Verify the Algebraic Identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

OBJECTIVE

To verify the algebraic identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.

Materials Required

1. Geometry box
2. Acrylic sheet
3. Scissors
4. Adhesive/Adhesive tape
5. Cutter

Prerequisite Knowledge

1. Concept of cuboid and its volume.
2. Concept of cube and its volume.

Theory

1. For concept of cuboid and its volume refer to Activity 7.
2. For concept of cube and its volume refer to Activity 7.

Procedure

1. By using acrylic sheet and adhesive tape/adhesive, make a cube of side $(a - b)$ units, where $a > b$. (see Fig. 8.1)

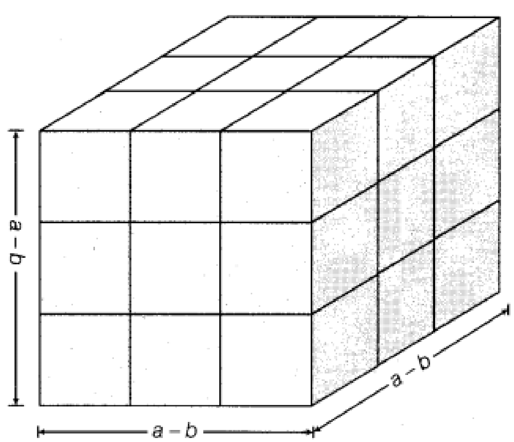

Fig. 8.1

2. By using acrylic sheet and adhesive tape, make three cuboids each of dimensions, (a-b) x a x b. (see Fig. 8.2)

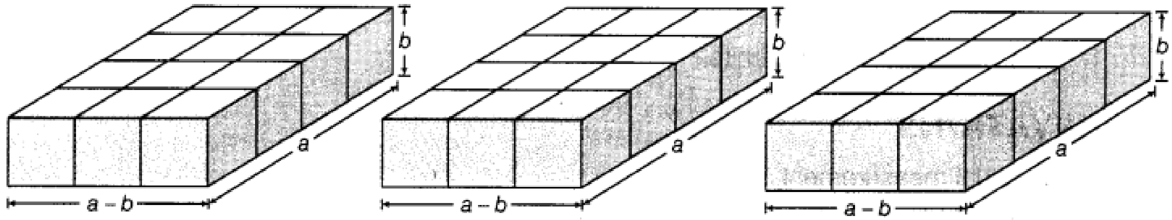

Fig. 8.2

3. By using acrylic sheet and adhesive tape make a cube of side b units, (see Fig. 8.3)

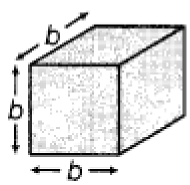

Fig. 8.3

4. Arrange all the cubes and cuboids as shown in Fig 8.4.

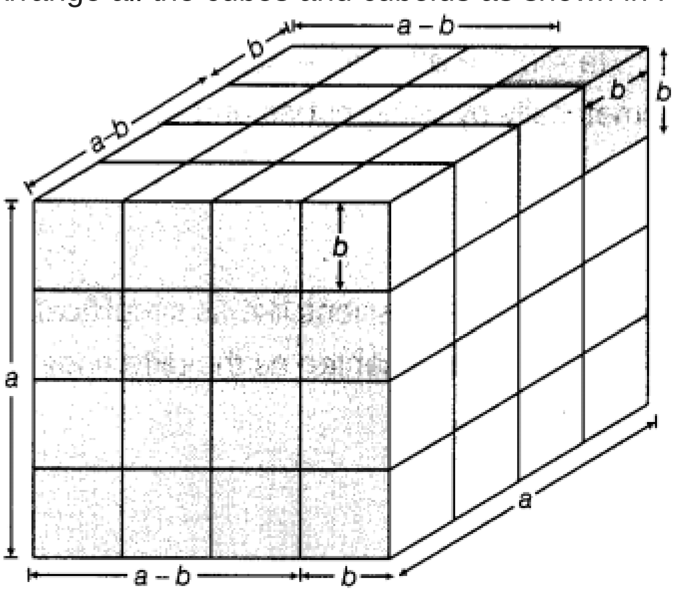

Fig. 8.4

Demonstration

In Fig. 8.1, volume of the cube of side (a – b) units = $(a - b)^3$
In Fig. 8.2, volume of a cuboid of sides (a – b) x a x b =(a – b)ab
In Fig. 8.2, volume of three cuboids = 3 x (a – b) ab In Fig. 8.3, volume of the cube of side b = b^3
In Fig. 8.4, volume of the solid = Sum of volume of all cubes and cuboids

$= (a- b)^3 + (a – b) . ab + (a – b) . ab + (a – b) ab + b^3$
$= (a – b)^3+3 (a – b) . ab + b^3$ …(i)
Also, the obtained solid in Fig. 8.4 is a cube of side a.
Therefore, its volume = a^3
From Eqs. (i) and (ii), we get
$(a – b)^3 + 3ab (a – b) + b^3= a^3$
$\Rightarrow (a – b)^3 = a^3 – b^3 – 3ab (a – b)$
Here, volume is in cubic units.

Observation

By actual measurement,
a = …….. , b = …….. , a-b =…….. ,
So, a^3 =…….. , ab = …….. ,
b^3 =…….. , ab(a – b) = …….. ,
3ab(a – b) = …….. , $(a – b)^3$ = ……..
Therefore, we observe that
$(a-b)^3 = a^3 – b^3 -3ab (a-b)$ or $(a-b)^3 =a^3-3a^2b + 3ab^2-b^3$

Result

From above observation, algebraic identity for any a, to, where (a > b) is $(a – b)^3 = a^3 – b^3 – 3ab(a – b)$
Has been verified geometrically by using cubes and cuboids.

Application

This identity is useful in

1. many operations of algebraic expressions like as simplification and factorization.
2. calculating cube of a number represented as the difference of two convenient numbers.

Viva Voce

Question 1:
What is the formula of the volume of a cube?
Answer:
Volume of a cube = side x side x side = $(\text{side})^3$

Question 2:
What is the formula of the volume of a cuboid?
Answer:
Volume of a cuboid = length x breadth x height

Question 3:
How would you expand $a^3 – b^3$, in the terms of $(a – b)^3$?
Answer:

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
$= a^3 - b^3 - 3a^2b + 3ab^2 \Rightarrow a^3 - b^3 = (a - b)^3 + 3a^2b - 3ab^2$

Question 4:
What is the expanded form of $(a - b)^3$?
Answer:
Expanded form of $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Question 5:
Does the resulted value of the product of $(a - b)^2$ and $(a - b)$ is same as $(a - b)^3$? Give reason.
Answer:
Yes, because $(a - b)^2 (a - b) = (a - b)^3 = a^3 - b^3 - 3ab(a - b)$ [$\therefore A^m \times A^n = (A)^{m+n}$]

Suggested Activity

Verify that $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$ by taking $x = 100$ and $y = 2$.