02

LCM and HCF

Factor and Multiple

If a number divides another number exactly, then the number which divides them is called the **factor** of that number and the number that has been divided is known as the **multiple** of that number.

In other words, we say that if a divides b exactly (without leaving any remainder), then a is the factor of b and b is the multiple of a.

e.g. 2 and 5 are factors of 10 and 10 is a multiple of 2 and 5.

Common Multiple

A common multiple of two numbers or more than two numbers is that number which is exactly divisible by both, without leaving any remainder.

e.g. 36 is a common multiple of 2, 3, 4, 6, 9, 12 and 18 as 36 is completely divisible by each of them.

Least Common Multiple (LCM)

The least common multiple of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

e.g. We can obtain LCM of 12 and 18 as follows
Multiple of 12 are 12, 24, 36, 48, 72, ...

Multiple of 18 are 18, 36, 54, 72, ...

Common multiple of 12 and 18 are 36, 72, ...

$\therefore$ LCM of 12 and 18 = 36

Methods for Finding LCM

To find the LCM of given numbers, two methods are used which are as follows.

(i) Prime Factorisation Method

In this method, following step are used.

***Step* I** Break the given number into their prime factors.

***Step* II** Find the product of highest powers of all the factors, which occur in the given numbers.

***Step* III** This product is the required LCM.

e.g. LCM of 6, 8 and 12 is

***Step* I**

2	6
3	3
	1

2	8
2	4
2	2
	1

2	12
2	6
3	3
	1

Factors of

$6 = 2 \times 3 = 2^1 \times 3^1$

$8 = 2 \times 2 \times 2 = 2^3$

$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$

Step II LCM of 6, 8, 12 = $2^3 \times 3$

Step III LCM of 6, 8, 12 = $8 \times 3 = 24$

(ii) Division Method

In this method, following step are used.

Step I Write down the given numbers is a line. Separating them by commas.

Step II Divide by anyone of the prime numbers which exactly divides atleast two of the given numbers.

Step III Write down the quotient and the undivided numbers in the line below the first.

Step IV Repeat this process until you get a line of numbers which are prime to one-another.

Step V The product of all divisors and the numbers in the last line is the required LCM.

e.g. LCM of 16, 24, 36 and 54 by division method is

2	16, 24, 36, 54
2	8, 12, 18, 27
2	4, 6, 9, 27
2	2, 3, 9, 27
3	1, 3, 9, 27
3	1, 1, 3, 9,
3	1, 1, 1, 3
	1, 1, 1, 1

$\therefore$ Required LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 16 \times 27 = 432$

Example 1 LCM of 16, 25 and 40 is

(a) 350 (b) 400 (c) 300 (d) 450

Sol. (*b*) **By prime factorisation method**

2	16
2	8
2	4
2	2
	1

5	25
5	5
	1

2	40
2	20
2	10
5	5
	1

So, $16 = 2^4$, $25 = 5^2$ and $40 = 2^3 \times 5$

$\therefore$ LCM $= 4^4 \times 5^2 = 16 \times 25 = 400$

By division method

2	16, 25, 40
2	8, 25, 20
2	4, 25, 10
2	2, 25, 5
5	1, 25, 5
5	1, 5, 1
	1, 1, 1

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 400$

LCM of Polynomials

A polynomial p(x) is called the LCM of two or more given polynomials, if it is a polynomial of smallest degree which is divided by each one of the given polynomials.

e.g. LCM of $15x^2y^4z$ and $21\,xy^2z^3$ will be as follows.

$15x^2y^4z = 3 \times 5 \times x^2 \times y^4 \times z$

and $21xy^2z^3 = 3 \times 7 \times x \times y^2 \times z^3$

Here, Maximum power of 3 is 1.

Maximum power of 5 is 1.

Maximum power of 7 is 1.

Maximum power of x is 2.

Maximum power of y is 4.

Maximum power of z is 3.

Hence, required LCM is given by

$= 3 \times 5 \times 7 \times x^2 \times y^4 \times z^3 = 105\,x^2y^4z^3$

Common Factor

A common factor of two or more numbers is a number that divides of them exactly.

e.g. 4 is a common factor of 12, 64, 128.

Highest Common Factor (HCF)

The HCF of two or more numbers is the greatest number that divides each of them exactly.

e.g. 5 is the HCF of 20 and 25.

The terms highest common divisor and greatest common divisor are often used in the sense of Highest Common Factor (HCF).

Methods for Finding HCF

To find the HCF of given numbers, two methods are used which are as follows.

(i) Prime Factorisation Method

Following are the steps for calculating HCF through prime factorisation method.

***Step* I** Resolve the given numbers into their prime factors.

***Step* II** Find the product of all the prime factors (with least power) common to all the numbers.

***Step* III** The product of common prime factors (with the least powers) gives HCF.

e.g. HCF of 12, 18, 24 by prime factorisation method will be as follows

$$12 = 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$24 = 2 \times 2 \times 2 \times 3$$

Now, we choose those factors which are common to all the members. Here, 2 and 3 are common. So, required HCF $= 2 \times 3 = 6$

(ii) Division Method

Following are the steps to obtain HCF through division method.

***Step* I** Divide the larger number by the smaller one.

***Step* II** Divide the divisor by the remainder.

***Step* III** Repeat step II till the remainder becomes zero. The last divisor is the required HCF.

☑ To calculate the HCF of more than two numbers, calculate the HCF of first two numbers then take the third number and HCF of first two numbers and calculate their HCF and so on. The resulting HCF will be the required HCF of numbers.

e.g. HCF of 12, 18 and 27 by division method will be as follow.

```
12) 18 (1
    12
    ---
     6) 12 (2
        12
        ---
         ×
        ---
```

Here, last divisor i.e HCF of 12 and 18 is 6.

Again, HCF of 6 and 27 is

```
6) 27 (4
   24
   ---
    3) 6 (2
       6
       ---
       ×
       ---
```

∴ HCF of 12, 18, 27 is 3.

Example 2 The HCF of 108, 360 and 408 is

(a) 4 (b) 8
(c) 12 (d) 16

Sol. (*c*) **By prime factorisation**

$$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$$
$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$$
$$408 = 2 \times 2 \times 2 \times 3 \times 17 = 2^3 \times 3 \times 17$$

∴HCF of 108, 360 and 408 $= 2^2 \times 3 = 12$

By Division Method

We, have, 108, 360 and 408

```
108) 360 (3
     324
     ---
      36) 108 (3
          108
          ---
           ×
          ---
```

Now, HCF of 36 and 408

```
36) 408 (11
    36
    ---
     48
     36
    ---
     12) 36 (3
         36
         ---
          ×
         ---
```

HCF of Polynomials

A polynomial $h(x)$ is called the HCF or GCD of two or more given polynomials, if $h(x)$ is a polynomial of highest degree dividing each one of the given polynomials.

e.g. HCF of $20x^3y^5z^2$ and $35x^5y^3z^3$ will be as follows.

$$20x^3y^5z^2 = 2 \times 2 \times 5 \times x^3 \times y^5 \times z^2$$
$$35x^5y^3z^3 = 5 \times 7 \times x^5 \times y^3 \times z^3$$

Hence, required HCF is given by

$$= 5 \times x^3 \times y^3 \times z^2$$
$$= 5x^3y^3z^2$$

LCM and HCF of Fractions

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\text{and HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

Example 3 The LCM of $\frac{72}{250}, \frac{126}{75}$ and $\frac{162}{165}$ is

(a) $149\frac{1}{25}$ (b) $151\frac{1}{5}$

(c) $154\frac{1}{10}$ (d) None of these

Sol. (*b*) LCM of $\frac{72}{250}, \frac{126}{75}$ and $\frac{162}{165}$

$$= \frac{\text{LCM of } 72, 126 \text{ and } 162}{\text{HCF of } 250, 75 \text{ and } 165} = \frac{756}{5} = 151\frac{1}{5}$$

Example 4 The HCF of $\frac{6}{7}, \frac{5}{14}$ and $\frac{10}{21}$ is

(a) $\frac{1}{42}$ (b) $\frac{1}{21}$ (c) $\frac{5}{7}$ (d) $\frac{30}{7}$

Sol. (*a*) HCF of $\frac{6}{7}, \frac{5}{14}$ and $\frac{10}{21} = \frac{\text{HCF of } 6, 5, 10}{\text{LCM of } 7, 14, 21}$

$$= \frac{1}{7 \times 2 \times 3} = \frac{1}{42}$$

HCF and LCM of Decimals

To find the HCF or LCM of given decimal numbers, first make equal number of places after decimal point in each number. Then, find their HCF or LCM without assuming the decimal point. At last put the decimal point leaving as many decimal places to the right as there are in each of the given numbers. This is the required HCF or LCM.

e.g. LCM of 0.6, 9.6 and 0.12

These numbers are equivalent to 0.60, 9.60 and 0.12. Now, we find the LCM of 60, 960 and 12.

12	60, 960, 12
5	5, 80, 1
	1, 16, 1

$\therefore$ LCM of 60, 960 and $12 = 12 \times 5 \times 16 = 960$

Hence, LCM of 0.6, 9.6 and $0.12 = 9.60$

Example 5 The HCF of 16.5, 0.90 and 15 is

(a) 0.3 (b) 0.6 (c) 0.5 (d) 0.15

Sol. (*a*) The given numbers are equivalent to 16.50, 0.90 and 15.00. Firstly, we have to find the HCF of 1650, 90 and 1500.

2	1650
3	825
5	275
5	55
	11

2	90
3	45
3	15
	5

2	1500
2	750
3	375
5	125
5	25
	5

$$1650 = 2 \times 3 \times 5 \times 5 \times 11 = 2^1 \times 3^1 \times 5^2 \times 11^1$$

$$90 = 2 \times 3 \times 3 \times 5 = 2^1 \times 3^2 \times 5^1$$

$$1500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 = 2^2 \times 3^1 \times 5^3$$

$\therefore$ HCF of 1650, 90 and $1500 = 2^1 \times 3^1 \times 5^1$

$$= 2 \times 3 \times 5 = 30$$

$\therefore$Required HCF of 16.50,0.90 and 15 is 0.3.

Points to Remember

1. Product of two numbers = HCF of the numbers × LCM of the numbers
2. The least number which when divided by x, y and z leaves the remainders a, b and c respectively such that $(x - a = y - b = z - c)$
   $= [\text{LCM of } (x, y, z)] - k$
   where, $k = (x - a) = (y - b) = (z - c)$
3. The least number which when divided by x, y and z leaves the same remainder k in each case $= [\text{LCM of } (x, y, z) + k]$.
4. The greatest number which divides the numbers x, y and z, leaving remainders a, b and c, respectively = HCF of $(x - a)$, $(y - b)$, $(z - c)$
5. The greatest number that will divide $x, y, z, \ldots$ leaving the same remainder in each case
   $= \text{HCF of } (x - y), (y - z), (z - x), \ldots$

Example 6 The LCM of two numbers is 2079, their HCF is 27. If the first number is 189, then the second number is

(a) 179 (b) 197 (c) 297 (d) 279

Sol. (*c*) According to the formula,

Product of two numbers = HCF × LCM

$\Rightarrow \quad 189 \times \text{second number} = 2079 \times 27$

$\therefore \quad \text{second number} = \frac{2079 \times 27}{189} = 297$

Example 7 The greatest number which divides 29, 60 and 103 leaving remainders 5,12 and 17 respectively is
(a) 26 (b) 32 (c) 24 (d) 36

Sol. (*c*) According to the question,

$x = 29, y = 60, z = 103$
$a = 5, b = 12, c = 17$

Now, according to the formula,
Required number = HCF of (29 – 5), (60 – 12), (103 – 17)
= HCF of 24, 48, 96

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$
$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3^1$

$\therefore$ Required HCF of 24, 48 and 96 $= 2^3 \times 3^1$
$= 8 \times 3 = 24$

Hence, 24 is the required number.

Example 8 The least number which divided by 24, 30 and 54 leaves 5 as remainder in each case, is
(a) 1080 (b) 1075 (c) 1085 (d) 1090

Sol. (*c*) According to the question,

$x = 24, y = 30, z = 54, k = 5$

According to the formula,
Required number = [LCM of (24, 30 and 54)] + 5

2	24, 30, 54
3	12, 15, 27
	4, 5, 9

$\therefore$ LCM $= 2 \times 3 \times 4 \times 5 \times 9 = 1080$
$\therefore$ Required number = 1080 + 5 = 1085

Example 9 Find the least number which when divided by 24,32 and 36 leaves the remainders 19, 27 and 31 respectively.
(a) 238 (b) 280 (c) 287 (d) 285

Sol. (*a*) Given that, $x = 24, y = 32, z = 36,$
$a = 19, b = 27, c = 31$

then, $24 - 19 = 5, 32 - 27 = 5, 36 - 31 = 5$ $\therefore$ $k = 5$

According to the formula,
Required number = (LCM of 24, 32 and 36) – 5
Now, LCM of 24,32 and 36 $= 2 \times 2 \times 2 \times 3 \times 3 \times 4$
= 288

$\therefore$ Required number = 288 – 5 = 283

Example 10 What is the greatest number that will divide 99, 123 and 183 leaving the same remainder in each case?
(a) 15 (b) 12 (c) 21 (d) 16

Sol. (*b*) Given that, $x = 99, y = 123$ and $z = 183$

According to the formula,
Required number = HCF of $(|x - y|, |y - z|, |z - x|)$

$|x - y| = |99 - 123| = 24$
$|y - z| = |123 - 183| = 60$
$|z - x| = |183 - 99| = 84$

2	24	2	60	2	84
2	12	2	30	2	42
2	6	3	15	3	21
3	3	5	5	7	7
	1		1		1

Therefore,
Factors of 24 $= 2^2 \times 3^1$
Factors of 60 $= 2^2 \times 3^1 \times 5^1$
Factors of 84 $= 2^2 \times 3^1 \times 7^1$

$\therefore$ HCF of 24, 60 and 84 $= 2^2 \times 3^1 = 4 \times 3 = 12$

Example 11 The traffic lights at three different road crossings change after every 48 s, 72 s and 108 s respectively. If they change simultaneously at 7 am, at what time will they change simultaneously again?
(a) In 7 min 12 s (b) In 8 min 12 s
(c) In 7 min 14 s (d) In 8 min 14 s

Sol. (*a*) The time period after which the traffic lights at three different road crossings changes simultaneously again will be the LCM of 48, 72 and 108.

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$

$= \frac{432}{60}$ min = 7 min 12s

[∵ 1 min = 60 s and and 1s = 1/60 min]

So, time when they will change again

$$\begin{array}{r} = 07:00:00 \\ + \quad 07:12 \\ \hline 07:07:12 \end{array}$$

i.e. 7 m 12 s past 7 am.

Practice Exercise

1. LCM of 15 and 20 is
(a) 50 (b) 60 (c) 70 (d) 80

2. LCM of 12, 15, 20, 27 is
(a) 540 (b) 520 (c) 570 (d) 510

3. LCM of $(a^3 + b^3)$ and $(a^4 - b^4)$ is
(a) $(a^3 + b^3)(a + b)(a^2 + b^2)$
(b) $(a + b)(a^2 + ab + b^2)(a^3 + b^3)$
(c) $(a^3 + b^3)(a^2 + b^2)(a - b)$
(d) None of the above

4. LCM of $(x + 2)^2 (x - 2)$ and $x^2 - 4x - 12$ is
(a) $(x + 2)(x - 2)$ (b) $(x + 2)(x - 2)(x - 6)$
(c) $(x + 2)(x - 2)^2$ (d) $(x + 2)^2(x - 2)(x - 6)$

5. HCF of 6, 8, 12 is
(a) 1 (b) 2 (c) 3 (d) 4

6. HCF of 132, 204 and 228 is
(a) 12 (b) 18 (c) 6 (d) 21

7. The HCF of $x^2 - 9$ and $x^2 - 5x + 6$ is
(a) $(x + 3)(x + 2)$ (b) $(x - 3)(x + 2)$
(c) $(x + 3)(x - 2)$ (d) $(x - 2)(x - 3)$

8. The HCF of $2(x^2 - y^2)$ and $5(x^3 - y^3)$ is
(a) $(x - y)$ (b) $2(x^2 - y^2)$
(c) $(x + y)$ (d) None of these

9. If the HCF of $(p^2 - p - 6)$ and $(p^2 + 3p - 18)$ is $(p - a)$. The value of a is
(a) 2 (b) 3
(c) 4 (d) 5

10. The LCM of $\left(\frac{5}{2}, \frac{8}{9}, \frac{11}{14}\right)$ is
(a) 280 (b) 360 (c) 420 (d) 440

11. LCM of $\frac{1}{3}, \frac{2}{9}, \frac{5}{6}$ and $\frac{4}{27}$ is
(a) $\frac{1}{54}$ (b) $\frac{10}{27}$ (c) $\frac{20}{3}$ (d) $\frac{3}{20}$

12. The HCF of $\left(\frac{35}{12}, \frac{49}{30}, \frac{21}{20}\right)$ is
(a) $\frac{7}{60}$ (b) $\frac{7}{12}$ (c) $\frac{105}{60}$ (d) $\frac{105}{12}$

13. The HCF of $\frac{1}{2}, \frac{3}{4}$ and $\frac{4}{5}$ is
(a) $\frac{1}{20}$ (b) $\frac{1}{40}$
(c) 20 (d) 40

14. The LCM of 2.5, 1.2, 20 and 7.5 is
(a) 60 (b) 65
(c) 70 (d) 50

15. The product of HCF and LCM of 14 and 16 is
(a) 2 (b) 12
(c) 224 (d) 112

16. The LCM and HCF of two numbers are 4125 and 25, respectively. One number is 375. Find by how much is the second number less than the first?
(a) 50 (b) 25
(c) 75 (d) 100

17. The HCF and LCM of two numbers are 12 and 72, respectively. If the sum of two numbers is 60, then one of the two numbers will be
(a) 12 (b) 24
(c) 60 (d) 70

18. The LCM of any two numbers is twelve times of their HCF. The sum of their HCF and LCM is 403. If one number is 93, find the other number.
(a) 31 (b) 93 (c) 124 (d) 105

19. The HCF of two numbers is 8. Which one of the following can never be their LCM?
(a) 24 (b) 48
(c) 56 (d) 60

20. The product of two non-zero expressions is $(x + y + z)\,p^3$. If their HCF is p^2, then their LCM is
(a) $(y + z)\,p$ (b) $(z + x)\,p$
(c) $(x + y + z)\,p$ (d) $(x + y)\,p$

21. The GCD of the polynomials is $(x + 3)$ and their LCM is $(x^3 - 7x + 6)$. If one of the polynomials is $(x^2 + 2x - 3)$, then the other is
(a) $(x^2 + x - 6)$ (b) $(x^2 - x + 6)$
(c) $(x^2 + x + 6)$ (d) None of these

22. The smallest 3-digit number, which is exactly divisible by 6, 8 and 12, is
(a) 120 (b) 105
(c) 130 (d) 110

23. The least number which when divided by 6, 15 and 18 leave remainder 5 in each case, is
(a) 95 (b) 90
(c) 85 (d) 97

24. The least number which when divided by 24, 32 and 36 leaves the remainders 19, 27 and 31 respectively, is
(a) 360 (b) 140
(c) 280 (d) 283

25. What will be the greatest number that divides 1023 and 750 leaving remainders 3 and 2 respectively?
(a) 60 (b) 68 (c) 65 (d) 62

26. Three brands A, B and C of biscuits are available in packets of 12, 15 and 21 biscuits, respectively. If a shopkeeper wants to buy an equal number of biscuits of each brand, what is the minimum number of packets of each brand, he should buy?
(a) 35, 28, 20 (b) 30, 28, 33
(c) 20, 28, 37 (d) 33, 35, 42

27. In a school, all the students can stand in a row, so that each row has 5, 9 or 10 students. The least number of students in the school is
(a) 90 (b) 60
(c) 80 (d) 70

28. Five bells begin to toll together at intervals of 9 s, 6 s, 4 s, 10 s and 58 s, respectively. How many times will they toll together in the span of 1 h?
(a) 10 (b) 9
(c) 15 (d) 12

29. Find the largest number which divides 1305, 4665 and 6905. Leaving same remainder in each case.
(a) 1210 (b) 1130 (c) 1120 (d) 1210

30. Three tankers contain 403 L, 434 L and 465 L of diesel, respectively. The maximum capacity of a container, that can measure the diesel of the three containers exact number of times, is
(a) 32 L (b) 31 L (c) 30 L (d) 33 L

Answers

1	(b)	2	(a)	3	(c)	4	(d)	5	(b)	6	(a)	7	(d)	8	(a)	9	(b)	10	(d)
11	(c)	12	(a)	13	(a)	14	(a)	15	(c)	16	(d)	17	(b)	18	(c)	19	(d)	20	(c)
21	(a)	22	(a)	23	(a)	24	(d)	25	(b)	26	(a)	27	(a)	28	(a)	29	(c)	30	(b)

Hints & Solutions

1. (b) We have, LCM of 15 and 20 is

5	15, 20
3	3, 4
4	1, 4
	1, 1

$\therefore$ LCM of 15 and 20 $= 5 \times 3 \times 4 = 60$

2. (a) We have, LCM of 12, 15, 20, 27 is

2	12, 15, 20, 27
2	6, 15, 10, 27
3	3, 15, 5, 27
5	1, 5, 5, 9
9	1, 1, 1, 9
	1, 1, 1, 1

LCM of 12, 15, 20, 27 $= 2 \times 2 \times 3 \times 5 \times 9 = 540$

3. (c) Now, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

and $a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$

$\because$ HCF $= (a + b)$.

$$\therefore \text{LCM} = \frac{(a^3 + b^3)(a - b)(a + b)(a^2 + b^2)}{(a + b)}$$

$$= (a^3 + b^3)(a^2 + b^2)(a - b)$$

4. (d) Now, $(x^2 - 4x - 12) = x^2 - 6x + 2x - 12$

$$= x(x - 6) + 2(x - 6)$$

$$= (x - 6)(x + 2)$$

and another expression is $(x + 2)^2 (x - 2)$.

$\therefore$ LCM of $(x + 2)^2(x - 2)$ and $x^2 - 4x - 12$

$= (x - 6)(x - 2)(x + 2)^2$

5. (b)

```
6) 12 (2
   12
   ×
```

```
6) 8 (1
   6
   2) 6 (3
      6
      ×
```

$\therefore$ HCF of 6, 8 and 12 = 2

6. (a) We have, numbers 132, 204 and 228.

HCF of 132 and 204,

```
132) 204 (1
     132
      72) 132 (1
           72
           60) 72 (1
               60
               12) 60 (5
                   60
                   ×
```

Again, HCF of 12 and 228,

```
12) 228 (19
    12
    108
    108
    ×
```

$\therefore$ HCF of 132, 204 and 208 = 12.

7. (d) $x^2 - 9 = (x + 3)(x - 3)$

and $x^2 - 5x + 6 = x^2 - 3x - 2x + 6$

$$= x(x - 3) - 2(x - 3) = (x - 3)(x - 2)$$

$\therefore$ Required HCF $= (x - 3)(x - 2)$

8. (a) Now, $2(x^2 - y^2) = 2(x - y)(x + y)$

and $5(x^3 - y^3) = 5(x - y)(x^2 + xy + y^2)$

Hence, HCF $= (x - y)$.

9. (b) $\because (p - a) = 0 \Rightarrow p = a$

On putting the value of p in $p^2 - p - 6$,

$$a^2 - a - 6 \quad \ldots(i)$$

Again putting the value of p in $p^2 + 3p - 18$,

$$a^2 + 3a - 18 \quad \ldots(ii)$$

Therefore, $a^2 - a - 6 = a^2 + 3a - 18$

$$\Rightarrow \quad -4a = -12 \Rightarrow a = \frac{12}{4} = 3$$

10. (d) $\text{LCM of} \left(\frac{5}{2}, \frac{8}{9}, \frac{11}{14}\right) = \frac{\text{LCM of } (5, 8, 11)}{\text{HCF of } (2, 9, 14)}$

$$= \frac{440}{1} = 440$$

11. (c) $\text{LCM of} \left(\frac{1}{3}, \frac{2}{9}, -\frac{5}{6}, \frac{4}{27}\right)$

$$= \frac{\text{LCM of } (1, 2, 5, 4)}{\text{HCF of } (3, 9, 6, 27)} = \frac{20}{3}$$

12. (a) $\text{HCF of} \left(\frac{35}{12}, \frac{49}{30}, \frac{21}{20}\right) = \frac{\text{HCF of } (35, 49, 21)}{\text{LCM of } (12, 30, 20)}$

$$= \frac{7}{60}$$

13. (a) HCF of $\left(\frac{1}{2}, \frac{3}{4}, \frac{4}{5}\right) = \frac{\text{HCF of } (1, 3, 4)}{\text{LCM of } (2, 4, 5)} = \frac{1}{20}$

14. (a) Required LCM

$$= (\text{LCM of 25, 12, 200 and 75}) \times 0.1$$

$\Rightarrow$ LCM of 25, 12, 200, 75,

2	25, 12, 200, 75
2	25, 6, 100, 75
3	25, 3, 50, 75
5	25, 1, 50, 25
5	5, 1, 10, 5
	1, 1, 2, 1

LCM of 25, 12, 200, 75 $= 2 \times 2 \times 3 \times 5 \times 5 \times 2 = 600$

$\therefore$ Required LCM $= 600 \times 0.1 = 60$

15. (c) Required product $= 14 \times 16 = 224$

16. (d) Since, LCM $\times$ HCF = Product of two numbers

$\therefore \quad 4125 \times 25 = 375 \times x$

$\Rightarrow \quad x = \frac{4125 \times 25}{375} = 275$

Hence, the required difference

$$= 375 - 275 = 100$$

17. (b) Let one of the number be x. Then, the other number will be $60 - x$.

Given, HCF = 12, LCM = 72 and sum of two numbers = 60.

$\therefore \quad x(60 - x) = 12 \times 72$

$\Rightarrow \quad 60x - x^2 = 864$

$\Rightarrow \quad x^2 - 60x + 864 = 0$

$\Rightarrow \quad (x - 36)(x - 24) = 0 \quad \Rightarrow \quad x = 36, 24$

18. (c) Let the HCF = x and LCM = 12x

$\Rightarrow \quad x + 12x = 403 \Rightarrow x = \frac{403}{13} = 31 = \text{HCF}$

and LCM $= 12 \times 31 = 372$

$\therefore$ Required number $= \frac{31 \times 372}{93} = 124$

19. (d) Since, HCF of two numbers is 8, then their LCM must be a multiple of 8.

Hence, option (d) is correct.

20. (c) LCM $= \frac{\text{Product of the expressions}}{\text{HCF}}$

$$= \frac{(x + y + z)\, p^3}{p^2} = (x + y + z)\, p$$

21. (a) Other polynomial $= \frac{\text{LCM} \times \text{HCF}}{\text{Given polynomial}}$

$$= \frac{(x + 3) \times (x^3 - 7x + 6)}{(x^2 + 2x - 3)} = \frac{(x + 3)(x^3 - 7x + 6)}{(x + 3)(x - 1)}$$

$$= \frac{(x - 1)(x^2 + x - 6)}{(x - 1)} = x^2 + x - 6$$

22. (a) Firstly, we have to find out the LCM of 6, 8 and 12.

3	465
5	155
31	31
	1

2	6, 8, 12
2	3, 4, 6
2	3, 2, 3
3	3, 1, 3
	1, 1, 1

$\therefore$ LCM $= 2 \times 2 \times 2 \times 3 = 24$

We know that, smallest three-digit number is 100.

On dividing 100 by 24, we get remainder 4.

Now, required 3-digit number which is exactly divisible by 24 = (Smallest 3-digit number + Divisor − Remainder)

$$= 100 + 24 - 4 = 120$$

23. (a) First, we have to find out the LCM of 6, 15, 18.

2	6, 15, 18
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

$\therefore$ LCM $= 2 \times 3 \times 3 \times 5 = 90$

Now, required number = LCM + Remainder

$$= 90 + 5 = 95$$

Hence, the required number is 95.

24. (d) Given that $x = 24$, $y = 32$, $z = 36$, $a = 19$, $b = 27$ and $c = 31$

Then, $24 - 19 = 5$

$32 - 27 = 5$

$36 - 31 = 5$

$\therefore \quad k = 5$

According to the formula,

Required number = (LCM of 24, 32 and 36) − 5

$\because$ LCM of 24, 32, 36 = 288

$\therefore$ Required number $= 288 - 5 = 283$

25. (b) Required number = HCF of $(1023-3, 750-2)$

$= \text{HCF of } (1020, 748)$

```
748) 1020 (1
     748
     272) 748 (2
          544
          204) 272 (1
               204
                68) 204 (3
                    204
                     ×
```

$\therefore$ HCf = 68

26. (a) In brand A, number of biscuits = 12

In brand B, number of biscuits = 15

In brand C, number of biscuits = 21

First of all, we find the LCM of 12, 15 and 21

3	12, 15, 21
	4, 5, 7

LCM of 12, 15 and 21 $= 3 \times 4 \times 5 \times 7$

$= 420$

Now, number of packets of brand A $= \frac{420}{12} = 35$

Number of packets of brand B $= \frac{420}{15} = 28$

Number of packets of brand C $= \frac{420}{21} = 20$

27. (a) We have to find the LCM of 5, 9, 10,

3	5, 9, 10
3	5, 3, 10
5	5, 1, 10
2	1, 1, 2
	1, 1, 1

LCM $= 3 \times 3 \times 5 \times 2 = 90$

$\therefore$ Total students in the school = 90

28. (a) We have to find the LCM of 9, 6, 4, 10 and 8,

2	9, 6, 4, 10, 8
2	9, 3, 2, 5, 4
3	9, 3, 1, 5, 2
2	3, 1, 1, 5, 2
3	3, 1, 1, 5, 1
5	1, 1, 1, 5, 1
	1, 1,1, 1, 1

LCM of 9, 6, 4, 10, 8 $= 2 \times 2 \times 3 \times 3 \times 5 \times 2 = 360$

In 1 h, the ring will toll together

$\frac{3600}{360}$ times = 10 times

29. (c) Given that, $x = 1305, y = 4665, z = 6405$

Then, $|x-y| = |1305-4665| = 3360$

$|y-z| = |4665-6905| = 2240$

$|z-x| = |6905-1305| = 5600$

$\therefore$ Required number

= HCF of 3360, 2240 and 5600 = 1120

30. (b) Here, maximum quantity of container will be equal to the HCF of 403, 434 and 465.

13	403
31	31
	1

2	434
7	217
31	31
	1

Prime factorisation of 403 $= 13 \times 31$

Prime factorisation of 434 $= 2 \times 7 \times 31$

Prime factorisation of 465 $= 3 \times 5 \times 31$

Common factor of 403, 434 and 465 is 31.

$\therefore$ Maximum capacity of container

= HCF of 403, 434 and 465 = 31 L