CHAPTER 11

Mensuration

Triangle, Quadrilaterals, Regular Polygons

In mensuration we often have to deal with the problem of finding the areas and perimeters of plane figures.

Triangle

1. Perimeter = 3 × side

2. Area = $\frac{1}{2} \times \text{base} \times \text{height}$, or

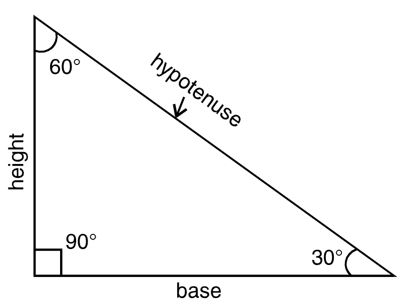
Area = $\sqrt{s(s-a)(s-b)(s-c)}$

where *a, b, c,* are the lengths of the sides of triangle and $s = \frac{a+b+c}{2}$

Right Angled Triangle : It is one where one of the angles is right angle, i.e., 90°.

1. $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

2. Area = $\frac{1}{2} \times \text{Base} \times \text{Perpendicular}$



N.B. : 30-60-90 triangle is a special case of right angled triangle, where the three angles are equal to 30°, 60° and 90°.

Here,

3. Side opposite to the angle 30°

$= \frac{1}{2} \times \text{Hypotenuse}$

4. Side opposite to the angle 60°

$= \frac{\sqrt{3}}{2} \times \text{Hypotenuse}$

Equilateral Triangle : All three sides are equal in length and all three angles are equal to 60°

1. Area = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$ **2.** Area = $\frac{(\text{Height})^2}{\sqrt{3}}$

3. Height = $\frac{\sqrt{3}}{2} \times \text{side}$ **4.** Perimeter = 3 × side

Isosceles Triangle : Two sides are equal in lengths.

1. Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

where a = lengths of opposite sides

b = length of unequal side

2. In an isosceles right triangle,

(*a*) Hypotenuse = $\sqrt{2} \times \text{congruent side}$ (a)

(*b*) Area = $\frac{1}{2} \times a^2$

(*c*) Perimeter = $\sqrt{2} \times a\left(\sqrt{2}+1\right)$

Quadrilaterals

Rectangle:

1. Area = length(l) × breadth(b)

2. Perimeter = $2(l + b)$

3. Diagonal = $\sqrt{l^2 + b^2}$

4. Area of the path (inside of the rectangle) $= 2x\,(l + b - 2x)$

where, x is the width of path.

5. Area of the path (Outside of the rectangle) = $2x\,(l + b + 2x)$

Square:

1. Area = $(\text{Side})^2$
2. Perimeter = 4 × side
3. Diagonal = $\text{side}\times\sqrt{2}$
4. Area of path (outside of square) = $4x \times (a + x)$ [where a = side, x = width of path]
5. Area of path (inside of square) = $4x \times (a - x)$

Parallelogram: In parallelogram opposite sides are parallel and equal. The two diagonals are not always equal but they bisect each other at the point of intersection.

Area = Base × Height.

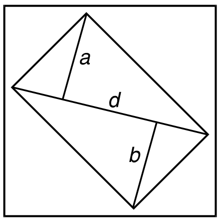
Trapezium : It is a quadrilateral whose one pair of opposite sides is parallel. Other two opposite sides are oblique.

1. Area = $\frac{1}{2}\times\text{Height}\times(\text{Sum of parallel sides})$. Here, height is the distance between the two parallel sides.
2. Median = $\left(\frac{1}{2}\times\text{Sum of parallel side}\right)$. Here, median is the segment joining the midpoints of oblique sides.
3. Height = $\frac{2}{K}\sqrt{s(s-k)(s-c)(s-d)}$
   (where $k = a - b$, *i.e.,* the difference between the parallel sides and c and d are the two non-parallel sides.)
   Also $s = \frac{k+c+d}{2}$

Rhombus : It is parallelogram whose all sides are equal and diagonals are bisect each other at right angle.

1. Area = $\frac{1}{2}\times\text{Product of diagonals}$
2. Side = $\sqrt{\left(\frac{d_1}{2}\right)^2+\left(\frac{d_2}{2}\right)^2}$,
   where d_1 and d_2 are diagonals
3. Perimeter = 4 × side
4. Area of a rhombus (with one side and one diagonal given)
   $$= \text{diagonal} \times \sqrt{(\text{side})^2-\left(\frac{\text{diagonal}}{2}\right)^2}$$
5. Other diagonal
   $$= 2\times\sqrt{(\text{side})^2-\left(\frac{\text{diagonal}}{2}\right)^2}$$

Quadrilateral : Area = $\frac{1}{2}\times\text{One diagonal}\times$



(Sum of perpendicular to it from the opposite vertices)

$= \frac{1}{2}\times d\times(a+b)$

Circle :

1. Diameter = 2 × Radius
2. Area = $\pi r^2 = \frac{\pi}{4}d^2$
   [where d = diameter = $\sqrt{\frac{4A}{\pi}}$
3. Circumference = $2\pi r = \pi d$
4. Radius = $\frac{\text{Circumference}}{2\pi} = \frac{\sqrt{\text{Area}}}{\pi}$
5. Length of an Arc = $\frac{\theta}{360°}\times 2\pi r$
6. Area of sector = $\frac{\theta}{360°}\times\pi r^2 = \frac{1}{2}\times\text{Arc}\times r$
7. Area of the ring or circular path = $\pi(R + r)(R - r)$

Remember some tricks for plane figures.

(*i*) If area related each side is increased by a%, then

(*a*) Percentage increase in the area
$$= 2a+\frac{a^2}{100}$$

(*b*) $\frac{\text{New Area}}{\text{Old Area}} = \left(1+\frac{a}{100}\right)^2$

(*ii*) If area related to each side is decreased by a%, then

(*a*) Percentage decrease in area = $2a-\frac{a^2}{100}$

(*b*) $\frac{\text{New Area}}{\text{Old Area}} = \left(1-\frac{a}{100}\right)^2$

(*iii*) If the perimeter of a rectangle, circle, quadrilateral and triangle are same, then area of the circle will be the largest.

Polygon

1. Interior angle + Exterior angle = 180°
2. Each interior angle = $\left(\frac{2n-4}{n}\right)\times 90°$
   where n = number of sides
3. Sum of Exterior angles = 360°
4. Perimeter = Number of sides × Length of side.

5. For an equilateral triangle of side 'a' *(a)* radius of inscribed circle = $\frac{a}{2\sqrt{3}}$ and side of the triangle = $2\sqrt{3}r$, *(b)* radius of circumcircle = $\frac{a}{\sqrt{3}}$

6. Area of regular polygon = $\frac{1}{2}$(No. of sides) (Radius of the incribed circle)

7. Area of regular hexagon

$= \frac{3\sqrt{3}}{2}(\text{side})^2 = 2.598\ (\text{side})^2$

8. Area of a regular octagon

$= 2\left(\sqrt{2}+1\right)(\text{side})^2 = 4.828\ (\text{side})^2$

9. Area of cyclic quadrilateral A

$= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$

where, $s = \frac{a+b+c+d}{2}$ and,

$\angle A + \angle C = \angle B + \angle D = 180°$

10. For circum-circle, if a is the length of each side of a regular polygon and R is the circum-radius, then

(*a*) $R = \frac{a}{2}\text{cosec}\left(\frac{180°}{n}\right)$

(*b*) Area of the polygon = $\frac{1}{4}na^2 \cot\left(\frac{180°}{n}\right)$, or

(*c*) Area of the polygon

$= nR^2 \sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$

(*d*) Area of the circum-circle of n-sided regular polygon

$= \frac{\pi}{4}a^2 \text{cosec}^2\left(\frac{180°}{n}\right)$

11. For in-circle, if a is the length of a side of a regular polygon and r is the radius of the in-circle, then

(*a*) $r = \frac{a}{2}\cot\left(\frac{180°}{n}\right)$

(*b*) Area of the polygon = $\pi r^2 \cot\left(\frac{180°}{n}\right)$

(*c*) Area of the in-circle of an n-sided regular polygon

$= \frac{\pi}{4}a^2 \cot^2\left(\frac{180°}{n}\right)$

(*d*) Radius of the in-circle of a regular hexagon = $\frac{\sqrt{3}}{4} \times a^2$

(*e*) Area of the in-circle = $\frac{3}{4}\pi a^2$

EXERCISE

1. A rectangle measures 50 cm × 25 cm. Its area is
A. 1150 sq. cm B. 1250 sq. cm
C. 1275 sq. cm D. 1280 sq. cm

2. A field is in the form of a square whose perimeter is 580 m. Area of this field is
A. 20025 sq. m B. 20225 sq. m
C. 30025 sq. m D. 19975 sq. cm

3. Find the area of the square whose each side measures 20 cm.
A. 300 sq. cm B. 380 sq. cm
C. 360 sq. cm D. 400 sq. cm

4. Area of a circle is 154 sq. cm. Its circumference will be
A. 44 cm B. 48 cm
C. 54 cm D. 68 cm

5. The base and the height of a triangle is 8 cm and 10 cm respectively. Its area will be
A. 40 sq. cm B. 20 sq. cm
C. 49 sq. cm D. 64 sq. cm

6. If it is given that the parallel sides of a trapezium are 15 m and 25 m while the distance between them is 10 m. its area will be
A. 150 sq. m B. 225 sq. m
C. 200 sq. m D. 270 sq. m

7. Perimeter of a rectangular field is 760 m and its length and breadth are in the ratio 11 : 8. Area of this rectangular field is
A. 35200 sq. m B. 34700 sq. m
C. 35600 sq. m D. 45200 sq. m

8. If side of a square is reduced by 50%, its area will be reduced by
A. 50% B. 75%
C. 80% D. 60%

9. If area of a square is equal to the area of a rectangle 6.4 m long and 2.5 m wide, then each side of this square measures
A. 8 m B. 5.4 m
C. 3.8 m D. 4 m

10. If perimeter of a right angled triangle is six times its smallest side, then the three sides of this triangle are in the ratio of
A. 13 : 5 : 12 B. 13 : 12 : 5
C. 12 : 5 : 13 D. 13 : 5 : 10

11. The perimeter of a square is 24 m and that of another is 32 m. Find the perimeter of a third square area of which is equal to sum of the areas of these two squares
A. 40 m B. 51 m
C. 37 m D. 42 m

12. If each side of a square is doubled, its area will become
A. double B. four times
C. three times D. eight times

13. The difference between the areas of two squares is 225 sq. metres and each side of the bigger square is 25 metres. The side of the smaller square is
A. 18 m B. 21 m
C. 20 m D. 22 m

14. If the perimeter of an equilateral triangle is 72 cm, its area will be
A. $144\sqrt{3}$ sq. cm B. $142\sqrt{3}$ sq. cm
C. $154\sqrt{2}$ sq. cm D. $144\sqrt{2}$ sq. cm

15. The radii of two circles are 5 cm and 12 cm respectively. Find the radius of a circle which is equal in area to these two circles.
A. 15 cm B. 13 cm
C. 10 cm D. 8 cm

16. If the circumference of a circle is equal to the perimeter of a square, then their areas are in the ratio of
A. 14 : 11 B. 7 : 8
C. 14 : 13 D. 13 : 11

17. Three sides of a triangle are in the ratio of 17 : 15 : 8. If the perimeter of this triangle is 40 m, find its area
A. 50 sq. m B. 49 sq. m
C. 60 sq. m D. 69 sq. m

18. If the length of a rectangle is increased by 20% and width is decreased by 15%, then its area
A. decreased by 4% B. increases by 2%
C. decreases by 2% D. increases by 3%

ANSWERS

1	2	3	4	5	6	7	8	9	10
B	A	D	A	A	C	A	B	D	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
A	B	C	A	B	A	C	B		

EXPLANATORY ANSWERS

1. Area of the rectangle
$= l \times b = 50 \times 25 = 1250$ sq. cm

2. Perimeter of the square = 4 × side
According to question :
$$4 \times \text{side} = 580 \Rightarrow \text{side} = \frac{580}{4} = 145 \text{ m}$$
$\therefore$ Area = $(\text{side})^2 = (145)^2 = 20025$ sq. m.

3. Area of the square = $(\text{side})^2 = (20)^2 = 400$ sq. cm.

4. Area of the circle = πr^2
$\therefore \quad \pi r^2 = 154$
$\Rightarrow \quad r^2 = \frac{154 \times 7}{22}$
$\Rightarrow \quad r^2 = 7 \times 7 \Rightarrow r = 7$ cm
$\therefore$ Circumference of the circle
$= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

5. Area of the triangle
$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 8 \times 10 = 40$ sq. cm.

6. Area of the trapezium
$= \frac{1}{2} \times (\text{sum of the parallel sides}) \times \text{distance between them}$
$= \frac{1}{2} \times (15 + 25) \times 10 = \frac{1}{2} \times 40 \times 10 = 200$ sq. m.

7. Suppose length and breadth of the rectangular field are $11x$ m and $8x$ m.
$\therefore$ Perimeter of the field
$= 2(11x + 8x) = 2 \times 19x = 38x$ m.
According to question :
Perimeter of the field = 760 m
$$\therefore \quad 38x = 760 \Rightarrow x = \frac{760}{38} = 20$$
$\therefore$ Length of the field = 11 × 20 = 220 m
Breadth of the field = 8 × 20 = 160 m
$\therefore$ Area of the field = 220 × 160 = 35200 sq.m.

8. Suppose side of the square = x m
In the first case :
Area of the square = x^2 sq. m.
In the second case :
On effecting 50% reduction in the side of the square,
Side of the new square = $x - 50\%$ of $x = \frac{x}{2}$ m
$\therefore$ Area of the new square $= \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$ sq.m.
$\therefore$ Reduction in area of the square
$= x^2 - \frac{x^2}{4} = \frac{3x^2}{4}$ sq. m.
$\therefore$ Percentage reduction $= \frac{3x^2/4}{x^2} \times 100 = 75\%$
Hence, area of the square will be reduced by 75%.

9. Area of the rectangle = 6.4 × 2.5 = 16.00 sq.m.
According to question :
Area of the square = Area of the rectangle
$\therefore$ Area of the square = 16 sq. m.
$\therefore$ Side of the square $= \sqrt{16} = \sqrt{4 \times 4} = 4$ m.

10. Suppose, the three sides of the triangle are *a, b* and *c* and *a* is the largest while *c* is the smallest side of the triangle.

$\therefore \quad a^2 = b^2 + c^2 \quad \ldots (i)$

Perimeter of the triangle = $a + b + c$

According to question :

$(a + b + c) = c \times 6$

$\Rightarrow \quad a + b = 5c \quad \ldots (ii)$

From equation *(i)*, $a^2 - b^2 = c^2$

$\Rightarrow (a + b)(a - b) = c^2$

$\Rightarrow \quad 5c(a - b) = c^2$

$\Rightarrow \quad a - b = \frac{c}{5} \quad [\because a + b = 5c] \quad \ldots (iii)$

From equation *(ii)* and *(iii)*,

$a + b = 5c, \; a - b = \frac{c}{5}$

$\Rightarrow \quad 2a = 5c + \frac{c}{5}$

$= \frac{26c}{5} \Rightarrow a = \frac{26c}{10} = \frac{13}{5}c$

$\Rightarrow \quad a : c = 13 : 5$

on substituting the value of a in equation *(ii)*

$b = 5c - \frac{13c}{5} = \frac{12c}{5}$

$\Rightarrow \quad b : c = 12 : 5$

$\therefore \quad a : b : c = 13 : 12 : 5$

Hence, the three sides of the right angled triangle are in the ratio of 13 : 12 : 5.

11. Perimeter of the 1st square = 24 m

$\therefore$ Side of the 1st square = $\frac{24}{4} = 6$ m

Perimeter of the 2nd square = 32 m

$\therefore$ Side of the 2nd square = $\frac{32}{4} = 8$ m

According to question :

Area of the third square = Area of the first two squares

$\therefore$ Area of the third square = Areas of the 1st square + Area of the 2nd square

$= 6^2 + 8^2 = 36 + 64 = 100$ sq. m.

$\therefore$ Side of the third square = $\sqrt{100} = 10$ m

$\therefore$ Perimeter of the third square

= 4 × side = 4 × 10 = 40 m.

12. In the first case:

Side of the square = x m

$\therefore$ Area of the square = x^2 sq. m

In the second case:

Side of the square = $2x$ m.

$\therefore$ Area of the square = $(2x)^2 = 4x^2$ sq. m

Hence, it is clear that if side of a square is doubled, its area becomes four times.

13. Suppose side of the smaller square = x m

$\because$ Area of the bigger square = $(25)^2 = 625$ sq. m

And area of the smaller square = x^2 sq. m

According to question :

Differene between the areas of two squares

= 225 sq. m

$\therefore \quad 625 - x^2 = 225$

$\Rightarrow \quad x^2 = 625 - 225 = 400$

$\Rightarrow \quad x = \sqrt{400} = 20$ m

$\therefore$ Side of the smaller side = 20 m.

14. Suppose side of the equilateral triangle = x cm

$\therefore$ Perimeter of the triangle = $3x$ cm

According to question :

$$3x = 72 \Rightarrow x = \frac{72}{3} = 24 \text{ cm.}$$

$\therefore$ Area of the equilateral triangle

$= \frac{\sqrt{3}}{4} \times x^2 = \frac{\sqrt{3}}{4} \times (24)^2$

$= 144\sqrt{3}$ sq. cm.

15. $\because$ Radius of the first circle = 5 cm

$\therefore$ Area of the first circle = $\pi \times 5^2 = 25\pi$ sq. cm and

Radius of the second circle = 12 cm

$\therefore$ Area of the second circle

$= \pi \times 12^2 = 144\pi$ sq. cm

According to question :

Area of the new circle

= Sum of the areas of the two circles

$\therefore$ Area of the new circle

$= 25\pi + 144\pi = 169\pi$ sq. cm $= \pi(13)^2$ sq. cm

Hence, it is clear that the radius of the new circle will be 13 cm.

16. Suppose radius of the circle = R cm

And side of the square = x cm

$\therefore$ Circumference of the circle = $2\pi R$ cm

and Perimeter of the square = $4x$ cm

According to question :

$$2\pi R = 4x \Rightarrow R = \frac{2x}{\pi}$$

$\therefore$ Ratio between the areas of the circle and the square

$= \pi R^2 : x^2$

$\Rightarrow x \times \left(\frac{2x}{\pi}\right)^2 : x^2 \Rightarrow \frac{4x^2}{\pi} : x^2 \Rightarrow 4 : \pi : \Rightarrow 4 : \frac{22}{7}$

$\Rightarrow \quad 14 : 11.$

18. Suppose the length and the width of the rectangle are x metre and y metre respectively.

$\therefore$ Area of the rectangle = xy sq. metre

According to question :

On effecting 20% increase in the length and 15% decrease in the width :

Length of the new rectangle

$= x + 20\%$ of $x = 1.2\,x$ metre

Width of the new rectangle

$= y - 15\%$ of $y = .85\,y$ metre

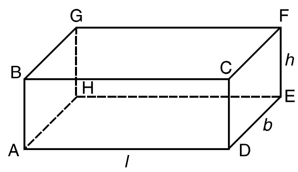
$\therefore$ Area of the new rectangle

$= 1.2x \times .85y = 1.020\,xy$ sq. metre

Prism; Right Circular Cone; Right Circular Cylinder; Sphere; Hemisphere; Right Pyramid; Rectangular Parallelepiped

In mensuration we often have to deal with the problem of finding the volume of solid figure.

Cuboid : A cuboid has six faces, each one a ractangle. It has 12 edges. For example, a rectangular brick.



Let length = l, Breadth = b and height = h, then,

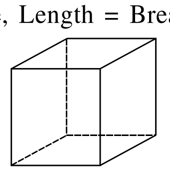
1. Volume = (Length × Breadth × Height)

2. Whole surface = $2(lb + bh + lh)$

3. Diagonal = $\sqrt{l^2 + b^2 + h^2}$

4. Area of 4 walls of a room = $2 \times h\ (l + b)$

Cube : In a cube, Length = Breadth = Height



1. Volume = $(l)^3$

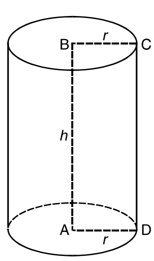
2. Length = $\sqrt[3]{\text{Volume}}$

3. Whole Surface area = $6\ l^2$

4. Diagonal = $l \times \sqrt{3}$

5. Lateral surface area = $4\ l^2$

Cylinder :



1. Volume = $\pi r^2 h$

2. Curved surface Area = $2\pi rh$

3. Total surface area = $2\pi r(r + h)$
where r = radius, h = height

4. Curved surface of hollow cylinder
= $2\pi h\ (r_1 + r_2)$

5. Total surface of hollow cylinder
= $2\pi h\ (r_1 + r_2) + 2\pi\ (r_1^2 - r_2^2)$

6. Volume of hollow cylinder

$= \pi h\left(r_1^2 - r_2^2\right)$

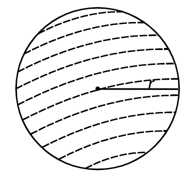
where r_1 = outer radius
r_2 = inner radius

Spherical Cell :

1. Volume = $\frac{4}{3}\pi\left(R^3 - r^3\right)$

2. Total surface area = $4\pi(R^2 - r^2)$
where R = Outer radius
r = Inner radius

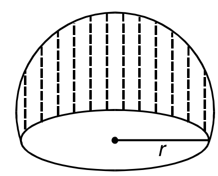
Sphere



1. Volume = $\frac{4}{3}\pi r^3$

2. Surface area = $4\pi r^2$

Semi-Sphere

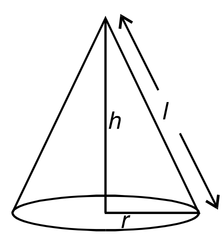


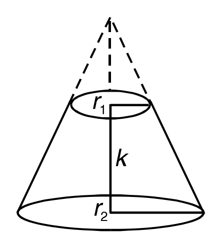
1. Volume = $\frac{2}{3}\pi r^3$

2. Curved surface area = $2\pi r^2$

3. Total surface area = $3\pi r^2$

Cone :





1. Slant height (l) = $\sqrt{r^2 + h^2}$

2. Volume = $\frac{1}{3}\pi r^2 h$

3. Curved surface area = πrl

4. Total surface area = $\pi r\ (l + r)$

5. If the thickness of the frustum of a cone be k and the radii of its ends are r_1 and r_2, then

(i) Slant height of the frustum of a cone
$= \sqrt{K^2 + (r_1 - r_2)^2}$

(ii) Curved surface of the frustum = $\pi(r_1 + r_2)\ l$.

(iii) Volume = $\frac{\pi K}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$

Right Pyramid

1. Volume = $\frac{1}{3}(\text{area of the base}) \times \text{height}$

2. Lateral surface area
$= \frac{1}{2}(\text{Perimeter of the base}) \times \text{Slant height}$

3. Total surface area = 2 (Area of one end) + Lateral surface area

Right Prism

1. Volume = Area of the base × Height
2. Lateral surface Area = Perimeter of the Base × Height
3. Total surface Area = 2 (Area of one end) + Lateral surface area.

Rectangular Parallelepiped

Sometimes also referred to as "Rhomboid", a parallelepiped is a 3-D shape moulded by 6 parallelograms. If observed more carefully, as a cube relates to a square, a cuboid relates to a rectangle, the same way a parallelepiped is related to parallelogram.

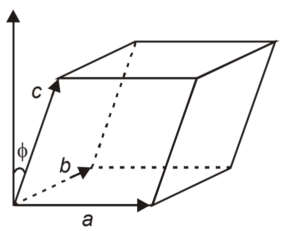
We have the following formula for finding out the volume, lateral surface area and surface area of rectangular parallelepiped.

The formulas are:

$$\text{Volume} = abc$$

$$\text{Surface area} = 2ab + 2bc + 2ac$$

$$\text{Diagonal} = \sqrt{a^2 + b^2}$$



Example: Counting 38 cu. ft. of coal to a ton, how many tons will a coal bin 19 ft. long, 6 ft. wide, and 9 ft. deep contain, when level full?

Solution: The volume of a rectangular parallelepiped is given by the formula

$$V = L \times W \times H$$

Substitute the values of length, width and height of a tin, we have

$$V = L \times W \times H$$
$$V = (19\text{ft})\ (6\text{R.})\ (9\text{ ft.})$$
$$V = 1026\ R^3$$

The density of a substance is given by the formula

$$\rho = \frac{W}{V}$$

where, ρ is the density, W is the weight, and V is the volume of a substance respectively.

$$W = V \times \rho$$

Therefore, the weight of a coal in a bin is:

$$W = (1026\,R^3)\left(\frac{1\text{ ton}}{38\,n^3}\right)$$

$$W = 27\text{ tons.}$$

Example: The base face of a parallelepiped has opposite sides measuring 5 inches and 10 inches. The height of the parallelepiped is 4 inches. Find the cost of painting its walls from outside at a cost of INR 1.5 per square inch.

Solution: We need to find the lateral surface area first, therefore;

LSA = Perimeter of base × height

LSA = 2 (5 + 10) + 6

LSA = 180 sq. inch

Cost of painting = Lateral surface area × cost per square inch

Cost of painting the walls = 180 × 1.5 = ₹ 270.

EXERCISE

1. A solid in the form of a cuboid is 4 cm × 3 cm × 2 cm. Its volume will be
   A. 20 cu cm B. 22 cu cm
   C. 28 cu cm D. 24 cu cm
2. A reservoir is 3 m long, 2 m wide and 1 m deep. Its capacity in litres is
   A. 8000 litres B. 10000 litres
   C. 6500 litres D. 6000 litres
3. Surface area of a cube is 1014 sq. cm. Its volume will be
   A. 2197 cu cm B. 2297 cu cm
   C. 2179 cu cm D. 2117 cu cm
4. If the volumes of two cubical blocks are in the ratio of 8 : 1, what will be the ratio of their edges?
   A. 1 : 2 B. 2 : 1
   C. 4 : 1 D. 2 : 3
5. Two spheres have their surface areas in the ratio 9 : 16. Their volumes are in the ratio of
   A. 64 : 27 B. 27 : 64
   C. 16 : 27 D. 11 : 27
6. The length of the longest rod that can be place in a room 12 m long, 9 m broad and 8 m high is
   A. 17 m *(b)* 18 m
   C. 25 m D. 16 m
7. The radius and the height of a right circular cone are in the ratio of 3 : 5. If its volume is 120π cu m, its slant height is
   A. $3\sqrt{34}$m B. $2\sqrt{28}$m
   C. $2\sqrt{44}$m D. $2\sqrt{34}$m
8. Circumference of the base of a cylinder is 88 cm and height of the cylinder is 42 cm. Its volume is
   A. 25872 cu cm B. 28572 cu cm
   C. 25870 cu cm D. 22584 cu cm
9. It fwo cubes each of 10 cm side are kept close to each other, then the cuboid so formed will have surface area equal to
   A. 1200 sq. cm B. 5000 sq. cm
   C. 1000 sq. cm D. 1250 sq. cm

10. A rectangular piece of paper is 30 cm long and 20 cm wide. How many ways can be adopted if one wants to give this rectangular piece of paper a cylindrical form?
A. Three B. Two
C. One D. Four

11. In the above question, the cylinders formed will have their volumes in the ratio of
A. 2 : 3 B. 3 : 1 : 1
C. 3 : 2 D. 1 : 3 : 1

12. If a solid sphere of 3 cm radius is melted and recast into a right circular cone whose base radius is same as that of the sphere, the height of the cone will be
A. 8 cm B. 12 cm
C. 6 cm D. 5 cm

13. Diameter of a roller is 2.4 m and it is 1.68 m long. If it takes 1000 complete revolutions once over to level a field, the area of the field is
A. 12672 sq. m B. 12671 sq. m
C. 12762 sq. m D. 11768 sq. m

14. If each edge of a cube is increased by 10%, then by how much percent will the surface area of this cube be increased?
A. 21% B. 18%
C. 15% D. 20%

15. Height and base radius of a solid cylinder are 14 m and 4 m respectively. It is melted and recast into a solid cone of the same base radius as that of the cylinder, what will be the height of the cone?
A. 21 m B. 42 m
C. 48 m D. 54 m

16. A room is in the form of a cube of side 10 m. How many bales of cotton can be kept in it if each bale covers 5 cu m space?
A. 100 B. 175
C. 200 D. 225

17. Three cubes having side 2 cm, 3 cm respectively are melted together to form a new cube. The side of the new cube will be
A. 3.526 cm B. 4.628 cm
C. 4.626 cm D. 4.528 cm

18. If base diameter of a cylinder is increased by 50%, then by how much percent its height must be decreased so as to keep its volume unaltered?
A. 45.56% B. 55.56%
C. 50.16% D. 62.33%

19. The surface area of a cube is 600 sq. m. Its diagonal is
A. $10\sqrt{3}$ cm B. $5\sqrt{3}$ cm
C. $4\sqrt{2}$ cm D. $10\sqrt{2}$ cm

20. The base diameter of a conical tomb is 28 m and its slant height is 50 m. Find the cost of white-washing its curved surface at the rate of 80 paise per sq. m?
A. ₹ 1860 B. ₹ 1760
C. ₹ 1950 D. ₹ 1875

ANSWERS

1	2	3	4	5	6	7	8	9	10
D	D	A	B	B	A	D	A	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	B	A	A	B	C	C	B	A	B

EXPLANATORY ANSWERS

1. Volume of the cuboid
$= l \times b \times h = 4 \times 3 \times 2 = 24$ cu. cm.

2. Volume of the reservoir
$= l \times b \times h = 3 \times 2 \times 1 = 6$ cu. cm
($\because$ 1 cu m = 1000 litre)
$\therefore$ Capacity of the reservoir
$= 6 \times 1000 = 6000$ litre.

3. Surface area of a cube $= 6 \times (\text{side})^2$
$\therefore \quad 6 \times (\text{side})^2 = 1014$
$\Rightarrow \quad (\text{side})^2 = \frac{1014}{6} = 169$
$\Rightarrow \quad \text{side} = \sqrt{169} = 13$ cm
$\therefore$ Volume of the cube $= (\text{side})^3 = (13)^3$
$= 2197$ cu cm.

4. Suppose sides of the two cubical blocks are a_1 and a_2 respectively
$\therefore$ Volume of the two cubical blocks will be $(a_1)^3$ and $(a_2)^3$ rspectively
According to question :
$$a_1^3 : a_2^3 = 8 : 1$$
$$\therefore \quad \frac{a_1^3}{a_2^3} = \frac{8}{1}$$
$$\therefore \quad \left(\frac{a_1}{a_2}\right)^3 = \left(\frac{2}{1}\right)^3 \Rightarrow a_1 : a_2$$
$$= 2 : 1$$
Therefore ratio of their edges = 2 : 1.

5. Suppose radii of the two spheres are r_1 and r_2 respectively.

$\therefore$ Surface areas of the two spheres are $4\pi r_1^2$ and $4\pi r_2^2$ respectively

According to question :

$4\pi r_1^2 : 4\pi r_2^2 = 9 : 16 \Rightarrow r_1^2 : r_2^2 = 9 : 16$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{9}{16} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2$$

$$\Rightarrow \quad r_1 : r_2 = 3:4 \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{3}{4}\right)^3$$

$$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{27}{64} \Rightarrow r_1^3 : r_2^3 = 27 : 64.$$

Therefore ratio of their volumes

$$= \frac{4}{3}\pi r_1^3 : \frac{4}{3}\pi r_2^3 = r_1^3 : r_2^3 = 27 : 64$$

6. The longest rod that can be placed in the cuboidal room = Length of the diagonal =

$$\sqrt{l^2 + b^2 + h^2} = \sqrt{(12)^2 + (9)^2 + (8)^2}$$

$$= \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ m}$$

7. Suppose the base radius and the height of the right circular cone are $3x$ m and $5x$ m respectively.

$\therefore$ Volume of the cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3x)^2 \times 5x \text{ cu m}$$

According to question :

Volume of the cone = 120π cu m (given)

$$\therefore \quad \frac{1}{3}\pi \times 9x^2 \times 5x = 120\pi$$

$$\Rightarrow \quad x^3 = \frac{120 \times 3}{9 \times 5}$$

$$\Rightarrow \quad x^3 = 8 \Rightarrow x^3 = (2)^3$$

$$\Rightarrow \quad x = 2 \text{ m}$$

$\therefore$ The radius and the height of the cone will be $3 \times 2 = 6$ m and $5 \times 2 = 10$ m respectively.

$\therefore$ Slant height of the cone

$$= \sqrt{r^2 + h^2} = \sqrt{(6)^2 + (10)^2}$$

$$= \sqrt{36 + 100} = \sqrt{136} = 2\sqrt{34} \text{m}.$$

8. Suppose the base radius of the cylinder = r cm

$\therefore$ Circumference of the base of the cylinder = $2\pi r$ cm

According to question :

$$2\pi r = 88$$

$$\therefore \quad r = \frac{88}{2\pi} = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

$\therefore$ Volume of the cylinder

$$= \pi r^2 h = \frac{22}{7} \times (14)^2 \times 42$$

$$= 22 \times 2 \times 14 \times 42$$

$$= 25872 \text{ cu cm.}$$

9. Candidates should note that in such type of questions where two cubes of equal edges are kept close to each other, only the length of the cuboid so form increases and breadth and height of the cuboid remain same as that of the cube.

$\therefore$ Length of the cuboid = Edge of the first cube + Edge of the second cube

$$= 10 + 10 = 20 \text{ cm}$$

$\therefore$ Surface area of the cuboid

$$= 2(20 \times 10 + 10 \times 10 + 10 \times 20)$$

$$= 2(200 + 100 + 200)$$

$$= 5 \times 500 = 1000 \text{ sq. cm.}$$

10. Obviously, two ways can be adopted if one wants to give the rectangular piece of paper a cylindrical form, *i.e.,*

1. *When the paper is bent towards its length.* In this case, the circumference of the base of the cylinder will be equal to the length of the rectangular piece of paper and the height of the cylinder will be equal to the breadth of the rectangular piece of paper.
2. *When the paper is bent towards its breadth.* In this case, the circumference of the base of the cylinder will be equal to the breadth of the rectangular piece of paper and the height of the cylinder will be equal to the length of the rectangular piece of paper.

11. In the first case :

$$2\pi r = 30$$

$$r = \frac{15}{\pi} \text{ cm and } h = 20 \text{ cm}$$

$\Rightarrow$ volume $(v_1) = \pi r^2 h$

$$= \frac{15 \times 15 \times 20}{\pi} = \frac{4500}{\pi} \text{ cu cm}$$

In the second case :

$$2\pi r = 20 \Rightarrow r = \frac{10}{\pi} \text{ cm and } h = 30 \text{ cm}$$

$\therefore$ Volume $(v_2) = \pi r^2 h$

$$= \frac{10 \times 10 \times 30}{\pi} = \frac{3000}{\pi} \text{ cu cm}$$

$\therefore$ Ratio of the two volumes

$$= v_1 : v_2 = \frac{4500}{\pi} : \frac{3000}{\pi} = 3 : 2$$

12. Volume of the sphere of radius 3 cm

$$= \frac{4}{3}\pi (3)^3 = \frac{4}{3}\pi \times 27 \text{ cu cm}$$

Suppose the height of the cone = h cm

∴ Volume of the cone having base radius equal to that of the sphere = $\frac{1}{3}\pi(3)^2 \times h$

∵ Volume of the cone = Volume of the sphere

$$\therefore \quad \frac{1}{3}\pi(3)^2 \times h = \frac{4}{3}\pi \times 27 \Rightarrow h = 12 \text{ cm}$$

∴ Height of the cone = 12 cm.

13. Diameter of the roller = 2.4 m

∴ Radius of the roller = 1.2 m

And height (length) of the roller = 1.68 m

∴ Surface area of the roller

$$= 2\pi rh = 2 \times \frac{22}{7} \times 1.2 \times 1.68 = 12.672 \text{ sq. m}$$

∴ In one complete revolution, the roller covers 2.672 sq. m.

∴ It will cover in 1000 revolutions = 12.672 × 1000 = 12672 sq. m

∴ Area of the field = 12672 sq. m.

14. The two edges which are included in surface area of the cube are increased by 10%.

$$\therefore \quad x\% = y\% = 10\%$$

and in case of percentage increase, values of x and y are positive

∴ Percentage increase in the surface area of the cube

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(10 + 10 + \frac{10 \times 10}{100}\right)\% = 21\%.$$

15. Volume of the solid cylinder

$= \pi r^2 h = \pi r^2 \times 14$ cu m

According to question :

Radius of the cone = Radius of the cylinder

= r m = 4 m

and volume of the cone

= Volume of the cylinder

$$\therefore \quad \frac{1}{3}\pi r^2 \times \text{height} = \pi r^2 \times 14$$

$$\therefore \quad \text{Height} = 14 \times 3 = 42 \text{ m}$$

∴ Height of the cone = 42 m.

16. Volume of the cubical room

$= (10)^3 = 1000$ cu m

Number of cotton bales which can be placed in the room

$$= \frac{\text{Volume of the room}}{\text{Volume of each cotton bale}} = \frac{1000}{5} = 200.$$

17. Edges of the three cubes are 2 cm, 3 cm and 4 cm respectively

∴ Their volumes will be $(2)^3 = 8$ cu cm, $(3)^3 = 27$ cu cm and $(4)^3 = 64$ cu cm respectively.

Volume of the new cube = Total volume of the three cubes

∴ Volume of the new cube

= 8 + 27 + 64 = 99 cu cm

∴ Side of the new cube = $\sqrt[3]{99}$ = 4.626 cm.

18. Suppose the height of the cylinder should be decreased by H%

∵ Volume of a cylinder comprises two radii (*i.e.,* two edges) and one height (as the third edge). Radius is increased by 50%.

It means $x\% = y\% = 50\%$ and percentage decrease in the third edge (*i.e.,* height) = H%.

Therefore, z% = H% and in case of percentage decrease value of z will be negative.

∴ Change in the volume of the cylinder

$$= \left(x + y + (-z) + \frac{xy + y(-z) + (-zx)}{100} + \frac{xy(-z)}{100^2}\right)\%$$

Since volume of the cylinder remains unchanged.

$$\therefore \quad \text{Change} = 0\%$$

$$\therefore \left(50 + 50 + (-\text{H}) + \frac{50 \times 50 - 50\text{H} - 50\text{H}}{100} + \frac{50 \times 50 \times (-\text{H})}{100^2}\right)\%$$

∴ 100 – H + 25 – H – .25H = 0

$$\Rightarrow \quad 2.25\text{H} = 125$$

$$\Rightarrow \quad \text{H} = \frac{125}{2.25} = 55.56$$

∴ Height of the cylinder should be decreased by 55.56%.

19. ∵ Surface area of the cube = 6 × (side)^2

$$\therefore \quad 6 \times (\text{side})^2 = 600$$

$$\Rightarrow \quad \text{side}^2 = 100$$

$$\Rightarrow \quad \text{side} = \sqrt{100} = 10 \text{ cm}$$

∴ Diagonal of the cube

$$= \sqrt{3} \times \text{side} = \sqrt{3} \times 10 = 10\sqrt{3} \text{ cm.}$$

20. ∵ Area of the curved surface of the cone = πrl

(where r = radius of the cone and l = slant height of the cone)

∴ Area of the curved surface of the cone

$$= \frac{22}{7} \times \frac{28}{2} \times 50 = 2200 \text{ sq. m.}$$

∴ Cost of whitewashing at 80 paise per sq. m

$$= 2200 \times \frac{80}{100} = ₹\ 1760.$$