Electrostatics

Coulomb's Law: The magnitude of the electrostatic force of interaction between two point charges is directly proportional to the scalar multiplication of the magnitudes of charges and inversely proportional to the square of the distance between them. The force is along the straight line joining them. If the two charges have the same sign, the electrostatic force between them is repulsive; if they have different sign, the force between them is attractive.

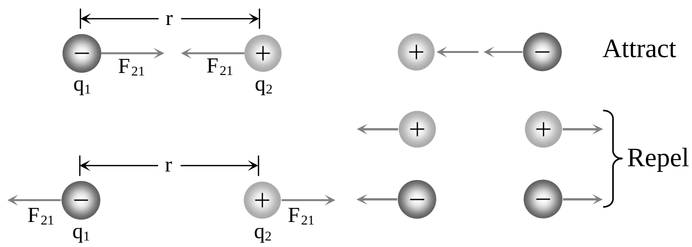


Figure: 14.1

The force is a vector quantity. While calculating the force form Coulomb's law, the sign of charge is not retained in formula; but the signs of charges indicate direction of force which is seen by inspection with the rule that the charge on which force is to be calculated is assumed to have the tendency of motion while the other charge due to which force is to be calculated is assumed at rest, unless otherwise stated.

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2} \text{ (in air)}$$

$$= \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q_1q_2}{r^2} \text{ (in medium of dielectric constant K)}$$

$$K = \frac{\varepsilon}{\varepsilon_0} = \frac{\text{permittivity of medium}}{\text{permittivity of free space}}$$

$$\varepsilon_0 = 8.855\times10^{-12} \text{ coul}^2/\text{N-m}^2$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times9^9 \text{ N-m}^2/\text{colul}^2$$

Charge is

- Transferable: It can be transferred from one body to another.
- Associated with mass: Charge cannot exist without mass but reverse is not true.
- Conserved: It can neither be created nor be destroyed.
- Quantisation of charge : q = + ne
- Invariant: Independent of velocity of charged particle.
- Electric charge produces electric field $(\vec{E})$, magnetic field $(\vec{B})$ and electromagnetic radiations.
- Linear charge distribution: Charge on a line e.g., charged straight wire, circular charged ring, etc.

  $$\lambda = \frac{\text{Charge}}{\text{Length}} = \text{Linear charge density}$$

  → S.I. unit is C / m
  → Dimension is $[L^{-1}TA]$
- Surface charge distribution: Charge distributed on a surface e.g. plane sheet of charge, conducting sphere, conducting cylinder, etc.

  $$\sigma = \frac{\text{Charge}}{\text{Area}} = \text{Surface charge density}$$

  → S.I. unit is C / m^2
  → Dimension is $[L^{-2}TA]$
- Volume charge density: Charge distributes through out the volume of the body e.g. charge on a dielectric sphere, etc.

  $$\rho = \frac{\text{Charge}}{\text{Volume}} = \text{volume charge density}$$

  → S.I. unit is C / m^3
  → Dimension is $[L^{-3}TA]$

Electric Field Strength : $\vec{E} = \lim_{q_0\to0}\frac{\vec{E}}{q_0}$ Newton/Coulomb

Electric field strength due to an isolated point charge $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

- Total electric flux $= \int_S \vec{E}\cdot d\vec{S}$
- Gauss Theorem : $\int_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\Sigma q$

  Total electric flux $= \frac{1}{\varepsilon_0}\times$ net charge enclosed by closed surface

- Electric potential: $V = \lim_{x\to\infty}\frac{W}{q_0} = -\int_{\infty}^{r}\vec{E}\cdot d\vec{r}$
- Relation between electric field and potential $E = -\frac{\Delta V}{\Delta r} = \frac{V}{d}$ (numerically)
- Electric field due to point charge, $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$ (Scalar quantity)
- Electric potential energy of two charges, $U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$
- Electric potential energy of a system of charges $E_S = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{i=1}^{n}\cdot\frac{q_i q_j}{r_{ij}}$

Electric Field Due to a Charged Conducting Sphere of Radius R

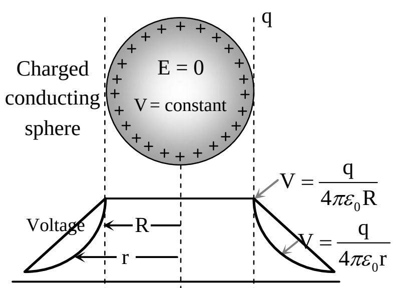


Figure: 14.2

- Electric field outside, $E_S = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}(r > R)$
- Electric field at surface, $E_S = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$
- Electric field inside the conductor, $E_{in} = 0(r < R)$
- Electric potential

$$V_{outside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \qquad (r > R)$$

$$V_{surface} = V_{inside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R} \qquad (r \le R)$$

- Electric Field Due to a Non-conducting Uniformly Spherical Charge

$$E_{inside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qr}{R^3} \quad (r < R)$$

$$E_{surface} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} \quad (\text{at } r = R)$$

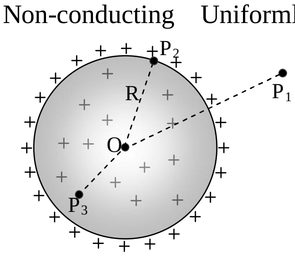


Figure: 14 .3

$$E_{outside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \quad (r > R)$$

- Electric potential

$$V_{inside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(3R^2 - r^2)}{2R^3} \quad (r < R)$$

$$V_{surface} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R} \quad (\text{at } r = R)$$

$$V_{outside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \quad (r > R)$$

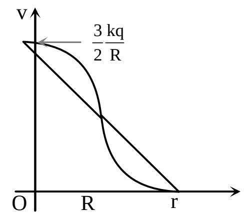


Figure: 14.4

Electric Potential and Electric field Intensity by Long Charged Wire

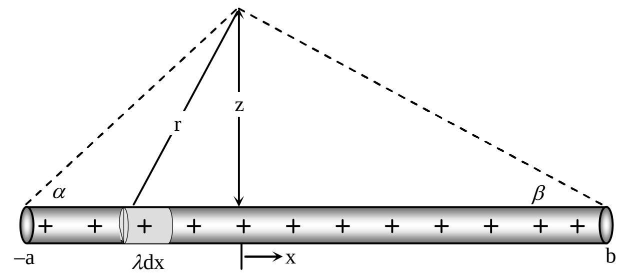


Figure: 14.5

$$V = \int\frac{kzq}{r} = \int_{-a}^{b}\frac{k\lambda\, zx}{r} = \int_{-a}^{b}\frac{k\lambda\, zx}{\sqrt{x^2+d^2}} = k\lambda\ln\left[\frac{b+\sqrt{b^2+z^2}}{-a+\sqrt{b^2+z^2}}\right]$$

$$E_z = k\lambda z\int_{-a}^{b}\frac{dx}{(z^2+x^2)^{3/2}} = \frac{k\lambda}{z}\left[\frac{x}{(z^2+x^2)}\right]$$

Note

- That as the limit is taken as a and b approach infinity, this approaches the infinite line change $E_Z = \frac{k\lambda}{z}\left[\frac{b}{(z^2+b^2)^{1/2}} + \frac{a}{(z^2+b^2)^{1/2}}\right]$
- That as the limit is taken as a and b approach infinity, this approaches the infinite line charge expression: $E_Z = \frac{2k\lambda}{z} = \frac{\lambda}{2\pi z\varepsilon_0}$

Electric Potential and Electric Field Intensity Due to Line Charge

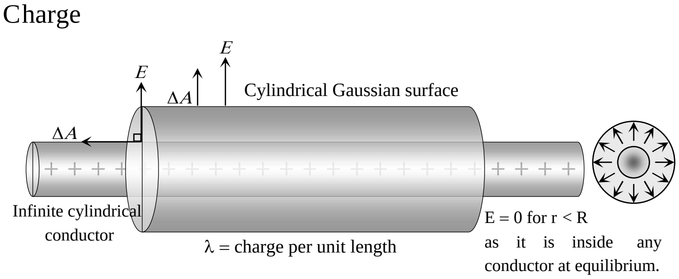


Figure: 14.6

- Electric field $E = \frac{2k\lambda}{r}$ Where λ = charge per unit length.
- Potential difference between two points distant r_1 and r_2 from line charge $\Delta V = \frac{2k\lambda}{r}\log_e \frac{r_2}{r_1}$

Electric Field at the Axis of a Charge of Ring of Radius R

$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qx}{\left(R^2+x^2\right)^{3/2}}$ along the axis

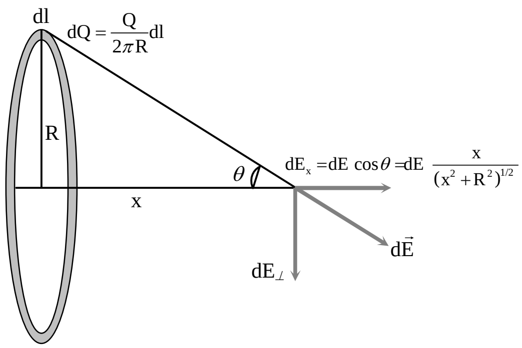


Figure: 14.7

Electric field Intensity due to a Non-conducting Thin Sheet of Charge

$E = \frac{\sigma}{2\varepsilon_0}$ (Where σ = charge per unit surface area)

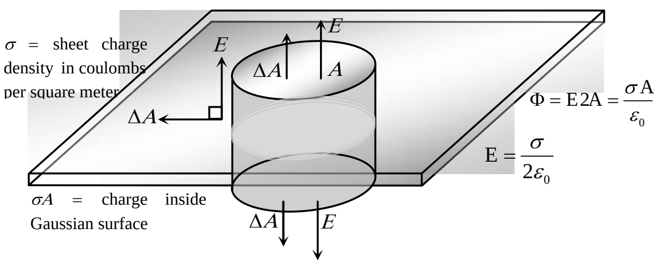


Figure: 14.8

- Electric field intensity due to a conductor of any shape $E = \frac{\sigma}{\varepsilon_0}$
- Electric field between two plates having equal and opposite charge densities (σ) $E = \frac{\sigma}{\varepsilon_0}$

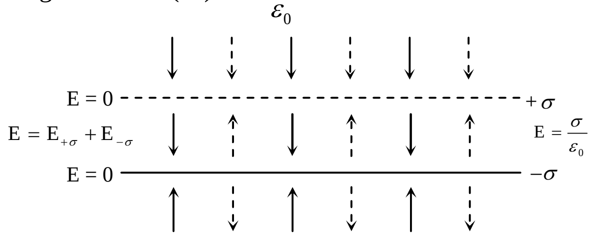


Figure: 14.9

- Electrostatic energy density $u_e = \frac{1}{2}K\varepsilon_0 E^2$ Joule/m^3

$= \frac{1}{2}\varepsilon_0 E^2$ (in air K = 1)

Electric Dipole

- Electric dipole moment $\vec{p} = q\cdot 2\vec{l}$ (from –q to +q)

 Where q = magnitude of each charge;

 $2l$ = Separation between two charges.

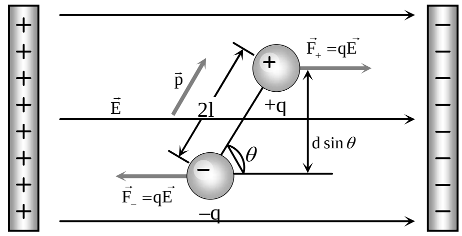


Figure: 14.10

Net force on a dipole in uniform electric field = 0.

- Torque on a dipole in a uniform electric field $\tau = pE\sin\theta = (\vec{p}\times\vec{E})$ [Vector form].

 In a uniform electric field an electric dipole experience no force but only torque, but in non -uniform field it experiences both force and torque.

- Potential energy of electric dipole $U = -pE\sin\theta = \vec{p}\cdot\vec{E}$ (Vector form).
- Work done in rotating the dipole form equilibrium position through an angle θ, $W = pE(1-\cos\theta)$.
- Electric potential and electric field Intensity due to electric dipole: $V = kq\left[\frac{1}{r_+}-\frac{1}{r_-}\right] = kq\left[\frac{r_- + r_+}{r_+ r_-}\right]$. For cases where $r >> d$, this can be approximated by $V = \frac{kp\cos\theta}{r^2}$.

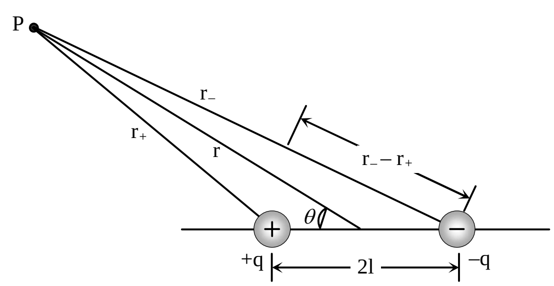


Figure: 14.11

Where, $\vec{p} = q2\vec{l}$ is defined as the dipole moment. The potential of dipole is of most interest where $r >> d$. The standard approximations are $r_- - r_+ \approx 2l\cos\theta$, $r_+ r_- \approx r^2$.

$$\vec{E} = -\nabla V = \left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial r}\hat{a}_\theta\right] = \frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\hat{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^3}\hat{a}_\theta$$

$$= \frac{Qd}{2\pi\varepsilon_0 r^3} = \left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)$$

$$\vec{E} = \frac{\vec{P}}{4\pi\varepsilon_0 r^3} = \left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)$$

Electric field due to a dipole along axis ($\theta = 0°$)

$$E_{axis} = \frac{\vec{P}}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}$$

At perpendicular bisector/equatorial ($\theta = 90°$)

$$E_{equitorial} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}$$

Obviously due to an electric dipole: $E \propto \frac{1}{r^3}$.

Electric potential due to an electric dipole : $V_{axis} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$,

$V_{equitorial} = 0$

- Binding energy of an electric dipole $B_n = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{2l}$
- Electrostatic force between two short dipoles of dipole moments p_1 and p_2 at separation is:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1p_2}{r^4} \text{ (when coaxial)}$$

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3p_1p_2}{r^4} \text{ (when mutually perpendicular)}$$

- Kinetic energy gained by a charge q accelerated through a potential difference of V volts is $E_k = qV$
- For equilibrium of charge between two horizontal plates having a potential difference V volts $qE = mg$ or $q\frac{V}{d} = mg$
- The electric lines of force never intersect; they are directed normally to an equipotential surface.

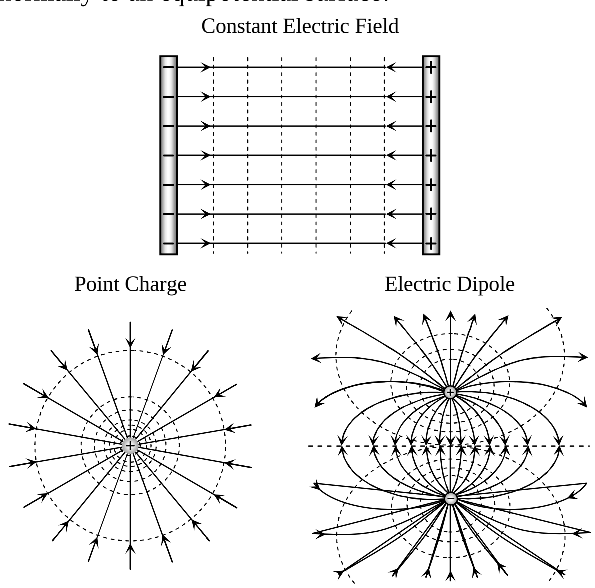


Figure 14.12: Dashed lines are Equipotential Lines while Solid Lines are Electric Field Lines.

A Van de Graaff Generator: It is an electrostatic generator which uses a moving belt to accumulate very high amounts of electrical potential on a hollow metal globe on the top of the stand.

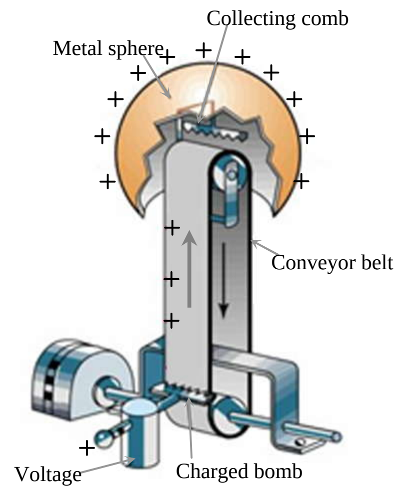
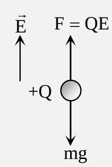
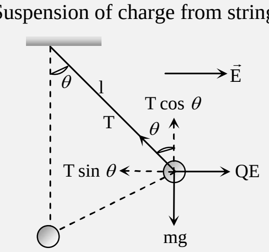
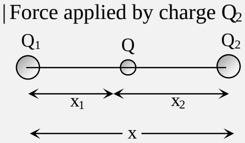


Figure: 14.13 Van de Graaff Generator

Table: 14.1 Different Cases of Equilibrium of Charge

Suspended charge	System of three collinear charge
Freely suspended charge In equilibrium $QE = mg$ $\Rightarrow E = \frac{mg}{Q}$  Suspension of charge from string  In equilibrium $T\sin\theta = QF$ ….(i) $T\cos\theta = mg$ ….(ii) From equations (i) and (ii) $T = \sqrt{(QE)^2 + (mg)^2}$ and $\tan\theta = \frac{QE}{mg}$	In the following figure three charges Q_1, Q and Q_2 are kept along a straight line, charge Q will be in equilibrium if and only if $\lvert$Force applied by charge $Q_1\rvert$ $= \lvert$Force applied by charge $Q_2\rvert$  i.e $\frac{QQ_1}{x_1^2} = \frac{Q_2Q}{x_2^2}$ $\Rightarrow \frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$ This is the necessary condition for Q to be in equilibrium. If all the three charges (Q_1, Q and Q_2) are similar, Q will be in stable equilibrium. If extreme charges are similar while charge Q is of different nature so Q will be in unstable equilibrium.

Time Period of Oscillation of a Charged Body

Case (i): If some charge say + Q is given to bob and an electric field E is applied in the direction as shown in figure then equilibrium position of charged bob (point charge) changes from O to O′.

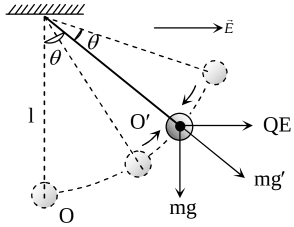


Figure: 14.14

On displacing the bob from its equilibrium position 0 ′. It will oscillate under the effective acceleration g′, where

$mg' = \sqrt{(mg)^2 + (QE)^2}$

$\Rightarrow\ g' = \sqrt{g^2 + (QE/m)^2}$.

Hence the new time period is $T_1 = 2\pi\sqrt{\frac{l}{g'}}$

$= 2\pi\sqrt{\frac{l}{(g^2(QE/m)^2)^{1/2}}}$

Since, g' > g, so $T_1 < T$, i.e. time period of pendulum will decrease.

Case (ii): If electric field is applied in the downward direction then.

Effective acceleration $g' = g + \frac{QE}{m}$

So new time period $T_2 = 2\pi\sqrt{\frac{l}{g + (QE/m)}} \Rightarrow T_2 < T$

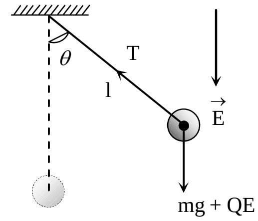


Figure: 14.15

Case (iii): In case 2 if electric field is applied in upward direction then, effective acceleration $g' = g - \frac{QE}{m}$

So new time period $T_3 = 2\pi\sqrt{\frac{l}{g - (QE/m)}} \Rightarrow T_3 > T$

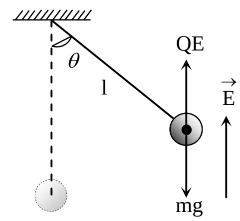
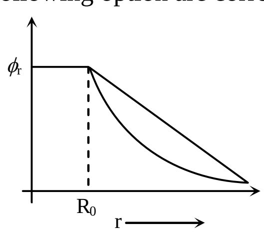


Figure: 14.16

Multiple Choice Questions

1. The electrostatic potential (ϕ_r) of a spherical symmetric system, kept at origin, is shown in the adjacent figure, and given $\phi_r = \frac{q}{4\pi\varepsilon_0 r}\ (r \geq R_0)$, , $\phi_r = \frac{q}{4\pi\varepsilon_0 R_0}\ (r \leq R_0)$.

   Which of the following option are correct?



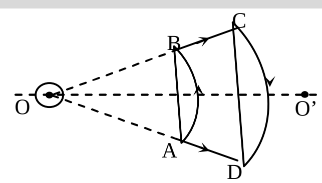
   (a) For spherical region $r \leq R_0$, total electrostatic energy stored is zero.

   (b) Within $r = 2R_0$, total charge is

   (c) There will be no charge anywhere except at $r = R_0$

   (d) Electric field is discontinuous at $r = R_0$

2. An infinite current carrying wire passes through point O and in perpendicular to the plane containing a current carrying loop ABCD as shown in the figure. Choose the correct option (s).



   (a) Net force on the loop is zero.

   (b) Net torque on the loop is zero

   (c) As seen from O, the loop rotates clockwise.

   (d) As seen from O, the loop rotates anticlockwise

3. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the c ylinders are initially electrically neutral.

   (a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.

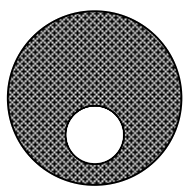
   (b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.

   (c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.

   (d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

4. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the spher e is then
(a) negative and distributed uniformly over the surface of the sphere
(b) negative and appears only at the point on the sphere closest to the point charge
(c) negative and distributed non -uniformly over the entire surface of the sphere
(d) zero

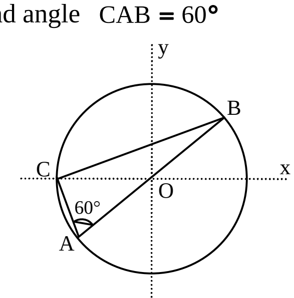
5. A spherical portion has been removed from a solid sphere having a change distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is :



(a) zero everywhere (b) non-zero and uniform
(c) non-uniform (d) zero only at its center

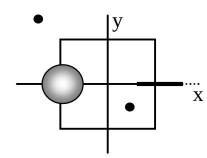
6. Positive and negative point charges of equal magnitude are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, \frac{-a}{2}\right)$, respectively. The work done by the electric field when another positive point charge is moved form $(-a, 0, 0)$ to $(0, a, 0)$ is:
(a) Positive
(b) Negative
(c) Zero
(d) Depends on the path connecting the initial and final positions

7. Consider a system of three charges $\frac{q}{3}, \frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle CAB = 60°



(a) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis
(b) The potential energy of the system is zero
(c) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$
(d) The potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$

8. A disk of radius a /4 having a uniformly distributed charge 6C is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges –7C and 3C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is

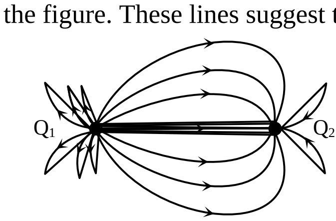


(a) $\frac{-2C}{\varepsilon_0}$ (b) $\frac{2C}{\varepsilon_0}$ (c) $\frac{10C}{\varepsilon_0}$ (d) $\frac{12C}{\varepsilon_0}$

9. Three concentric metallic spherical shells of radii R, 2R, 3R, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is:
(a) 1 : 2 : 3 (b) 1 : 3 : 5 (c) 1 : 4 : 9 (d) 1 : 8 : 18

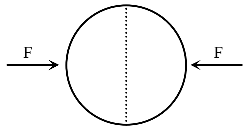
10. Under the influence of the coulomb field of charge +Q, a charge –q is moving around it in an elliptical orbit. Find out the correct statement(s).
(a) The angular momentum of the charge – q is constant
(b) The linear momentum of the charge – q is constant
(c) The angular velocity of the charge – q is constant
(d) The linear speed of the charge – q is constant

11. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(a) $|Q_1| > |Q_2|$ (b) $|Q_1| < |Q_2|$
(c) at a finite distance to the left of Q_1 the electric field is zero
(d) at a finite distance to the right of Q_2 the electric field is zero

12. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$

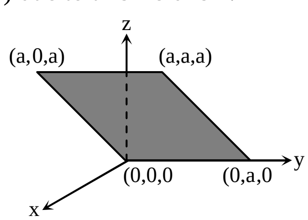
(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

13. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times 10^5\ Vm^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given g = 9.8 ms $^{-2}$, viscosity of the air $= 1.8\times 10^{-5}$ Ns m^{-2} and the density of oil $= 900$ kg m^{-3}, the magnitude of q is

(a) 1.6×10^{-19}C (b) 3.2×10^{-19}C

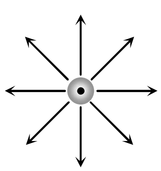
(c) 4.8×10^{-19}C (d) 8.0×10^{-19}C

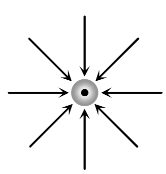
14. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is :

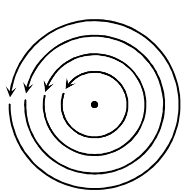


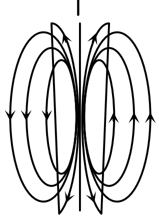
(a) $2E_0a^2$ (b) $\sqrt{2}E_0a^2$ (c) E_0a^2 (d) $\frac{E_0a^2}{\sqrt{2}}$

15. Which of the file patterns given below is valid for electric field as well as for magnetic field?

(a) 

(b) 

(c) 

(d) 

16. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept far apart and each is given charge + Now they are connected by a thin metal wire. Then

(a) $E_A^{inside} = 0$ (b) $Q_A > Q_B$

(c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (d) $E_A^{on\ surface} < E_B^{on\ surface}$

17. Which of the following statement(s) is/are correct ?

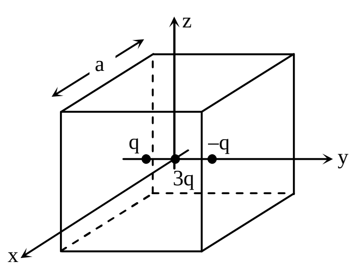
(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid

(b) The Gauss law can be used to calculate the field distribution around an electric dipole

(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point Bat potential V_B is $(V_B - V_A)$

18. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, – q at (0, – a/4, 0), + 3q at (0, 0, 0) and – q at (0, +a/4, 0). Chooses the correct option (s) ?



(a) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$

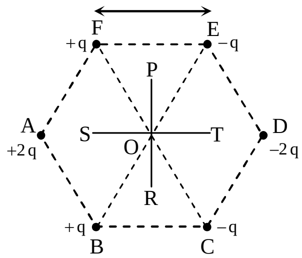
(b) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$

(c) The net electric flux crossing the entire region is q/ε_0

(d) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$

19. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure.

Given that : $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement (s) is (are) correct ?

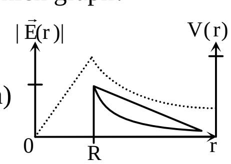


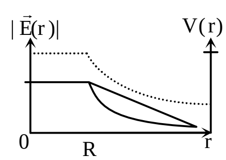
(a) The electric field at O is 6K along OD
(b) The potential at O is zero
(c) The potential at all points on the line PR is same
(d) The potential at all points on the line ST is same

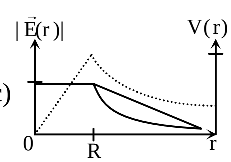
20. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly.

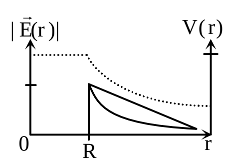
(a) 1×10^{-5}V (b) 1×10^{-7}V
(c) 1×10^{-9}V (d) 1×10^{-10}V

21. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

(a)


(b)


(c)


(d)


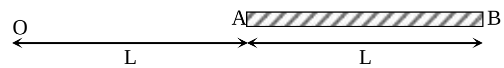
22. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$
(c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

23. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $(y << a)$ along the y-axis, the net force acting on the particle is proportional to

(a) y (b) −y (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

24. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is:

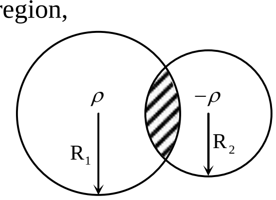


(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$ (c) $\frac{Q}{4\pi\varepsilon_0 L\ell n2}$ (d) $\frac{Q\ell n2}{4\pi\varepsilon_0 L}$

25. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centre of the spheres is zero. The ratio ρ_1/ρ_2 can be

(a) – 4 (b) $-\frac{32}{25}$ (c) $\frac{32}{25}$ (d) 4

26. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



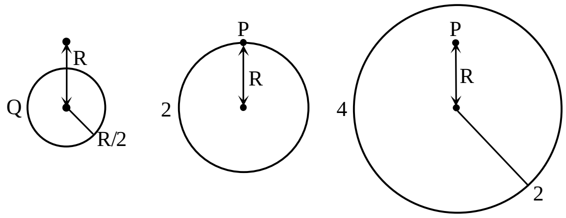
(a) the electrostatic field is zero
(b) the electrostatic potential is constant
(c) the electrostatic field is constant in magnitude
(d) the electrostatic field has same direction

27. Let $E_1(r), E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(a) $Q = 4\sigma\pi r_0^2$ (b) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(c) $E_1(r_0/2) = 2E_2(r_0/2)$ (d) $E_2(r_0/2) = 4E_3(r_0/2)$

28. Charges $Q, 2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2, R$ and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then



Sphere 1 Sphere 2 Sphere 3

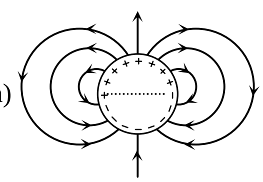
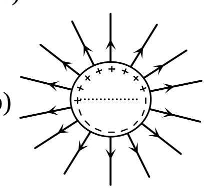
(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$

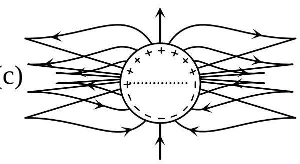
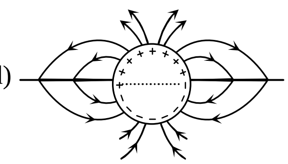
(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

29. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_o$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is:

(a) –80 J (c) 80 J (c) 120 J (d) –120 J

30. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in: (figures are schematic and not drawn to scale)

(a)  (b) 

(c)  (d) 

ANSWERS and SOLUTIONS

1. (a, b, c, d) The potential shown is for charged spherical conductor.

2. (a, c) Magnetic force on wire BC would be perpendicular to the plane of the loop along the outward direction and on wire DA the magnetic force would be along the inward normal, so net force on the wire loop is zero and torque on the loop would be along the clockwise sense as seen from O.

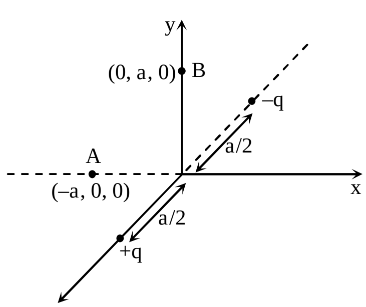
3. (a) $dV = -\vec{E}\cdot d\vec{r}$ and $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

Where, r is distance from the axis of cylindrical charge distribution (r is equal to or greater than radius of cylindrical charge distribution).

4. (d)

5. (b)

6. (c)



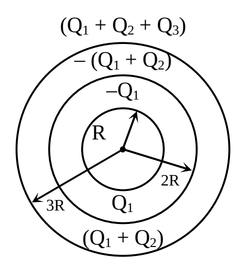
7. (c) $F_{BC} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\left(\dfrac{q}{3}\right)\left(\dfrac{2q}{3}\right)}{(R\sqrt{3})^2}$

$= \dfrac{q^2}{54\pi\varepsilon_0 R^2}$

8. (a) Total charge enclosed by cube is $-2C$.

Hence electric flux through the cube is $\dfrac{-2C}{\varepsilon_0}$.

9. (b) $\dfrac{Q_1}{4\pi R^2} = \dfrac{Q_1 + Q_2}{4\pi(2R)^2} = \dfrac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$



$\Rightarrow$ $Q_1 : Q_2 : Q_3 :: 1:3:5$

10. (a)

11. (a, d) No. of electric field lines of forces emerging from Q_1 are larger than terminating at Q_2

12. (a) Pressure $=\dfrac{\sigma^2}{2\varepsilon_0}$ and force $=\dfrac{\sigma^2}{2\varepsilon_0}\times\pi R^2$

13. (d) $\dfrac{4}{3}\pi R^3\rho g = qE = 6\pi\eta R v_T$

$\therefore \quad q = 8.0\times10^{-19}C$

14. (c) $\vec{A} = (a\hat{i} + a\hat{k})\times(a\hat{j}) = [a^2\hat{k} - a^2\hat{i}]$

$\phi = \vec{E}\cdot\vec{A} = E_0\hat{i}\cdot[a^2\hat{k} - a^2\hat{i}] = -E_0 a^2$

15. (c)

16. (a, b, c, d) On connecting $V_A = V_B$

$\Rightarrow \dfrac{kQ_A}{R_A} = \dfrac{kQ_B}{R_B}$

Also σR = constant

$\Rightarrow \dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$

17. (c, d) (d) is correct if we assume it is work done against electrostatic force.

18. (a, c, d) As $q_{en} = 3q - q - q = q$

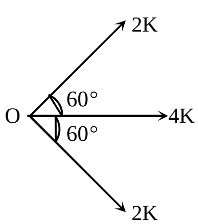
$\therefore$ flux $\phi = \dfrac{q}{\varepsilon_0}$ (c)

Obtain (a) is correct as for the two given faces charges are symmetrically located.

Option (b) is not correct as for $y = +\dfrac{a}{2}$ and $y = -\dfrac{a}{2}$ location of charge is symmetrical so fluxes will be equal.

Option (d) is correct as for $z = +\dfrac{a}{2}$ and $x = +\dfrac{a}{2}$ location of charge is symmetrical.

19. (a, b, c) $K = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{L^2}$



Potential at $O = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{q}{L} + \dfrac{2q}{L} + \dfrac{q}{L} - \dfrac{q}{L} - \dfrac{2q}{L} - \dfrac{q}{L}\right] = 0$

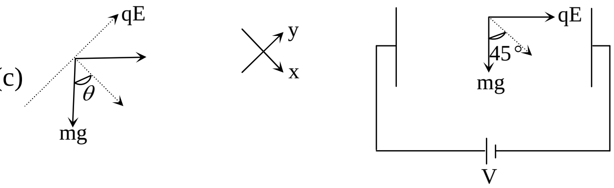
Any point on line PR is equidistant from the charges F and E; A & D; B and C. Each mentioned pair of charges are of equal magnitudes and opposite signs. Thus I by superposition principle; all points of PR have O potential.

Considering the electric field at O;

Net field along OD

= 4K + 2K cos60 + 2K cos60 = 6K

20. (c)



Since the angle $\theta = 45°$

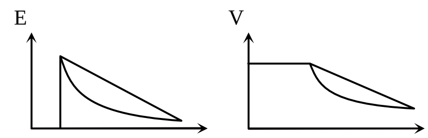
$\therefore \quad mg = qE$

$\Rightarrow \quad q\dfrac{V}{d} = mg$

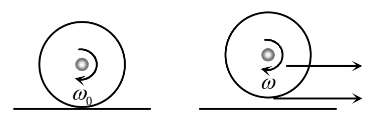
$\Rightarrow \quad V = \dfrac{mgd}{q} = \dfrac{1\cdot\times10^{-27}\times10\times10^{-2}}{1\cdot6\times10^{-19}} = 1\times10^{-9}$ Volts

21. (d) $E = 0 \; (r < R) = \dfrac{KQ}{r}(r > R)$

$V = \dfrac{KQ}{R}\;(r < R)$ (Constant) $= \dfrac{KQ}{r}\;(r > R)$



22. (c) COAM about the contact point $I\omega_0 = I\omega + mvr$



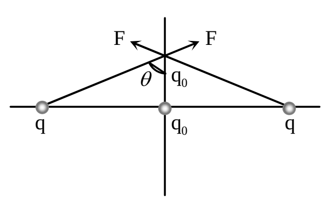
$mr^2\omega_0 = mr^2\omega + mvr = 2mvr$

$\therefore \quad v = \dfrac{r\omega_0}{2}$

23. (a) $F_{net} = \dfrac{1}{4\pi\varepsilon_0}\times\dfrac{2qq_0}{y^2 + a^2}\cos\theta$

$= \dfrac{2qq_0 y}{4\pi\varepsilon_0(y^2 + a^2)^{3/2}}$

$= \dfrac{2qq_0}{4\pi\varepsilon_0}\times\dfrac{y}{a^3}$ as $(y << a)$

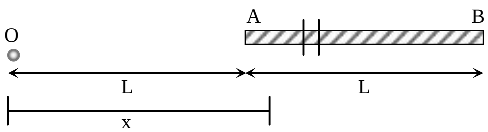


$\therefore \quad F \propto y$

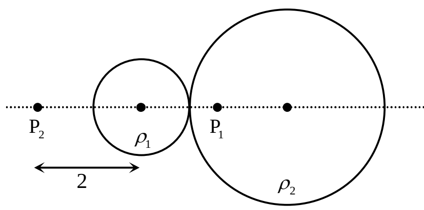
24. (d) $V = \int \frac{K\,dQ}{x}$

$$= K\int\left(\frac{Q}{L}\right)\frac{1}{x}\,dx = \frac{KQ}{L}\int_L^{2L}\frac{1}{x}\,dx$$

$$= \frac{KQ}{L}(\ell n x)_L^{2L} = \frac{KQ}{L}\ell n 2 = \frac{Q\ell n 2}{4\pi\varepsilon_0 L}$$



25. (b, d) At point P_1, $\frac{1}{4\pi\varepsilon_0}\frac{\rho_1(4/3)\,\pi R^3}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$

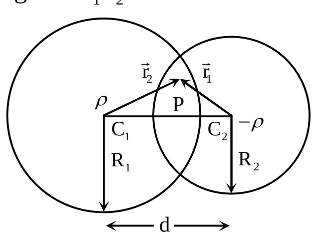


$\Rightarrow \quad \frac{\rho_1 R}{12} = \frac{\rho_2 R}{3} \Rightarrow \frac{\rho_1}{\rho_2} = 4$

At point P_2, $\Rightarrow \frac{\rho_1(4/3)\,\pi R^3}{(2R)^2} + \frac{\rho_2(4/3)\,\pi 8R^3}{(5R)^2} = 0$

$\therefore \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}$

26. (c, d) In triangle PC_1C_2



$\vec{r}_2 = \vec{d} + \vec{r}_1$

The electrostatic field at point P is:

$$\Rightarrow \quad \vec{E} = \frac{K\left(\rho\frac{4}{3}\pi R_1^3\right)\vec{r}_2}{R_1^3} + \frac{K\left(\rho\frac{4}{3}\pi R_2^3\right)(-\vec{r}_1)}{R_2^3}$$

$$\Rightarrow \quad \vec{E} = K\,\rho\frac{4}{3}\pi(\vec{r}_2 - \vec{r}_1)$$

$$\Rightarrow \quad \vec{E} = \frac{\rho}{3\varepsilon_0}\vec{d}$$

27. (c) $\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\,\varepsilon_0}$

$$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{\pi\varepsilon_0 r_0^2}, E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{\pi\varepsilon_0 r_0}, E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\,\varepsilon_0}$$

$$\therefore \quad E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$$

28. (c) For point outside dielectric sphere $E = \frac{Q}{4\pi\varepsilon_0 R^2}$

For point inside dielectric sphere $E = E_s\frac{r}{R}$

Exact Ratio $E_1 : E_2 : E_3 : = 2:4:1$

29. (a) $\vec{E} = 30x^2\hat{i}$

$$\Rightarrow \quad V_A - V_B = -\int_0^2 30\,x^2.\,dx = -80 \text{ J}$$

30. (a) Nature of electric field lines.

❑❑❑