Chapter

2 FRACTIONS AND DECIMALS

KEY FACTS

1. Number of the form $\frac{a}{b}$ where $b \neq 0$ are called fractions.

2. Types of fractions:

(a) **Proper fraction**: A fraction $\frac{a}{b}$ in which $b \neq 0$ and $a < b$.

(b) **Improper fraction** : A fraction $\frac{a}{b}$ in which $b \neq 0$ and $a = b$ or $a > b$.

(c) **Mixed fraction** : A fraction which can be expressed as the sum of a natural number and a fraction.

(d) **Decimal fraction** : A fraction whose denominator is a multiple of 10 is called a **decimal fraction**.

(e) **Vulgar fractions** : Fractions having denominators as whole numbers other than a power of 10, i.e., 10, 100, 1000 ..... etc.

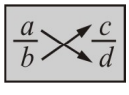
3. Fractions obtained on multiplying or dividing both numerator and denominator of a given fraction by the same non-zero number, and the given fraction are called **equivalent fractions**.

4. A fraction of the form $\frac{a}{b}, b \neq 0$, where a and b are whole numbers is called a **simple fraction**, whereas a fraction of the form $\frac{a}{b}$ where a and b are fractions is called a **complex fractions.**

5. A fraction $\frac{a}{b}$ is said to be in its lowest form if the HCF of a and b is 1.

6. Comparing fractions $\frac{a}{b}$ and $\frac{c}{d}$:

(*i*) If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$ (*ii*) If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$ (*iii*) If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$



Note. Before applying the rule of cross-multiplication, put the given fractions in standard from. Thus, $\frac{a}{-b}$ should be written as $\frac{-a}{b}$.

7. (*i*) For addition or subtraction of like fractions, numerators are added (or subtracted), denominator remaining the same.

(*ii*) For addition or subtraction of unlike fractions, first change them to equivalent like fractions (by finding the LCM of denominators) and then do as in (*i*).

Note. To add or subtract mixed fractions, you may add or subtract the whole numbers parts separately and fractional parts separately.

8. If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions, then., $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$

9. If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions, then $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$. Where $\frac{d}{c}$ is the **reciprocal** of $\frac{c}{d}$.

10. (a) The sequence of operations followed while simplifying a numerical expression is **BODMAS**.

(b) In case of brackets the sequence followed is '—', ( ), { }, [ ], i.e., from the innermost to the outer most.

11. A decimal number has two parts, the **whole number part** on the left of the decimal point and the **decimal part** on the right of the decimal point. E.g. 12.56.

The **number of digits in the decimal part** is the **number of decimal places**. E.g., the number 12.56 has two decimal places.

12. To *add* or *subtract* decimal numbers write the numbers one below the other so that the decimal points are in one vertical line. Then add or subtract.

13. To multiply :

(*i*) **a decimal by 10 or powers of 10 :** Shift the decimal point to the right by as many places as there are number of zeros,e.g.,4.952×10 = 45.92, 4.952 × 100 = 495.2 etc.

(*ii*) **a decimal by a decimal :** Ignore the decimal point and multiply them as whole numbers. The number of decimal places in the product is the sum of the number of decimal places both in the multiplicand and multiplier.

14. To divide :

(*i*) **a decimal by 10 or powers of 10 :** Shift the decimal point to the left by as many places as there are numbers of zeros, e.g., 65.958 ÷ 10 = 6.5958, 65.958 ÷ 1000 = 0.065958 etc.

(*ii*) **a decimal by a decimal** : Convert the divisor to a whole number by shifting the decimal point to the right by the same number of decimal places as there are in the divisor. Then shift the decimal point of the dividend also by the same number of places to the right, e.g., 16.403 ÷ 6.98 = 1640.3 ÷ 698.

15. A fraction $\frac{p}{q}$ is a terminating decimal if in its lowest form the denominator has factors as 2 or 5 or both 2 and 5, otherwise $\frac{p}{q}$ is expressible as a non-terminating repeating or recurring decimal. For example : $\frac{7}{20}=0.35$, $\frac{5}{9}=0.5555$.....

16. To change decimals to fractions :

(*i*) **When the decimals are terminating** : Remove the decimal point and write the resulting number in the numerator. In the denominator write 1 followed by as many zeros as the number of decimal places in the given decimal number $45.93=\frac{4593}{100}$.

(*ii*) **When the decimals are repeating** :

(a) In a **pure recurring decimal** (a decimal number in which all the digits after the decimal point are repeated). Write the repeated figure only once in the numerator and write as many nines in the denominator as is the number of repeated digits, e.g., $0.\overline{7}=\frac{7}{9}$, $0.\overline{59}=\frac{59}{99}$, $2.\overline{63}=2+\frac{63}{99}=2\frac{7}{11}$.

(b) In a **mixed recurring decimal** (a decimal in which at least one digit after the decimal point is not repeated and then some digits are repeated), consider the decimal point. Subtract the non-repeating part from the whole and divide by (as many nines as there are repeating digits × as many tens as are non-repeating digits), e.g., $5.1\overline{925}=5+\frac{1925-1}{9990}=5+\frac{1924}{9990}=5+\frac{26}{135}=\frac{701}{135}$.

(Three repeating digits (925) and one non-repeating digit (1))

Solved Examples

Ex. 1. *Evaluate* $8-8\times\dfrac{2\frac{1}{5}-1\frac{2}{7}}{2-\frac{1}{6-\frac{1}{6}}}$.

Sol. $8-8\times\dfrac{\frac{11}{5}-\frac{9}{7}}{2-\frac{1}{\frac{35}{6}}}=8-8\times\dfrac{\frac{77-45}{35}}{2-\frac{6}{35}}=8-8+\dfrac{\frac{32}{35}}{\frac{64}{35}}=8-8\times\dfrac{1}{2}=8-4=\mathbf{4}.$

Ex. 2. *What is the value of* $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right).....\left(1+\frac{1}{120}\right)$*?*

Sol. Given exp. $=\dfrac{3}{2}\times\dfrac{4}{3}\times\dfrac{5}{4}\times.....\times\dfrac{120}{119}\times\dfrac{121}{120}=\dfrac{121}{2}=\mathbf{60.5}$

Ex. 3. *If* $\dfrac{2x}{1+\frac{1}{1+\frac{x}{1-x}}}=1$*, then find the value of x.*

Sol. Given, $\dfrac{2x}{1+\frac{1}{\frac{1-x+x}{1-x}}}=1\Rightarrow\dfrac{2x}{1+\frac{1}{\frac{1}{1-x}}}=1\Rightarrow\dfrac{2x}{1+(1-x)}=1\Rightarrow 2x=2-x\Rightarrow 3x=2\Rightarrow x=\mathbf{\frac{2}{3}}.$

Ex. 4. *If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is* $18\frac{26}{27}$*. What is the original fraction?*

Sol. Let the fraction be x. Then,

$$\frac{x\times x}{\frac{1}{x}}=18\frac{26}{27}\quad\Rightarrow\quad x^3=\frac{512}{27}\quad\Rightarrow\quad x=\sqrt[3]{\frac{512}{27}}=\frac{8}{3}=\mathbf{2\frac{2}{3}}.$$

Ex. 5. *The fluid contained in a bucket can fill four large bottles or seven small bottles. A full large bottle is used to fill an empty small bottle. What fraction of the fluid is left over in the large bottle when the small one is full ?*

(a) $\frac{2}{7}$ *(b)* $\frac{3}{7}$ *(c)* $\frac{4}{7}$ *(d)* $\frac{5}{7}$

Sol. Let the capacity of the bucket be x litres. Then,

Capacity of 1 large bottle $=\dfrac{x}{4}$

Capacity of 1 small bottle $=\dfrac{x}{7}$

Fluid left in large bottle $= \frac{x}{4} - \frac{x}{7} = \frac{3x}{28}$

$\therefore$ Required fraction $= \frac{3x/28}{x/4} = \frac{3x}{28} \times \frac{4}{x} = \mathbf{\frac{3}{7}}$.

Ex. 6. ***A man has divided his total money in his will in such a way that half of it goes to his wife, $\frac{2}{3}$ rd of the remaining among his three sons equally and the rest among his four daughters equally. If each daughter gets Rs 20,000, how much money will each son get ?***

Sol. Let the total money, the man had = Rs x

Money gone to wife = Rs $\frac{x}{2}$

Remaining money = $\frac{x}{2}$

$\therefore$ Sons' share = $\frac{2}{3}$ of $\frac{x}{2} = \text{Rs}\frac{x}{3}$

Each son's share = $\frac{1}{3}$ of $\text{Rs}\frac{x}{3} = \text{Rs}\ \frac{x}{9}$

Daughters' share = $\frac{x}{2} - \frac{x}{3} = \frac{x}{6}$

Each daughter's share $= \frac{1}{4} \times \text{Rs}\frac{x}{6} = \text{Rs}\frac{x}{24}$

Given, $\frac{x}{24} = 20000 \Rightarrow x = \text{Rs } 480000$

$\therefore$ Each son's share = $\frac{480000}{9} = \mathbf{Rs\ 53,333.33}$

Ex. 7. ***Simplify :*** $\dfrac{\left(3\frac{2}{3}\right)^2 - \left(2\frac{1}{2}\right)^2}{\left(4\frac{3}{4}\right)^2 - \left(3\frac{1}{3}\right)^2} \div \dfrac{3\frac{2}{3} - 2\frac{1}{2}}{4\frac{3}{4} - 3\frac{1}{3}}$.

Sol. Given exp. $= \dfrac{\left(\frac{11}{3}\right)^2 - \left(\frac{5}{2}\right)^2}{\left(\frac{19}{4}\right)^2 - \left(\frac{10}{3}\right)^2} \times \dfrac{\left(\frac{19}{4} - \frac{10}{3}\right)}{\left(\frac{11}{3} - \frac{5}{2}\right)} = \dfrac{\left(\frac{11}{3} + \frac{5}{2}\right)\cancel{\left(\frac{11}{3} - \frac{5}{2}\right)}}{\left(\frac{19}{4} + \frac{10}{3}\right)\cancel{\left(\frac{19}{4} - \frac{10}{3}\right)}} \times \dfrac{\cancel{\left(\frac{19}{4} - \frac{10}{3}\right)}}{\cancel{\left(\frac{11}{3} - \frac{5}{2}\right)}}$

$= \dfrac{\frac{22+15}{6}}{\frac{57+40}{12}} = \dfrac{\frac{37}{6}}{\frac{97}{12}} = \dfrac{37 \times 2}{97} = \mathbf{\frac{74}{97}}$.

Ex. 8. ***If 1.5x = 0.04y then what is the value of*** $\frac{y-x}{y+x}$ ***?***

Sol. $1.5x = 0.04\,y \Rightarrow \frac{y}{x} = \frac{1.5}{0.04} = \frac{150}{4} = 37.5$

$\therefore \frac{y-x}{y+x} = \frac{\frac{y}{x}-1}{\frac{y}{x}+1} = \frac{37.5-1}{37.5+1} = \frac{36.5}{38.5} = \frac{\mathbf{73}}{\mathbf{77}}.$

Ex. 9. ***Evaluate :*** $\frac{(6.4)^2 - (5.4)^2}{(8.9)^2 + (8.9 \times 2.2) + (1.1)^2}$

Sol. Given exp. $= \frac{(6.4+5.4)(6.4-5.4)}{(8.9+1.1)^2} = \frac{11.8 \times 1}{100} = \mathbf{0.118}.$

Ex. 10. ***Simplify :*** $\frac{(0.06)^2 + (0.47)^2 + (0.079)^2}{(0.006)^2 + (0.047)^2 + (0.0079)^2}$

Sol. Given exp. $= \frac{(0.06)^2 + (0.47)^2 + (0.079)^2}{\left(\frac{0.06}{10}\right)^2 + \left(\frac{0.47}{10}\right)^2 + \left(\frac{0.079}{10}\right)^2} = 100\left[\frac{(0.06)^2 + (0.47)^2 + (0.079)^2}{(0.06)^2 + (0.47)^2 + (0.079)^2}\right] = \mathbf{100}.$

Ex. 11. ***The value of*** $\frac{3.157 \times 4126 \times 3.198}{63.972 \times 2835.121}$ ***is closest to***

(a) 0.002 ***(b) 0.02*** ***(c) 0.2*** ***(d) 2***

Sol. (c) The expression approximately $= \frac{3.2 \times 4126 \times 3.2}{64 \times 2835}$

$= 0.232$

$= \mathbf{0.2}$ (approx)

Ex. 12. ***Which among the following numbers is the greatest ?***

$0.07 + \sqrt{0.16}$, $\sqrt{1.44}$, 1.2×0.83, $1.02 - \frac{0.6}{24}$

Sol. $0.07 + \sqrt{0.16} = 0.07 + 0.4 = 0.47$

$\sqrt{1.44} = 1.2$

$1.2 \times 0.83 = 0.996$

$1.02 - \frac{0.6}{24} = 1.02 - \frac{\cancel{6}^{1}}{\cancel{240}_{40}} = 1.02 - 0.025 = 0.995$

$\therefore$ The greatest number is 1.2, i.e., $\sqrt{\mathbf{1.44}}$.

Question Bank–2

1. Find the value of $\frac{2}{3}\times\frac{3}{\frac{5}{6}\div\frac{2}{3}\text{ of }1\frac{1}{4}}$

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) 1 (d) 2

2. $\frac{3\frac{1}{4}-\frac{4}{5}\text{ of }\frac{5}{6}}{4\frac{1}{3}\div\frac{1}{5}-\left(\frac{3}{10}+21\frac{1}{5}\right)}-\left(1\frac{2}{3}\text{ of }1\frac{1}{2}\right)$ is equal to

(a) 9 (b) $11\frac{1}{2}$

(c) 13 (d) $15\frac{1}{2}$

3. Evaluate $\frac{1\frac{1}{7}-\frac{2}{3}+\frac{\frac{2}{5}}{1-\frac{1}{25}}}{1-\frac{1}{7}\left(\frac{1}{3}+\frac{\frac{2}{5}}{1-\frac{2}{5}}\right)}$

(a) $\frac{3}{4}$ (b) $\frac{24}{25}$

(c) 1 (d) $1\frac{1}{24}$

4. Simplify: $\frac{1}{1+\frac{\frac{2}{3}}{1+\frac{2}{3}+\frac{\frac{8}{9}}{1-\frac{2}{3}}}}$

(a) $\frac{11}{13}$ (b) $\frac{13}{15}$

(c) $\frac{13}{11}$ (d) $\frac{15}{13}$

5. If $\frac{37}{13}=2+\frac{1}{x+\frac{1}{y+\frac{1}{z}}}$, where x, y, z are natural numbers, then x, y, z are

(a) 1, 2, 5 (b) 1, 5, 2

(c) 5, 2, 11 (d) 11, 2, 5

6. The sum of the first 25 terms of the series $\frac{1}{3}+\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\frac{1}{3}+\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\ldots\ldots$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

7. $\frac{17}{15}\times\frac{17}{15}+\frac{2}{15}\times\frac{2}{15}-\frac{17}{15}\times\frac{4}{15}$ is equal to

(a) 0 (b) 1

(c) 10 (d) 11

8. $\left(4\frac{11}{15}+\frac{15}{71}\right)^2-\left(4\frac{11}{15}-\frac{15}{71}\right)^2$ is equal to

(a) 1 (b) 2

(c) 3 (d) 4

9. The square root of $\frac{\left(3\frac{1}{4}\right)^4-\left(4\frac{1}{3}\right)^4}{\left(3\frac{1}{4}\right)^2-\left(4\frac{1}{3}\right)^2}$ is

(a) $7\frac{1}{2}$ (b) $5\frac{5}{12}$

(c) $1\frac{1}{12}$ (d) $1\frac{7}{12}$

10. What is the value of $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\ldots\ldots\ldots+\frac{17}{5184}+\frac{19}{8100}$?

(a) 0.95 (b) 1

(c) 0.99 (d) 0.98

11. $\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)\times\left(1+\frac{1}{10+\frac{1}{10}}\right)-\left(1-\frac{1}{10+\frac{1}{10}}\right)\times\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]\div\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)+\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]$ simplifies to

(a) $\frac{100}{101}$ (b) $\frac{90}{101}$

(c) $\frac{20}{101}$ (d) $\frac{101}{100}$

12. A student was asked to simplify the following :

$$\frac{7}{5-2\frac{2}{3}} \div \frac{3-\frac{2}{3-1\frac{1}{2}}}{4-1\frac{1}{2}} - \frac{5}{7} \times \left[\frac{7}{10} + 1\frac{1}{5} \times \frac{3\frac{1}{3} - 2\frac{1}{2}}{2\frac{5}{21} - 2} \right] + \frac{\frac{3}{1.6} + \frac{5}{3.2}}{\frac{5}{4.8} + \frac{1}{9.6}}$$

His answer was $3\frac{1}{5}$. Find the per cent error in the answer.

(a) 10% (b) 20%
(c) 25% (d) 30%

13. In a school, $\frac{3}{7}$ of the students are girls and the rest are boys. $\frac{1}{4}$ of the boys are below ten years of age and $\frac{5}{6}$ of the girls are also below ten years of age. If the number of students above ten years of age is 500, then find the total number of students in the school.

(a) 600 (b) 1000
(c) 900 (d) 1100

14. A man first sold $\frac{2}{3}$rd of his total quantity of rice and 100 kg. Again he sold $\frac{1}{2}$ of the remaining quantity and 100 kg. If the total remaining quantity of the stock is 150 kg. Then, what was the original stock of rice ?

(a) 2100 kg (b) 1800 kg
(c) 2400 kg (d) 2000 kg

15. The value of $1 - \frac{1}{20} + \frac{1}{20^2} - \frac{1}{20^3} + ...$ correct to 5 places of decimal is

(a) 1.05 (b) 0.95238
(c) 0.95239 (d) 10.5

16. $\left(\frac{1}{1\times4} + \frac{1}{4\times7} + \frac{1}{7\times10} + \frac{1}{10\times13} + \frac{1}{13\times16} \right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{5}{16}$
(c) $\frac{3}{8}$ (d) $\frac{41}{7280}$

17. If $\frac{a}{b} = \frac{1}{3}, \frac{b}{c} = 2, \frac{c}{d} = \frac{1}{2}, \frac{d}{e} = 3$ and $\frac{e}{f} = \frac{1}{4}$, then what is the value of $\frac{abc}{def}$?

(a) $\frac{3}{8}$ (b) $\frac{27}{8}$
(c) $\frac{3}{4}$ (d) $\frac{27}{4}$

18. Evaluate : 515.15 – 15.51 – 1.51 – 5.11 – 1.11.

(a) 491.91 (b) 419.91
(c) 499.19 (d) 411.19

19. Simplify : 12.28 × 1.5 – 36 ÷ 2.4

(a) 3.24 (b) 3.42
(c) 4.32 (d) 4.23

20. Which of the following is equal to 1 ?

(a) $\frac{(0.11)^2}{(1.1)^2 \times 0.1}$ (b) $\frac{(1.1)^2}{11^2 \times (0.01)^2}$

(c) $\frac{(0.011)^2}{1.1^2 \times 0.012}$ (d) $\frac{(0.11)^2}{11^2 \times 0.01}$

21. When the number $N = 0.73\overline{545}$ is written as a fraction in its lowest terms, the denominator exceeds the numerator by

(a) 199 (b) 299
(c) 109 (d) 219

22. What is the value of $(7.5 \times 7.5 + 37.5 + 2.5 \times 2.5)$?

(a) 30 (b) 60
(c) 80 (d) 100

23. Evaluate $0.\overline{142857} \div 0.\overline{285714}$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) 2 (d) 10

24. If $1^3 + 2^3 + ... + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + ... + (0.99)^3$ is close to

(a) 0.2695 (b) 0.3695
(c) 2.695 (d) 3.695

25. $\frac{5.42 \times 6 + 5.42 \times 24}{32.71 \times 32.71 - 27.29 \times 27.29} \div \frac{6.54 \times 6.54 - 3.46 \times 3.46}{3.08 \times 5 + 3.08 \times 45}$

is equal to

(a) 0.3 (b) 0.4
(c) 0.7 (d) 2.5

26. The value of $\frac{(67.542)^2-(32.458)^2}{75.458-40.374}$ is

(a) 1 (b) 10

(c) 100 (d) 0.1

27. $\frac{0.1\times0.1\times0.1+0.02\times0.02\times0.02}{0.2\times0.2\times0.2+0.04\times0.04\times0.04}$ is equal to

(a) 0.0125 (b) 0.125

(c) 0.25 (d) 0.5

28. $\sqrt{(0.798)^2+0.404\times0.798+(0.202)^2}+1$ is equal to

(a) 0 (b) 2

(c) 1.596 (d) 0.404

29. Which of the following numbers is the least ?

$(0.5)^2$, $\sqrt{0.49}$, $\sqrt[3]{0.008}$, 0.23

(a) $(0.5)^2$ (b) $\sqrt{0.49}$

(c) $\sqrt[3]{0.008}$ (d) 0.23

30. The value of $\frac{489.1375\times0.0483\times1.956}{0.0873\times92.581\times99.749}$ is closest to

(a) 0.006 (b) 0.06

(c) 0.6 (d) 6

Answers

1. (d)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (c)
11. (c)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (c)
21. (b)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (c)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (b)

Hints and Solutions

1. (d) $\frac{2}{3}\times\frac{3}{\frac{5}{6}\div\frac{2}{3}\text{ of }1\frac{1}{4}}$

$=\frac{2}{3}\times\frac{3}{\frac{5}{6}\div\frac{2}{3}\times\frac{5}{4}}=\frac{2}{3}\times\frac{3}{\frac{5}{6}\div\frac{5}{6}}$

$=\frac{2}{3}\times\frac{3}{1}=\mathbf{2}.$

2. (c) $\frac{3\frac{1}{4}-\frac{4}{5}\text{ of }\frac{5}{6}}{4\frac{1}{3}\div\frac{1}{5}-\left(\frac{3}{10}+21\frac{1}{5}\right)}-\left(1\frac{2}{3}\text{ of }1\frac{1}{2}\right)$

$=\frac{\frac{13}{4}-\frac{4}{5}\times\frac{5}{6}}{\frac{13}{3}\div\frac{1}{5}-\left(\frac{3}{10}+\frac{106}{5}\right)}-\left(\frac{5}{3}\times\frac{3}{2}\right)$

$=\frac{\frac{13}{4}-\frac{2}{3}}{\frac{13}{3}\times\frac{5}{1}-\left(\frac{3+212}{10}\right)}-\frac{5}{2}$

$=\frac{\frac{39-8}{12}}{\frac{65}{3}-\frac{215}{10}}-\frac{5}{2}=\frac{\frac{31}{12}}{\frac{650-645}{30}}-\frac{5}{2}$

$=\frac{31}{12}\div\frac{5}{30}-\frac{5}{2}=\frac{31}{12}\times\frac{30}{5}-\frac{5}{2}$

$=\frac{31}{2}-\frac{5}{2}=\frac{26}{2}=\mathbf{13}.$

3. (d) $\frac{1\frac{1}{7}-\frac{2}{3}+\frac{\frac{2}{5}}{1-\frac{1}{25}}}{1-\frac{1}{7}\left(\frac{1}{3}+\frac{\frac{2}{5}}{1-\frac{2}{5}}\right)}=\frac{\frac{8}{7}-\frac{2}{3}+\frac{\frac{2}{5}}{\frac{24}{25}}}{1-\frac{1}{7}\left(\frac{1}{3}+\frac{\frac{2}{5}}{\frac{3}{5}}\right)}$

$=\frac{\frac{8}{7}-\frac{2}{3}+\frac{2}{5}\times\frac{25}{24}}{1-\frac{1}{7}\left(\frac{1}{3}+\frac{2}{3}\right)}$

$=\frac{\frac{8}{7}-\frac{2}{3}+\frac{5}{12}}{1-\frac{1}{7}}=\frac{\frac{96-56+35}{84}}{\frac{6}{7}}$

$=\frac{75}{84}\times\frac{7}{6}=\frac{25}{24}=\mathbf{1\frac{1}{24}}$

4. (b) $\frac{1}{1+\frac{\frac{2}{3}}{1+\frac{2}{3}+\frac{\frac{8}{9}}{1-\frac{2}{3}}}}=\frac{1}{1+\frac{\frac{2}{3}}{1+\frac{2}{3}+\frac{\frac{8}{9}}{\frac{1}{3}}}}$

$$= \frac{1}{1+\frac{\frac{2}{3}}{1+\frac{2}{3}+\frac{8}{3}}} = \frac{1}{1+\frac{\frac{2}{3}}{1+\frac{13}{3}}}$$

$$= \frac{1}{1+\frac{2}{13}} = \frac{1}{\frac{15}{13}} = \mathbf{\frac{13}{15}}.$$

5. (b) $2+\frac{1}{x+\frac{1}{y+\frac{1}{z}}} = \frac{37}{13} = 2\frac{11}{13} = 2+\frac{11}{13}$

$$\Rightarrow \frac{1}{x+\frac{1}{y+\frac{1}{z}}} = \frac{11}{13} \Rightarrow x+\frac{1}{y+\frac{1}{z}} = \frac{13}{11}$$

$$\Rightarrow x+\frac{1}{y+\frac{1}{z}} = 1+\frac{2}{11}$$

$$\Rightarrow x = 1,\ y+\frac{1}{z} = \frac{11}{2} = 5\frac{1}{2} = 5+\frac{1}{2}$$

$\Rightarrow x = \mathbf{1}, y = \mathbf{5}, z = \mathbf{2}$

6. (c) Sum of the first 6 terms

$$= \frac{1}{3}+\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\frac{1}{2}-\frac{1}{4} = 0$$

$\therefore$ Sum of first 24 terms = 0

$\therefore$ Required sum = 25th term = $\mathbf{\frac{1}{3}}$.

7. (b) Given exp. $= \frac{17}{15}\times\frac{17}{15}+\frac{2}{15}\times\frac{2}{15}-2\times\frac{17}{15}\times\frac{2}{15}$

$$= \left(\frac{17}{15}\right)^2+\left(\frac{2}{15}\right)^2-2\times\frac{17}{15}\times\frac{2}{15}$$

$$= \left(\frac{17}{15}-\frac{2}{15}\right)^2 \text{ [Using } a^2+b^2-2ab = (a-b)^2\text{]}$$

$$= \left(\frac{15}{15}\right)^2 = 1^2 = \mathbf{1}$$

8. (d) $\left(4\frac{11}{15}+\frac{15}{71}\right)^2-\left(4\frac{11}{15}-\frac{15}{71}\right)^2$

$$= \left(4\frac{11}{15}+\frac{15}{71}+4\frac{11}{15}-\frac{15}{71}\right)$$

$$\left(4\frac{11}{15}+\frac{15}{71}-4\frac{11}{15}+\frac{15}{71}\right)$$

[Using $(a^2-b^2) = (a+b)(a-b)$]

$$= \left(8\frac{22}{15}\right)\times\left(\frac{30}{71}\right) = \frac{142}{15}\times\frac{30}{71} = \mathbf{4}.$$

9. (b) $\frac{\left(3\frac{1}{4}\right)^4-\left(4\frac{1}{3}\right)^4}{\left(3\frac{1}{4}\right)^2-\left(4\frac{1}{3}\right)^2}$

$$= \frac{\left(\left(3\frac{1}{4}\right)^2+\left(4\frac{1}{3}\right)^2\right)\left(\left(3\frac{1}{4}\right)^2-\left(4\frac{1}{3}\right)^2\right)}{\left(3\frac{1}{4}\right)^2-\left(4\frac{1}{3}\right)^2}$$

$$= \left(\frac{13}{4}\right)^2+\left(\frac{13}{3}\right)^2 = \frac{169}{16}+\frac{169}{9}$$

$[a^4-b^4 = (a^2+b^2)(a^2-b^2)]$

$$= 169\left(\frac{1}{16}+\frac{1}{9}\right) = 169\times\left(\frac{9+16}{144}\right)$$

$$= \frac{169\times25}{144}$$

$\therefore$ Square root of given exp. = $\sqrt{\frac{169\times25}{144}}$

$$= \frac{13\times5}{12} = \frac{65}{12} = \mathbf{5\frac{5}{12}}.$$

10. (c) $\frac{3}{4}+\frac{5}{6}+\frac{7}{144}+\ldots\ldots+\frac{17}{5184}+\frac{19}{8100}$

$$= \frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+\ldots\ldots+\frac{17}{8^2.9^2}+\frac{19}{9^2.10^2}$$

$$= \left(1-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{3^2}-\frac{1}{4^2}\right)+\ldots\ldots\cdot$$

$$+\left(\frac{1}{8^2}-\frac{1}{9^2}\right)+\left(\frac{1}{9^2}-\frac{1}{10^2}\right)$$

$$= 1-\frac{1}{10^2} = 1-\frac{1}{100} = \frac{99}{100} = \mathbf{0.99}.$$

11. (c) Given exp. $= \dfrac{\left(1+\dfrac{1}{10+\dfrac{1}{10}}\right)^2-\left(1-\dfrac{1}{10+\dfrac{1}{10}}\right)^2}{\left(1+\dfrac{1}{10+\dfrac{1}{10}}\right)+\left(1-\dfrac{1}{10+\dfrac{1}{10}}\right)}$

$$\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)+\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]$$

$$= \frac{\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)-\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]}{\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)+\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]}$$

$$= \left(1+\frac{1}{10+\frac{1}{10}}\right)-\left(1-\frac{1}{10+\frac{1}{10}}\right)$$

$$= \left(1+\frac{1}{\frac{101}{10}}\right)-\left(1-\frac{1}{\frac{101}{10}}\right)$$

$$= \left(1+\frac{10}{101}\right)-\left(1-\frac{10}{101}\right)$$

$$= \left(\frac{101+10}{101}\right)-\left(\frac{101-10}{101}\right)$$

$$= \left(\frac{111}{101}\right)-\left(\frac{91}{101}\right) = \mathbf{\frac{20}{101}}.$$

12. (c) Given exp. $= \dfrac{7}{5-\dfrac{8}{3}} \div \dfrac{3-\dfrac{2}{3-\dfrac{3}{2}}}{4-\dfrac{3}{2}}$

$$-\frac{5}{7}\left[\frac{7}{10}+\frac{6}{5}\times\frac{\frac{10}{3}-\frac{5}{2}}{\frac{47}{21}-2}\right]+\frac{\frac{30}{16}+\frac{50}{32}}{\frac{50}{48}+\frac{10}{96}}$$

$$= \frac{7}{\frac{7}{3}} \div \frac{3-\frac{2}{\frac{3}{2}}}{\frac{5}{2}} - \frac{5}{7}\left[\frac{7}{10}+\frac{6}{5}\times\frac{\frac{20-15}{6}}{\frac{47-42}{21}}\right]+\frac{\frac{15}{8}+\frac{25}{16}}{\frac{25}{24}+\frac{5}{48}}$$

$$= 3 \div \left[\left(3-\frac{4}{3}\right)\times\frac{2}{5}\right]-\frac{5}{7}\left[\frac{7}{10}+\frac{6}{5}\times\frac{\frac{5}{6}}{\frac{5}{21}}\right]+\frac{\frac{55}{16}}{\frac{55}{48}}$$

$$= 3 \div \left[\frac{5}{3}\times\frac{2}{5}\right]-\frac{5}{7}\left[\frac{7}{10}+\frac{6}{5}\times\frac{21}{6}\right]+\frac{48}{16}$$

$$= 3 \div \frac{2}{3}-\frac{5}{7}\left[\frac{7}{10}+\frac{21}{5}\right]+3$$

$$= \frac{9}{2}-\frac{5}{7}\times\frac{7}{10}-\frac{5}{7}\times\frac{21}{5}+3$$

$$= \frac{9}{2}-\frac{1}{2}-3+3 = \frac{8}{2} = 4$$

$\therefore$ Percentage error $= \left(\dfrac{4-3\frac{1}{5}}{3\frac{1}{5}}\times 100\right)\%$

$$= \left(\frac{4-\frac{16}{5}}{\frac{16}{5}}\times 100\right)\% = \left(\frac{\frac{4}{5}}{\frac{16}{5}}\times 100\right)\% = \mathbf{25\%}.$$

13. (b) Let the total number of students in the school be x.

Then, Number of girls $= \dfrac{3x}{7}$

Number of boys $= \dfrac{4x}{7}$

Number of boys below ten years of age

$$= \frac{1}{4}\times\frac{4x}{7} = \frac{x}{7}$$

Number of girls below ten years of age

$$= \frac{5}{6}\times\frac{3x}{7} = \frac{5x}{14}$$

$\therefore$ Total number of students below 10 years of age

$$= \frac{x}{7}+\frac{5x}{14} = \frac{7x}{14} = \frac{x}{2}$$

$\therefore$ Total number of students above 10 years of age

$$= x-\frac{x}{2} = \frac{x}{2}$$

Given, $\dfrac{x}{2} = 500 \Rightarrow x = \mathbf{1000}.$

14. (b) Let the original stock of rice be x kg.

$$\text{Parts of the stock sold first time} = \left(\frac{2x}{3}+100\right)\text{kg}$$

$$\therefore \text{ Remaining stock } = \left[x-\left(\frac{2x}{3}+100\right)\right]\text{kg}$$

$$= \left(\frac{x}{3}-100\right)\text{kg}$$

Part of the stock sold second time

$$= \left[\frac{1}{2}\left(\frac{x}{3}-100\right)+100\right]$$

$$= \left(\frac{x}{6}-50+100\right)\text{kg} = \left(\frac{x}{6}+50\right)\text{ kg}$$

$$\therefore \text{Remaining stock} = \left(\frac{x}{3}-100\right)-\left(\frac{x}{6}+50\right)$$

$$= \left(\frac{x}{3}-\frac{x}{6}-100-50\right)\text{kg}$$

$$= \left(\frac{x}{6}-50\right)\text{kg}$$

Given, $\frac{x}{6}-150=150 \Rightarrow \frac{x}{6}=300 \Rightarrow x = \mathbf{1800\ kg}$.

15. (b) Given exp. $= 1-\frac{1}{20}+\frac{1}{20^2}-\frac{1}{20^3}+.....$

$$= 1-\frac{1}{20}+\frac{1}{400}-\frac{1}{8000}+.....$$

$$= 1 - 0.05 + 0.0025 - 0.000125 +.....$$

$$= 1.0025 - 0.050125 = 0.952375 = \mathbf{0.95238.}$$

16. (b) $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\frac{1}{10\times13}+\frac{1}{13\times16}$

$$= \frac{1}{3}\left[\left(1-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)+\left(\frac{1}{10}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{16}\right)\right]$$

$$= \frac{1}{3}\left(1-\frac{1}{16}\right) = \frac{1}{3}\times\frac{15}{16} = \mathbf{\frac{5}{16}}$$

17. (a) $\frac{a}{b}=\frac{1}{3} \Rightarrow a=\frac{b}{3}$

$$\frac{b}{c}=2 \Rightarrow c=\frac{b}{2}$$

$$\frac{c}{d}=\frac{1}{2} \Rightarrow d=2c \Rightarrow d=2\times\frac{b}{2}=b$$

$$\frac{d}{e}=3 \Rightarrow e=\frac{d}{3} \Rightarrow e=\frac{b}{3}$$

$$\frac{e}{f}=\frac{1}{4} \Rightarrow f=4e \Rightarrow f=\frac{4b}{3}$$

$$\therefore \frac{abc}{def}=\frac{b/3\times b\times b/2}{b\times b/3\times 4b/3}=\mathbf{\frac{3}{8}}.$$

18. (a) Given exp.

$$= 515.15 - (15.51 + 1.51 + 5.11 + 1.11)$$

$$= 515.15 - 23.24$$

$$= \mathbf{491.91.}$$

19. (b) $12.28\times1.5-\frac{36}{2.4} = 18.42 - 15 = \mathbf{3.42}$

20. (c) $\frac{(0.11)^2}{(1.1)^2\times0.1}=\frac{0.0121}{1.21\times0.1}=\frac{0.0121}{0.121}=0.1;$

$$\frac{(1.1)^2}{11^2\times(0.01)^2}=\frac{1.21}{121\times0.0001}=\frac{0.01}{0.0001}=100;$$

$$\frac{(0.011)^2}{(1.1)^2\times(0.01)^2}=\frac{0.000121}{1.21\times0.0001}=1;$$

$$\frac{(0.11)^2}{11^2\times0.01}=\frac{0.0121}{121\times0.01}=\frac{0.0121}{1.21}=\mathbf{0.01}.$$

Hence, option (c) is the correct answer.

21. (b) $N = 0.73\overline{545} = \frac{73545-735}{99000}=\frac{72810}{99000}=\frac{809}{1100}$

$\therefore$ Required difference = 1100 – 809 = **299.**

22. (d) Given exp. $= (7.5)^2 + 2 \times 7.5 \times 2.5 + (2.5)^2$

$$= (7.5 + 2.5)^2 = 10^2 = \mathbf{100.}$$

23. (a) $0.\overline{142857} \div 0.\overline{285714}$

$$= \frac{142857}{999999}\div\frac{285714}{999999}$$

$$= \frac{\overset{1}{\cancel{1421857}}}{\cancel{999999}}\div\frac{\cancel{999999}}{\overset{2}{\cancel{2857124}}}=\mathbf{\frac{1}{2}}.$$

24. (c) $(0.11)^3 + (0.22)^3 +..... + (0.99)^3$

$$= (0.11)^3\ [1^3 + 2^3 + 3^3 +..... + 9^3]$$

$$= 0.001331 \times 2025 = 2.695275 = \mathbf{2.695}\ \text{(approx.)}$$

25. (d) Given exp.

$$= \frac{5.42\times(6+24)}{(32.71)^2-(27.29)^2}\div\frac{(6.54)^2-(3.46)^2}{3.08\times(5+45)}$$

$$= \frac{5.42\times30}{(32.71+27.29)(32.71-27.29)} \div \frac{(6.54+3.46)(6.54-3.46)}{3.08\times50}$$

$$= \frac{5.42\times30}{60\times5.42}\div\frac{10\times3.08}{3.08\times50}$$

$$= \frac{1}{2}\div\frac{1}{5}=\frac{5}{2}=\mathbf{2.5}.$$

26. (c) Given exp.

$$= \frac{(67.542+32.458)(67.542-32.458)}{35.084}$$

$$= \frac{100\times 35.084}{35.084} = \mathbf{100.}$$

27. (b) Given exp.

$$= \frac{0.1\times0.1\times0.1+0.02\times0.02\times0.02}{8(0.1\times0.1\times0.1)+8(0.02\times0.02\times0.02)}$$

$$= \frac{1}{8}\left(\frac{0.1\times0.1\times0.1+0.02\times0.02\times0.02}{0.1\times0.1\times0.1+0.02\times0.02\times0.02}\right)$$

$$= \frac{1}{8} = \mathbf{0.125}.$$

28. (b) $\sqrt{(0.798)^2+0.404\times0.798+(0.202)^2}+1$

$$= \sqrt{(0.798)^2+2\times0.202\times0.798+(0.202)^2}+1$$

$$= \sqrt{(0.798+0.202)^2}+1 = \sqrt{1}+1 = 1+1 = \mathbf{2}.$$

29. (c) $(0.5)^2 = 0.25$; $\sqrt{0.49} = 0.7$;

$\sqrt[3]{0.008} = 0.2$; 0.23

Arranging in ascending order the numbers are 0.2, 0.23, 0.25, 0.7.

$\therefore$ $\sqrt[3]{0.008} = 0.2$ is the least.

30. (b) $\dfrac{489.1375\times0.0483\times1.956}{0.0873\times92.581\times99.749} = \dfrac{489\times0.05\times2}{0.09\times93\times100}$

$$= \frac{489}{9\times93\times10} = \frac{163}{279}\times\frac{1}{10} = \frac{0.58}{10} = 0.058 = \mathbf{0.06}.$$

Self Assessment Sheet–2

1. If $4\frac{1}{a}\times b\frac{2}{3} = 7$, find the values of a and b.

(a) 1, 5 (b) 2, 3
(c) 3, 2 (d) 5, 1

2. Simplify : $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{3\frac{1}{2}+1\frac{1}{7}} \div \dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{5-\frac{1}{5}}}}$

(a) 0 (b) -1
(c) $3\frac{1}{24}$ (d) 1

3. Find the number which when multiplied by

$\dfrac{0.0016\times0.025}{0.325\times0.05} \div \dfrac{0.1216\times0.105\times0.002}{0.08512\times0.625\times0.039}$ yields the product 20?

(a) 25 (b) 100
(c) 80 (d) 200

4. The value of $0.\overline{2}+0.\overline{3}+0.\overline{4}+0.\overline{9}+0.\overline{39}$ is

(a) $0.\overline{57}$ (b) $1\frac{20}{33}$
(c) $2\frac{13}{39}$ (d) $2\frac{13}{33}$

5. If $8.5-\{5\frac{1}{2}-(7\frac{1}{2}+2.8\div x)\}\times4.25\div(0.2)^2 = 306$, the value of x is

(a) 1.75 (b) 3.5
(c) 7 (d) 1.4

6. A student was asked to simplify

$\dfrac{0.6\times0.6\times0.6+0.5\times0.5\times0.5+0.1\times0.1\times0.1-0.09}{0.6\times0.6+0.5\times0.5+0.1\times0.1-0.41}$

and his answer was 0.6. By what per cent was his answer wrong.

(a) 25% (b) 100%
(c) 50% (d) 120%

7. The simplest value of

$\dfrac{\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right).....\left(1+\frac{1}{50}\right)}{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right).....\left(1-\frac{1}{50}\right)}$ is

(a) $\frac{7}{32}$ (b) $\frac{4}{7}$
(c) $1\frac{87}{256}$ (d) $\frac{256}{343}$

8. $\dfrac{(0.22)^3+(0.11)^3+(0.32)^3}{(0.66)^3+(0.96)^3+(0.33)^3}+\dfrac{(0.32)^3+(0.45)^3-(0.77)^3}{81(0.32)(0.45)(0.77)}$ equals

(a) 1 (b) $\frac{1}{11}$
(c) 0 (d) -1

9. Find the value of

$\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right).....\left(1-\frac{1}{11^2}\right)\left(1-\frac{1}{12^2}\right)$

(a) $\frac{17}{18}$ (b) $\frac{13}{18}$
(c) $\frac{1}{144}$ (d) $\frac{1}{9}$

10. A student was asked to simplify the following:

$$\frac{7}{5-2\frac{2}{3}} \div \frac{3-\dfrac{2}{3-1\frac{1}{2}}}{4-1\frac{1}{2}} - \frac{5}{7} \times \left[\frac{7}{10} + 1\frac{1}{5} \times \frac{3\frac{1}{3}-2\frac{1}{2}}{2\frac{5}{21}-2} \right] + \frac{\dfrac{3}{1.6}+\dfrac{5}{3.2}}{\dfrac{5}{4.8}+\dfrac{1}{9.6}}$$

His answer was $3\frac{1}{5}$. Find the per cent error.

(a) 10% (b) 20%
(c) 25% (d) 50%

Answers

1. (d)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (b)