2 DETERMINACY AND INDETERMINACY

The structure that can be analysed by the equations of static equilibrium alone.

2.1. Equation of Static Equilibrium

2D Planar structure

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

3D Space Structure

$$\Sigma F_x = 0 \qquad \Sigma m_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma m_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma m_z = 0$$

Statically indeterminate structure are the one which cannot be found by equations of static equilibrium.

2.1.1. Degree of Static Indeterminacy

Can be termed as equations in addition to static equilibrium equation required to completely analyse the structure.

D_S = Number of unknown forces in member or at support – equation of static equilibrium available.

$$D_S = D_{Si} + D_{Se}$$

Total Internal External

2.1.2. Support Reaction

Plane

– Fix $R_x,\ R_y,\ M_z$ Reactions

$\delta_x,\ \delta_y,\ \theta_z$ Restrains

– Pin $R_x,\ R_y,$ Reactions

$\delta_x,\ \delta_y,$ Restrains

– Roller R_y Reactions

δ_y Restrains

Space Structure

- Fix R_x, R_y, R_z, M_x, M_y, M_z Reactions
- Pin R_x, R_y, R_z Reactions
- Roller R_y Reactions

2.1.3. External Indeterminacy

$$D_{se} = R - 3 \quad \text{Plane}$$
$$= R - 6 \quad \text{Space}$$

2.1.4. Internal Indeterminacy

$$D_{si} = m - (2j - 3) \quad \text{Truss plane}$$
$$= m - (3j - 6) \quad \text{Truss space}$$
$$= 3C \quad \text{Rigid plane frame}$$
$$= 6C \quad \text{Rigid space frame}$$

j = No. of Joints

C = No. of cuts required for obtaining open configuration.

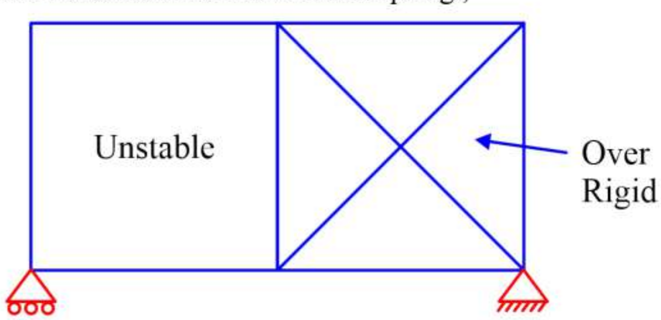
m = No. of members

Simplified Formula of D_S

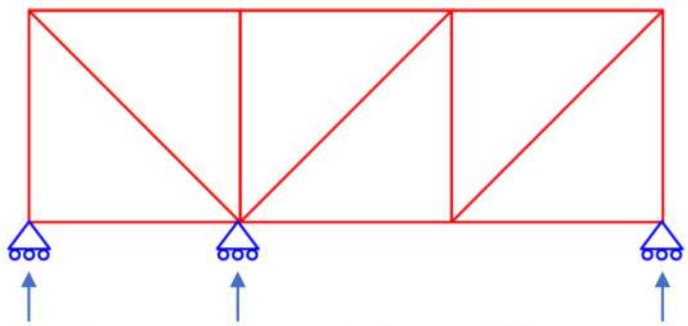
- Plane truss $D_S = (m + R) - 2j$
- Space truss $D_S = (m + R) - 3j$
- Rigid plane frame $D_S = (3m + R) - 3j$
- Rigid space frame $D_S = (6m + R) - 6j$

2.2. Truss (Static Indeterminacy)

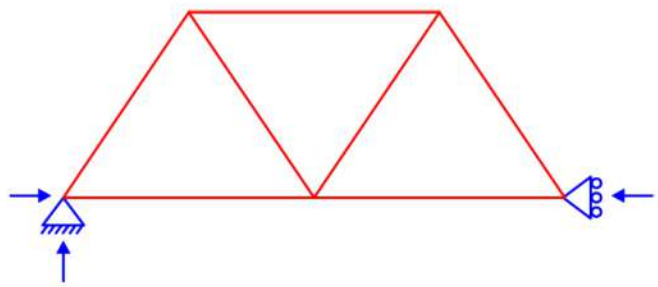
- Every joint is a Hinge Joint.
- Each joint has 2 nos. of equation (planar)
- Internal indeterminacy should be checked for individual loop. e.g.,



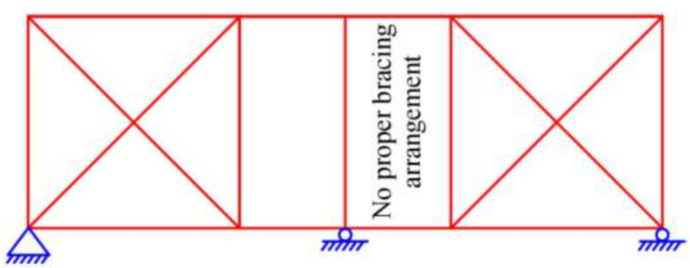
- All support reactions should not be parallel as may lead to unstability e.g.,



- All support reactions should not be concurrent as may lead to unstability. e.g.,



- If a truss is unstable we never discuss SD, or SID.
- If in any truss are appreciable deformation which can be due to no proper bracing it makes the structure unstable. e.g.,



Simple Truss

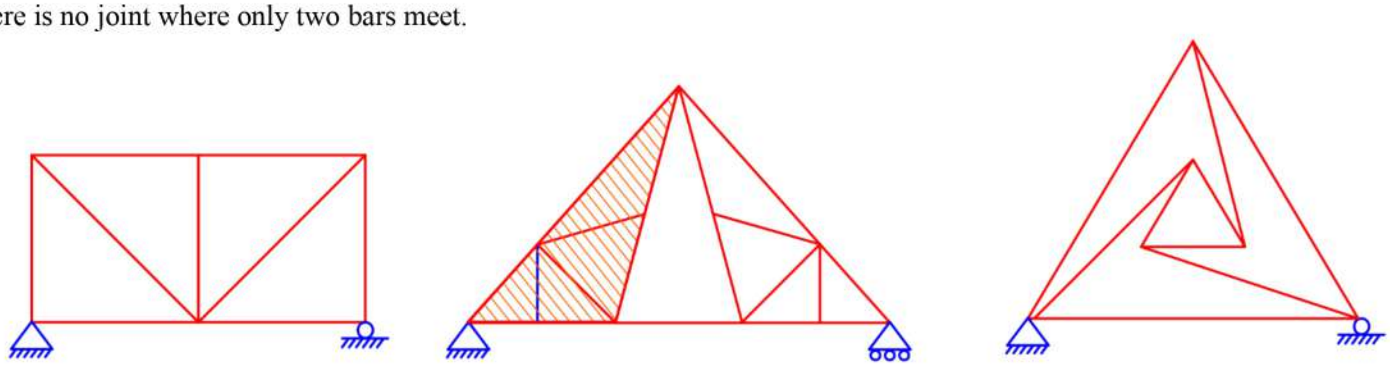
In a triangle when two bar and one joint are progressively added to form a truss.

Compound Truss

Two simple truss connected by a set of joints and bars.

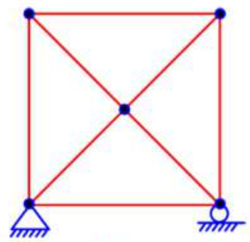
Complex Truss

There is no joint where only two bars meet.



Simple Truss **Compound Truss** **Complex Truss**

- Truss having members which crossover each other or member that serves as side for more than 2 triangle are likely to be indeterminate.



Difference between SD & SI

	Statically Determinate	Statically Indeterminate
(1)	Equilibrium equation sufficient to analyse	Insufficient
(2)	BM independent of material	Dependent
(3)	BM is independent of sections Area	Dependent
(4)	Stresses are not caused due to temperature change & lack of fit.	Stresses caused

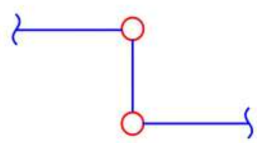
2.3. Frames/Beams

2.3.1. Internal Pin/Hinge

Pin cannot transmit moment from one part to other. Thus, provides extra condition $\Sigma M = 0$.

Internal Link

Bar with pin @ each end



incapable of transmitting moment as well as horizontal force.

Two additional conditions are, $\Sigma M = 0$, $\Sigma H = 0$

Loading Type

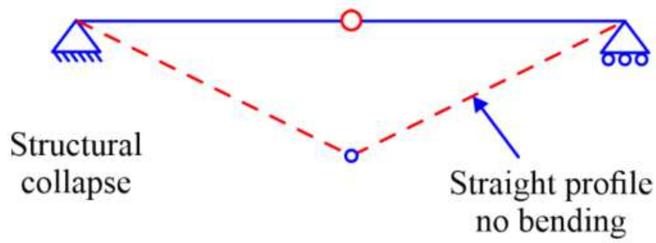
- General Loading has both vertical and horizontal component.
- Vertical loading condition is important for beams but not frames.

Open tree like Structure Concept used for Finding Indeterminacy

- trees have only one root
- trees cannot have closed looped branch.

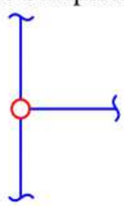
Unstable or Deficient Structure

If there are not sufficient number of Restraint the structure under go Rigid Body movement upon application of a small displacement. e.g.,



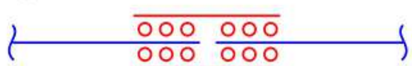
2.4. Restraining Members/Joint

The concept relates to making structure completely rigid and then analyzing it for indeterminacy.

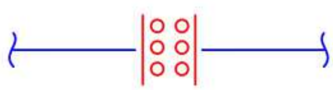
- Plane frame = $(m' - 1)$  m' = no. of members joining the Hinged Joint.
- Space frame = $3(m' - 1)$

  Concept = Rotation of Members
- R_H & M are the restrains required to make rigid.

- R_H are the restrains required to make rigid.



- R_v is restraint required to make rigid.



2.4.1. Rigid Frames

In a plane frame, every member carries 3 forces. (BM, SF, Axial)

$$\text{Total no. of unknown} = \underset{\text{member}}{3m} + \underset{\text{Support reaction}}{R}$$

At each joint equilibrium equation = $3j$

$$\Sigma f_x = 0$$

$$\Sigma f_y = 0$$

$$\Sigma M = 0$$

$$D_S = 3m + R - 3j$$

$$D_S = 3m + R - 3j - \Sigma(m' - 1)$$

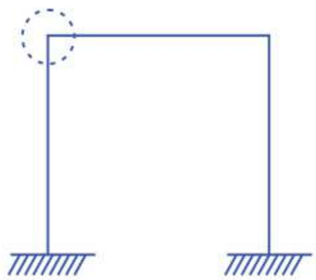
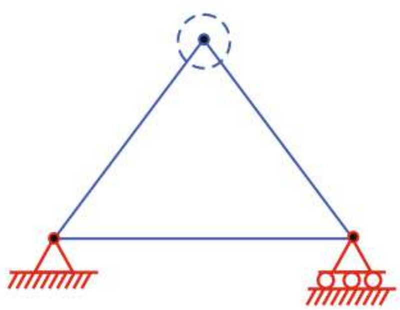
where m' is the number of members @ hinge it.

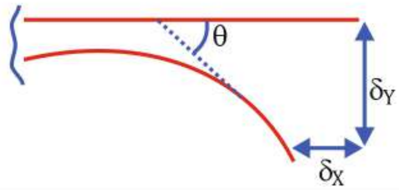
$$D_S = 6m + R - 6j - \Sigma 3(m' - 1)$$

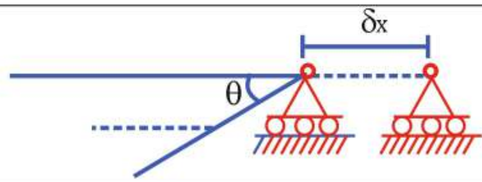
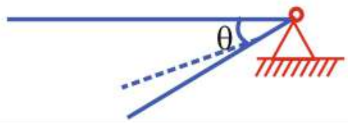
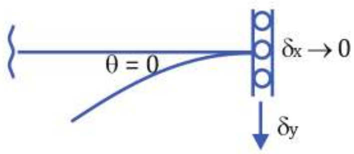
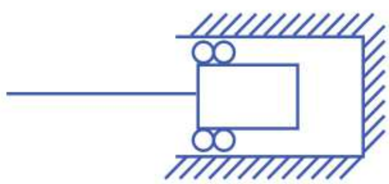
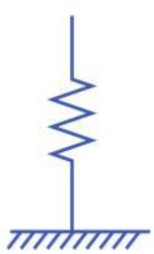
Denote by D_K.

- Also called as degrees of freedom (DOF)
- Kinematic indeterminacy:

The no of unknown joint displacements is called degrees of freedom

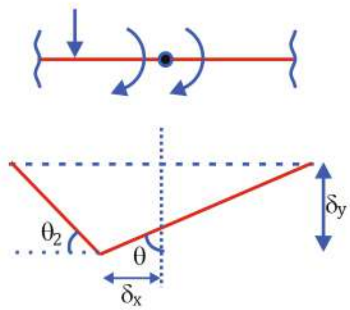
	Types of joints	Degrees of freedom
1	Rigid joint of a plane frame  **Note:** At a rigid joint, the included angle remains the same before and after displacement	3 $(\delta_x, \delta_y, \theta)$.
2	Rigid joint of a space frame	6 (3 rotations θ_{xy}, θ_{yz}, θ_{xz} and 3 translations $\delta_x, \delta_y \& \delta_z$)
3	Pin joint of a plane frame  **NOTE:** As moments are not present, the design rotations are not considered in trusses.	
4	Pin joint of a space frame	3 $(\delta_x, \delta_y, \delta_z)$

	Types of support	Degrees of Freedom
1	Free end. 	3 $\left(\delta_x, \delta_y, \theta\right)$
2	Roller support.	2 $\left(\theta, \delta_x\right)$.

		
3	Hinged/Pinned support 	1 (θ)
4	Joined support 	0
5	Vertical shear hinge 	1 (δ_y)
6	Horizontal shear hinge 	1 (δ_x)
7	Damper support 	1 (δ_x).
8	Spring support  **Note:** Reactions will resist displacements. Vertical, $dv = 0$ Horizontal reaction, $\delta h = 0$ Moment reaction, $\theta = 0$	2 (θ,δ_x).

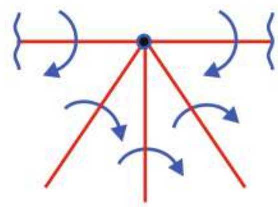
2.3. Effect of force release on D.O.F.

1. **Internal moment hinge.**



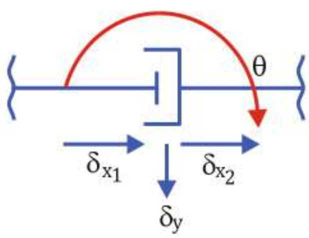
Note: Each member connected to a hinge can have its own notation, in addition to δx and δy.

Example:

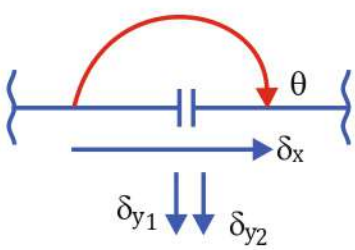


5 notations and 2 translations.

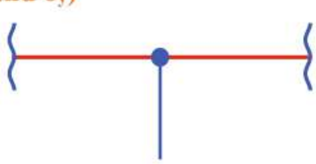
2. **Horizontal shear releases.**



3. **Vertical, shear release**



4. **(2 rotations - θ_1 and θ_2 2 translations - δ_x and δ_y)**



2θ, δx, δy

4. D.O.F

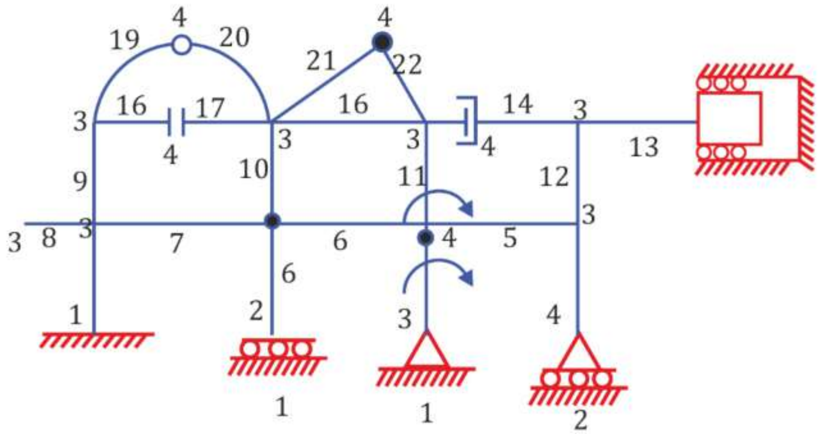
(2 horizontal trans- δ_{x1} and δ_{y1} vertical trans - δ_y and θ.)

4. D.O.F

(δ_{y1}, δ_{y2}, δx & θ).

D_k of rigid jointed plane frame:

D_k of rigid jointed plane Frame:



D_k = 52 (Considering axial deformations)

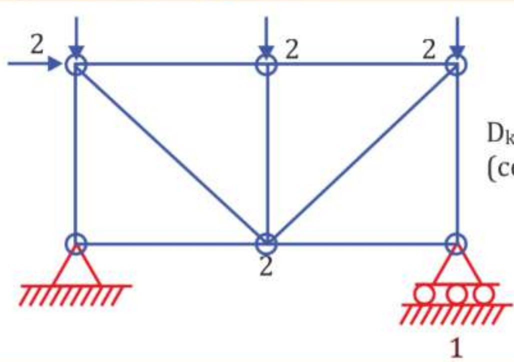
For a rigid joint with infinite member, there is only single rotation for a hinged joint, there will be infinite rotations:

> **Note:** Practically the axial deformations of members or rigid jointed structures are negligire.
> Assume axial deformation of all members are neglected then Dk = 52 – total no. of members.
> = 52 – 22 = 30 (neglection axial deformations)

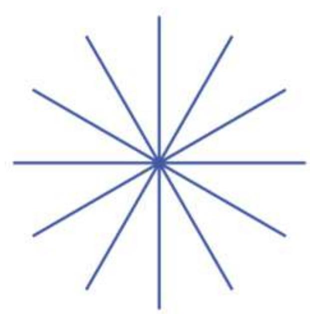
Axial deformation neglected or
members are rigid or
members are stiff or
members incotenoible

} neglect axial deformation

D_K of Pin jointed Plane Frames:



D_k = 9
(count the no: of members)



> **Note:** Rotations are not considered in trusses the only possible D.O.F in trusses are axial deformations. Hence the equation of neglecting AD do not arise in pin-jointed trusses

Formula for D_K:

$D_K = NJ - C$ where

N = 3 ; rigid jointed plane frame

N = 6: rigid jointed space frame

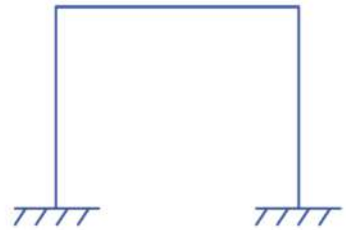
N = D.O.F at a joint

J = no of joints

C = compatibility equations

N = 2; pin jointed plane frame

N = 3 ; pin jointed space frame



J = 4 (supports are also considered as jointed)

O = Reactions, if actual deformations consider

= m + r; if axial deformations are neglect

Where $m \rightarrow$ no of members.

$D_K = 3 \times 4 - 6 = 6$

$D_K = 6 \times 4$ (joints)

$= 24$ (considering AD)

$D_K = 24 - 8$ (member)

$= 16$ (neglecting AD).