^{th}$ term = 106

$a_{50} = 106$

$a + (50-1)d = 106$

$a + 49d = 106$ .................. (2)

By subtracting (1) from (2)

$$\begin{array}{rcl} a + 49d &=& 106 \\ a + 2d &=& 12 \\ - \quad - & & - \\ \hline 47d &=& 94 \end{array}$$

$$d = \frac{94}{47}$$

$d = 2$ .................(3)

Putting the value of d from equation (3)in equation (1)

$a + 2(2) = 12$

$a + 4 = 12$

$a = 12 - 4$

$a = 8$ ...............(4)

$29^{th}$ term of the arithmetic progression $= a + (29-1)d$

$= 8 + (28)(2)$

$= 8 + 56$

$= 64$

Therefore, the $29^{th}$ term of arithmetic progression is 64.

## Try These

1. An arithmetic progression whose third term is 12 and the last term is 106, contains 50 terms. Find the $21^{st}$ term of this arithmetic progression.
2. THe first term of an arithmetic progression is 10 and common difference is -3. Find the $11^{th}$ term.

Arithmetic progressions can be used to solve many daily life problems. Let us understand by some examples:-

**Example-8.** How many two digit numbers are divisible by 5?

**Solution:** The list of two digit numbers divisible by 5 is :-

10, 15, 20, ............., 95

This is an arithmetic progression, whose first term is a = 10, the common difference d = 5 and $n^{th}$ term $a_n = 95$

Since $n^{th}$ term

$95 = 10 + (n-1) .5$

$95 = 10 + 5n - 5$

$95 = 5+5n$

$5n = 95 - 5$

$$n = \frac{90}{5}$$

$n = 18$

Therefore, 18 two digit numbers are divisible by 5.

**Example-9.** Jyoti started to work in 1997 at a monthly salary of Rs.5000 and gets an annual increment of Rs.200 in her salary. In which year did her salary become Rs.7000/-?

**Solution:**

Monthly salaries (in ₹) for years 1997, 1998, 1999, 2000 ……..are

5000, 5200, 5400, 5600…..

This is an A.P., because difference of any two successive terms is 200, therefore the common difference d = 200 and the first term a = 5000.

Suppose that Jyoti's salary becomes 7000 in n years.

Then,

$$a_n = 7000$$

$$a + (n-1)d = 7000$$

$$5000 + (n-1)\, 200 = 7000$$

$$(n-1)\, 200 = 7000 - 5000$$

$$(n-1)\, 200 = 2000$$

$$n - 1 = \frac{2000}{200}$$

$$n - 1 = 10$$

$$n = 11$$

Therefore in eleventh year i.e. in 2007 Jyoti's salary will become Rs.7000.

Till now you have solved examples in which series of numbers formed arithmetic progressions. Now we will solve examples where letter combinations (p, q, r etc.) form arithmetic progressions.

**Example-10.** If in an arithmetic progression the $p^{th}$ term is q and the $q^{th}$ term is p, then find the $m^{th}$ term.

**Solution:**

Let the first term be a and the common difference be d of the arithmetic progression.

Now,

$p^{th}$ term of the arithmetic progression = q

$\therefore a+(p-1)d = q$ .............(1)

$q^{th}$ term of the arithmetic progression = p

$\therefore a+(q-1)d = p$ .............(2)

By subtracting (2) from (1)

$$a + (p-1)d = q$$
$$a + (q-1)d = p$$

By subtracting – – –

$$[(p-1)-(q-1)]d = q-p$$

$$[p-1-q+1]d = q-p$$

$$(p-q)\,d = q-p$$

$$d = \frac{-(p-q)}{(p-q)}$$

$$d = -1 \qquad .....(3)$$

Putting the value of d from equation (3) in (1)

$$a + (p-1)(-1) = q$$

$$a = q + (p-1)$$

$$a = q + p - 1 \qquad .....(4)$$

$m^{th}$ term of the progression $a_m = a + (m-1)d$

$= (p+q-1) + (m-1)(-1)$

$= p + q - 1 - m + 1$

$= p + q - m$

Therefore, $m^{th}$ term of the progression $= p + q - m$

## Exercise - 1

Q.1 Choose the correct option and give reasons:

(i) First term and common difference of given arithmetic progression are:-

$\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots\ldots\ldots\ldots$

(a) $\frac{1}{2}, -\frac{1}{2}$ (b) $\frac{3}{2}, -1$ (c) $\frac{3}{2}, -\frac{1}{2}$ (d) $\frac{-3}{2}, -1$

(ii) If first term is -2 and common difference is -2 for an arithmetic progression, then fourth term is __________.

(a) 0 (b) $-2$ (c) $-4$ (d) $-8$

(iii) $15^{th}$ term in the arithmetic progression 7, 13, 19, ..... is

(a) 91 (b) 97 (c) 112 (d) 90

(iv) First term is 4 and common difference is -4 of an arithmetic progression, then nth term is:-

(a) 8 - 2n (b) 4 - 2n (c) 8 - 4n (d) 8 - 8n

(v) 78 is which term of arithmetic progression 3, 8, 13, 18, .....

(a) $15^{th}$ (b) $16^{th}$ (c) $17^{th}$ (d) $18^{th}$

Q.2 Which is an arithmetic progression from the following progressions, give reasons:-

(a) $a, a^2, a^3, a^4, \ldots\ldots$

(b) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots.$

(c) $1^2, 3^2, 5^2, 7^2, 9^2, \ldots.$

(d) $0, -4, -8, -12, \ldots.$

(e) $16, 18\frac{1}{2}, 20\frac{1}{2}, 23, \ldots\ldots\ldots\ldots\ldots\ldots$

Q.3 Find the $10^{th}$ term of the arithmetic progression 9, 5, 1, -3, .....

Q.4 Find the $40^{th}$ term of arithmetic progression 100, 70, 40……..

Q.5 Find the $n^{th}$ term of the arithmetic progression $\frac{1}{9}, \frac{4}{9}, \frac{7}{9}, \ldots\ldots\ldots$

Q.6 Find the $m^{th}$ term of arithmetic progression 950, 900, 850, .....

Q.7 Last term of the arithmetic progression 8, 15, 22, ..... is 218. Find the number of terms.

Q.8 Which term is 0 of the AP 27, 24, 21, .....?

Q.9 Common difference of two arithmetic progressions is same. If the difference of their 99th terms is 99, then what will be the difference of their 999th term?

Q.10 In a flower bed there are 23 rose plants in the first row, 21 in the second, 19 in the third and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Q.11 Sanjay saved Rs. 5 in the first week of a year and then increased his weekly savings by Rs. 1.75. If in the nth week, his weekly saving was Rs. 20.75, find n.

Q.12 Is-47 a term of the arithmetic progression $18, 15\frac{1}{2}, 13, ........$ If yes. then which term?

Q.13 Find the 31$^{st}$ term of an arithmetic progression whose 11$^{th}$ term is 38 and the 16$^{th}$ term is 73.

Q.14 Find the m$^{th}$ term of an Arithmetic progression whose 12$^{th}$ term exceeds the 5$^{th}$ term by 14 and the sum of both terms is 36.

Q.15 Which term of the arithmetic progression 3, 15, 27, 39, ..... will be 132 more than its 54$^{th}$ term?

Q.16 The sum of the 4$^{th}$ and 8$^{th}$ terms of an arithmetic progression is 24 and the sum of the 6$^{th}$ and 10$^{th}$ terms is 44. Find the first three terms of the arithmetic progression.

Q.17 How many three-digit numbers are divisible by 3?

Q.18 Find the 10$^{th}$ term from the last in the arithmetic progression 3, 8, 13, ..... 253.

Q.19 If in an arithmetic progression pth term is $\frac{1}{q}$ and qth term is $\frac{1}{p}$, then prove that (pq)$^{th}$ term of arithmetic progression is 1.

Q.20 If p$^{th}$ term is q and q$^{th}$ term is p of an arithmetic progression, then prove that (p+q)$^{th}$ term is zero.

Q.21 p$^{th}$, q$^{th}$ and r$^{th}$ terms of an arithmetic progression are a, b, c respectively then prove that a (q - r) + b (r - p) + c (p - q) = 0.

Q.22 For what value of n, the n$^{th}$ terms of two arithmetic progression 63, 65, 67, ..... and 3, 10, 17, ..... will be equal?

## Arithmetic Mean

Suppose three quantities a, A, b are in arithmetic progression, then the middle quantity A is said to be arithmetic mean of the two quantities a and b.

Since a, A, b are in arithmetic progression.

therefore,

$$A - a = b - A$$

$$A + A = b + a$$

$2A = a + b$

$A = \frac{a+b}{2}$

Therefore, you can say that the arithmetic mean of two quantities is the half of the sum of those two quantities. Let us understand with the help of some examples:-

**Example-11.** Find the arithmetic mean of $\sqrt{2}+1$ and $\sqrt{2}-1$

**Solution:**

Arithmetic mean $= \frac{(\sqrt{2}+1)+(\sqrt{2}-1)}{2}$

$= \frac{(\sqrt{2}+1)+(\sqrt{2}-1)}{2}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

## Forming an arithmetic progression between two quantities a and b

We can insert new numbers between any two numbers and form an AP. We will have to take the common difference d depending on the number of terms in the series. Suppose we want to insert n terms, $A_1, A_2, A_3 \ldots\ldots A_n$ between two values a and b. Then a, $A_1, A_2, A_3 \ldots\ldots A_n$, b will form the AP where the first term is a, the last term is b and the number of terms is (n+2).

Suppose the common difference of an arithmetic progression is d,

then the last term $b = a + (\overline{n+2}-1)d$ [$\because$ $n^{th}$ term, $a_n = a + (n-1)d$ ]

$b = a + (n+1)d$

$b - a = (n+1)d$

$d = \frac{b-a}{n+1}$

therefore, $A_1 = a + d = a + \frac{b-a}{n+1}$

$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$

$A_3 = a + 3d = a + 3\left(\frac{b-a}{n+1}\right)$

By looking at the above pattern we can say that

$$n^{th} \text{ term} = \quad A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right)$$

Let us understand it by some examples:-

**Example-12.** Form an AP by inserting 3 terms between 11 and -5.

**Solution:**

Let $A_1, A_2, A_3$ be 3 terms between 11 and - 5. Therefore 11, $A_1$, $A_2$, $A_3$, -5 are in an arithmetic progression. First term of this arithmetic progression is a = 11, $5^{th}$ term = - 5. Suppose common difference of the arithmetic progression is d.

$$5^{th} \text{ term} = a + 4d \qquad [\because n^{th} \text{ term}, \ a_n = a + (n-1)d\ ]$$

$$-5 = 11 + 4d$$

$$-5-11 = 4d$$

$$4d = -16$$

$$d = \frac{-16}{4}$$

$$d = -4$$

Therefore, $A_1 = a + d$

$= 11 + (-4)$

$= 7$

$A_2 = a + 2d$

$= 11 + 2\,(-4)$

$= 11 - 8$

$= 3$

$A_3 = a + 3d$

$= 11 + 3\,(-4)$

$= 11 - 12$

$= -1$

Therefore, three terms between 11 and -5 are 7, 3, -1 which form the AP 11,7,3,-1,-5.

**Example-13.** *n* terms lie between 2 and 41. The ratio between the fourth term and the $(n - 1)^{th}$ term is 2 : 5, then find the value of n.

**Solution:**

Suppose $A_1, A_2, A_3, \ldots\ldots A_n$ are n terms between 2 and 41. Therefore 2, $A_1, A_2, A_3 \ldots\ldots A_n$, 41 are in arithmetic progression, where the first term a = 2 and the $(n + 2)^{th}$ term is 41.

Suppose common difference of progression is d.

then $(n+2)^{th}$ term = 41

$$2+(\overline{n+2}-1)d = 41 \qquad [n^{th} \text{ term } a_n = a+(n-1)d,]$$

$$2+(n+1)d = 41$$

$$(n+1)d = 41-2$$

$$d = \frac{39}{n+1}$$

According to the question

$$\frac{4^{th} \text{ term of the series } A_4}{n^{th} \text{ term of the series } A_{n-1}} = \frac{2}{5}$$

$$\frac{a+4d}{a+(n-1)d} = \frac{2}{5}$$

$$5a+20d = 2a+2(n-1)d$$

$$5a-2a = 2(n-1)d-20d$$

$$3a = (2n-2)d-20d$$

$$3a = (2n-2-20)d$$

$$3a = (2n-22)d$$

$$3(2) = (2n\text{-}22)\left(\frac{39}{n+1}\right)$$

(On putting the value of a and d)

$$6(n+1) = 39(2n-22)$$

$$6n+6 = 78n-858$$

$$6+858 = 78n-6n$$

$$864 = 72n$$

$$n = \frac{864}{72}$$

$$n = 12$$

## Exercise - 2

Q.1 Find the arithmetic mean of $\frac{1}{2}$ and $-\frac{1}{2}$

Q.2 Find the arithmetic mean of $x^2+3xy$ and $y^2-3xy$.

Q.3 Arithmetic mean and product of two numbers are 7 and 45 respectively. Find the numbers.

Q.4 Arithmetic mean and sum of squares of two numbers are 6 and 90. Find the numbers.

Q.5 Form an AP by inserting 6 terms between -4 and 10.

Q.6 Form an AP by inserting 5 terms 11 and -7.

Q.7 If in an arithmetic progression the mean of $p^{th}$ and $q^{th}$ term is equal to mean of $r^{th}$ and $s^{th}$ term then prove that p + q = r + s.

Q.8 *n* terms lie between 7 and 49 in an AP. If ratio of the fourth term and the $(n - 1)^{th}$ term is 5 : 4, then find the value of *n*.

## Sum of Arithmetic Progression

If the sum of the first three term of the arithmetic progression 5, 7, 9, 11, 13, .....is denoted by $S_3$ then,

$$S_3 = 5+7+9=21$$

To know the sum of the first four terms of this arithmetic progression we will add the first four terms i.e. 5, 7, 9 and 11. In this way we will get 32 as the sum of first four terms. But if you want to know the sum of first 90 terms of progression then you will have to add first 90 terms of progression. This process will be very long. If you want to know the sum of first n terms of an AP, you will use first term a, common difference d and no of terms n.

Suppose the first term of an arithmetic progression is a and the common difference is d. Therefore,

$$a,\ a+d,\ a+2d,\ ............$$

is the arithmetic progression.

Suppose the sum of first three terms of the arithmetic progression is $S_3$ then,

$$S_3 = a+(a+d)+(a+2d) \qquad .....(1)$$

If we write the sum of terms in the reverse order

$$S_3 = (a+2d)+(a+d)+a \qquad .....(2)$$

By adding equation (1) and (2) according to terms

$$2S_3 = [a+(a+2d)]+[(a+d)+(a+d)]+[(a+2d)+a]$$

$$2S_3 = [2a+2d]+[2a+2d]+[2a+2d]$$

$$2S_3 = 3[2a+2d]$$

$$S_3 = \frac{3}{2}[2a+(3-1)d] \qquad .....(3)$$

Suppose the sum of the first four terms is $S_4$, then

$$S_4 = a+(a+d)+(a+2d)+(a+3d) \qquad .....(4)$$

If we write the sum of terms in the reverse order

$S_4=(a+3d)+(a+2d)+(a+d)+a$ .....(5)

By adding equations (4) and (5) according to the terms

$$2S_4=[a+(a+3d)]+[(a+d)+(a+2d)]+[(a+2d)+(a+d)]+[(a+3d)+a]$$

$$2S_4=[2a+3d]+[2a+3d]+[2a+3d]+[2a+3d]$$

$$2S_4=4[2a+3d]$$

$$S_4=\frac{4}{2}[2a+(4-1)d] \quad .....(6)$$

Similarly,

$$S_5=\frac{5}{2}[2a+(5-1)d] \quad .....(7)$$

$$S_6=\frac{6}{2}[2a+(6-1)d] \quad .....(8)$$

By looking at the pattern of equations (3), (6), (7), (8) we can say that in an arithmetic progression whose first term is a and the common difference is d, the sum of n terms $S_n$ is given by:

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

## Deduction Method for finding the sum

Suppose the first term is a and the common difference is d of an arithmetic progression, therefore

a, a+d, a+2d, ............

is an arithmetic progression.

The n[th] term of arithmetic progression is a + (n - 1) d. Let Sn be the sum of n terms of this arithmetic progression. Therefore,

$S_n=a+(a+d)+(a+2d)+ .......+[a+(n-2)d, +[a+(n-1)d,$ .....(1)

If we write the terms in the reverse order-

$S_n= [a+(n-1)d,+[a+(n-2)d,+ .......+(a+d)+a$ .....(2)

By adding equations (1) and (2) according to terms-

$$S_n+S_n=[a+\overline{a+(n-1)d}]+[(a+d)+\overline{a+(n-2)d}]+$$

$$[(a+2d)+\overline{a+(n-3)d}]+......................................$$

$$..........+[\overline{a+(n-2)d}+(a+d)]+[\overline{a+(n-1)d}+a]$$

$$2S_n = \{2a+(n-1)d\}+[2a+(n-1)d]+..........\ ..........+[2a+(n-1)d]$$

In the above equation, number of terms in the right side is n (Why?)

$$2S_n = n[2a+(n-1)d]$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Therefore sum of first n terms of arithmetic progression is

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

This can also be written in the following way:-

$$S_n = \frac{n}{2}[a+a+(n-1)d]$$

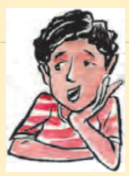
$$= \frac{n}{2}[a+a_n]$$

If there are only n terms in an arithmetic progression, then the $n^{th}$ term $a_n$ is the last term i.e. $a_n = l$, here the letter l will be used for last term.

Sum of n terms of arithmetic progression in this condition will be given by:

$$S_n = \frac{n}{2}(a+l)$$

## Try These



1. Is the $n^{th}$ term of a progression is equal to the difference between sum of the first n terms ($S_n$) and sum of the first (n - 1) terms, (Sn-1)?
2. If the sum of the first n terms of an arithmetic progression is $S_n = 4n - n^2$, then can you find the value of the first term? Is this $S_1$? What is the sum of first two terms of this arithmetic progression? What is the second term? In this way find the third, the fourth, the fifteenth and the $n^{th}$ terms.

**Example:-14.** Find the sum of 17 terms of the arithmetic progression 5, 1, -3, ......

**Solution:**

Here, first term a = 5, common difference d = 1 - 5 = -4, number of terms n = 17.

We know that,

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{17} = \frac{17}{2}[2(5)+(17-1)(-4)]$$

$$= \frac{17}{2}[10+(16)(-4)]$$

$$= \frac{17}{2}(10-64)$$

$$= \frac{17}{2}(-54)$$

$$= -459$$

Therefore, sum of first 17 terms of the given arithmetic progression is -459.

**Example-15.** Sum of first 14 term of an arithmetic progression is 1050 and its first term is 10, then find the 20$^{th}$ term.

**Solution:**

Here a=10, n=14, $S_{14}$=1050

We know that,

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$1050 = \frac{14}{2}[2(10)+(14-1)d]$$

$$1050 = 7(20+13d)$$

$$1050 = 140 + 91d$$

$$91d = 1050 - 140$$

$$91d = 910$$

$$d = \frac{910}{91}$$

$$d = 10$$

Therefore 20$^{th}$ term, $a_{20}$=10+(20-1)(10)

[$\because$ n$^{th}$ term, $a_n$=a+(n-1)d]

$a_{20}$=10+190=200

Therefore 20$^{th}$ term is 200.

**Example-16.** Find the sum of all odd numbers between 100 and 200.

**Solution:**

Odd numbers between 100 and 200 are

101, 103, 105, ---------------------------199

This list of numbers is an arithmetic progression (Why?)

First term of this arithmetic progression is a = 101, last term $\ell$ = 199, Common difference d=2.

Suppose number of terms is n of this arithmetic progression, then

$$n^{th} \text{ term} = 199$$

$$a+(n-1)d = 199$$

$$101+(n-1)(2) = 199$$

$$2n-2 = 199-101$$

$$2n-2 = 98$$

$$2n = 98+2$$

$$n = \frac{100}{2}$$

$$n = 50$$

Therefore, sum of odd numbers between 100 and 200

$$S_n = \frac{n}{2}(a+l)$$

$$S_{50} = \frac{50}{2}(101+199)$$

$$= 25\ (300)$$

$$= 7500$$

Therefore, sum of odd numbers between 100 and 200 is 7500.

**Example-17.** How many terms of arithmetic progression 17, 15, 13, ..... must be taken to get a sum of 72.

**Solution:**

Here first term a = 17, common difference d = 15 - 17 = -2

Suppose sum of n term is 72, then $S_n$ = 72.

We know that,

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$72 = \frac{n}{2}[2(17)+(n-1)(-2)]$$

$$72 = \frac{n}{2}(34 - 2n + 2)$$

$$72 \times 2 = n(36-2n)$$

$$144 = 36n-2n^2$$

$$2n^2-36n+144 = 0$$

$$n^2-18n+72 = 0$$

$$n^2-6n-12n+72 = 0$$

$$n(n-6)-12(n-6) = 0$$

$$(n-6)(n-12) = 0$$

$$n=6, n=12$$

Both values of n are possible and acceptable therefore the number of terms is either 6 or 12.

**Comment:-**

(i) In this situation sum of first 6 term = sum of first 12 terms = 72

(ii) Reason that there are two possible answers is that the sum of $7^{th}$ term to $12^{th}$ term is zero.

**Example-18.** In a school, students thought of planting trees in and around the school campus to reduce air pollution. It was decided that the number of trees that each section of each class would plant would be the same as the class in which they are studying, e.g., a section of class I would plant 1 tree, a section of class II would plant 2 trees and so on till class XII. There are three sections for each class. How many trees will be planted by the students?

**Solution:** Since there are three sections for each class, so number of trees planted by class I, Class II, Class III, ..... Class XII respectively is given by:

$1\times3,\ 2\times3,\ 3\times3,\ \ldots\ldots\ldots\ldots 12\times3$

Or

3, 6, 9, ------------, 36

This is an arithmetic progression (Why?)

First term of this arithmetic progression a = 3, Common difference d = 6 - 3 = 3, Number of terms n = 12, Last term $l$ = 36

Therefore number of trees planted by school students will be equal to sum of all terms of the arithmetic progression. Thus,

Total Number of trees planted by school students

$$S_n = \frac{n}{2}(a+l)$$

$$S_{12} = \frac{12}{2}(3+36)$$
$$= 6 \times 39$$
$$= 234$$

Therefore 234 plants are planted by the school students.

**Example-19.** A spiral is made up of successive semicircles of radii 0.5 cm, 1.0 cm., 1.5 cm., 2.0 cm......, with centers alternately at A and B, starting with center at A as shown in the figure. What is the total length of such a spiral made up of thirteen consecutive semicircles?

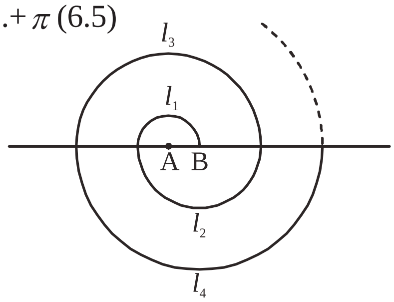
(Take $\pi = \frac{22}{7}$)

**Solution:**

We know that length of semicircle with radius r is $\pi r$.

Therefore, total length of a spiral made up of thirteen consecutive semicircles

$= \pi(0.5) + \pi(1.0) + \pi(1.5) + \pi(2.0) + ...... + \pi(6.5)$




$= \pi(0.5)[1+2+3+........+13]$

$= \pi(0.5)[\frac{13}{2}\{2(1)+(13-1)\cdot 1\}]$

$= \pi(0.5)[\frac{13}{2}(2+12)]$

$= \pi(0.5)(\frac{13}{2}\times 14)$

$= \pi(0.5)(91)$

$= \frac{22}{7}\times\frac{5}{10}\times 91$

$= 143$ cm

> 1+2+3....... + 13 is an AP whose first term is 1, common difference is 1 and number of terms is 13..................
>
> $= \frac{n}{2}[2a+(n-1)d]$

Therefore, total length of spiral made up of thirteen consecutive semicircles is 143cm.

## Exercise-3

Q.1 Find the sum of the following arithmetic progressions:-

(i) 9, 12, 15, ............... up to 16 terms.

(ii) 8, 3, -2, ............... up to 22 terms.

(iii) 0.6, 1.7, 2.8, ...............up to 100 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ...............up to 11 terms.

(v) $\frac{n^2+1}{n}, n, \frac{n^2-1}{n},$ ............... up to 20 terms.

(vi) $\left(1-\frac{1}{n}\right), \left(1-\frac{2}{n}\right), \left(1-\frac{3}{n}\right),$ .............. up to n terms.

Q.2 How many terms will it take to get the sum 1046.5 in AP 7, $10\frac{1}{2}$, 14, ............

Q.3 How many terms of the arithmetic progression 24, 21, 18, ..... must be taken so that their sum is 78?

Q.4 First term of an arithmetic progress is 1, last term is 11 and sum 36. Find the number of terms and common difference.

Q.5 First term is 17 and last term is 350 in an AP. If common difference is 9, then how many terms are in it? Find the sum of this progression.

Q.6 Find the sum of all multiples of 3 natural numbers between 1 and 100.

Q.7 Find the sum of all odd numbers lying between 0 and 50.

Q.8 Find the sum of first 51 terms of that arithmetic progression whose second term 14 and third term is 18.

Q.9 If the sum of first 7 terms of an arithmetic progression is 49 and that of 17 terms is 289, find the sum of first n terms.

Q.10 If first, second and last terms are respectively a, b and 2a of an arithmetic progression, then prove that sum of progression will be $\frac{3ab}{2(b-a)}$

Q.11 Sum of the first n terms of an arithmetic progression is $n^2 + 4n$. Find the $15^{th}$ term of the progression.

Q.12 Find the sum of first 24 terms of the list of numbers whose $n^{th}$ term is given by $a_n = 3 + 2n$.

Q.13 Sum of $p^{th}$, $q^{th}$, $r^{th}$ terms of an arithmetic progression are a, b, c respectively, then prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Q.14 Sums of n term of three arithmetic progressions are $s_1$, $s_2$, $s_3$ respectively. If first term is 1 for every progression and common differences are 1, 2, 3 respectively then prove that $s_1 + s_3 = 2s_2$.

Q.15 If the sum of n, 2n, 3n terms of an arithmetic progression are $s_1$, $s_2$, $s_3$ respectively then prove that $s_3 = 3(s_2 - s_1)$.

Q.16 A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find:

(i) The number of TV sets produced in the $1^{st}$ year.

(ii) The number of TV sets produced in the $9^{th}$ year.

(iii) The total production in first 7 years.

Q.17 A contract on construction job specifies a penalty for delay of completion beyond a certain dates as follows : ₹ 200/- for the first day, ₹ 250/- for the second day, ₹ 300/- for the third day, etc., the penalty for each successive day being ₹ 50/- more than the preceding day. How much money will the contractor have to pay as penalty if he has delayed the work by 30 days?

Q.18 A sum of ₹ 700/- is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 10/- less than its preceding prize, find the value of each of the prizes.

Q.19 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see fig.)

Find out in how many rows are the 200 logs placed and how many logs are in the top row?

Q.20 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (See. fig.)

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: To pick up the first potato and the second potato, the total distance run by a competitor is $(2 \times 5) + 2(5 + 3)$.

## What We Have Learnt

1. An Arithmetic Progression (AP) is a list of numbers in which each term except the first term is obtained by adding a fixed number d to the preceding term. The fixed number d is called the common difference. If first term is a, then the general form of an arithmetic progression is

   a, a + d, a + 2d, a + 3d, ............

2. A given list of numbers $a_1, a_2, a_3,$ ..... is an arithmetic progression if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3,$ .........., ......... give the same value, i.e. the value of $a_{k+1} - a_k$ is the same, where k = 1, 2, 3, .....

3. First term is a and common difference is d of an arithmetic progression, then n^th term of this arithmetic progression-

   $$a_n = a + (n - 1)\, d$$

   This n^th term of the arithmetic progression is known as the general term.

4. If a, A, b are in arithmetic progression, then $A = \frac{a+b}{2}$ and A is said to be arithmetic mean of a and b.

5. If n terms $A_1, A_2, A_3,$ ..... $A_n$, are taken between two terms a and b such that $a, A_1, A_2, A_3,$ ........., $A_n$, b are in arithmetic progression whose first term is a, last term is b and the number of term is (n + 2).

6. The sum $S_n$ of n term of an arithmetic progression can be obtained by using the following formulae:

   (i) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

   (ii) $S_n = \frac{n}{2}\left[a + l\right]$

   where the first term of arithmetic progression is a, the common difference is d, the number of *terms is n and the last term is l.*

# ANSWER KEY

## Exercise-1

(1) (i) (b) (ii) (d) (iii) (a) (iv) (c) (v) (b)

(2) (b), (d) (3) –27 (4) –1070 (5) $\frac{3n-2}{9}$

(6) $1000 - 50m$ (7) 31 (8) $10^{th}$ term

(9) 99 (10) 10 (11) 10

(12) Yes, $27^{th}$ term (13) 178 (14) 2m+1

(15) $65^{th}$ term (16) –13, –8, –3 (17) 300

(18) 208 (22) 13

## Exercise-2

(1) 0 (2) $\frac{x^2+y^2}{2}$ (3) 5 and 9

(4) 3, 9 (5) –2, 0, 2, 4, 6, 8 (6) 8, 5, 2, –1, –4

(8) n = 5

## Exercise-3

(1) (i) 504 (ii) –979 (iii) 5505 (iv) $\frac{33}{20}$

(v) $\frac{10(2n^2-17)}{n}$ (vi) $\frac{1}{2}(n-1)$

(2) 23 (3) 4 or 13 (4) $n = 6, d = 2$

(5) 38, 6973 (6) 1683 (7) 625

(8) 5610 (9) $n^2$ (11) 33

(12) 672 (16) (i) 550 (ii) 750 (iii) 4375

(17) ₹ 27750 (18) Amount of prizes (in ₹) 130, 120, 110, 100, 90, 80, 70

(19) 16 rows, 5 logs are kept in the top row.

**Hint :** S = 200, a = 20, d = -1, substuting in the formula $S_n = \frac{n}{2}[2a+(n-1)d]$ we get two value of n, 16 and 25. Now $a_{25}$ = a + 24d = –4 i.e. number of logs in $25^{th}$ row is negative, which is not possible. So n = 25 is not acceptable. For n = 16, $a_{16}$ = a + 15d = 5, so the number of rows is 16 and 5 logs are kept in the top row.

(20) 370 meter.