Applications of Vectors in Mechanics

Type – 1

Choose the most appropriate option (a, b, c or d).

Q 1. P is a point in the plane of the ΔABC whose orthocenter is H and the circumcentre is O. Forces $\vec{AP}, \vec{BP}, \vec{CP}$ and $\vec{PH}$ act at P. The force that will keep the given forces in equilibrium is

(a) $2\vec{OP}$ (b) $3\vec{OP}$ (c) $2\vec{PO}$ (d) none of these

Q 2. Three forces $\vec{P}, \vec{O}$ and $\vec{R}$ each of 15 units, act along AB, BC and CA respectively. The position vectors of A, B and C are $\vec{OA} = 2\vec{i} - j + 3\vec{k}$, $\vec{OB} = 5\vec{i} + 3j - 2\vec{k}$ and $\vec{OC} = 2\vec{i} + 2j + 3\vec{k}$. The resultant force vector is

(a) $\left(12\frac{9}{\sqrt{2}} - 7\sqrt{3}\right)\vec{i} - (9 - 6\sqrt{2} + \sqrt{3})\vec{j} + \left(5\sqrt{3} - \frac{15}{\sqrt{2}}\right)\vec{k}$

(b) $\left(12 + \frac{9}{\sqrt{2}} - 7\sqrt{3}\right)\vec{i} + (9 - 6\sqrt{2} + \sqrt{3})\vec{j} + \left(\frac{15}{\sqrt{2}} - 5\sqrt{3}\right)\vec{k}$

(c) $75\vec{i} + 60\vec{j} + 60\vec{k}$

(d) none of these

Q 3. A ship is sailing towards north at a speed of 1.25 m/s. The current is taking it towards east at the rate of 1 m/s. A sailor is climbing a vertical pole on the ship at the rate of 0.5 m/s. The magnitude of the velocity of the sailor in space is

(a) 2.75 m/s (b) $\frac{3\sqrt{5}}{4}$ m/s (c) $\frac{3\sqrt{5}}{2}$ m/s (d) none of these

Q 4. A force $10\vec{i} - 5\vec{j} + 7\vec{k}$ displaces a particle from the point A to the point B. The position vectors of A and B are $3\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{i} + 3\vec{j} + 2\vec{k}$ respectively. Then the work done is

(a) 40 (b) 20 (C) 60 (d) none of these

Q 5. Constant forces $\vec{P} = \vec{i} - 2\vec{j} + 3\vec{k}, \vec{Q} = -\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{R} = 2\vec{i} - 4\vec{j} + 3\vec{k}$ act on a particle. The work $4\vec{i} - 3\vec{j} - 2\vec{k}$ to the point B with position vector $6\vec{i} + \vec{j} - 3\vec{k}$ is

(a) 15 (b) 13 (c) $\sqrt{13}$ (d) none of these

Q 6. The vertices of a triangle ABC are A(–1, 0, 2), B(1,2, 0) and C(2, 3, 4). The moment of a force of magnitude 10 acting at A along AB about C is

(a) $\frac{50\sqrt{6}}{3}$ (b) $20\sqrt{6}$ (c) $\frac{50}{\sqrt{3}}$ (d) none of these

Q 7. The vector moment about the point $\vec{i}+2\vec{j}+3\vec{k}$ of the resultant of the forces $\vec{i}-2\vec{j}+5\vec{k}$ and $3\vec{j}-4\vec{k}$ acting at the point $2\vec{i}+3\vec{j}-\vec{k}$ is

(a) $5\vec{i}+\vec{j}-4\vec{k}$ (b) $5\vec{i}-\vec{j}-4\vec{k}$ (c) $3\vec{i}+\vec{j}-4\vec{k}$ (d) none of these

Q 8. A rigid body is rotating at 5 radians per second about an axis AB, where A and B are points whose position vectors are $2\vec{i}+\vec{j}+\vec{k}$ and $8\vec{i}-2\vec{j}+3\vec{k}$ respectively. The velocity of the particle of the body at the point whose position vector is $5\vec{i}+2\vec{j}-\vec{k}$ is

(a) $\frac{15}{7}(2\vec{i}+6\vec{j}+3\vec{k})$ (b) $\frac{5}{7}(4\vec{i}-6\vec{j}-3\vec{k})$ (c) $\frac{15}{7}(4\vec{i}-6\vec{j}-3\vec{k})$ (d) none of these

Type 2

Choose the correct options. One or more options may be correct.

Q 9. A particle is in equilibrium when the forces

$\vec{F}_1=-10\vec{k},\ \vec{F}_2=\frac{u}{13}(4\vec{i}-12\vec{j}+3\vec{k}),\vec{F}_3=\frac{v}{13}(-4\vec{i}-12\vec{j}+3\vec{k})$ and $\vec{F}=w(\cos\theta\vec{i}+\sin\theta\vec{j})$ act on it.

Then

(a) $u = 65(1 - 3\cot\theta)$ (b) $v=\frac{65}{3}+65\cot\theta$ (c) $w = 40\operatorname{cosec}\theta$ (d) none of these

Q 10. The resolved parts of the force vector $5\vec{i}+4\vec{j}+2\vec{k}$ along and perpendicular to the vector $3\vec{i}+4\vec{j}-5\vec{k}$ are $\vec{u}$ and $\vec{v}$ respectively. Then

(a) $\vec{u}=\frac{21}{50}(3\vec{i}+4\vec{j}-5\vec{k})$ (b) $\vec{u}=\frac{21}{5\sqrt{2}}(3\vec{i}+4\vec{j}-5\vec{k})$

(c) $\vec{v}=\frac{1}{50}(187\vec{i}+116\vec{j}+5\vec{k})$ (d) $\frac{1}{5\sqrt{2}}(\vec{i}-2\vec{j}-\vec{k})$

Answers

1c 2a 3b 4a 5b 6a 7b 8a 9b,c 10a,c