Level - II

Chapter 2

Units and Measurements

Solutions

SECTION - A

Objective Type Questions

(System of Units)

1. Which of the following practical units of length is not correct?

(1) 1 fermi = 10^{-15} m
(2) 1 astronomical unit = 1.496×10^{11} m
(3) 1 parsec = 3.26 light year
(4) 1 light year = 9.46×10^{12} m

Sol. Answer (4)

1 light year = 9.46×10^{15} m

2. The unit of length, velocity and force are doubled. Which of the following is the correct change in the other units?

(1) Unit of time is doubled
(2) Unit of mass is doubled
(3) Unit of momentum is doubled
(4) Unit of energy is doubled

Sol. Answer (3)

As $t = \frac{1}{v}$, so on doubling units of length and velocity, unit of time remains same.

$p = F \times t$

$p' = 2F \times t$

$\boxed{p' = 2p}$

3. The unit of "impulse per unit area" is same as that of

(1) Viscosity
(2) Surface tension
(3) Bulk modulus
(4) Force

Sol. Answer (1)

$$\frac{\text{Impulse}}{\text{Area}} = \frac{MLT^{-1}}{L^2} \Rightarrow [ML^{-1}T^{-1}]$$

Coefficient of viscosity $\Rightarrow \eta = [ML^{-1}T^{-1}]$

So, $\frac{\text{Impulse}}{\text{Area}}$ = coefficient of viscosity

4. In a practical unit if the unit of mass becomes double and that of unit of time becomes half, then 8 joule will be equal to .... unit of work.

(1) 6
(2) 4
(3) 1
(4) 10

Sol. Answer (3)

Work $\rightarrow$ $[ML^2T^{-2}]$

$n_1u_1 = n_2u_2$

$$\frac{(8)M_1L_1^2T_1^{-2}}{M_2L_2^2T_2^{-2}} = n_2$$

$$\Rightarrow \quad 8\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2} = n_2$$

$$\Rightarrow \quad 8\left[\frac{M_1}{2M_1}\right]\left[\frac{L_1}{L_1}\right]^2\left[\frac{2T_1}{T_1}\right]^{-2} = n_2$$

$$\Rightarrow \quad 8\times\frac{1}{2}\times\frac{1}{4} = n_2$$

$$\Rightarrow \quad \boxed{n_2 = 1}$$

So, unit of 8 joule = $\boxed{1\times\text{new units}}$

5. The equation of a stationary wave is $y = 2A\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$. Which of the following statements is incorrect?

(1) The unit of ct is same as that of λ

(2) The unit of x is same as that of λ

(3) The unit of $\frac{2\pi c}{\lambda}$ is same as that of $\frac{2\pi x}{\lambda t}$

(4) The unit of $\frac{c}{\lambda}$ is same as that of $\frac{x}{\lambda}$

Sol. Answer (4)

$$y = 2A\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

$$\frac{ct}{\lambda} = \text{dimensionless} \Rightarrow \boxed{ct = \lambda}$$

$$\frac{x}{\lambda} = \text{dimensionless} \Rightarrow \boxed{x = \lambda}$$

(Errors)

6. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be

(1) 2%

(2) 4%

(3) 6%

(4) 8%

Sol. Answer (3)

Volume of sphere = $\frac{4}{3}\pi R^3$

$$\Rightarrow \quad \frac{\Delta V}{V}\times 100\% = 3\times\frac{\Delta R}{R}\times 100\% = 3 \times 2\%$$

$$\Rightarrow \quad \boxed{\frac{\Delta V}{V}\times 100\% = 6\%}$$

7. A set of defective observation of weights is used by a student to find the mass of an object using a physical balance. A large number of readings will reduce

(1) Random error

(2) Systematic error

(3) Random as well as systematic error

(4) Neither random nor systematic error

Sol. Answer (1)

Random errors can be reduced by taking a large number of observations.

8. A force F is applied on a square area of side L. If the percentage error in the measurement of L is 2% and that in F is 4%, what is the maximum percentage error in pressure?

(1) 2% (2) 4% (3) 6% (4) 8%

Sol. Answer (4)

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\frac{\Delta P}{P}\times 100\% = \frac{\Delta F}{F}\times 100\% + \frac{2\Delta L}{L}\times 100\% = 4\% + 2 \times 2\%$$

$$\boxed{\frac{\Delta P}{P}\times 100\% = 8\%}$$

9. The radius of a sphere is (5.3 ± 0.1) cm. The percentage error in its volume is

(1) $\frac{0.1}{5.3}\times 100$ (2) $3\times\frac{0.1}{5.3}\times 100$ (3) $\frac{3}{2}\times\frac{0.1}{5.3}\times 100$ (4) $6\times\frac{0.1}{0.3}\times 100$

Sol. Answer (2)

$$r = (5.3 \pm 0.1)\text{ cm}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\Delta V}{V}\times 100\% = \frac{3\Delta r}{r}\times 100\%$$

$$\boxed{\frac{\Delta V}{V}\times 100\% = \frac{3\times 0.1}{5.3}\times 100}$$

10. If the percentage error in the measurement of momentum and mass of an object are 2% and 3% respectively, then maximum percentage error in the calculated value of its kinetic energy is

(1) 2% (2) 1% (3) 5% (4) 7%

Sol. Answer (4)

$$\text{KE} = \frac{\text{Momentum}}{\text{Mass}} = \frac{p^2}{2m}$$

$$\frac{\Delta K}{K}\times 100\% = \left(\frac{2\Delta p}{p}\times 100\right)\% + \left(\frac{\Delta m}{m}\times 100\right)\%$$

$$= 2 \times 2\% + 3\%$$

$$\boxed{\frac{\Delta K}{K}\times 100\% \Rightarrow 7\%}$$

11. The acceleration due to gravity is measured on the surface of earth by using a simple pendulum. If α and β are relative errors in the measurement of length and time period respectively, then percentage error in the measurement of acceleration due to gravity is

(1) $\left(\alpha+\frac{1}{2}\beta\right)\times 100$ (2) $(\alpha - 2\beta)$ (3) $(2\alpha + \beta) \times 100$ (4) $(\alpha + 2\beta) \times 100$

Sol. Answer (4)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow \quad T^2 = 4\pi^2 \frac{L}{g}$$

$$\frac{\Delta g}{g}\times 100\% = \frac{\Delta L}{L}\times 100\% + \frac{2\Delta T}{T}\times 100\%$$

$$\boxed{\frac{\Delta g}{g}\times 100\% = (\alpha + 2\beta)\times 100}$$

12. A public park, in the form of a square, has an area of $(100.0 \pm 0.2)\ m^2$. The side of park is

(1) (10.00 ± 0.01) m (2) (10.0 ± 0.1) m (3) (10.00 ± 0.02) m (4) (10.0 ± 0.2) m

Sol. Answer (1)

$$A = (100 \pm 0.2)\ m^2$$

$$100 = l^2 \Rightarrow \boxed{l = 10\ m}$$

$$\frac{\Delta A}{A} = \frac{2\Delta l}{l}$$

$$\frac{0.2}{100} = 2\times\frac{\Delta l}{10}$$

$$\Rightarrow \quad \boxed{\Delta l = 0.01\ m}$$

So, length = (10 ± 0.01) m

13. A physical quantity is represented by $X = [M^aL^bT^{-c}]$. If percentage error in the measurement of M, L and T are α%, β% and γ% respectively, then maximum percentage error in measurement of X should be (Given that α, β and γ are very small)

(1) $(\alpha a - \beta b + \gamma c)$% (2) $(\alpha a + \beta b + \gamma c)$% (3) $(\alpha a - \beta b - \gamma c)$% (4) $(\alpha a + \beta b - \gamma c)$%

Sol. Answer (2)

$$X = [M^aL^bT^{-c}]$$

$$\frac{\Delta X}{X}\times 100\% = \frac{a\Delta M}{M}\times 100\% + \frac{b\Delta L}{L}\times 100\% + \frac{c\Delta T}{T}\times 100\%$$

$$\Rightarrow \quad \boxed{\frac{\Delta X}{X}\times 100\% = (a\alpha + b\beta + c\gamma)\%}$$

14. The least count of a stop watch is $\frac{1}{5}$ second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The maximum percentage error in the measurement of time will be

(1) 0.1% (2) 0.8% (3) 1.8% (4) 8%

Sol. Answer (2)

Least count = $\Delta T = \frac{1}{5}$ s = 0.2 s

T = 25 s

Percentage error = $\frac{\Delta T}{T} \times 100\% = \frac{0.2}{25} \times 100\% = 0.8\%$

15. A student measures the distance traversed in free fall of a body, initially at rest in a given time. He uses this data to estimate g, the acceleration due to gravity. If the maximum percentage errors in measurement of the distance and the time are e_1 and e_2 respectively, the maximum percentage error in the estimation of g is

(1) $e_2 - e_1$ (2) $e_1 + 2e_2$ (3) $e_1 + e_2$ (4) $e_1 - 2e_2$

Sol. Answer (2)

$g = LT^{-2}$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} = \frac{2\Delta T}{T} \Rightarrow \boxed{\frac{\Delta g}{g} = e_1 + 2e_2}$$

(Dimensions of Physical Quantities, Formulae and Equations)

16. A dimensionally consistent relation for the volume V of a liquid of coefficient of viscosity 'η' flowing per second, through a tube of radius r and length l and having a pressure difference P across its ends, is

(1) $V = \frac{\pi P r^4}{8\eta l}$ (2) $V = \frac{\pi\eta}{8Pr^4}$ (3) $V = \frac{8P\eta}{\pi r^4}$ (4) $V = \frac{\pi P\eta}{8r^4}$

Sol. Answer (1)

On checking the dimensionality the correct relation is

$$V = \frac{\pi P r^4}{8\eta l}$$

17. E, m, J and G denote energy, mass, angular momentum and gravitational constant respectively. The dimensions of $\frac{EJ^2}{m^5G^2}$ are same as of

(1) Angle (2) Length (3) Mass (4) Time

Sol. Answer (1)

$$\frac{EJ^2}{m^5G^2} \Rightarrow \frac{ML^2T^{-2} \cdot (ML^2T^{-1})^2}{M^5 \cdot (M^{-1}L^3T^{-2})^2} = \frac{ML^2T^{-2}M^2L^4T^{-2}}{M^5M^{-2}L^6T^{-4}}$$

$\Rightarrow$ $[M^0L^0T^0]$ = Angle (Dimensionless)

18. If y represents pressure and x represents velocity gradient, then the dimensions of $\frac{d^2y}{dx^2}$ are

(1) $[ML^{-1}T^{-2}]$ (2) $[M^2L^{-2}T^{-2}]$ (3) $[ML^{-1}T^0]$ (4) $[M^2L^{-2}T^{-4}]$

Sol. Answer (3)

$\frac{d^2y}{dx^2}$ will have dimensions of $\frac{y}{x^2}$

$y \to$ pressure, $x \to$ velocity gradient

$$x \to \frac{V}{L} \Rightarrow \frac{LT^{-1}}{L} \Rightarrow T^{-1}$$

$$\frac{y}{x^2} = \frac{ML^{-1}T^{-2}}{T^{-2}} \Rightarrow [ML^{-1}]$$

(Application of Dimensions)

19. The dimensions of $\frac{\alpha}{\beta}$ in the equation $F = \frac{\alpha - t^2}{\beta v^2}$, where F is the force, v is velocity and t is time, is

(1) $[MLT^{-1}]$ (2) $[ML^{-1}T^{-2}]$ (3) $[ML^3T^{-4}]$ (4) $[ML^2T^{-4}]$

Sol. Answer (3)

$$F = \frac{\alpha - t^2}{\beta v^2}$$

Dimensionally, $\alpha = [T^2]$

$$[MLT^{-2}] = \frac{[T^2]}{\beta[L^2T^{-2}]}$$

$$\beta = \frac{T^2}{[MLT^{-2} \cdot L^2T^{-2}]}$$

$\Rightarrow \beta = [M^{-1}L^{-3}T^6]$

Dimensions of $\frac{\alpha}{\beta} = \frac{T^2}{M^{-1}L^{-3}T^6} = [ML^3T^{-4}]$

20. Even if a physical quantity depends upon three quantities, out of which two are dimensionally same, then the formula cannot be derived by the method of dimensions. This statement

(1) May be true (2) May be false (3) Must be true (4) Must be false

Sol. Answer (3)

This statement is completely correct. If a quantity depends upon two other quantities which are dimensionally same then formula's validity can be checked but it can't be derived by the method of dimensions.

21. In a new system of units energy (E), density (d) and power (P) are taken as fundamental units, then the dimensional formula of universal gravitational constant G will be

(1) $[E^{-1}d^{-2}P^2]$ (2) $[E^{-2}d^{-1}P^2]$ (3) $[E^2d^{-1}P^{-1}]$ (4) $[E^1d^{-2}P^{-2}]$

Sol. Answer (2)

$G = [E^a\, d^b\, P^c]$

$E = [ML^2T^{-2}]$

$d = [ML^{-3}]$

$P = [ML^2T^{-3}]$

$G = [M^{-1}L^3T^{-2}]$

$[M^{-1}L^3T^{-2}] = [ML^2T^{-2}]^a\ [ML^{-3}]^b\ [ML^2T^{-3}]^c$

$a + b + c = -1$

$2a - 3b + 2c = 3$

$-2a - 3c = -2 \Rightarrow 2a + 3c = 2$

On solving,

$a = -2$

$b = -1$

$c = 2$

So, $\boxed{G = [E^{-2}d^{-1}P^2]}$

22. In equation $y = x^2 \cos^2 2\pi \frac{\beta\gamma}{\alpha}$, the units of x, α, β are m, s^{-1} and $(ms^{-1})^{-1}$ respectively. The units of y and γ are

(1) m^2, ms^{-2} (2) m, ms^{-1} (3) m^2, m (4) m, ms^{-2}

Sol. Answer (1)

$y = x^2 \cos^2 2\pi\left(\frac{\beta\gamma}{\alpha}\right)$

The argument of a trigonometric ratio is always dimensionless.

$\frac{\beta\gamma}{\alpha} = [M^0L^0T^0]$ or $\beta\gamma = \alpha \Rightarrow \gamma = \frac{T^{-1}}{L^{-1}T} \Rightarrow [LT^{-2}]$

and $y = x^2 \Rightarrow [L^2]$

$\alpha = s^{-1} \Rightarrow [T^{-1}]$, $\beta = [LT^{-1}]^{-1} \Rightarrow [L^{-1}T]$

$\boxed{y = m^2}$ $\boxed{\gamma = ms^{-2}}$

23. Let P represent radiation pressure, c represent speed of light and I represent radiation energy striking a unit area per second, then $P^xI^yc^z$ will be dimensionless for

(1) $x = 0$, $y = z$ (2) $x = y = z$ (3) $x = z = -y$ (4) $x = y = -z$

Sol. Answer (3)

$P^xI^yc^z$

$P \rightarrow$ Pressure $\rightarrow [ML^{-1}T^{-2}]$

$I \rightarrow$ Intensity $\rightarrow \frac{E}{AT} \Rightarrow \frac{ML^2T^{-2}}{L^2T} \Rightarrow [MT^{-3}]$

$c \rightarrow$ Speed of light $= [LT^{-1}]$

$[M^0L^0T^0] = [ML^{-1}T^{-2}]^x\ [MT^{-3}]^y\ [LT^{-1}]^z$

$\boxed{x = -y} \Rightarrow x + y = 0,\ -x + z = 0 \Rightarrow \boxed{x = z}$

$\boxed{x = z = -y}$

24. The number of particles crossing per unit area perpendicular to Z axis per unit time is given by $N = -D\dfrac{(N_2 - N_1)}{(Z_2 - Z_1)}$, where N_2 and N_1 are the number of particles per unit volume at Z_2 and Z_1 respectively. What is the dimensional formula for D?

(1) $[M^0L^{-1}T^2]$ (2) $[M^0L^{-1}T^{-1}]$ (3) $[M^0L^2T^{-1}]$ (4) $[M^0L^2T^2]$

Sol. Answer (3)

$$N = -D\frac{(N_2 - N_1)}{(Z_2 - Z_1)}$$

Dimensionally,

$$D = \frac{N(Z_2 - Z_1)}{(N_2 - N_1)}$$

Given,

N_2, N_1 $\rightarrow$ Number of particles per unit volume.

$$N_2, N_1 \rightarrow \frac{N}{V} \Rightarrow [L^{-3}]$$

$$Z_2 - Z_1 \rightarrow [L]$$

$$N \rightarrow \frac{\text{Number of particles}}{\text{Area}\cdot(T)}$$

$$N \rightarrow [L^{-2}T^{-1}]$$

So, $D = \dfrac{L^{-2}T^{-1} \times L}{L^{-3}} \Rightarrow [L^2T^{-1}]$

25. The frequency of vibrations f of a mass m suspended from a spring of spring constant K is given by a relation of type $f = cm^xK^y$, where c is a dimensionless constant. The values of x and y are

(1) $x = \frac{1}{2},\ y = \frac{1}{2}$ (2) $x = \frac{-1}{2},\ y = \frac{-1}{2}$ (3) $x = \frac{1}{2},\ y = \frac{-1}{2}$ (4) $x = \frac{-1}{2},\ y = \frac{1}{2}$

Sol. Answer (4)

$f \rightarrow$ Frequency $\rightarrow [T^{-1}]$

$m \rightarrow$ Mass $\rightarrow [M]$

$c \rightarrow$ Constant

$$K = \frac{f}{x} = \frac{MLT^{-2}}{L} \Rightarrow [MT^{-2}]$$

$$[M^0L^0T^{-1}] = c[M^x\, M^y\, T^{-2y}]$$

$$x + y = 0, \qquad -2y = -1$$

$$\Rightarrow \boxed{x = \frac{-1}{2}} \qquad \Rightarrow \boxed{y = \frac{1}{2}}$$

26. If energy E, velocity V and time T are taken as fundamental units, the dimensional formula for surface tension is

(1) $[EV^{-2}T^{-2}]$ (2) $[E^{-2}VT^{-2}]$ (3) $[E^{-2}V^{-2}T]$ (4) $[E^{-2}V^{-2}T^{-2}]$

Sol. Answer (1)

Surface tension = $\dfrac{\text{Force}}{\text{Length}} = \dfrac{MLT^{-2}}{L} \Rightarrow [MT^{-2}]$

Surface tension = $[MT^{-2}]$

$E \rightarrow [ML^2T^{-2}]$

$V \rightarrow [LT^{-1}]$

$T \rightarrow [T]$

Surface tension = $[E^a\ V^b\ T^c]$

$[MT^{-2}] = [ML^2T^{-2}]^a\ [LT^{-1}]^b\ [T]^c$

On comparing,

$\boxed{a=1}$, $2a + b = 0$

$\Rightarrow\ 2 + b = 0$

$\Rightarrow\ \boxed{b=-2}$

$-2a - b + c = -2$

$\Rightarrow\ -2 + 2 + c = -2$

$\Rightarrow\ \boxed{c=-2}$

Surface tension = $[EV^{-2}T^{-2}]$

27. If force F, area A and density D are taken as the fundamental units, the representation of Young's modulus 'Y' will be

(1) $[F^{-1}A^{-1}D^{-1}]$ (2) $[FA^{-2}D^2]$ (3) $[FA^{-1}D]$ (4) $[FA^{-1}D^0]$

Sol. Answer (4)

Young's modulus = $\dfrac{\text{Stress}}{\text{Strain}} = [ML^{-1}T^{-2}]$

$F \rightarrow [MLT^{-2}]$

$A \rightarrow [L^2]$

$D \rightarrow [ML^{-3}]$

$[ML^{-1}T^{-2}] = [MLT^{-2}]^a\ [L^2]^b\ [ML^{-3}]^c$

$a + c = 1,\quad a + 2b - 3c = -1$

$\Rightarrow\ \boxed{a=1-c}\quad \Rightarrow\ -2 = -2a - 3c$

$\Rightarrow\ 2 = 2a + 3c$

$\Rightarrow\ 2 = 2 - 2c + 3c$

$\Rightarrow\ 0 = +c \Rightarrow \boxed{c=0}$

$\therefore\ \boxed{a=1}$

$1 + 2b = -1$

$2b = -2$

$\Rightarrow\ \boxed{b=-1}$

Young's modulus = $[FA^{-1}D^0]$

SECTION - B

Previous Years Questions

1. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of –0.004 cm, the correct diameter of the ball is **[NEET-2018]**

(1) 0.521 cm (2) 0.525 cm (3) 0.529 cm (4) 0.053 cm

Sol. Answer (3)

Diameter of the ball

= MSR + CSR × (Least count) – Zero error

= 0.5 cm + 25 × 0.001 – (–0.004)

= 0.5 + 0.025 + 0.004

= 0.529 cm

2. A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and e is charge] **[NEET-2017]**

(1) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{\frac{1}{2}}$ (2) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{\frac{1}{2}}$ (3) $\frac{1}{c^2}\left[\frac{e^2}{G4\pi\varepsilon_0}\right]^{\frac{1}{2}}$ (4) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$

Sol. Answer (1)

Let $\frac{e^2}{4\pi\varepsilon_0} = A = ML^3T^{-2}$

$l = C^xG^y(A)^z$

$L = [LT^{-1}]^x\,[M^{-1}L^3T^{-2}]^y\,[ML^3T^{-2}]^z$

$-y + z = 0 \Rightarrow y = z$ ...(i)

$x + 3y + 3z = 1$ ...(ii)

$-x - 4z = 0$ ...(iii)

From (i), (ii) & (iii)

$z = y = \frac{1}{2},\ x = -2$

3. Planck's constant (h), speed of light in vacuum (c) and Newton's gravitational constant (G) are three fundamental constants. Which of the following combinations of these has the dimension of length? **[NEET(Phase-2) 2016]**

(1) $\frac{\sqrt{hG}}{c^{3/2}}$ (2) $\frac{\sqrt{hG}}{c^{5/2}}$ (3) $\sqrt{\frac{hc}{G}}$ (4) $\sqrt{\frac{Gc}{h^{3/2}}}$

Sol. Answer (1)

$L \propto h^ac^bG^c$

$[L]^1 = [M^1L^2T^{-1}]^a\,[LT^{-1}]^b\,[M^{-1}L^3T^{-2}]^c$

Solving, $a = \frac{1}{2},\ c = \frac{1}{2},\ b = -\frac{3}{2}$

$\Rightarrow\ L = \frac{\sqrt{hG}}{c^{3/2}}$

4. If dimensions of critical velocity v_c of a liquid flowing through a tube are expressed as $[\eta^x \rho^y r^z]$ where η, ρ and r are the coefficient of viscosity of liquid, density of liquid and radius of the tube respectively, then the values of x, y and z are given by **[Re-AIPMT-2015]**

(1) 1, 1, 1 (2) 1, – 1, –1 (3) – 1, – 1, 1 (4) –1, – 1, – 1

Sol. Answer (2)

Equation of critical velocity, $v_c = \dfrac{R\eta}{\rho D}$

$v_c \propto \eta^1 \rho^{-1} D^{-1}$

$\therefore$ $x = 1$, $y = -1$, $z = -1$

5. If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities, the dimensional formula of surface tension will be **[AIPMT-2015]**

(1) $[E^{-2}\ V^{-1}\ T^{-3}]$ (2) $[E\ V^{-2}\ T^{-1}]$ (3) $[E\ V^{-1}\ T^{-2}]$ (4) $[E\ V^{-2}\ T^{-2}]$

Sol. Answer (4)

6. If force (F), velocity (V) and time (T) are taken as fundamental units, then the dimensions of mass are **[AIPMT-2014]**

(1) $[F\ V\ T^{-1}]$ (2) $[F\ V\ T^{-2}]$ (3) $[F\ V^{-1}\ T^{-1}]$ (4) $[F\ V^{-1}\ T]$

Sol. Answer (4)

$M = F^x\ V^y\ T^z$

$M = (MLT^{-2})^x\ (LT^{-1})^y\ (T)^z$

$M = M^x\ L^{x+y}\ T^{-2x-y+z}$

Equating powers of M, L and T both sides

$x = 1$, $x + y = 0$, $-2x - y + z = 0$

Solving equations $x = 1$, $y = -1$, $z = 1$

$M = F\ V^{-1}\ T$

7. In an experiment four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows : $P = \dfrac{a^3b^2}{cd}$. % error in P is **[NEET-2013]**

(1) 10% (2) 7% (3) 4% (4) 14%

Sol. Answer (4)

$P = \dfrac{a^3b^2}{cd}$

$$\frac{\Delta P}{P}\times 100\% = \left(\frac{3\Delta a}{a}+\frac{2\Delta b}{b}+\frac{\Delta c}{c}+\frac{\Delta d}{d}\right)\times 100\% = 14\%$$

8. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are **[AIPMT (Prelims)-2012]**

(1) kgs^{-1} (2) kgs (3) $kgms^{-1}$ (4) $kgms^{-2}$

Sol. Answer (1)

$$F \propto v \quad \Rightarrow \quad F = bv \quad \Rightarrow \quad b = \frac{F}{v} = \frac{kgms^{-2}}{ms^{-1}} = kgs^{-1}$$

9. The dimensions of $(\mu_0\varepsilon_0)^{-½}$ are **[AIPMT (Prelims)-2011 & (Mains)-2012]**

(1) $[L^{-½} T^{½}]$ (2) $[L^{½} T^{-½}]$ (3) $[L^{-1} T]$ (4) $[L T^{-1}]$

Sol. Answer (4)

Speed of light $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow c = (\mu_0\varepsilon_0)^{-1/2}$

So, dimensional formula of $(\mu_0\varepsilon_0)^{-1/2}$

10. The density of a material in CGS system of units is 4 g/cm^3. In a system of units in which unit of length is 10 cm and unit of mass is 100 g, the value of density of material will be **[AIPMT (Mains)-2011]**

(1) 400 (2) 0.04 (3) 0.4 (4) 40

Sol. Answer (4)

Density, $n_1u_1 = n_2u_2 \Rightarrow \dfrac{4\text{ g}}{\text{cm}^3} = n_2 \times \dfrac{100\text{ g}}{10^3\text{ cm}^3} \Rightarrow \boxed{n_2 = 40}$

11. A student measures the distance traversed in free fall of a body, initially at rest in a given time. He uses this data to estimate *g*, the acceleration due to gravity. If the maximum percentage errors in measurement of the distance and the time are e_1 and e_2 respectively, the percentage error in the estimation of *g* is

[AIPMT (Mains)-2010]

(1) $e_2 - e_1$ (2) $e_1 + 2e_2$ (3) $e_1 + e_2$ (4) $e_1 - 2e_2$

Sol. Answer (2)

12. The dimension of $\frac{1}{2}\varepsilon_0 E^2$, where ε_0 is permittivity of free space and *E* is electric field, is

[AIPMT (Prelims)-2010]

(1) $ML^2 T^{-2}$ (2) $ML^{-1} T^{-2}$ (3) $ML^2 T^{-2}$ (4) MLT^{-1}

Sol. Answer (2)

Energy density $= \dfrac{E}{V} = \dfrac{1}{2}\varepsilon_0 E^2 \Rightarrow \dfrac{ML^2T^{-2}}{L^3} \Rightarrow \boxed{[ML^{-1}T^{-2}] = \frac{1}{2}\varepsilon_0 E^2}$

13. If the dimensions of a physical quantity are given by $M^a L^b T^c$, then the physical quantity will be

[AIPMT (Prelims)-2009]

(1) Velocity if $a = 1$, $b = 0$, $c = -1$
(2) Acceleration if $a = 1$, $b = 1$, $c = -2$
(3) Force if $a = 0$, $b = -1$, $c = -2$
(4) Pressure if $a = 1$, $b = -1$, $c = -2$

Sol. Answer (4)

Pressure = $[ML^{-1}T^{-2}]$

14. Which two of the following five physical parameters have the same dimensions? **[AIPMT (Prelims)-2008]**

(a) Energy density
(b) Refractive index
(c) Dielectric constant
(d) Young's modulus
(e) Magnetic field

(1) (a) and (e)
(2) (b) and (d)
(3) (c) and (e)
(4) (a) and (d)

Sol. Answer (4)

Refractive index and dielectric constant are dimensional constant

Energy density = $\dfrac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$

Young's modulus = $\dfrac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$

So, (d) & (a)

15. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be **[AIPMT (Prelims)-2008]**

(1) 2%
(2) 4%
(3) 6%
(4) 8%

Sol. Answer (3)

Volume of sphere = $\dfrac{4}{3}\pi R^3$

$$\Rightarrow \quad \frac{\Delta V}{V}\times 100\% = 3\times\frac{\Delta R}{R}\times 100\%$$

$$= 3 \times 2\%$$

$$\Rightarrow \quad \boxed{\frac{\Delta V}{V}\times 100\% = 6\%}$$

16. Dimensions of resistance in an electrical circuit, in terms of dimension of mass M, of length L, of time T and of current I, would be **[AIPMT (Prelims)-2007]**

(1) $[ML^2T^{-3}I^{-2}]$ (2) $[ML^2T^{-3}I^{-1}]$ (3) $[ML^2T^{-2}]$ (4) $[ML^2T^{-1}I^{-1}]$

Sol. Answer (1)

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{W}{qI} = \frac{ML^2T^{-2}}{AT \cdot A} \Rightarrow \boxed{R = [ML^2T^{-3}A^{-2}]}$$

17. The velocity v of a particle at time t is given by, $v = at + \frac{b}{t+c}$, where a, b and c are constants, The dimensions of a, b and c are respectively : **[AIPMT (Prelims)-2006]**

(1) $[LT^{-2}]$, [L] and [T] (2) $[L^2]$, [T] and $[LT^2]$ (3) $[LT^2]$, [LT] and [L] (4) [L], [LT] and $[T^2]$

Sol. Answer (1)

$$v = at + \frac{b}{t+c}$$

By the principle of homogeneity, $c = t = [T]$

$at = v \Rightarrow a = [LT^{-2}]$

$$\frac{b}{T} = LT^{-1} \Rightarrow b = [L]$$

18. The ratio of the dimensions of Planck's constant and that of the moment of inertia is the dimension of

[AIPMT (Prelims)-2005]

(1) Frequency (2) Velocity (3) Angular momentum (4) Time

Sol. Answer (1)

$$\frac{h}{I} = \frac{ML^2T^{-1}}{ML^2} \Rightarrow [T^{-1}] \rightarrow \text{Frequency}$$

19. The pair of quantities having same dimensions is

(1) Young's modulus and Energy (2) Impulse and Surface Tension

(3) Angular momentum and Work (4) Work and Torque

Sol. Answer (4)

Work = Force × Displacement

$$\boxed{W = [ML^2T^{-2}}$$

Torque = Perpendicular distance × Force = $[ML^2T^{-2}]$

20. The dimensions of μ_0 are

(1) $\left[M^1 L^{-\frac{1}{2}} T^{\frac{1}{2}}\right]$ (2) $\left[M^1 L^{\frac{1}{2}} T^{-\frac{1}{2}}\right]$ (3) $[L^{-1}T]$ (4) $[M^1L^1T^{-2}A^{-2}]$

Sol. Answer (4)

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = LT^{-1} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = L^2T^{-2} \Rightarrow \mu_0 = \frac{1}{L^2T^{-2}\varepsilon_0} \Rightarrow \mu_0 = \frac{1}{L^2T^{-2}}[ML^3T^{-4}A^{-2}] \Rightarrow \boxed{\mu_0 = [MLT^{-2}A^{-2}}$$

21. What is the dimension of surface tension?

(1) $[ML^1T^0]$ (2) $[ML^1T^{-1}]$ (3) $[ML^0T^{-2}]$ (4) $[M^1L^0T^{-2}]$

Sol. Answer (3, 4)

$$\text{Surface tension} = \frac{F}{L} = \frac{MLT^{-2}}{L} = [MT^{-2}]$$

22. Which of the following has the dimensions of pressure?

(1) $[MLT^{-2}]$ (2) $[ML^{-1}T^{-2}]$ (3) $[ML^{-2}T^{-2}]$ (4) $[M^{-1}L^{-1}]$

Sol. Answer (2)

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2} \Rightarrow [ML^{-1}T^{-2}]$$

$$\boxed{P = [ML^{-1}T^{-2}}$$

23. Percentage errors in the measurement of mass and speed are 2% and 3% respectively. The error in the estimate of kinetic energy obtained by measuring mass and speed will be

(1) 8% (2) 2% (3) 12% (4) 10%

Sol. Answer (1)

$$KE = \frac{1}{2}MV^2 \Rightarrow \frac{\Delta K}{K}\times 100\% = \frac{\Delta M}{M}\times 100\% + \frac{2\Delta V}{V}\times 100\% = 2\% + 2\times 3\% \Rightarrow \boxed{\frac{\Delta K}{K}\times 100\% = 8\%}$$

24. Which of the following is a dimensional constant?

(1) Relative density (2) Gravitational constant (3) Refractive index (4) Poisson's ratio

Sol. Answer (2)

Dimensional constant $[G] = [M^{-1}L^3T^{-2}]$

25. The dimensions of RC is

(1) Square of time (2) Square of inverse time (3) Time (4) Inverse time

Sol. Answer (3)

RC = Time

26. The dimensions of impulse are equal to that of

(1) Pressure (2) Linear momentum (3) Force (4) Angular momentum

Sol. Answer (2)

Impulse = $\Delta p \Rightarrow [MLT^{-1}]$

27. The density of a cube is measured by measuring its mass and length of its sides. If the maximum error in the measurement of mass and lengths are 3% and 2% respectively, the maximum error in the measurement of density would be

(1) 12% (2) 14% (3) 7% (4) 9%

Sol. Answer (4)

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\frac{\Delta d}{d}\times 100\% = \frac{\Delta m}{m}\times 100\% + \frac{3\Delta l}{l}\times 100\% = 3\% + 3 \times 2\%$$

$$\boxed{\frac{\Delta d}{d}\times 100\% = 9\%}$$

28. An equation is given here $\left(P+\frac{a}{V^2}\right) = b\frac{\theta}{V}$ where P = Pressure, V = Volume and θ = Absolute temperature.

If a and b are constants, the dimensions of a will be

(1) $[ML^{-5}\,T^{-1}]$ (2) $[ML^{5}\,T^{1}]$ (3) $[ML^{5}\,T^{-2}]$ (4) $[M^{-1}\,L^{5}T^{2}]$

Sol. Answer (3)

$$\left(P+\frac{a}{V^2}\right) = b\frac{\theta}{V}$$

Dimensionally, $P = \frac{a}{V^2}$

$ML^{-1}T^{-2} \times L^6 = a \Rightarrow \boxed{a = [ML^5T^{-2}]}$

29. Which of the following dimensions will be the same as that of time?

(1) $\frac{L}{R}$ (2) $\frac{C}{L}$ (3) LC (4) $\frac{R}{L}$

Sol. Answer (1)

$\frac{L}{R}$ = Time

30. The dimensional formula of magnetic flux is

(1) $[M^0L^{-2}T^2A^{-2}]$ (2) $[ML^0T^{-2}A^{-2}]$ (3) $[ML^2T^{-2}A^{-1}]$ (4) $[ML^2T^{-1}A^3]$

Sol. Answer (3)

$$\phi = BA = \frac{F}{qv} \times A \qquad [F = qvB]$$

$$= \frac{MLT^{-2}}{AT \cdot LT^{-1}} \times L^2$$

$$= [ML^2T^{-2}A^{-1}]$$

31. Which pair do not have equal dimensions?

(1) Energy and torque

(2) Force and impulse

(3) Angular momentum and Planck's constant

(4) Elastic modulus and pressure

Sol. Answer (2)

Force = $[MLT^{-2}]$

Impulse = Force × Time

$\Rightarrow$ $[MLT^{-1}]$

32. The dimensions of Planck's constant equals to that of

(1) Energy (2) Momentum (3) Angular momentum (4) Power

Sol. Answer (3)

$E = h\nu$

$$\frac{ML^2T^{-2}}{T^{-1}} = h$$

$$\Rightarrow \boxed{h = [ML^2T^{-1}]}$$

Angular momentum $= mvr = MLT^{-1}L$

$$\boxed{L = [ML^2T^{-1}]}$$

33. The dimensions of universal gravitational constant are

(1) $[M^{-1}L^3T^{-2}]$ (2) $[ML^2T^{-1}]$ (3) $[M^{-2}L^3T^{-2}]$ (4) $[M^{-2}L^2T^{-1}]$

Sol. Answer (1)

Gravitational constant = $[M^{-1}L^3T^{-2}]$

SECTION - C

Assertion-Reason Type Questions

1. A : Shake and light year, both measure time.

   R : Both have dimension of time.

Sol. Answer (4)

Shake $\rightarrow$ Unit of time

Light year $\rightarrow$ Unit of length

2. A : Displacement gradient is a dimensionless quantity.

   R : Displacement is dimensionless quantity.

Sol. Answer (3)

Displacement gradient = $\dfrac{\text{Displacement}}{\text{Length}} \Rightarrow$ Dimensionless

But displacement is not dimensionless.

3. A : Absolute error in a physical quantity can be positive, negative or zero.

   R : Absolute error is the difference in measured value and true value of physical quantity.

Sol. Answer (4)

Absolute error is always positive as it is $|\text{true value} - \text{measured value}|$

4. A : A unitless physical quantity must be dimensionless.

   R : A pure number is always dimensionless.

Sol. Answer (2)

If a quantity doesnot have units so definitely it will be dimensionless but reverse is not true.

Pure number $\rightarrow$ also dimensionless.

5. A : Absolute error is unitless and dimensionless.

   R : All type of errors are unitless and dimensionless.

Sol. Answer (4)

Absolute error is not dimensionless rather it will having dimensions of the measured quantity.

6. A : Higher is the precision of measurement, if instrument have smaller least count.

   R : Smaller the percentage error, higher is the accuracy of measurement.

Sol. Answer (2)

Smaller least count means higher precisions.

So, error will be very smaller.

Low least count means low error and hence high accuracy.

7. A : The maximum possible error in a reading is taken as least count of the measuring instrument.

   R : Error in a measurement cannot be greater than least count of the measuring instrument.

Sol. Answer (3)

The assertion is true as least count is the maximum possible error in the measurement.

But the error can be greater than least count it will depend upon power of quantity.

8. A : In a measurement, two readings obtained are 20.004 and 20.0004. The second measurement is more precise.

R : Measurement having more decimal places is more precise.

Sol. Answer (1)

The precisions is decided by the more number of decimal places so, 20.0004 is more precise.

9. A : Out of the measurements *A* = 20.00 and *B* = 20.000, *B* is more accurate.

R : Percentage error in *B* is less than the percentage error in *A*.

Sol. Answer (1)

Out of 20.00 and 20.000

The second measurement is more precise and more accurate also. The percentage error in second reading is less.

$$\frac{0.01}{20.00}\times 100 \Rightarrow \frac{1}{20} = 0.05\%$$

$$\frac{0.001}{20.000}\times 100 \Rightarrow 0.0005\%$$

10. A : When we change the unit of a measurement of a quantity, its numerical value changes.

R : The product of numerical value of the physical quantity and unit for a quantity remain constant.

Sol. Answer (1)

Numerical value × Unit = constant

11. A : All physically correct equations are dimensionally correct.

R : All dimensionally correct equations are physically correct.

Sol. Answer (3)

If an equation is physically correct it has to be dimensionally correct also.

But the reverse is not true.

12. A : Physical relations involving addition and subtraction cannot be derived by dimensional analysis.

R : Numerical constants cannot be deduced by the method of dimensions.

Sol. Answer (2)

Those equations carrying multiplication and divisions of physical quantities can be derived but not valid for addition or subtraction.

13. A : If displacement *y* of a particle executing simple harmonic motion depends upon amplitude *a* angular frequency ω and time *t* then the relation $y = a \sin\omega t$ cannot be dimensionally achieved.

R : An equation cannot be achieved by dimensional analysis; if it contains dimensionless expressions.

Sol. Answer (1)

Assertion and reason is correct and correctly explains assertion.

14. A : An exact number has infinite number of significant digits.

R : A number, which is a measured value has finite number of significant digits.

Sol. Answer (2)

An exact number contains infinite number of significant figures.

15. A : A dimensionless quantity may have unit.

R : Two physical quantities having same dimensions, may have different units.

Sol. Answer (2)

Dimensionless quantity may have unit. for example, angle.

Also two quantities having same dimensions may have different units.

Work $\rightarrow ML^2T^{-2} \rightarrow$ Joule

Torque $\rightarrow ML^2T^{-2} \rightarrow$ Nm

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