

Numeric Response

Questions Definite Integration

Q.1 If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg(x(1-x))dx$ and $I_2 = \int_{r(-a)}^{r(a)} g(x(1-x))dx$, then find the value of $\frac{I_2}{I_1}$.

Q.2 Find the value of $\int_{-2}^2 |[x]|dx$ (where $[.]$ denotes greatest integer function)

Q.3 Evaluate: $\int_{-1}^1 \log(x + \sqrt{x^2 + 1})dx$

Q.4 Evaluate: $\int_{\pi}^{10\pi} |\sin x|dx$

Q.5 If $\int_0^{\pi/5} \cos^3 \theta d\theta$ is equal to $\frac{2}{\lambda}$ then find λ .

Q.6 Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

Q.7 If $I_n = \int_0^{\pi/4} \tan^n x dx$ and $I_5 + I_6 = \frac{1}{\lambda}$ then find λ .

Q.8 If $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx = \frac{\pi^2}{k}$ then find value of k .

Q.9 Find the value of integral $\int_{-1}^1 |1-x|dx$.

Q.10 If $\int_0^a \frac{x^7 dx}{\sqrt{a^2-x^2}} = \frac{k}{\lambda} k^i$ then find value of $\lambda - k$.

Q.11 If $\int_0^{\pi/2} \sin^4 x \cos^3 x dx = \frac{7\pi}{k}$ then find value of k .

Q.12 Evaluate: $\int_0^{200x} \sqrt{\frac{1-\cos 2x}{2}} dx$

Q.13 If $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{\lambda}$ then find the value of λ .

Q.14 Find the value of $\int_0^{2014} \frac{2^x}{2^x + 2^{2014-x}} dx$.

Q.15 If $\int_0^1 x^6 \sqrt{1-x^2} dx = \frac{\lambda\pi}{k}$ then find $\lambda + k$.

ANSWER KEY

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|------------|---------|-----------|-------------|------------|----------|-------------|
| 1. 2.00 | 2. 4.00 | 3. 0.00 | 4. 18.00 | 5. 12.00 | 6. 1.00 | 7. 7.00 |
| 8. 32.00 | 9. 2.00 | 10. 19.00 | 11. 2048.00 | 12. 400.00 | 13. 4.00 | 14. 1007.00 |
| 15. 261.00 | | | | | | |

Hints & Solutions

1. Note : $f(a) + f(-a) = 1$

$$I_1 = \int_{f(-a)}^{f(a)} xg(x(1-x))dx$$

$$I_1 = \int_{f(-a)}^{f(a)} (1-x)g((1-x)x)dx$$

$$\Rightarrow I_1 + I_1 = \int_{f(-a)}^{f(a)} g(x(1-x))dx$$

$$\Rightarrow 2I_1 = I_2$$

$$\Rightarrow \frac{I_2}{I_1} = 2$$

2. $I = \int_{-2}^{-1} |-2| dx + \int_{-1}^0 |-1| dx + \int_0^1 |0| dx + \int_1^2 |1| dx$
 $\Rightarrow 2(x)_{-2}^{-1} + 1.(x)_{-1}^0 + 0 + (x)_1^2$
 $= 4$

3. as $\log(x + \sqrt{1+x^2})$ is an odd function. So

$$\int_{-1}^1 \log(x + \sqrt{1+x^2}) dx = 0$$

4. $\int_{\pi}^{10\pi} |\sin x| dx = 9 \int_0^{\pi} |\sin x| dx$
 $= 9 \int_0^{\pi} \sin x dx = 18$

5. $4\theta = t$

$$\therefore I = \frac{1}{4} \int_0^{\pi/2} \cos^3 t dt$$

$$= \frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$$

6. Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$\text{We get } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^{-x}} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^{-x}} dx$$

$$\therefore 2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2$$

$$\Rightarrow I = 1$$

7. $I_n = \int_0^{\pi/4} \tan^n x dx$
 $= \int_0^{\pi/4} \tan^{n-2} x (\sec^2 x - 1) dx$
 $= \frac{1}{n-1} - I_{n-2}$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$$

$$\Rightarrow I_8 + I_6 = \frac{1}{8-1} = \frac{1}{7}$$

8. $I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

Put $\tan^{-1} x = t$ also, when $x = 0, t = 0$
 $\frac{1}{1+x^2} dx = dt$ when $x = 1, t = \pi/4$

$$I = \int_0^{\pi/4} t dt$$

$$I = \left[\frac{t^2}{2} \right]_0^{\pi/4}$$

$$I = \frac{(\pi/4)^2}{2} - 0$$

$$I = \frac{\pi^2}{32}$$

$$9. \quad I = \int_{-1}^1 |1-x| dx$$

$$I = \int_{-1}^1 (1-x) dx$$

$$I = \left(x - \frac{x^2}{2} \right)_{-1}^1$$

$$I = \left(1 - \frac{1}{2} \right) - \left(-1 - \frac{1}{2} \right)$$

$$I = 2$$

$$10. \quad I = \int_0^{\pi/2} \frac{a^7 \sin^7 \theta a \cos \theta d\theta}{a \cos \theta}$$

$$\text{put } x/a = \sin \theta$$

$$dx = a \cos \theta d\theta$$

$$I = a^7 \cdot \frac{6.4.2}{7.5.3.1} = \frac{16}{35} a^7$$

$$11. \quad I = \int_0^{\pi/2} \sin^4 x \cos^8 x dx$$

$$= \frac{(4-1)(4-3) \times (8-1)(8-3)(8-5)(8-7)}{12 \cdot 10 \cdot 8 \cdot 6 \cdot 4 \cdot 2 \cdot 1} \cdot \frac{\pi}{2}$$

$$I = \frac{7\pi}{2048}$$

$$12. \quad I = \int_0^{200\pi} \sqrt{\frac{1-\cos 2x}{2}} dx = \int_0^{200\pi} |\sin x| dx$$

$$= 200 \int_0^{\pi} |\sin x| dx = 200 \int_0^{\pi} \sin x dx$$

$$= 200 \times 2 = 400$$

$$13. \quad \text{Let } I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Here the lower limit is zero hence, we can replace x by $(a-x)$ i.e. by $\pi/2 - x$

$$\therefore I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)} + \sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$\text{Adding } 2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

$$14. \quad \int_0^{2014} \frac{2^x}{2^x + 2^{2014-x}} dx = \frac{2014}{2} = 1007$$

$$15. \quad \text{Let } x = \sin \theta \Rightarrow dx = \cos \theta d\theta$$

$$\int_0^{\pi/2} \sin^6 \theta \cdot \cos^2 \theta d\theta$$

$$= \frac{5 \cdot 3 \cdot 1 \cdot 1}{8 \cdot 6 \cdot 4 \cdot 2} \times \frac{\pi}{2} = \frac{5\pi}{256}$$