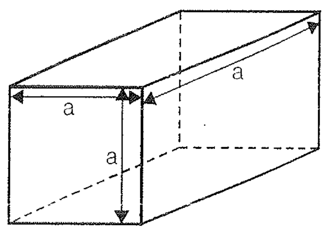
3.3 CHAPTER

Cubes and Dices

Cubes

A type of cuboid in which all the sides i.e. length, breadth & height are equal. All faces of cubes are of same area.



In a cube there are 8-corners/vertices
There are 6-faces [all equal in area]
There are 12-edges [all equal in length]
No. of edges = No. of vertices + No. of face – 2

Primarily question in cubes are based on painting of faces of cubes & then cutting the painted cube into identical cubelets. Because questions are based on the way cube is painted & how cuts are being made, hence we don't recommend any thumb rule. Still however because some basic rules are always applicable which we'll be using as building blocks for answering questions.

- If n equidistanced cuts are made (all parallel to same surface), cube will be divided into (n + 1) identical cuboidal pieces with each such cut there will $2a^2$ new surface area will be generated which will be unpainted.
- If I want to cut my bigger cube into identical n^3 cubelets, using minimum number cuts, I need total 3(n – 1) cuts, such that (n – 1) cuts parallel to each of these faces which are joining to corner.
- If number of cuts are not multiple of three then cube can never be cut into identical cubes but still it can be cut into maximum number of identical cuboidal pieces. To maximize such number of pieces we need to split the number of cuts into three parts which are closest. That we'll see in upcoming examples.

I Practice Exercise: I

Directions for Questions (1 - 9):
If a cube is cut into n^3 identical cubelets using minimum no. of cuts, after painting all faces of cube with white color, then answer the following questions.

1. What is minimum number cuts required?
2. What is maximum number of possible painted faces in once of such of cubelets?
3. How many cubelets will have exactly 3 faces painted?
4. How many cubelets will have exactly 2 faces painted?
5. How many cubelets will have exactly 1 face painted?
6. How many cubelets will have no faces painted?
7. How many cubelets will have at most 2 faces painted?
8. How many cubelets will have at least 1 face painted?
9. If initially V liters of paints was required to paint all faces of cube than how much extra paint will be required such that all the unpained faces of all cubelets will be painted.

Direction (Qs. 10 to 12) : A cube is divided into 216 identical cubelets. Each cut is made parallel to some surface of cube. But before cutting the cube is colored with green color on one set of opposite faces, red on the other set of opposite faces and blue on the third set.

10. How many cubelets are there which are painted with exactly one color?
    (a) 96 (b) 108
    (c) 124 (d) 48
11. How many cubelets are there which are painted with at least two coluor?
    (a) 96 (b) 56
    (c) 108 (d) 124

12. How many cubelets are there which are painted with at most one color?
(a) 124 (b) 108
(c) 160 (d) 96

Direction (Qs. 13 to 17): A cube is divided into 343 identical cubelets. Each cut is made parallel to some surface of the cube. But before doing that the cube is colored with green color on one set of adjacent faces, red on the second and blue on the third set.

13. How many cubelets are there which are colored with exactly are color?
(a) 180 (b) 150
(c) 165 (d) 148

14. How many minimum cuts you have made?
(a) 15 (b) 18
(c) 21 (d) 9

15. How many cubelets are colored with exactly two colors?
(a) 48 (b) 36
(c) 51 (d) 50

16. How many cubelets are thin which are colored with exactly three colors?
(a) 6 (b) 4
(c) 2 (d) 8

17. How many cubelets are painted with multiple colors?
(a) 51 (b) 53
(c) 115 (d) 120

Direction (Qs. 18 to 22): A cuboid is divided into 192 identical cubelets. This is done by making minimum no. of cuts possible. All cuts are parallel to some of the face. But before doing so. The cube is painted with green color on one set of opposite faces. Blue on other set of opposite faces and red on remaining their pair of opposite faces.

18. What is the maximum number of cubelets possible which one colored with green colored only?
(a) 48 (b) 64
(c) 96 (d) 72

19. What is the minimum no. of cuts I have made?
(a) 15 (b) 18
(c) 21 (d) 12

20. What is the minimum no. of cubelets possible which are painted with green & blue colors?
(a) 16 (b) 24
(c) 32 (d) 48

21. What is the number of cubelets which are painted with exactly 3 colors?
(a) 4 (b) 6
(c) 8 (d) 2

22. What is the number of cubelets possible which are painted with none of the color?
(a) 16 (b) 24
(c) 36 (d) 48

□□□□

Solution

1. Total number cuts required in $3(n - 1)$. $(n - 1)$ equidistant cuts parallel to each of 3 face which are joining to corner.

2. Maximum faces painted will be three in any such cubelets, which will be in case of cubelets coming out of corners of big cube after cutting.

3, 4, 5, 6.

Cubes painted from 3 faces = 8

Cubes painted from 2 faces = $12(n - 2)$

Cubes painted from 1 face = $6(n - 2)^2$

Cubes painted from none face = $(n - 2)^2$

Where 'n' is number of part into which each side is divided.

7. To find out the number of cubelets with at most 2 faces painted, I need to remove all those cubes which have exactly 3 faces painted

= Total no. of cubes – No. of cubes with 3 faces painted

$= (n^3 - 8)$

8. To find out the number of cubelets with at least one face painted, I need to remove all the cubelets which have no face painted

$= n^3 - (n - 2)^3$

9. As with every cut unpainted area of $2a^2$ is created Total no. of cuts used is $3(n - 1)$.
Hence total unpainted area generated is, (which is equal to areas of all unpainted faces of all small cubelets).
$= 2a^2 \times 3(n - 1) = 6a^2(n - 1)$

As the total surface area of cube was $6a^2$ for which V liters of paint was required, hence total paint required to paint, all unpainted faces of small cubelets is = $\frac{6a^2(n-1)}{6a^2} \times V$ = (n – 1)V liters

10, 11 & 12.

Because cube was painted with same color on opposite set of faces. We can still used formula.

$[(n-2)+2]^3 = {}^3C_0(n-2)^3 + {}^3C_1(n-2)^2 \times 2 + {}^3C_2(n-2) \times 2^2 + {}^3C_3 \times 2^3$

10. Exactly one color painted

$= {}^3C_1(n-2)^2 \times 2$

$= 3 \times (n-2)^2 \times 2$

$= 3 \times 4^2 \times 2 = 96$

Option (a)

11. Basically we are looking for no. of cubes which colored on two faces or three face

$= {}^3C_2(n-2) \times 2^2 + {}^3C_3 \times 2^3$

$= 3 \times (n-2) \times 4 + 8$

$= 3 \times 4 \times 4 + 8 = 56$

Option (b)

12. Basically in this question we are looking for no. of cubelet, which are uncolored & no. of cubelets which are colored on just one face.

$= {}^3C_0(n-2)^3 + {}^3C_1(n-2)^2 \times 2$

$= (n-2)^3 + 3 \times 2 \times (n-2)^2$

$= 4^3 + 6 \times 4^2$

$\Rightarrow 64 + 96 = 160$. Option (c)

Alternatively: Total no. cubelets which are colored an at most are face = total no. of cubelets – total no. of cubelets which are colored on multiple faces.

= 216 – 56 = 160. option (c)

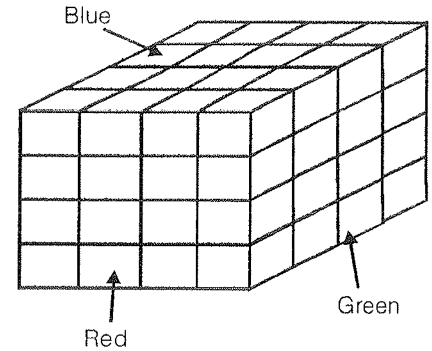
13. From each face $5 \times 5 = 25$ cubelets are painted is one color. Such total = $25 \times 6 = 150$ cubelets. Further because each face is having one similar color adjacent face and 3 other different coloured faces hence $5 \times 3 = 15$ extra cubelets will come from three edges, where adjacent faces of similar color are meeting hence total = 150 + 15 = 165 cube will be then colored in exactly one color?

Hence option (c)

14. $n^3 = 343 = 7^3 \Rightarrow n = 7$

Minimum number of cuts = 3(n – 1)

= 3(7 – 1) = 3×6 = 18. Option (b)



As we can see in above figure. 3 faces are visible in 3-diff. colours, out of hidden faces, bottom is red, one is green & another is blue.

15. Number of cubelet with no face painted will

$= (n-2)^3 = (7-2)^3 = 125$

No. of cubelets with 2 color

= Total no. of cubelet – [cubelets with one color + cubelets with no color + cubelets with three color]

= 343 – [125 + 2 + 165] = 51

Option (c)

16. Only two cubelets one is visible in figure & 2nd one exactly diagonally opposite corner of it will be colored with 3 three colors. Option (c)

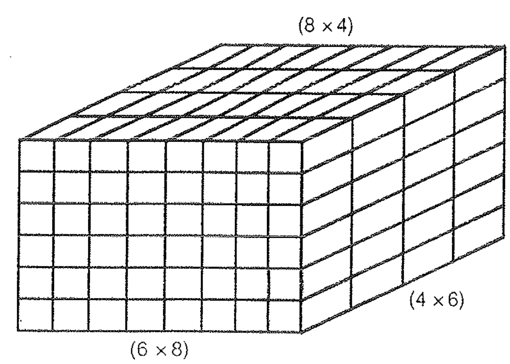
17. No. of cubelets with multiple color

= No. of cubelets with three color + No. cubelets with two color

= 2 + 51 = 53. Option (b)

18. As we want maximize number of cubelets with green in color only cuboid has to painted with green color on set of opposite faces of 6×8. Hence no. of green colored only cubelets will $(6-2) \times (8-4) = 4 \times 6 = 24$, from face. There are two such faces hence maximum total no. of only green painted cubelets will be = $24 \times 2 = 48$

Hence option (a)



19. The no. of cuts will be minimum when 192 will be factorized into 3 closest factors. $192 = 4 \times 6 \times 8$, so we have two opposite faces of 4×6 and two opposite faces of 6×8 and two opposite faces of 4×8. Hence the minimum number of cuts will be $= (4-1) + (6-1) + (8-1) = 3 + 5 + 7 = 15$. Hence option (a)

20. To minimize the number of cubelets with blue & green blue should be selected on (4×6) or (6×8) & green should be colored on (6×8) or (4×6) respectively cubelets from the edges where green & blue faces will be meeting will give the number of desired cubelets $= 4 + 4 + 4 = 16$
Hence option (a)

21. Again only the cubelets from corners of cuboid will be painted with 3 colors; hence 8 cubelets will be colored in 3 colors. Hence option (c)

22. Number of unpainted cubelets will be
$(4-2) \times (6-2) \times (8-2) = 2 \times 4 \times 6 = 48$
Hence option (d)

❑❑❑❑

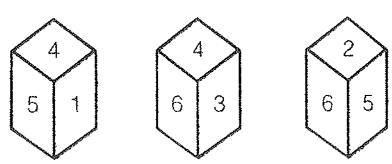
DICES

Dices are cubical structure with number of points from 1 to 6 are marked on faces. Dices are of two kinds.

1. **Symmetric:** Where sum of numbers on opposite pair of faces is same.
   Naturally the faces opposite to each other will be
   $1 \to 6$, $2 \to 5$, $3 \to 4$
2. **Asymmetric:** When sum of numbers on opposite pair of faces is different.

Example:

A dice with it's faces numbered 1 to 6 is shown in three different positions x, y, z

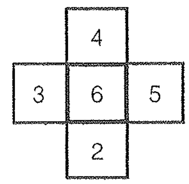


(x) (y) (z)

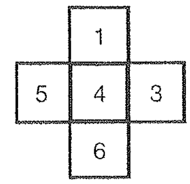
Find opposite faces pairs.

Solution.

Hence pair
$1 \to 6$
$2 \to 4$
$5 \to 3$



Hence 1 is marked opposite to 6



2 is marked opposite to 4

❑❑❑❑

Practice Exercise: I

Direction (Qs. 1 to 5) : A cube is painted & then divided cut into 336 smaller but identical pieces by making the minimum number of cuts possible. All cuts are parallel to some face.

1. What is the minimum no. of cuts required?
   (a) 21 (b) 18
   (c) 24 (d) None of these
2. How many smaller pieces have exactly 3 painted faces?
   (a) 20 (b) 8
   (c) 12 (d) 6
3. How many smaller pieces have at least 2 face painted?
   (a) 64 (b) 68
   (c) 72 (d) 76
4. How many smaller pieces have at most one face painted?
   (a) 248 (b) 264
   (c) 268 (d) None of these
5. How many smaller pieces have no face painted?
   (a) 144 (b) 124
   (c) 120 (d) 108

Direction (Qs. 6 to 13) : Questions are based on the following information.

Three adjacent faces of one cube is painted in pink, one adjacent pair of faces is painted in black & the remaining faces are painted in violet. The cube is the cut into 216 identical cubelets.

6. How many of cubelets will have all three colors on them?
   (a) 1 (b) 2
   (c) 3 (d) 4

7. How many of the smaller cubes have only pink & black colors on them?
(a) 16 (b) 18
(c) 19 (d) 24

8. How many smaller cubes or cubelets have exactly two colors on them?
(a) 32 (b) 36
(c) 37 (d) 42

9. How many the cubelets have exactly two painted surface in two different colors?
(a) 36 (b) 32
(c) 48 (d) 38

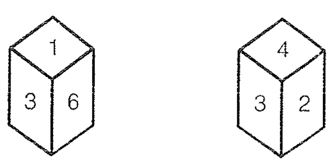
10. How many of the cubelets have exactly one color on them?
(a) 107 (b) 109
(c) 96 (d) 113

11. How many of the cubelets have exactly one painted surface of exactly one color?
(a) 84 (b) 108
(c) 96 (d) 102

12. How many cubelets do not have pink color on them?
(a) 120 (b) 100
(c) 150 (d) 125

13. How many cubelets have black or violet color on them but not pink color on them?
(a) 60 (b) 59
(c) 51 (d) 64

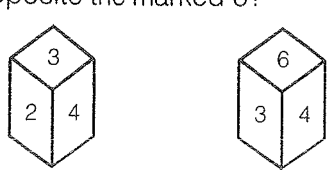
14. The six faces of a cube are marked as 1, 2, 3, 4, 5 & 6. Given below are two different view of same cube.



(I) (II)

Which face is opposite the face marked 5?
(a) 2 (b) 4
(c) 3 (d) 6

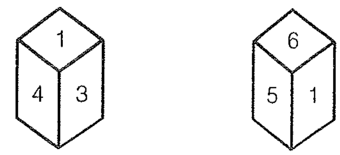
15. The six faces of a cube are marked as faces 1, 2, 3, 4, 5 & 6. Given below are two different views. Which face is opposite the marked 6?



(i) (ii)

(a) 4 (b) 5
(c) 1 (d) 2

16. The six faces of a cube as marked 1, 2, 3, 4, 5 & 6. Given below are two different view of same cube.



(I) (II)

Which face is opposite the face mark as 6?
(a) 3 (b) 4
(c) 2 (d) None of these

17. How many cuboids of dimensions $4 \times 5 \times 6$ are required to form a cube of least size if cuboids have to be placed adjacent, above or below each other?
(a) 1600 (b) 1800
(c) 1200 (d) None of these

18. A cube has been cut into cuboids of size $2 \times 3 \times 4$. What is least possible integer length of the edge of cube and how many such cuboids are obtained from this cube?
(a) 96 (b) 72
(c) 60 (d) 84

Direction (Qs. 19 to 22) : Read the following information carefully & answer the question below it.

A dice is prepared in following manner?
(i) 1 should lie between 2 & 3
(ii) 2 should lie opposite to 3
(iii) 4 should lie between 5 & 6
(iv) 5 & 6 should lie opposite to each other
(v) 4 should lie face down

19. The face opposite to 1 is
(a) 2 (b) 4
(c) 6 (d) 5

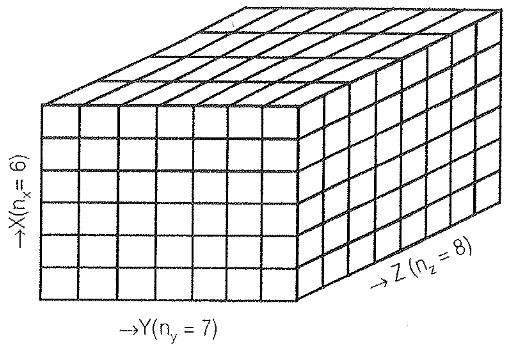
20. The upper face is
(a) 1 (b) 6
(c) 2 (d) 5

21. The face adjacent to 5 are
(a) 2, 6, 1, 4 (b) 1, 3, 4, 6
(c) 3, 4, 2, 1 (d) 2, 6, 4, 5

22. The face adjacent to 3 are
(a) 5, 4, 1, 2 (b) 1, 2, 5, 6
(c) 5, 6, 4, 1 (d) 2, 6, 4, 5

Solutions

Answer 1 to 5: Number of identical pieces $336 = 8 \times 7 \times 6$. Hence we need 7 cut's in Z-direction, 6 cuts in Y-direction, 5-cuts in X-directions.



Means our cube cut into 8 parts along Z-directions say $n_Z = 8$, similarly $n_Y = 7$, $n_X = 6$.
For total number of identical pieces we can say
$n_X \times n_Y \times n_Z = 336$
$\{(n_x - 2) + 2\} \times \{(n_Y - 2) + 2\} \times \{(n_Z - 2) + 2\}$

Now look at table below:

No. of pieces	Formula	Answers for $6 \times 7 \times 8$ cube
3-face painted (Corner pieces)	2^3	8
2-faces painted piece	$4[(n_x - 2) + (n_y - 2) + (n_z - 2)]$	$4[(8-2) + (7-2) + (6-2)] = 60$
Only one face painted pieces	$2[(n_x - 2)(n_y - 2) + (n_y - 2)(n_z - 2) + (n_z - 2)(n_x - 2)]$	$2[(8-2)(7-2) + (7-2)(6-2) + (8-2) \times (6-2)]$
Pieces no face painted	$(n_x - 2)(n_y - 2)(n_z - 2)$	$(8-2)(7-2)(6-2) = 120$

1. (b) Minimum no. of cuts
$(8 - 1) + (7 - 1) + (6 - 1) = 18$

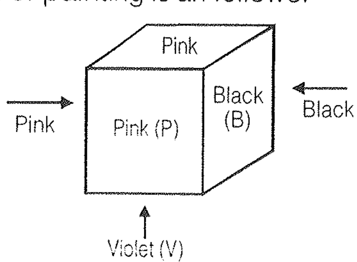
2. (b) All pieces from corners of cube = 8

3. (b) 2 faces painted + 3 faces painted
$= 8 + 60 = 68$

4. (c) No. faces painted + one face painted
$= 120 + 148 = 268$

5. (c) $(8 - 2)(7 - 2)(6 - 2) = 120$

Answer 6 to 13: Cube is cut in $6 \times 6 \times 6 = 216$ cubelets
Pattern of painting is an follows.



Let us analysis the color combination

Corners	Edges	Faces
PPP – 1, BBP = 1	PP = 3, PB = 4	P = 3
PPB – 2, BBV = 1	BB = 1, PV = 2	B = 2
PPV – 1, PVB = 2	BV = 2	V = 1

6. (b) The small pieces with three color will from corners. Only 2-such diagonl opposite y cubelets are possible.

7. (c) The cubelets with only pink & black are found at corners & along the edges PB
= 2 PPB cubelets + 1 BBP cubelet + 4 P edges $\times (6 - 2) = 19$ cubelets.

8. (c) The cubelets with exactly two colors are found at corners & along the edges having different colors on either side = 5 corner pieces + 8 edges $\times (6 - 2) = 37$ cubelets.

9. (b) In the previous question there is no. restriction on the number of painted surface. But in this question there is a restriction. The paint should be only on two faces only. Hence we'll not consider cubelets from corner. Hence only 32 cubelets.

10. (d) The cubelets with exactly one color are
    (a) One exactly at corner (PPP) = 1
    (b) At the three edges each of PP category $= 3(6 - 2) = 12$
    (c) At the one edge of BB category $= 1(6 - 2) = 4$
    (d) At the middle of each of the six faces $= 6 \times (6 - 2) = 96$
    So total cublets with exactly one colour are
    $1 + 12 + 4 + 96 = 113$

11. (c) In this question only the cubelets from middle of faces of cube will be considered $= 6 \times (6 - 2)^2 = 96$ cubelets.

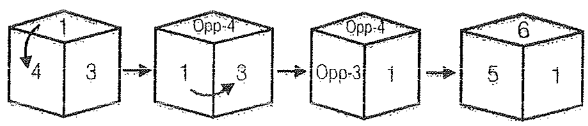
12. (d) The number of cubelets with no pink color
$= 216 - $ (36 from one pink surface + 30 from second + 25 from third)
= 125 cubelets.

13. (c) Here cubelets without any face colored has to be removed from previous question number. The number of cubelets with black or violet but not pink
$= 125 - (6 - 2)^3 = 61$

14. (c) As 3 is adjacent to (1 & 6) as well as (4 & 2) hence 3 is opposite to the remaining face 5.

15. (d) 2 & 6 both are adjacent to both 3 & 4. Hence 2 & 6 have to be opposite.

16. (b) From (I) & (II) we know that 1 is adjacent to 3, 4, 5, 6. Hence 1 is opposite to 2. We can find the other two pairs of opposite faces by rotating one of the view to reach another.



⇒ Opp to 4 is 6 & opp. to 2 is 5.

17. (b) Least possible dimension of cube is LCM of (4, 5, 6) = 60

The number of cuboids required

$$= \frac{60 \times 60 \times 60}{4 \times 5 \times 6} = 1800$$

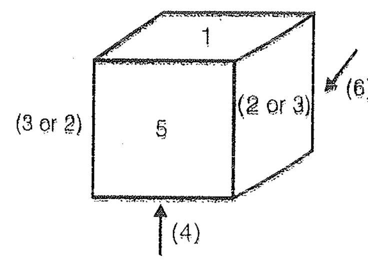
18. (b) The least possible dimension of cubie

LCM of 2, 3, 4 = 12

No. of cuboids that can be obtained

$$= \frac{12 \times 12 \times 12}{2 \times 3 \times 4} = 72$$

Answer 19 to 22:



19. (b) The face opposite to 1 will be 4.

20. (a) The upper face is 1.

21. (c) 6 is opposite to 5, hence 1, 2, 3, 4 must be adjacent to 5.

22. (c) 3 is opposite to 2 hence 1, 4, 5, 6 are adjacent to 3.