15. Linear Inequations

(Including Number Lines)

Exercise 15 (A)

Question 1.
If the replacement set is the set of natural numbers, solve.
(i) x – 5 < 0
(ii) x + 1 < 7
(iii) 3x – 4 > 6
(iv) 4x + 1 > 17
Solution:
(i) x – 5 < 0
x – 5 + 5 <0 + 5 ……(Adding 5)
=> x < 5
Required answer = {1, 2, 3, 4}
(ii) x + 1 ≤ 7 => x + 1 – 1 ≤ 7 – 1 (Subtracting 1)
=> x ≤ 6
Required answer = {1, 2, 3, 4, 5, 6}
(iii) 3x – 4 > 6
3x – 4 + 4 > 6 + 4 (Adding 4)
=> 3x > 10
$\frac{3x}{3} > \frac{10}{3}$ …(Dividing by 3)
=> $x > \frac{10}{3}$
=> $x > 3\frac{1}{3}$
Required answer = { 4, 5, 6, …}
(iv) 4x + 1 ≥ 17
=> 4x + 1 – 1 ≥ 17 – 1 (Subtracting)
=> 4x ≥ 16
=> $\frac{4x}{4} \geq \frac{16}{4}$ (Dividing by 4)
=> x ≥ 4
Required answer = {4, 5, 6, …}

Question 2.
If the replacement set = {-6, -3, 0, 3, 6, 9}; find the truth set of the following:
(i) 2x – 1 > 9
(ii) 3x + 7 < 1
Solution:
(i) 2x – 1 > 9
⇒ 2x – 1 + 1 > 9 + 1 (Adding 1)
⇒ 2x > 10
⇒ x > 5 (Dividing by 2)
⇒ x > 5

Required answer = {6, 9}
(ii) $3x + 7 \leq 1$
$\Rightarrow 3x + 7 - 7 \leq 1 - 7$ (Subtracting 7)
$\Rightarrow 3x \leq -6$
$\Rightarrow x \leq -2$
Required Answer = {-6, -3}

Question 3.
Solve $7 > 3x - 8$; $x \in N$
Solution:
$7 > 3x - 8$
$\Rightarrow 7 - 3x > 3x - 3x - 8$ (Subtracting 3x)
$\Rightarrow 7 - 7 - 3x > 3x - 3x - 8 - 7$ (Subtracting 7)
$\Rightarrow -3x > -15$
$\Rightarrow x < 5$ (Dividing by -3)
Required Answer = {1, 2, 3, 4}
Note : Division by negative number reverses the inequality.

Question 4.
$-17 < 9y - 8$; $y \in Z$
Solution:
$-17 < 9y - 8$
$\Rightarrow -17 + 8 < 9y - 8 + 8$ (Adding 8)
$\Rightarrow -9 < 9y$
$\Rightarrow -1 < y$ (Dividing by 9)
Required number = {0, 1, 2, 3, 4, …}

Question 5.
Solve $9x - 7 \leq 28 + 4x$; $x \in W$
Solution:
$9x - 1 \leq 28 + 4x$
$\Rightarrow 9x - 4x - 7 \leq 28 + 4x - 4x$ (Subtracting 4x)
$\Rightarrow 5x - 7 \leq 28$
$\Rightarrow 5x - 7 + 7 \leq 28 + 7$ (Adding 7)
$\Rightarrow 5x \leq 35$
$\Rightarrow x \leq 7$ (Dividing by 5)
Required answer = {0, 1, 2, 3, 4, 5, 6, 7}

Question 6.
Solve : $\frac{2}{3}x + 8 < 12$; $x \in W$

Solution:

Sol $\frac{2}{3}x + 8 < 12$

$\Rightarrow \frac{2}{3}x + 8 - 8 < 12 - 8$

$\Rightarrow \frac{2}{3}x < 4$

$\Rightarrow \frac{2}{3}x \times \frac{3}{2} < 4 \times \frac{3}{2}$ (Multiplying by $\frac{3}{2}$)

$\Rightarrow x < 6$

$\therefore$ Required answer = {0, 1, 2, 3, 4, 5}

Question 7.

Solve -5 (x + 4) > 30 ; x ∈ Z

Solution:

Sol. $-5(x+4) > 30$

$\Rightarrow \frac{-5(x+4)}{-5} < \frac{30}{-5}$ ...(Dividing by –5)

Note : *Division by a negative number reverses the equality*

$\Rightarrow x + 4 < -6$

$\Rightarrow x + 4 - 4 < -6 - 4$ (Subtracting 4)

$\Rightarrow x < -10$

$\therefore$ Required Answer = {–11, –12, –13, ...}

Question 8.

Solve the inquation 8 – 2x > x – 5 ; x ∈ N.

Solution:

Sol. $8 - 2x \geq x - 5; x \in N$

$\Rightarrow 8 + 5 \geq 2x + x$

$\Rightarrow 13 \geq 3x \Rightarrow 3x \leq 13$

$\Rightarrow x \leq \frac{13}{3} = 4\frac{1}{3}$

x = 1, 2, 3, 4 (x ∈ N)

Solution set = {1, 2, 3, 4}

Question 9.

Solve the inequality 18 – 3 (2x – 5) > 12; x ∈ W.

Solution:

Sol. $18 - 3(2x - 5) > 12;\ x \in W$

$\Rightarrow 18 - 6x + 15 \quad 33 - 12 > 6x$

$\Rightarrow 21 > 6x$

$\Rightarrow 6x < 21 \Rightarrow x < \frac{21}{6} + \frac{7}{2} = 3\frac{1}{2}$

But $x \in W,\ x = 0, 1, 2, 3$

$\therefore$ Solution set = $\{0, 1, 2, 3\}$

Question 10.

Solve : $\frac{2x+1}{3}$ + 15 < 17; x ∈ W.

Solution:

Sol. $\frac{2x+1}{3} + 15 \leq 17;\ x \in W$

$\Rightarrow \frac{2x+1}{3} \leq 17 - 15 = 2$

$\Rightarrow 2x + 1 \leq 6 \Rightarrow 2x \leq 5$

$\Rightarrow x \leq \frac{5}{2} = 2\frac{1}{2}$

But $x \in W$

$\therefore x = 0, 1, 2$

$\therefore$ Solution set is = $\{0, 1, 2\}$

Question 11.

Solve : -3 + x < 2, x ∈ N

Solution:

Sol. $-3 + x < 2,\ x \in N$

$\Rightarrow x < 2 - (-3)$

$\Rightarrow x < 2 + 3$

$\Rightarrow x < 5$

$\therefore x = 1, 2, 3, 4 \qquad (\because x \in N)$

$\therefore$ Solution set = $\{1, 2, 3, 4\}$

Question 12.

Solve : $4x - 5 > 10 - x$, $x \in \{0, 1, 2, 3, 4, 5, 6, 7\}$

Solution:

Sol. $4x - 5 > 10 - x, \; x \in N$

$\Rightarrow 4x + x > 10 + 5$

$\Rightarrow 5x > 15$

$\Rightarrow x > \frac{15}{5} = 3$

$\therefore x = 4, 5, 6, 7$

Solution set = {4, 5, 6, 7}

Question 13.

Solve : $15 - 2(2x - 1) < 15$, $x \in Z$.

Solution:

Sol. $15 - 2(2x - 1) < 15, \; x \in Z$

$\Rightarrow 15 - 4x + 2 < 15$

$\Rightarrow 17 - 4x < 15$

$\Rightarrow -4x < 15 - 17$

$\Rightarrow -4x < -2$

$\Rightarrow \frac{-4}{-4}x > \frac{-2}{-4} = \frac{1}{2}$ (Dividing by -4)

$\therefore x = 1, 2, 3, 4, 5, ...$

$\therefore$ Solution set $= \{1, 2, 3, 4, 5, ...\}$

Question 14.

Solve : $\frac{2x+3}{5} > \frac{4x-1}{2}$, $x \in W$.

Solution:

Sol. $\frac{2x+3}{5} > \frac{4x-1}{2}, \; x \in W$

$\Rightarrow 2(2x + 3) > 5(4x - 1)$

$\Rightarrow 4x + 6 > 20x - 5$

$\Rightarrow 4x - 20x > -5 - 6$

$\Rightarrow -16x > -11$

$\Rightarrow x < \frac{-11}{-16}$ (Dividing by -16)

$\Rightarrow x < \frac{11}{16}$

$\therefore x = 0$

$\therefore$ Solution set $= \{0\}$

Exercise 15 (B)

Solve and graph the solution set on a number line :

Question 1.

$x - 5 < -2$; $x \in N$

Solution:

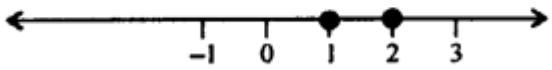
Sol. $x - 5 < -2$

$\Rightarrow x - 5 + 5 < -2 + 5$

(Adding 5 to both sides)

$\Rightarrow x < 3$

$\therefore$ The required graph is :



Question 2.

$3x - 1 > 5$; $x \in W$

Solution:

Sol. $3x - 1 > 5$

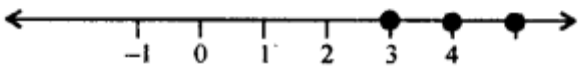
$\Rightarrow 3x - 1 + 1 > 5 + 1$ (Adding 1 to both sides)

$\Rightarrow 3x > 6$

$\Rightarrow \frac{3x}{3} > \frac{6}{3}$ (Dividing both sides by 3)

$\Rightarrow x > 2$

$\therefore$ The required graph is :



Question 3.

$-3x + 12 < -15$; $x \in R$.

Sol. $-3x + 12 < -15$

$\Rightarrow -3x + 12 - 12 < -15 - 12$

(Subtracting 12 from both sides)

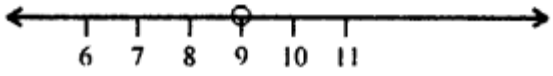
$\Rightarrow -3x < -27$

$\Rightarrow \frac{-3x}{-3} > \frac{-27}{-3}$ (Dividing both sides by -3)

Note : *Division by a negative number reverses the inequality.*

$\Rightarrow x > 9$

$\therefore$ The required graph is :



Question 4.

$7 > 3x - 8$; $x \in W$

Solution:

Sol. $7 \geq 3x-8$

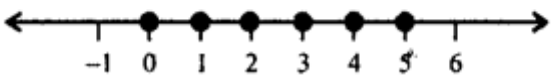
$\Rightarrow 7+8 \geq 3x-8+8$ (Adding 8 to both sides)

$\Rightarrow 15 \geq 3x$

$\Rightarrow \frac{15}{3} \geq \frac{3x}{3}$ (Dividing both sides by 3)

$\Rightarrow 5 \geq x$

$\therefore$ The required graph is :



Question 5.

$8x - 8 < -24$; $x \in Z$

Solution:

Sol. $8x-8 \leq -24$

$\Rightarrow 8x-8+8 \leq -24+8$

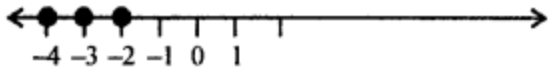
(Adding 8 to both sides)

$\Rightarrow 8x \leq -16$

$\Rightarrow \frac{8x}{8} \leq \frac{-16}{8}$ (Dividing both sides by 8)

$\Rightarrow x \leq -2$

$\therefore$ The required graph is :



Question 6.

$8x - 9 > 35 - 3x$; $x \in N$

Solution:

Sol. $8x-9 \geq 35-3x$

$\Rightarrow 8x+3x-9 \geq 35-3x+3x$

(Adding $3x$ to both sides)

$\Rightarrow 11x-9 \geq 35$

$\Rightarrow 11x-9+9 \geq 35+9$

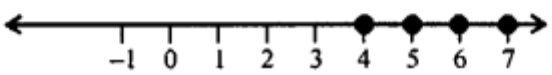
(Adding 9 to both sides)

$\Rightarrow 11x \geq 44$

$\Rightarrow \frac{11x}{11} \geq \frac{44}{11}$ (Dividing both sides by 11)

$\Rightarrow x \geq 4$

$\therefore$ The required graph is :



Question 7.

$5x + 4 > 8x - 11$; $x \in Z$

Solution:

Sol. $5x + 4 > 8x - 11$

$\Rightarrow 5x - 5x + 4 > 8x - 5x - 11$

(Subtracting $5x$ from both sides)

$\Rightarrow 4 > 3x - 11$

$\Rightarrow 4 + 11 > 3x - 11 + 11$

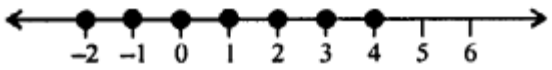
(Adding 11 to both sides)

$\Rightarrow 15 > 3x$

$\Rightarrow \frac{15}{3} > \frac{3x}{3}$ (Dividing both sides by 3)

$\Rightarrow 5 > x$

$\therefore$ The required graph is



Question 8.

$\frac{2x}{5} + 1 < -3$; $x \in R$

Solution:

Sol. $\frac{2x}{5} + 1 < -3$

$\Rightarrow \frac{2x}{5} + 1 - 1 < -3 - 1$

(Subtracting 1 from both sides)

$\Rightarrow \frac{2x}{5} < -4$

$\Rightarrow \frac{2x}{5} \times 5 < -4 \times 5$

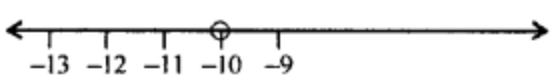
(Multiplying both sides by 5)

$\Rightarrow 2x < -20$

$\Rightarrow \frac{2x}{2} < \frac{-20}{2}$ (Dividing both sides by 2)

$\Rightarrow x < -10$

$\therefore$ The required graph is :



Question 9.

$\frac{x}{2} > -1 + \frac{3x}{4}$; $x \in N$

Solution:

Sol. $\frac{x}{2} > -1 + \frac{3x}{4}$

$\Rightarrow \frac{x}{2} \times 4 > -1 \times 4 + \frac{3x}{4} \times 4$

(Multiplying both sides by 4)

$\Rightarrow 2x > -4 + 3x$

$\Rightarrow 2x - 2x > -4 + 3x - 2x$

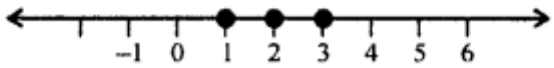
(Subtracting $2x$ from both sides)

$\Rightarrow 0 > -4 + x$

$\Rightarrow 0 + 4 > -4 + 4 + x$ (Adding 4 to both sides)

$\Rightarrow 4 > x$

$\therefore$ The required graph is :



Question 10.

$\frac{2}{3}x + 5 \leq \frac{1}{2}x + 6$; $x \in W$

Sol. $\frac{2}{3}x + 5 \leq \frac{1}{2}x + 6$

$\Rightarrow \frac{2}{3}x \times 6 + 5 \times 6 \leq \frac{1}{2}x \times 6 + 6 \times 6$

(Multiplying both sides by 6)

$\Rightarrow 4x + 30 \leq 3x + 36$

$\Rightarrow 4x - 3x + 30 \leq 3x - 3x + 36$

(Subtracting $3x$ from both sides)

$\Rightarrow x + 30 \leq 36$

$\Rightarrow x + 30 - 30 \leq 36 - 30$

(Subtracting 30 from both sides)

$x \leq 6$

$\therefore$ The required graph is :

Question 11.

Solve the inequation 5(x – 2) > 4 (x + 3) – 24 and represent its solution on a number line.

Given the replacement set is {-4, -3, -2, -1, 0, 1, 2, 3, 4}.

Solution:

Sol. $5(x - 2) > 4(x + 3) - 24$

$\Rightarrow 5x - 10 > 4x + 12 - 24$

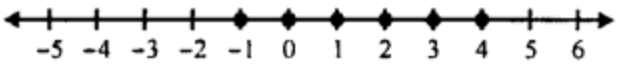
$\Rightarrow 5x - 4x > 10 - 12$

$\Rightarrow x > -2$

Since replacement set = {–4, –3, –2, –1, 0, 1, 2, 3, 4}

$\therefore$ Solution set = {–1, 0, 1, 2, 3, 4}

$\therefore$ Solution set on a number line is shown below.



Question 12.

Solve $\frac{2}{3}(x - 1) + 4 < 10$ and represent its solution on a number line.

Given replacement set is {-8, -6, -4, 3, 6, 8, 12}.

Solution:

Sol. $\frac{2}{3}(x - 1) + 4 < 10$

$\Rightarrow \frac{2}{3}(x - 1) < 10 - 4$

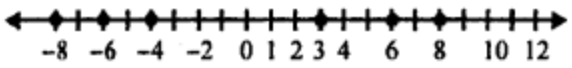
$\Rightarrow \frac{2}{3}(x - 1) < 6$

$\Rightarrow 2(x - 1) < 18$

$\Rightarrow x - 1 < 9$

$\Rightarrow x - 1 + 1 < 9 + 1$ (Adding 1 to both sides)

$\Rightarrow x < 10$



Thus $x < 10$

Since, replacement set = {–8, –6, –4, 3, 6, 8, 12}

$\Rightarrow$ Solution set = {–8, –6, –4, 3, 6, 8}

Question 13.

For each inequation, given below, represent the solution on a number line :

(i) $\frac{5}{2} - 2x \geq \frac{1}{2}$; $x \in W$

(ii) $3(2x - 1) \geq 2(2x + 3)$, $x \in Z$

(iii) $2(4 - 3x) \leq 4(x - 5)$, $x \in W$

(iv) $4(3x + 1) > 2(4x - 1)$, x is a negative integer

(v) $\frac{4-x}{2} < 3$, $x \in R$

(vi) $-2(x + 8) \leq 8$, $x \in R$

Solution:

$$(i)\ \frac{5}{2} - 2x \geq \frac{1}{2}, x \in W$$

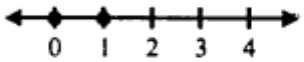
$$\Rightarrow -2x \geq \frac{1}{2} - \frac{5}{2}$$

$$\Rightarrow -2x \geq \frac{-4}{2}$$

$$\Rightarrow -2x > -2$$

$$\Rightarrow x \leq 1$$

$$\therefore x = \{0, 1\}$$



$$(ii)\ 3(2x - 1) \geq 2(2x + 3), x \in Z$$

$$\Rightarrow 6x - 3 \geq 4x + 6, x \in Z$$

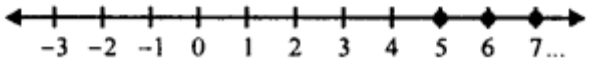
$$\Rightarrow 6x - 4x \geq 6 + 3$$

$$\Rightarrow 2x \geq 9$$

$$\Rightarrow x \geq \frac{9}{2}$$

$$\Rightarrow x \geq 4\frac{1}{2}$$

$$\therefore x = \{5, 6, 7, ...\}$$



$$(iii)\ 2(4 - 3x) \leq 4(x - 5), x \in W$$

$$8 - 6x \leq 4x - 20$$

$\Rightarrow -6x - 4x \leq -20 - 8$

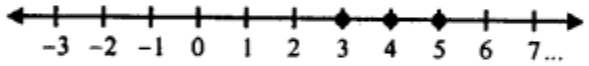
$\Rightarrow -10x \leq -28$

$\Rightarrow 10x \geq 28$

$\Rightarrow x \geq \frac{28}{10}$

$\Rightarrow x \geq 2.8$

$\therefore x = \{3, 4, 5, ...\}$



(*iv*) $4(3x + 1) > 2(4x - 1)$, x is a negative integer

$\Rightarrow 12x + 4 > 8x - 2$

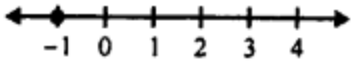
$\Rightarrow 12x - 8x > -2 - 4$

$\Rightarrow 4x > -6$

$\Rightarrow x > \frac{-6}{4}$

$\Rightarrow x > -1.5$

$\therefore x = \{-1\}$



(*v*) $\frac{4 - x}{2} < 3, x \in R$

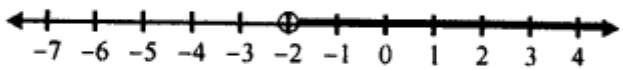
$\Rightarrow 4 - x < 6$

$\Rightarrow -x < 6 - 4$

$\Rightarrow -x < 2$

$\Rightarrow x > -2$

$\therefore x > -2$



(*vi*) $-2(x + 8) \leq 8, x \in R$

$-2x - 16 \leq 8$

$\Rightarrow -2x \leq 8 + 16$

$\Rightarrow -2x \leq 24$

$\Rightarrow x \geq \frac{-24}{2}$

$x \geq -12$