Chapter 3 Current Electricity

Question 1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?
Solution: E = 12V
r = 0.4 Ω
Maximum current is drawn when external resistance of the circuit is zero, i.e. R = 0

$$\therefore \quad I_{max} = \frac{E}{r} = \frac{12}{0.4} = 30A$$

Question 2. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?
Solution: E = 10 V, r = 3 Ω, I = 0. 5 A
E = V + Ir = IR + Ir

$$R = \frac{E - Ir}{I}$$
$$= \frac{10 - 0.5 \times 3}{0.5}$$
$$= \frac{8.5}{0.5}$$
$$= 17\ \Omega$$

V = IR = 0.5 x 17 = 8.5 V

Question 3. (a) Three resistors IΩ, 2Ω, and 3Ω are combined in series. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.
Solution: (a) In series, $R = R_1 + R_2 + R_3 = 1 + 2 + 3 = 6\Omega$

(b) $V = IR$, $I = \frac{V}{R} = \frac{12}{6} = 2A$

Potential drop across $I\Omega = V_1 = IR_1 = 2 \times 1 = 2\ V$

Potential drop across $2\Omega = V_2 = 2 \times 2 = 4\ V$

Potential drop across $3\Omega = V_3 = 2 \times 3 = 6V$

Question 4. (a) Three resistors 2Ω, 4Ω, and 5Ω are combined in parallel. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor and the total current drawn from the battery.
Solution:

a. $\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{10+5+4}{20} = \frac{19}{20}$ $\quad \therefore R = \frac{20}{19}\Omega$

b. Current through $2\Omega = \frac{20}{2} = 10A$

Current through $4\Omega = \frac{20}{4} = 5A$ Current through $5\Omega = \frac{20}{5} = 4A$

Hence total current, $I = 10 + 5 + 4 = 19A$

Or $I = \frac{E}{R} = \frac{20}{\left(\frac{20}{19}\right)} = 19A$

Question 5. At room temperature (27.0°C) the resistance of a heating element is 100 Ω,. What is the temperature of the element if the resistance is found to be 117 Ω,. Given that the temperature coefficient of the material of the resistor is $1.70 \times 1o^{-4}°C^{-1}$.
Solution:

T_1 = 27°C, R_1 = 100Ω, RΩ = 117 Ω, α = 1.7 x $1o^{-4}$°C^{-1}

$R_1 = R_2(1+a\ ?T)$

$$\Delta T = \frac{R_2 - R_1}{R_1 \alpha}$$

$$= \frac{117-100}{100 \times 1.7 \times 10^{-4}}$$

= 1000°

$T_2 = T_1 + \Delta T$ = 27 + 1000=1027°C

Question 6. A negligibly small current is passed through a wire of length 15m and uniform crosssection 6.0 x 10^{-7} m^2 and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

Solution: R=5Ω, l = 115m, A = 6 x 10^{-7} m^2

$$\text{resistivity, } \rho = \frac{RA}{l}$$

$$= \frac{5 \times 6 \times 10^{-7}}{15}$$

= 2 x 10^{-7} Ω m

Question 7. A silver wire has a resistance of 2.1 Ω at 27.5°C and a resistance of 2.7 ? at 100°C. Determine the temperature coefficient of resistivity of silver.

Solution: R_1 = 2.1 Ω, T_1 = 27.5°C

R_2 = 2.7 Ω, T_1 = 100°C

$$\alpha = \frac{R_2 - R_1}{R_1 \cdot \Delta T}$$

$$= \frac{2.7-2.1}{2.1(100-27.5)}$$

= 0.0039°C^{-1}

Question 8. A heating element using nichrome connected to a 230V supply draws an initial current of 3.2 A which settles after a few seconds to steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0°C? The temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70 x 10^{-4}°C^{-1}.

Solution:

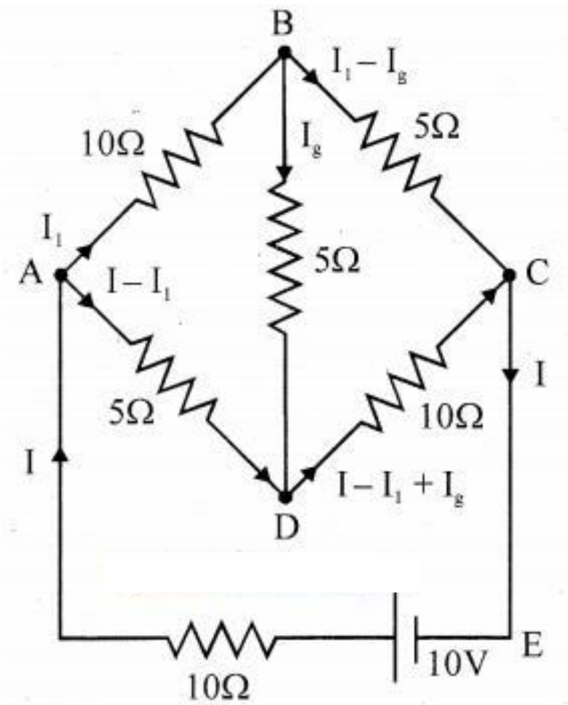
$$R_1 = \frac{230\text{V}}{3.2\text{A}} = 71.88\,\Omega \qquad R_2 = \frac{230\text{V}}{2.8\text{A}} = 82.14\,\Omega$$

$$\alpha = 1.7 \times 10^{-4}\ {}^\circ\text{C}^{-1},\ T_1 = 27^\circ\text{C} \qquad R_2 = R_1(1 + \alpha.\Delta T)$$

$$\Delta T = \frac{R_2 - R_1}{R_1\alpha} = \frac{82.14 - 71.88}{71.88 \times 1.7 \times 10^{-4}} = 840^\circ\text{C}$$

$$T_2 = T_1 + \Delta T = 27 + 840 = 867^\circ\text{C}$$

Question 9. Determine the current in each branch of the network shown in Fig.



Solution: For the mesh ABDA−→−−−, $10I_1 + 5I_g - 5(I - I_1) = 10$
$15I_1 + 5I_g - 5I = 0$
$3I_1 + I_g - I = 0$ …. (1)
For the mesh ABDA−→−−−, $10I_1 + 5I_g - 5(I - I_1) = 10$
$15I_1 + 5I_g - 5I = 0$
$3I_1 + I_g - I = 0$ …. (1)
For the mesh BDCB−−→−−−,
$5I_g + 10(I - I_1 + I_g) - 5(I_1 - I_g) = 0$
$-15I_1 + 20I_g + 10I = 0$
i.e., $3I_1 - 4I_g - 2I = 0$ … (2)

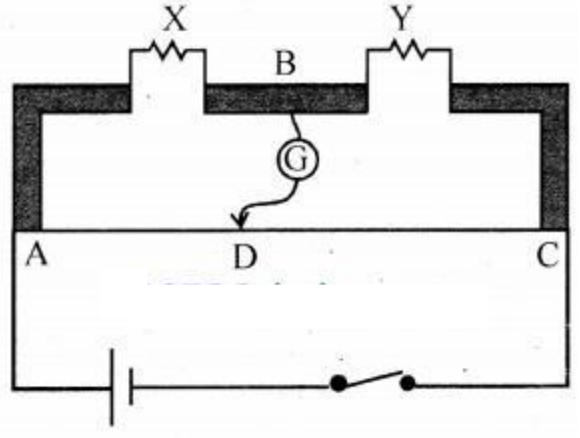
For the mesh ABCEA——→————, $10I_1 + 5(I_g - (I - I_1 + 101 = 10$
$15I_1 - 5I_g + 10I = 10$
$3I_1 - I_g + 10I = 10$ .... (3)
Equation (1) + (3) ⇒, $6I_1 + I = 2$ .... (4)
Equation (1) x 4 + (2) ⇒ $15I_1 - 6I = 0$ .... (5)

Solving equation (4) and (5) ⇒ $I_1 = \frac{4}{17}$ A and $I = \frac{10}{17}$
Substituting for I_1 and I in equation (3) ⇒ $I_g = -\frac{2}{17}$A

The negative sign shows that the direction of current in the branch BD is opposite to that shown in the figure.

Current through AB = $\frac{4}{17}$ A
Current through BC = $\frac{6}{17}$ A
Current through AD = $\frac{6}{17}$ A
Current through DC = $\frac{4}{17}$ A
Current through BD = $-\frac{2}{17}$ A

Question 10. (a) In a metre bridge the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5 Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?
(b) Determine the balance point of the bridge if X and Y are interchanged.
(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?



Solution:

(a) $\frac{X}{Y} = \frac{l}{100-l}$

$\frac{X}{12.5} = \frac{39.5}{60.5}$

$\therefore X = 8.16\ \Omega$

Thick wires minimise the resistance of the connections and hence they will not affect the bridge formula.

(b) $\frac{Y}{X} = \frac{l'}{100-l'}$

$\frac{12.5}{8.16} = \frac{l'}{100-l'}$

Solving we get, l′ = 60.5 cm

(c) The galvanometer will not show any current.

Question 11. A storage battery, of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?
Solution: r = 0.5 Ω, R = 15.5 Ω
when the storage battery emF 8 V is charged with a d.c supply of 120V the net EMF of the circuit
E= 120 – 8 = 112V
Therefore the current in the circuit during charging,

$$I = \frac{E}{R+r} = \frac{112}{15.4+0.5} = 7A$$

The terminal voltage of the storage battery would be equal to the sum of its EMF and the potential difference across its internal resistance i.e. terminal voltage = 8 + 0.5 x 7 = 11.5 V

Question 12. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second

cell?
Solution: $E \propto l$

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$
$$\therefore E_2 = \frac{l_2}{l_1} \times E_1 = \frac{63}{35} \times 12.5$$

i.e., $E_2 = 2.25$ V

Question 13. The number density of free electrons in a copper conductor is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire of 3.0 m long to its other end? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.
Solution: I = 3A
$n = 8.5 \times 10^{28}$ m^3
$e = 1.6 \times 10^{-19}$ C
$A = 2 \times 10^{-6}$ m^2
l = 3 m

$$v_d = \frac{I}{neA} = \frac{3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-6}} = 1.1 \times 10^{-4} \text{ m/s}$$

$$t = \frac{l}{v_d} = \frac{3}{1.1 \times 10^{-4}} = 2.72 \times 10^4 \text{ sec}$$

Question 14. The earth's surface has a negative surface charge of 10^{-9} Cm^{-2}. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different parts of the globe). (Radius of earth = 6.37×10^6m.)
Solution: $\sigma = 10^{-9}$ Cm^{-2}
$V = 400 \times 10^3$
I = 1800 A
$R_e = 6.37 \times 10^6$m
Total charge, $Q = \pi R^2_e \sigma$

Current, $I = \frac{Q}{t}$ $\quad \therefore t = \frac{Q}{I} = \frac{4\pi R_e^2 \sigma}{I} = \frac{4 \times 3.14 \times (6.37 \times 10^6)^2 \times 10^{-9}}{1800}$ $\quad t \approx 283$ sec

Question 15. (a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What is the current drawn from the supply and its terminal voltage?

(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?

Solution:

a. $I = \frac{nE}{R + m} = \frac{6 \times 2V}{8.5\Omega + (6 \times 0.015\Omega)} = 1.397$ A

Terminal voltage, V = 1 R = 1.397 × 8.5 = 11.88 V

b. $I_{max} = \frac{E}{r} = \frac{1.9}{380} = 0.005$ A

The cell cannot drive the starting motor of the car.

Question 16. Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. (ρ_{Al} = 2.63 x 10^{-8}Ωm, ρ_{cu} = 1.72 x 10^{-8}Ωm, Relative density of Al = 2.7 and that of Cu = 8.9.)

Solution:

We have, $M = \frac{\rho l^2 d}{R}$

For Cu, $M_{cu} = \frac{1.72 \times 10^{-8} \times l^2 \times 8.9}{R}$ ——— (1)

For Al, $M_{Al} = \frac{2.63 \times 10^{-8} \times l^2 \times 2.7}{R}$ ——— (2)

$\frac{(1)}{(2)} \Rightarrow \frac{M_{Cu}}{M_{Al}} = 2.2$

∴ Aluminium is lighter than copper. Since Al is lighter, it is preferred for overhead power cables.

Question 17. What conclusion can you draw from the following observations on a resistor made of alloy manganin?

Current A	Voltage V	Current A	Voltage V
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	118.5
1.0	19.7	7.0	138.2
2.0	39.4	8.0	158.0

Solution: This represents Ohm's Law.
The resistivity of manganin remains almost the same with the change in temperature.

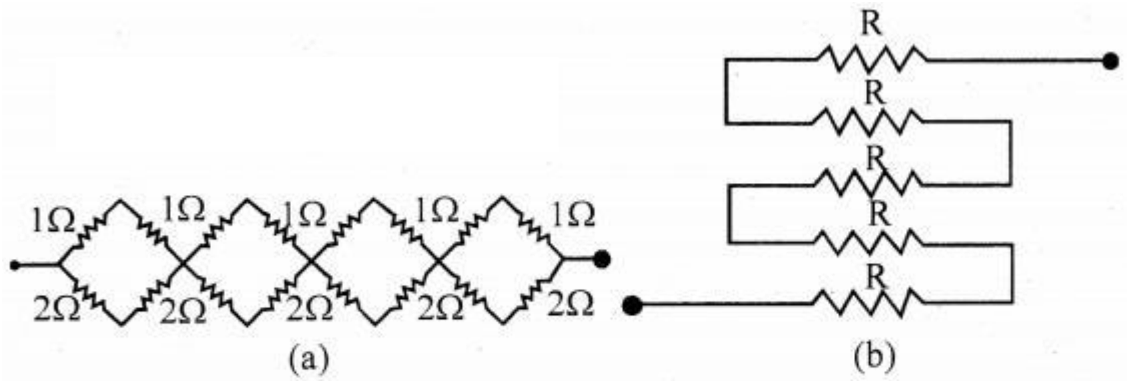
Question 18. Answer the following questions.
(a) A steady current flows in a metallic conductor of the non-uniform cross-section. Which of these quantities is constant along the conductor – current, current density, electric field, drift speed?
(b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.
(c) A low voltage supply from which one needs high currents must have

very low internal resistance. Why?
(d) A high tension (HT) supply of, say, 6 kV must have a very large internal resistance. Why?
Solution: (a) Except current the values of all the other quantities depend upon the area of cross-section of the conductor. Hence, only current remains constant, when it flows through a conductor of the non-uniform area of crosssection.

(b) No, ohm's law is not obeyed by all the elements. For example, vacuum diode tube and semiconductor diode.

(c) The maximum current that can be drawn from a voltage supply is given by,
Imax=Er Obviously, Imax will be large, if r is small.
(d) If the circuit containing the H.T supply gets short-circuited accidently, the current in the circuit will not exceed the safe limit, in case the internal resistance of the H.T supply is very large.

Question 19. Choose the correct alternative:
(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.
(b) Alloys usually have much (lower/higher) temperature coefficient of resistance than pure metals.
(c) The resistivity' of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.
(d) The resistivity' of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of ($10^{22}/10^{3}$).
Solution: (a) greater
(b) lower
(c) nearly independent
(d) 10^{22}
Question 20.
(a) Given n resistors each of resistance R. How will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum effective resistance?
(b) Given the resistances of 1Ω, 2Ω, 3Ω. How will you combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5)Ω, (iii) 6Ω, (iv) (6/11)Ω?
(c) Determine the equivalent resistance of networks shown in the figure.



Solution: (a)

$R_{max} = R + R + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ n times} = nR$

$\frac{1}{R_{min}} = \frac{1}{R} + \frac{1}{R} + \ldots\ldots\ldots\ldots \text{n times} = \frac{n}{R} \qquad \therefore R_{min} = \frac{R}{n}$

$\frac{R_{max}}{R_{min}} = \frac{nR}{\left(\frac{R}{n}\right)} = n^2$

(b) $R_1\Omega = 10$, $R_2 = 2\Omega$, $R_3 = 3\Omega$

i. $(R_1||R_2)$ series with R_3

$$R = \frac{R_1R_2}{R_1 + R_2} + R_3 = \frac{1 \times 2}{1 + 2} + 3 = \frac{11}{3}\Omega$$

ii. $(R_2||R_3)$ series with R_1

$$R = \frac{R_2R_3}{R_2 + R_3} + R_1 = \frac{2 \times 3}{2 + 3} + 1 = \frac{11}{5}\Omega$$

iii. $R = R_1 + R_2 + R_3 = 1 + 2 + 3 = 6\,\Omega$

iv. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6 + 3 + 2}{6} = \frac{11}{6} \qquad \therefore R = \frac{6}{11}\Omega$

(c) The network takes the form

i. R = (1 + 1) || (2 + 2) + (1 + 1)|| (2 + 2) + (1 + 1) || (2 + 2) + (1 + 1) || (2 + 2)

= (2×42+4) x 4

$= 86 \times 4$

$= 163\Omega$

ii. Being series, $R = R + R + R + R + R = 5\ R$