[SINGLE ANSWER CORRECT TYPE]

1. If the sides a,b,c are the roots of the equation $x^3 - 18x^2 + 104x - 192 = 0$, then the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is equal to -

(A) $\frac{3}{64}$ (B) $\frac{29}{48}$ (C) $\frac{29}{96}$ (D) $\frac{3}{128}$

2. In a ΔABC, if $\tan A + 3\tan C = 0$, then angle B lies in -

(A) $\left(0, \frac{\pi}{6}\right]$ (B) $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ (C) $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$ (D) $\left[\frac{5\pi}{6}, \pi\right)$

3. In ΔABC, if $a^2\cos 2A = 2b^2 + 2c^2 - a^2$, then A belongs to

(A) $\left(0, \frac{\pi}{6}\right)$ (B) $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$ (C) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{2}, \pi\right)$

4. In ΔABC, if $\cos A + \sin A - \frac{2}{\cos B + \sin B} = 0$, then $\frac{a+b}{c}$ is equal to

(A) $\sqrt{2}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $2\sqrt{2}$

5. If sides of ΔABC are connected with relation $4a^2 + 9b^2 + 16c^2 = 6ab + 12bc + 8ac$, then $\cos A$ is equal to

(A) 0 (B) $-\frac{1}{2}$ (C) $\frac{6}{7}$ (D) $-\frac{11}{24}$

6. Two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$ and the angle between the sides is $\frac{\pi}{3}$. Then perimeter of the triangle is

(A) $6+\sqrt{3}$ (B) $2\sqrt{3}+\sqrt{6}$ (C) $2\sqrt{3}+\sqrt{10}$ (D) none of these

7. If in $\Delta ABC, \frac{\sin A}{3} = \frac{\sin B}{3} = \frac{\sin C}{2}$, then the value of $\cos A + \cos B + \cos C$ is equal to

(A) $\frac{13}{9}$ (B) $\frac{12}{13}$ (C) $\frac{14}{9}$ (D) $\frac{9}{13}$

8. In any triangle ABC, $\frac{a^2\sin(B-C)}{\sin B + \sin C} + \frac{b^2\sin(C-A)}{\sin C + \sin A} + \frac{c^2\sin(A-B)}{\sin A + \sin B} =$

(A) a + b + c (B) a + b – c (C) a – b + c (D) 0

[MULTIPLE ANSWER CORRECT TYPE]

9. In ΔABC, which of the following is/are possible (where notations have usual meaning)

(A) sinA : sinB : sinC = 1 : 2 : 3 (B) $\Delta = \frac{bc}{4}$

(C) (a + b + c)(a + b – c) = 3ab (D) $b^2 - c^2 = aR$

10. In a triangle if the length of two longer sides are 8 and 7 and its angles are in A.P., then smaller side can be

(A) 3 (B) 4 (C) 5 (D) 6

[SUBJECTIVE TYPE]

11. Given a = 13, b = 14, and c = 15, then find the sines of the angles.

12. Prove that: $a \cos \frac{B-C}{2} = (b+c)\sin\frac{A}{2}$

13. Prove that: $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$

14. Prove that: $a (b \cos C - c \cos B) = b^2 - c^2$

15. Prove that: $\frac{\sin(B-C)}{\sin(B+C)} = \frac{b^2-c^2}{a^2}$

16. Prove that: $\frac{a+b}{a-b} = \tan\frac{A+B}{2}\cot\frac{A-B}{2}$

17. Prove that: $a^2 + b^2 + c^2 = 2 (bc \cos A + ca \cos B + ab \cos C)$

18. If in any triangle the angles be to one another as 1 : 2 : 3 prove that the corresponding sides are as $1 : \sqrt{3} : 2$

19. In any triangle ABC, prove that : $\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B} = \frac{a^2+b^2}{a^2+c^2}$

20. In any triangle ABC, prove that: $a \sin(B - C) + b \sin(C - A) + c \sin(A - B) = 0$

21. In any triangle ABC, prove that : $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$.

22. In any triangle ABC, prove that: $a (\sin B - \sin C) + b (\sin C - \sin A) + c (\sin A - \sin B) = 0$

23. If in a ΔABC, $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, prove that a^2, b^2, c^2 are in A.P.

24. In any triangle ABC, prove that : $\frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}+\sqrt{\sin B}} = \frac{a+b-2\sqrt{ab}}{a-b}$.

25. In any triangle ABC, prove that : $b \cos B + c \cos C = a \cos (B - C)$

Answers

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1. (C) **2.** (A) **3.** (D) **4.** (A) **5.** (D) **6.** (B) **7.** (A) **8.** (D) **9.** (BCD)

10. (AC) **11.** $\frac{4}{5}, \frac{56}{65}$ and $\frac{12}{13}$