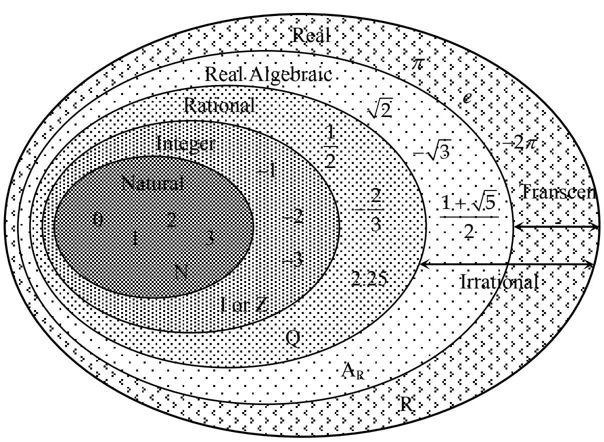
1 Numbers and Operations

Number theory is a vast and fascinating field of mathematics, sometimes called "higher arithmetic," consisting of the study of the properties of whole numbers. Primes and prime factorization are especially important in number theory, as are a number of functions such as the divisor function, Riemann zeta function, etc.

The great difficulty in proving relatively simple results in number theory prompted no less an authority than Gauss to remark that "it is just this which gives the higher arithmetic that magical charm which has made it the favourite science of the greatest mathematicians, not to mention its inexhaustible wealth, wherein it so greatly surpasses other parts of mathematics." Gauss, often known as the "prince of mathematics," called mathematics the "queen of the sciences" and considered number theory the "queen of mathematics".

Real numbers: Number which can represent actual physical quantities in a meaningful way are known as real numbers. These can be represented on the number line. Number line in geometrical straight line with arbitrarily defined zero (origin).

Classification of Numbers



Natural number: Set of all non-negative and non-fractional number from 1 to + ,N = {1,2,3,4,....}.

Whole number: Set of numbers from 0 to $+\infty$,
W = {0,1,2,3,4,.....}.

Integers: Set of all-non fractional numbers from $-\infty$ to $+\infty$,I or Z = (...., –3, –2, –1,0,1,2,3,....}.
Integers consist of the whole numbers and their negatives (including zero).
…., –3, –2, –1, 0, 1, 2, 3, 4, …

Integers extend infinitely in both negative and positive directions. Integers do not include fractions or decimals. The integer zero (0) is neither positive nor negative.

Odd numbers: Integers those are not divisible by 2
…, –5, –3, –1, 1, 3, 5, …

Even numbers: Integers those are divisible by 2; The integer zero (0) is an even number.
…, –4, –2, 0, 2, 4, …

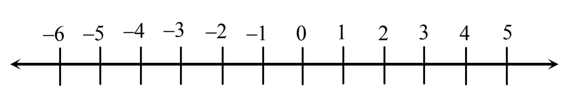
Consecutive integers: Integers that follow in sequence, where the difference between two successive integers is 1, are consecutive integers. An expression representing consecutive integers:
$n, n+1, n+2, n+3, \ldots,$ where n is any integer; examples of some consecutive integers : e.g.: (i) –1, 0, 1, 2, 3 and (ii) 1001, 1002, 1003, 1004

- **Addition of integers**
  even + even = even
  odd + odd = even
  odd + even = odd
  Adding zero (0) to any number doesn't change the value:
  9 + 0 = 9
  –1 + 0 = –1

- **Multiplication of integers**
  even × even = even
  odd × odd = odd
  odd × even = even

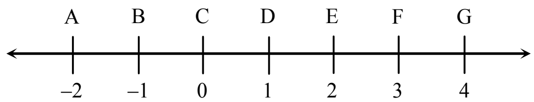
Multiplying any number by one (1) doesn't change the value:
$8 \times 1 = 8$
$-10 \times 1 = -10$

Number lines: A number line is used to graphically represent the relationships betweennumbers: integers, fractions, or decimals.



Numbers on a number line always increase as you move to the right. Negative numbers are always shown with a negative sign (–). For positive numbers, the plus sign (+) is usually not shown. The lengths and the ratios of the lengths of line segments represented on the number line.

Example 1. Here is an example of a number line question:



On the number line above, the ratio of AC to AG is equal to the ratio of CD to which of the following?

a. AD **b.** BD **c.** CG **d.** DF

Solution: In this question, the number line is used to determine lengths: AC = 2,
AG = 6, CD = 1. Once you have these lengths, the question becomes a ratio and proportion problem.

The ratio of AC to AG is 2 to 6.
AC is to AG as CD is to what?

$$\frac{2}{6}=\frac{1}{x} \text{ or } x=3$$

Now you have to go back to the number line to find which of the given segments has length 3. Because AD = 3, the answer is (A).

Fractions and rational Numbers: You should know how to do basic operations with fractions:

- Adding, subtracting, multiplying, and dividing fractions.
- Reducing to lowest terms.
- Finding the least common denominator.
- Expressing a value as a mixed number $\left(2\frac{1}{3}\right)$ and as an improper fraction $\left(\frac{7}{3}\right)$.
- Working with complex fractions – ones that have fractions in their numerators or denominators.

Fractions:

- **Common fraction:** Fractions whose denominator is not 10.
- Decimal fraction: Fractions whose denominator is 10 or any power of 10.
- Proper fraction: Numerator < Denominator i.e. $\frac{3}{5}$.
- Improper fraction: Numerator > Denominator i.e. $\frac{5}{3}$.
- Mixed fraction: Consists of integral as well as fractional part i.e. $3\frac{2}{7}$.
- Compound fraction: Fraction whose numerator and denominator themselves are
- Fractions. i.e. $\frac{2/3}{5/7}$.

Note: Improper fraction can be written in the form of mixed fractions.

You should know that a **rational number** is a number that can be represented by a fraction whose numerator and denominator are both integers (and the denominator must be nonzero). A fraction is a part of a whole or generally, any number of equal parts. e.g.: $\left(\frac{7}{3}\right)$. is a fraction and $\frac{6}{2}=3$ is a rational number.

Decimal fractions equivalents: You may have to work with decimal fraction equivalents. That is, you may have to be able to recognise common fractions as decimals and vice-versa.

Fraction	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Decimal	0.25	$0.\overline{3}$*	0.5	$0.\overline{6}$*	0.75

Reciprocals: The reciprocal of a number is 1 divided by that number; the reciprocal of 5 is $\frac{1}{5}$. Note that $5\times\frac{1}{5}=1$. The product of a number and its reciprocal is always 1.

e.g.: The reciprocal of $\frac{2}{3}$ is 1 divided by $\frac{2}{3}$, which is equal to $\frac{3}{2}$.

the reciprocal of any nonzero fraction by switching its numerator and denominator.

Note: The number zero (0) has no reciprocal. The number 1 is its own reciprocal. Also, the number –1 is its own reciprocal.

Rational numbers: These are real numbers which can be expressed in the form of p/q, where **p** and **q** are integers and $q\neq 0$.

Example: 2/3, 37/15, –17/19.

- All natural numbers, whole numbers and integers are rational.
- Rational numbers include all Integers (without any decimal part to it), terminating fractions (fractions in which the decimal parts terminating e.g. 0.75, –0.02 etc.) and also non-terminating but recurring decimals e.g. 0.666...., –2.333....., etc.

Properties of Rational Number

If a,b,c are three rational numbers.

- Commutative property of addition. $a+b=b+a$
- Associative property of addition $(a+b)+c=a+(b+c)$
- Additive inverse $a+(-a)=0$

  0 is identity element, –a is called additive inverse of a.
- Commutative property of multiplications a.b. = b.a.
- Associative property of multiplication $(a.b).c=a.(b.c)$
- Multiplicative inverse $(a)\times\left(\frac{1}{a}\right)=1$

  1 is called multiplicative identity and $\frac{1}{a}$ is called multiplicative inverse of a or reciprocal of a.
- Distributive property $a.(b+c)=a.b+a.c$

Example 2. Find three rational no's between a and b $(a < b)$.

Solution: $a<b$

$$a+a<b+a$$

$$2a<a+b \quad \text{or} \quad a<\frac{a+b}{2}$$

Again, $a<b$

$$a+b<b+b.$$

$$a+b<2b \quad \text{or} \quad \frac{a+b}{2}<b.$$

$$a<\frac{a+b}{2}<b$$

i.e. $\frac{a+b}{2}$ lies between a and b.

Hence 1st rational number between a and b is $\frac{a+b}{2}$.

For next rational number $\frac{a+\frac{a+b}{2}}{2}=\frac{\frac{2a+a+b}{2}}{2}=\frac{3a+b}{4}$

or $\quad a<\frac{3a+b}{4}<\frac{a+b}{2}<b$

Next, $\frac{\frac{a+b}{2}+b}{2}=\frac{a+b+2b}{2\times 2}=\frac{a+3b}{4}$

$a<\frac{3a+b}{4}<\frac{a+b}{2}<\frac{a+3b}{4}<b$, and continues like this.

Irrational Numbers: All real number which are not rational are irrational numbers. These are non-recurring as well as non-terminating type of decimal numbers.

e.g. $\sqrt{2},\sqrt[3]{4},2+\sqrt{3},\sqrt{2+\sqrt{3}},\sqrt[4]{\sqrt[7]{3}}$ etc.

Properties of Irrational Number

- Negative of an irrational number is an irrational number e.g. $a \quad \frac{b}{3}$ are irrational.
- Sum and difference of a rational and an irrational number is always an irrational number.
- Sum and difference of two irrational numbers is either rational or irrational number.
- Product of a non-zero rational number with an irrational number is either rational or irrationals
- Product of an irrational with a irrational is not always irrational.

Note

- $\sqrt{-2}\neq-\sqrt{2}$, it is not a irrational number.
- $\sqrt{-2}\times\sqrt{-3}\neq\left(\sqrt{-2\times-3}\right)=\sqrt{6}$ Instead $\sqrt{-2},\sqrt{-3}$ are called Imaginary numbers.

  $\sqrt{-2}=i\sqrt{2}$, where i (= iota) $=\sqrt{-1}$
- $i^2=-1$
- $i^3=i^2\times i=(-1)\times i=-i$
- $i^4=i^2\times i^2=(-1)\times(-1)=1$
- Numbers of the type $(a+ib)$ are called complex numbers where $(a,b)\in$ Reg. $2+3i,-2+4i,-3i,11-4i,$ are complex numbers.

Prime Numbers: A prime number is a positive integer greater than 1 that has exactly two whole number factors – itself and the number 1. The number 1 itself is not prime. Prime numbers include: 2, 3, 5, 7, 11, 13, 17, 19... (Note that 2 is the only even prime number).

Prime factors are the factors of a number that are prime numbers. That is, the prime factors of a number cannot be further divided into factors.
e.g.: the prime factors of the number 24 are 2 and 3.
$24=2\times2\times2\times3$

Co-prime numbers: If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers. e.g. 4, 9, are co-prime as H.C.F. of (4, 9) = 1. Any two consecutive numbers will always be co-prime.

Composite numbers: All natural number, which are not prime are composite numbers. If C is the set of composite number then C = {4,6,8,9,10,12,.....}.

Note: 1 is neither prime nor composite number.

Imaginary Numbers: All the numbers whose square is negative are called imaginary numbers. e.g. 3i, 4i, i ... Where i $=\sqrt{-1}$.

Complex Numbers: The combined form of real and imaginary numbers is known as complex numbers. It is denoted by $Z=A+iB$ where A is real part and B is imaginary part of Z and $A,B\in R.$ The set of complex number is the super set of all the sets of numbers.

Identification prime number

- Find approximate square root of given number.
- Divide the given number by prime numbers less than approximate square root of number. If given number is not divisible by any of this prime number then the number is prime otherwise not.

Example 3. 571, is it a prime?

Solution: Approximate square root of 571 = 24.

Prime number < 24 are 2, 3, 5, 7, 11, 13, 17, 19, & 23. But 571 is not divisible by any of these prime numbers so 571 is a prime number.

Example 4. Is 1 prime or composite number?

Solution: 1 is neither prime nor composite number.

Example 5. Find 4 rational numbers between 2 and 3.

Solution: (a) Write 2 and 3 multipling in N[r] and D[r] with (4+1).

(b) i.e. $2\ \frac{2\times(4+1)}{(4+1)}=\frac{10}{5}\ \&\ 3=\frac{3\times(4+1)}{(4+1)}=\frac{15}{5}$

(c) So, the four required numbers are $\frac{11}{5},\frac{12}{5},\frac{13}{5},\frac{14}{5}$.

Example 6. Find 3 rational numbers between $\frac{1}{3}\ \&\ \frac{1}{2}$.

Solution: 1st Method $\dfrac{\frac{1}{3}+\frac{1}{2}}{2}=\dfrac{\frac{2+3}{6}}{2}=\frac{5}{12}$

or $\frac{1}{3},\frac{5}{12},\frac{1}{2}$

$\dfrac{\frac{1}{3}+\frac{5}{12}}{2}=\dfrac{\frac{4+5}{12}}{2}=\frac{9}{24}$

or $\frac{1}{3},\frac{9}{24},\frac{5}{12},\frac{1}{2}=\dfrac{\frac{5}{12}+\frac{1}{2}}{2}=\dfrac{\frac{5}{12}+\frac{6}{12}}{2}=\frac{11}{24}$

or $\frac{1}{3},\frac{9}{24},\frac{5}{12},\frac{11}{24},\frac{1}{2}$

Verify: $\frac{8}{24}<\frac{9}{24}<\frac{10}{24}<\frac{11}{24}<\frac{12}{24}.\ \left(\text{as}\frac{8}{24}=\frac{1}{3}\ \&\ \frac{1}{2}\right)$

2nd Method: Find n rational numbers between a and b $(a<b)$.

- Find $d=\frac{b-a}{n+1}$
- 1st rational number will be $a+d$.

  2nd rational number will be $a+2d$.

  3rd rational number will be $a+3d$ and so on....

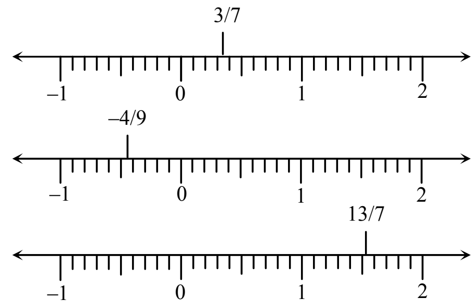
  nth rational number is $a+nd$.

Representation of Rational Number on Number Line

(i) 3/7 Divide a unit into 7 equal parts.

(ii) $-\frac{4}{9}$

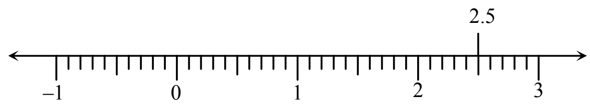
(iii) $\frac{13}{7}$

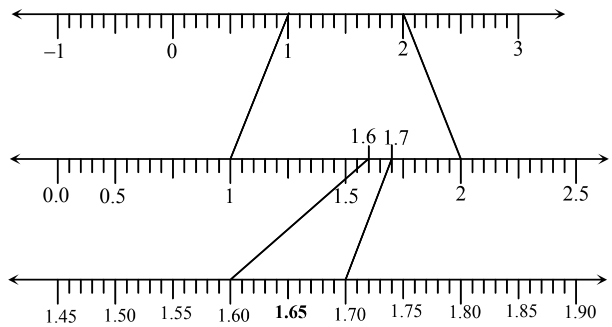


Decimal Number (Terminating)

(iv) 2.5

(v) 1.65 (process of magnification)





To convert a decimal into a vulgar fraction, follow these steps:

- Remove the number left to the decimal point, if any.
- Write the repeated figures only once in the numerator without the decimal point.
- Write as many nines in the denominator as the number of repeating figures.
- Add the number removed in step 1(if any) with the fraction obtained in the above steps.

Example 7. (i) $0.3=\frac{3}{9}=\frac{1}{3}$ **(ii)** $0.7=\frac{7}{9}$

To convert a mixed recurring decimal into a vulgar fraction, follow these steps.

- Remove the number left to the decimal point, if any.
- Numerator is the difference between the number formed by all the digits (taking repeated digits only once) and that formed by the digits which are not repeated.
- Denominator is the number formed by taking as many nines as the number of repeating figures followed by as many zeros as the number of non-repeating digits.
- Add the number removed in step 1 (if any) with the fraction obtained in the above steps.

Example 8.

(i) $0.54\overline{29} = \dfrac{(5429-54)}{9900} = \dfrac{5375}{9900} = \dfrac{215}{396}$

(ii) $0.1\overline{6} = \dfrac{(16-1)}{90} = \dfrac{15}{90} = \dfrac{1}{6}$

Rational Number in Decimal Representation

Terminating Decimal in this a finite number of digit occurs after decimal.

i.e. $\frac{1}{2} = 0.5, 0.6875, 0.15$ etc.

Non-terminating and Repeating (Recurring Decimal); In this a set of digits or a digit is repeated continuously.

Examples 9. $\frac{5}{11} = 0.454545\ldots\ldots\ldots\ldots = 0.\overline{45}.$

Irrational Number in Decimal Form

$\sqrt{2} = 1.414213\ldots$ i.e. it is not-recurring as well as non-terminating.

$\sqrt{3} = 1.732-50807\ldots$ i.e. it is non-recurring as well as non-terminating.

Examples 10. Insert an irrational number between 2 and 3.

Solution: $\sqrt{2\times 3} = \sqrt{6}$

Examples 11. Find two irrational number between 2 and 2.5.

Solution: 1st Method: $\sqrt{2\times 2.5} = \sqrt{5}$

Since there is no rational number whose square is 5.

So, $\sqrt{5}$ is irrational..

Also, $\sqrt{2\times\sqrt{5}}$ is a irrational number.

2nd Method: 2.101001000100001.... is between 2 and 2.5 and it is non-recurring as well as non-terminating.

Also, 2.201001000100001......... and so on.

Example 12. Find two irrational number between $\sqrt{2}$ and $\sqrt{3}$.

Solution: 1st Method: $\sqrt{\sqrt{2}\times\sqrt{3}} = \sqrt{\sqrt{6}} = \sqrt[4]{6}$

Irrational number between $\sqrt{2}$ and $\sqrt[4]{6}$

$\sqrt{\sqrt{2}\times\sqrt[4]{6}} = \sqrt[4]{2}\times\sqrt[8]{6}$

2nd Method: As $\sqrt{2} = 1.414213562\ldots$

and $\sqrt{3} = 1.732050808\ldots$

As, $\sqrt{3} > \sqrt{2}$ and $\sqrt{2}$ has 4 in the 1st place of decimal while $\sqrt{3}$ has 7 is the 1^{st} place of decimal.

1.501001000100001..., 1.601001000100001... etc. are in between $\sqrt{2}$ and $\sqrt{3}$.

Example 13. Find two irrational number between 0.3030030003..... and 0.3010010001 ....

Solution: 0.302020020002.... 0.302030030003... etc.

Example 14. Prove that $\sqrt{2}$ is an irrational number.

SolutionLet assume on the contrary that $\sqrt{2}$ is a rational number.

Then, there exists positive integer a and b such that $\sqrt{2} = \frac{a}{b}$ where, a and b are co primes i.e. their HCF is 1.

$\Rightarrow \left(\sqrt{2}\right)^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow 2 = \frac{a^2}{b^2}$

$\Rightarrow a^2 = 2b^2$

a^2 is multiple of 2

$a$is a multiple of 2 . . . (i)

$a = 2c$ for some integer c.

$\Rightarrow a^1 = 4c^1$

$\Rightarrow 2b^2 = 4c^1b^2$

$\Rightarrow b^2 = 2c^2$ is a multiple of 2

b is a multiple of 2 . . . (ii)

From *(i)* and *(ii)*, a and b have at least 2 as a common factor. But this contradicts the fact that a and b are co-prime. This means that $\sqrt{2}$ is an irrational number.

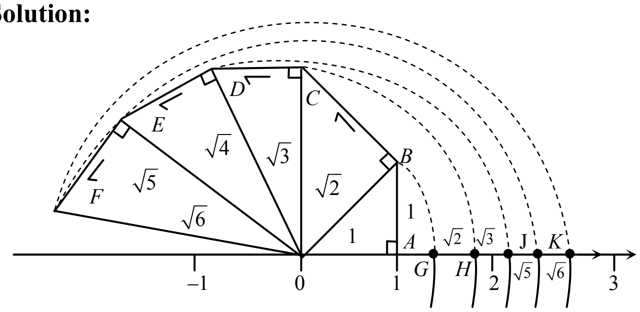
Example 15. Two irrational numbers are $\sqrt{3}, -\sqrt{3}$, , then

Solution: Sum $= \sqrt{3} + (-\sqrt{3}) = 0$ which is rational.Difference $= \sqrt{3} - (-\sqrt{3}) = 2\sqrt{3},$ which is irrational.

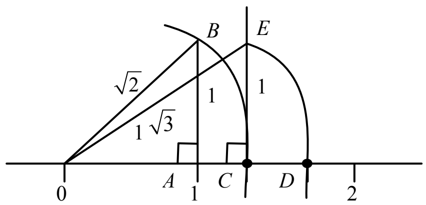
Irrational Number on a Number Line:

Example 16. Plot $\frac{3}{4}$ on a number line.

Solution:



- Plot $\sqrt{2}, \sqrt{3}$
- So, $OC = \sqrt{2}$ and $OD = \sqrt{3}$

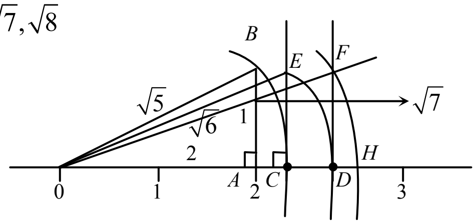


- Plot $\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$

  $OC = \sqrt{5}$

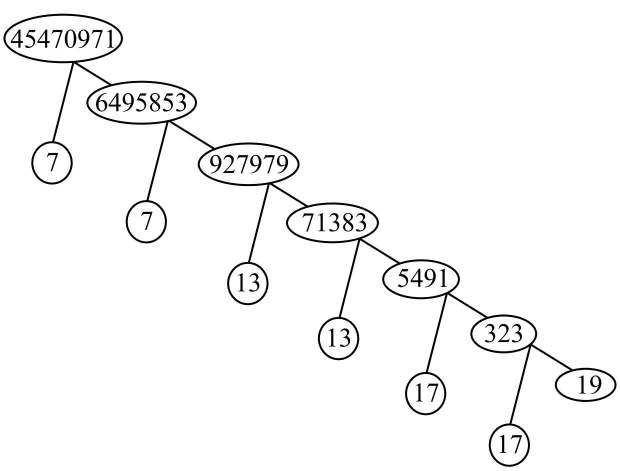
  $OD = \sqrt{6}$

  $OH = \sqrt{7} ...$



Fundamental Theorem of Arithmetic: Every composite number can be expressed as a product of primes, and this factorisation is unique, except for the order in which the prime factors occur.

Example 17. Determine the prime factors of 45470971.
Solution:



$45470971 = 7^2 \times 13^2 \times 17^2 \times 19.$

Example 18. What can you say about the prime factorisations of the denominators of the following rationales:

(i) 43.123456789 **(ii)** $43.\overline{12345678}$

Solution: (i) Since, 43.123456789 has terminating decimal, so prime factorisations of the denominator is of the form $2^m \times 5^n$, where m, n are non-negative integers.

(ii) Since, $43.\overline{12345678}$ has non-terminating repeating decimal expansion. So, its denominator has factors other than 2 or 5.

Simplification: It is a rule of inference in logic. To simplify some operations before the others in an expression we use brackets which indicate the order of simplification. There are four types of brackets.

(i) "—" is called a bar or vinculum
(ii) "( )" is called a round or curved brackets or parentheses.
(iii) "{ }" is called a curly brackets or braces.
(iv) "[ ]" is called box brackets or square brackets.

Some Rules

There are some rules to simplify an expression containing brackets.

- **Rule 1:** When an expression contains only addition and Subtraction: work from left to right within the brackets.
- **Rule 2:** When an expression contains only multiplication and division: work from left to right within the brackets.
- **Rule 3:** When an expression contains any three or all four operations: use the BODMAS rules.

Vinculum (or Bar)

When an expression contains vinculum before applying the "BODMAS" rule, we simplify the expression under the vinculum.

BODMAS Rule

We know that addition, subtraction, multiplication and division are four basic operation of mathematics. To simplify an expression which contains all these operations, we use a rule, called simplification rule.

According to this rule the four operations must be performed in the following order but after removing the brackets.

BO	for	Brackets of	{[()]}
D	for	Division	÷
M	For	Multiplication	×
A	For	Addition	+
S	For	Subtraction	–

Modulus of a Real Number

Modulus or absolute **value for any real number x is denoted by $|x|$ (a vertical bar on each side of the quantity) and is defined as:** $|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0. \end{cases}$

As can be seen from the above definition, the absolute value of x is always either positive or zero, but never negative.

Multiple Choice Questions

1. Which of the following statement is false?

a. Every fraction is a rational number
b. Every rational number is fraction
c. Every integer is a rational number
d. All the above

2. Express $0.3\overline{58}$ as rational number:

a. $\frac{358}{1000}$ **b.** $\frac{358}{999}$ **c.** $\frac{355}{990}$ **d.** All

3. Which step in the following problem is wrong?

$a = b = 1 \quad a = b$

Step 1: $= a^2 = ab$

Step 2: $= a^2 - b^2 = ab - b^2$

Step 3: $= (a+b)(a-b) = b(a-b)$

Step 4: $a + b = \frac{b(a-b)}{a-b}$

$a + b = b \quad 1+1=1 \quad 2=1$

a. Step 4 **b.** Step 3 **c.** Step 2 **d.** Step 1

4. The value of $\frac{3}{2}x \quad \frac{27}{5}$ is:

a. $\frac{40}{31}$ **b.** $\frac{4}{9}$ **c.** $\frac{1}{8}$ **d.** $\frac{31}{40}$

5. The value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5...}}}}}$ is:

a. 0 **b.** 5
c. can't be determined **d.** none

6. The number $(6+\sqrt{2})(6-\sqrt{2})$ is:

a. Rational **b.** Irrational **c.** Can't say **d.** None

7. Which of the following number has the terminal decimal representation?

a. $\frac{1}{7}$ **b.** $\frac{1}{3}$ **c.** $\frac{3}{5}$ **d.** $\frac{17}{3}$

8. The number $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$ where $x, y > 0$ is

a. rational **b.** irrational **c.** both **d.** none

9. The domain of the function $f(x) = \sqrt{x-4} + \sqrt{x-5} + |x| + x^2$

a. $R - \{4\}$ **b.** $R - \{4, 5\}$ **c.** $(5, \infty)$ **d.** R

10. If P: All integers are rational number and
Q: Every rational number is an integer, then which of the following is correct?

a. P is False and Q is True **b.** P is True and Q is False
c. Both P and Q are True **d.** Both P and Q are False

11. For any two rational numbers x and y, which of the following properties are correct?

(i) $x < y$ (ii) $x = y$ (iii) $x > y$

a. Only (i) and (ii) are correct
b. Only (ii) and (iii) are correct
c. Only (ii) is correct
d. All (i), (ii) and (iii) are correct

12. If A:The quotient of two integers is always a rational number and
R: $\frac{1}{0}$ is not rational
Then which of the following statements is true?

a. A is True and R is the correct explanation of A
b. A is False and R is the correct explanation of A
c. A is True and R is False
d. A is False and R is True

13. If A: Every whole number is a natural number and
R: 0 is not a natural number
Then which of the following statements is true?

a. A is True and R is the correct explanation of A
b. A is False and R is the correct explanation of A
c. A is True and R is False
d. A is False and R is True

14. 4 + 44 + 444 + 4444 + 44444 = ?

a. 49308 **b.** 49380 **c.** 40398 **d.** 49083

15. $\frac{2}{3}$ of $\frac{5}{4}$ of $\frac{6}{7}$ of 2170 =?

a. 1050 **b.** 1550 **c.** 1005 **d.** 1555

16. If $\frac{2x}{1+\frac{1}{1+\frac{x}{1-x}}} = 1$, then find the value of x.

a. $\frac{2}{3}$ **b.** $\frac{1}{3}$ **c.** $\frac{5}{6}$ **d.** $\frac{1}{4}$

17. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}$ is equal to:

a. 2 **b.** 2.5 **c.** 3 **d.** 3.5

18. $\frac{5}{6}\div\frac{6}{7}\times?-\frac{8}{9}\div1\frac{3}{5}+\frac{3}{4}\times3\frac{1}{3}=2\frac{7}{9}$

a. $\frac{7}{8}$ **b.** $\frac{6}{7}$ **c.** 1 **d.** 0

19. $\dfrac{3\frac{1}{4}-\frac{4}{5}\text{ of }\frac{5}{6}}{4\frac{1}{3}\div\frac{1}{5}-\left(\frac{3}{10}+21\frac{1}{5}\right)}$

a. $\frac{1}{6}$ **b.** $2\frac{7}{12}$

c. $15\frac{1}{2}$ **d.** $21\frac{1}{2}$

20. If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$, then the value of $\frac{a+b+c}{c}$ is:

a. $\frac{1}{\sqrt{7}}$ **b.** $\sqrt{2}$ **c.** 2 **d.** 7

21. The value of $\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2-\frac{1}{2}}}}$

a. $\frac{3}{8}$ **b.** $\frac{19}{8}$ **c.** $\frac{8}{3}$ **d.** $\frac{8}{19}$

22. The value of $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{120}\right)$ is equal to:

a. 30 **b.** 40.5

c. 60.5 **d.** 121

23. The sum of the first 35 terms of the series

$\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$

a. $-\frac{1}{2}$ **b.** $-\frac{1}{4}$ **c.** $\frac{1}{4}$ **d.** 0

24. Simplify: $10\frac{1}{2}-\left[8\frac{1}{2}+\left\{7-\overline{6-4}\right\}\right]\overline{6-4}=?$

a. –3 **b.** –2 **c.** 1 **d.** 4

25. $0.75\times0.75+0.25\times0.75\times2+0.25\times0.25$

a. 2 **b.** 2.5 **c.** 1 **d.** 3

26. Simplify: $\dfrac{(0.87)^3+(0.13)^3}{(0.87)^2+(0.13)^2-(0.87\times0.13)}$

a. 2 **b.** 2.5 **c.** 1 **d.** 3

27. $\dfrac{(63+36)^2+(63-36)^2}{63^2+36^2}=?$

a. 1 **b.** 2 **c.** 3 **d.** 4

28. $\dfrac{0.1+0.75}{2.5+0.05}\div\left(0.125+\dfrac{1}{4.8}\right)=?$

a. 1 **b.** 3 **c.** 2 **d.** 4

29. Which of the following is not a rational number?

a. $\sqrt{2}$ **b.** $\sqrt{4}$ **c.** $\sqrt{9}$ **d.** $\sqrt{16}$

30. Set of natural number is a subset of

a. Set of even numbers **b.** Set of odd numbers

c. Set of composite numbers **d.** Set of real numbers

ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	c	a	c	b	a	c	a	c	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
d	d	a	b	b	a	a	b	c	c
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
d	c	b	a	5	c	b	a	a	d

SOLUTIONS

1. **(a)** Every rational number is not a fraction. Therefore, in rational numbers, we use integers and in fractions, we use only natural numbers.

2. **(c)** $0.3\overline{58}=\frac{358-3}{990}=\frac{355}{990}$

3. **(a)** $\because\ a=b=1\Rightarrow a-b=0$

Division by zero is not defined. So, we cannot divide by $(a-b)$.

4. **(c)** $4-\dfrac{5}{1+\dfrac{1}{3+\dfrac{1}{2+\frac{1}{4}}}}=4-\dfrac{5}{1+\dfrac{1}{3+\dfrac{1}{\frac{8+1}{4}}}}=$

$4-\dfrac{5}{1+\dfrac{1}{3+\frac{4}{9}}}=4-\dfrac{5}{1+\dfrac{1}{\frac{27+4}{9}}}=4-\dfrac{5}{1+\frac{9}{31}}$

$=4-\dfrac{5}{\frac{31+9}{31}}=4-\dfrac{5}{\frac{40}{31}}=4-\frac{155}{40}=\frac{160-155}{40}=\frac{5}{40}=\frac{1}{8}$

5. **(b)** Let $x=\sqrt{5\sqrt{5\sqrt{5\ldots}}}$

$\Rightarrow\ x=\sqrt{5x}\quad\Rightarrow\ x^2-5x=0$

$\Rightarrow\ x(x-5)=0\Rightarrow x=0$ (or) $x=5$

$\therefore$ x can only be 5.

6. **(a)** $\left(6+\sqrt{2}\right)\left(6-\sqrt{2}\right)=36-2=34$

7. **(c)** $\frac{3}{5}=0.6$ where as other number have non-terminating decimals.

8. **(a)**

9. **(c)**

10. **(b)** Since every integer is having the denominator 1 can be expressed in $\frac{p}{q}$ form, p is True. Since a rational number with denominator other than 1 is not as integer (e.g. $\frac{3}{5}$), q is False.

11. **(d)** By the properties of rational numbers, all (i), (ii), (iii) are correct.

12. **(d)** Since $\frac{1}{0}$ is not rational, the quotient of two integers is not rational.

13. **(a)** Zero (0) is a whole number but not a natural number.

14. **(b)** $12345\times 4=49380$

15. **(b)** $\frac{2}{3}\times\frac{5}{4}\times\frac{6}{7}\times 2170=1550$

16. **(a)** We have: $\Rightarrow \dfrac{2x}{1+\dfrac{1}{1+\dfrac{(1-x)+x}{1-x}}}=1\Leftrightarrow\dfrac{2x}{1+\dfrac{1}{[1/(1-x)]}}$

$$=1\Leftrightarrow\frac{2x}{1+(1-x)}=1\ \Rightarrow\ 2x=2-\times\Leftrightarrow 3x=2\Leftrightarrow\times=\frac{2}{3}$$

17. **(a)** $\frac{28+14+7+4+2+1}{28}=\frac{56}{28}=2$

18. **(b)** $\frac{5}{6}\div\frac{6}{7}\times x-\frac{8}{9}\div\frac{8}{5}+\frac{3}{4}\times\frac{10}{3}=\frac{25}{9}$

$$\Rightarrow\ \frac{35}{36}x-\frac{5}{9}+\frac{5}{2}=\frac{25}{9}$$

$$\Rightarrow\frac{35}{36}x=\frac{5}{9}\Leftrightarrow x=\left(\frac{5}{6}\times\frac{36}{35}\right)=\frac{6}{7}$$

19. **(c)** $\dfrac{\frac{13}{4}-\frac{4}{5}\text{ of }\frac{5}{6}}{\frac{13}{3}\div\frac{1}{5}-\left(\frac{3}{10}+\frac{106}{5}\right)}=\dfrac{\frac{13}{4}-\frac{2}{3}}{\frac{13}{3}\times 5-\frac{215}{10}}$

$$=\frac{\frac{31}{12}}{\frac{65}{3}-\frac{43}{2}}=\left(\frac{31}{12}\times 6\right)=\frac{31}{2}=15.5$$

20. **(c)** $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=\text{k}(\text{say})$ Then, $a=3k,b=4k,c=7k$

$$\frac{a+b+c}{c}=\frac{3k+4k+7k}{7k}=\frac{14k}{7k}=2$$

21. **(d)** $\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2-\frac{1}{2}}}},=\dfrac{1}{2+\dfrac{1}{2+\frac{2}{3}}}=\dfrac{1}{2+\dfrac{1}{(8/3)}}$

$$=\frac{1}{2+\frac{3}{8}}=\frac{1}{(19/8)}=\frac{8}{19}$$

22. **(c)** $\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\ldots\times\frac{121}{120}=\frac{121}{2}=60.5$

23. **(b)** Clearly, sum of first 6 terms is zero. So, sum of first 30 terms = 0

Required sum $=\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{2}-\frac{1}{3}\right)=-\frac{1}{4}$

24. **(a)** $x=10\frac{1}{2}-\left[8\frac{1}{2}+(7-2)\right]=10\frac{1}{2}-\left[8\frac{1}{2}+5\right]$

$$=10\frac{1}{2}-13\frac{1}{2}=-3$$

25. **(c)** Let $0.75=a$ and $0.25=b$

We have $a^2+2ab+b^2=(a+b)^2=(0.75+0.25)^2=1$

26. **(c)** Let $0.87=a$ and $0.13=b$

Formula

$$\therefore\ \frac{a^3+b^3}{a^2+b^2-ab}=\frac{(a+b)(a^2-ab+b^2)}{(a^2+b^2-ab)}=a+b$$

$$=0.87+0.13=1$$

27. **(b)** $\dfrac{(63+36)^2+(63-36)^2}{(63^2+36^2)}=?$

Putting $63=a$ and $36=b$ in the given expression, we get

$$\Rightarrow\quad x=\frac{(a+b)^2+(a-b)^2}{a^2+b^2}\ \Rightarrow\ x=\frac{2(a^2+b^2)}{a^2+b^2}$$

$\left(\text{since }(a+b)^2+(a-b)^2=2(a^2+b^2)\right)$

$\Rightarrow\ x=2$

28. **(a)** $\dfrac{0.1+0.75}{2.5+0.05}\div\left(0.125+\dfrac{1}{4.8}\right)=?$

Putting x for (?)

$$\Rightarrow\ x=\frac{0.85}{2.55}\div\left(\frac{1}{8}+\frac{10}{48}\right)\Rightarrow x=\frac{1}{3}\div\left(\frac{16}{48}\right)$$

$\Rightarrow\ x=1$

29. **(a)** $\sqrt{2}$ is not a rational number. It can't be expresses in the fractional form.

30. **(d)**

□□□