# CHAPTER 10
# Simple Harmonic Motion (Oscillations)

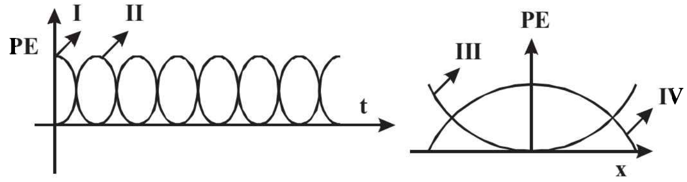
## Section-A JEE Advanced/ IIT-JEE

### A Fill in the Blanks

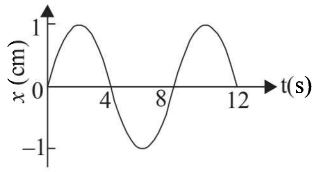
1. An object of mass 0.2 kg executes simple harmonic oscillation along the $x$-axis with a frequency of $(25/\pi)$ Hz. At the position $x = 0.04$, the object has Kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is .......m. *(1994 - 2marks)*

### C MCQs with One Correct Answer

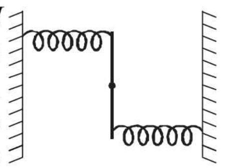
1. Two bodies $M$ and $N$ of equal masses are suspended from two separate massless springs of spring constants $k_1$ and $k_2$ respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of $M$ to that of $N$ is *(1988 - 1mark)*
   (a) $\frac{k_1}{k_2}$ (b) $\sqrt{k_1/k_2}$ (c) $\frac{k_2}{k_1}$ (d) $\sqrt{k_2/k_1}$
2. A particle free to move along the $x$-axis has potential energy given by $U(x) = k[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$, where $k$ is a positive constant of appropriate dimensions. Then *(1999S - 2marks)*
   (a) at points away from the origin, the particle is in unstable equilibrium
   (b) for any finite nonzero value of $x$, there is a force directed away from the origin
   (c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin.
   (d) for small displacements from $x = 0$, the motion is simple harmonic
3. The period of oscillation of a simple pendulum of length $L$ suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination $\alpha$, is given by *(2000S)*
   (a) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$
   (c) $2\pi\sqrt{\frac{L}{g}}$ (d) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$
4. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is $T_1$ and to go from $A/2$ to $A$ is $T_2$. Then *(2001S)*
   (a) $T_1 < T_2$ (b) $T_1 > T_2$
   (c) $T_1 = T_2$ (d) $T_1 = 2T_2$
5. For a particle executing SHM the displacement $x$ is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy ($PE$) as a function of time $t$ and displacement $x$ *(2003S)*




   (a) I, III (b) II, IV (c) II, III (d) I, IV
6. A simple pendulum has time period $T_1$. The point of suspension is now moved upward according to the relation $y = Kt^2$, ($K = 1$ m/s$^2$) where $y$ is the vertical displacement. The time period now becomes $T_2$. The ratio of $\frac{T_1^2}{T_2^2}$ is ($g = 10$ m/s$^2$) *(2005S)*
   (a) 5/6 (b) 6/5 (c) 1 (d) 4/5
7. The $x$-$t$ graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is *(2009)*

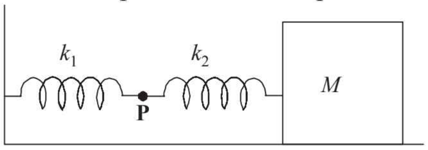



   (a) $\frac{\sqrt{3}}{32}\pi^2$ cm/s$^2$ (b) $\frac{-\pi^2}{32}$ cm/s$^2$
   (c) $\frac{\pi^2}{32}$ cm/s$^2$ (d) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s$^2$
8. A uniform rod of length $L$ and mass $M$ is pivoted at the centre. Its two ends are attached to two springs of equal spring constants $k$. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle $\theta$ in one direction and released. The frequency of oscillation is *(2009)*



   (a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$
   (c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

**9.** The mass $M$ shown in the figure oscillates in simple harmonic motion with amplitude $A$. The amplitude of the point $P$ is *(2009)*

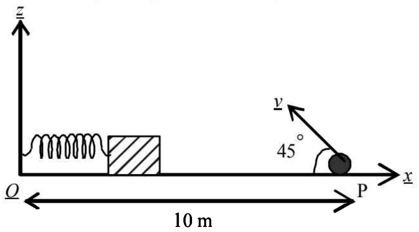



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$ (c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

**10.** A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and $\phi$ are *(2011)*

(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$ (c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

**11.** A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed $v$ form point $P$ at an angle of 45° as shown in the figure. Point $P$ is at a horizontal distance of 10 m from $O$. If the pebble hits the block at $t = 1$ s, the value of $v$ is (take $g = 10$ m/s$^2$) *(2012)*




(a) $\sqrt{50}$ m/s (b) $\sqrt{51}$ m/s
(c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

## D MCQs with One or More than One Correct

**1.** A particle executes simple harmonic motion with a frequency. $f$. The frequency with which its kinetic energy oscillates is *(1987 - 2marks)*

(a) $f/2$ (b) $f$
(c) $2f$ (d) $4f$

**2.** A linear harmonic oscillator of force constant $2 \times 10^6$ N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its *(1989 - 2 Mark)*

(a) maximum potential energy is 100 J
(b) maximum kinetic energy is 100 J
(c) maximum potential energy is 160 J
(d) maximum potential energy is zero

**3.** A uniform cylinder of length $L$ and mass $M$ having cross sectional area $A$ is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half- submerged in a liquid of density $\rho$ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with small amplitude. If the force constant of the spring is $k$, the frequency of oscillation of the cylinder is *(1990 - 2mark)*

(a) $\frac{1}{2\pi}\left(\frac{k - A\rho g}{M}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{M}\right)^{1/2}$

(c) $\frac{1}{2\pi}\left(\frac{k + \rho g L}{M}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{A\rho g}\right)^{1/2}$

**4.** A highly rigid cubical block $A$ of small mass $M$ and side L is fixed rigidly on to another cubical block $B$ of the same dimensions and of low modulus of rigidity $\eta$ such that the lower face of $A$ completely covers the upper face of $B$. The lower face of $B$ is rigidly held on a horizontal surface. A small force $F$ is applied perpendicular to one of the sides faces of $A$. After the force is withdrawn, block $A$ executes small oscillations the time period of which is given by *(1992 - 2mark)*

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$ (c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

**5.** One end of a long metallic wire of length $L$ is tied to the ceiling. The other end is tied to a massless spring of spring constant $K$. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are $A$ and $Y$ respectively. If the mass is slightly pulled down and released, it will oscillate with a time period $T$ equal to: *(1993-2marks)*

(a) $2\pi(m/K)^{1/2}$ (b) $2\pi\sqrt{\frac{m(YA + KL)}{YAK}}$

(c) $2\pi[(mYA/KL)]^{1/2}$ (d) $2\pi[(mL/YA)]^{1/2}$

**6.** A particle of mass $m$ is executing oscillations about the origin on the $x$ axis. Its potential energy is $V(x) = k|x|^3$ where $k$ is a positive constant. If the amplitude of oscillation is a, then its time period $T$ is *(1998S - 2marks)*

(a) proportional to $1/\sqrt{a}$ (b) independent of $a$
(c) proportional to $\sqrt{a}$ (d) proportional to $a^{3/2}$

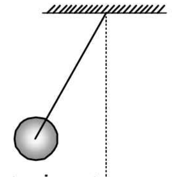
**7.** Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then. *(1999S - 3marks)*

(a) the resultant amplitude is $(1 + \sqrt{2})a$
(b) the phase of the resultant motion relative to the first is 90°
(c) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion
(d) the resulting motion is not simple harmonic.

**8.** The function $x = A\sin^2 \omega t + B\cos^2 \omega t + C \sin \omega t \cos \omega t$ represent SHM for which of the option(s)

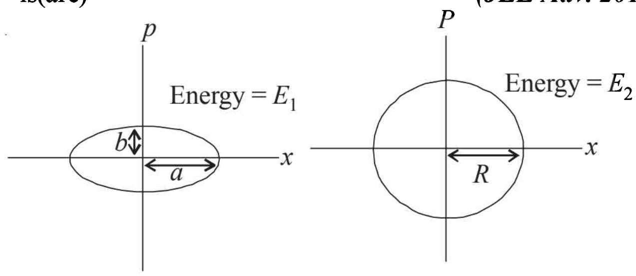
(a) for all value of $A$, $B$ and $C$ $(C \neq 0)$ *(2006 - 5M, –1)*
(b) $A = B, C = 2B$
(c) $A = -B, C = 2B$
(d) $A = B, C = 0$

9. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disc of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? ***(2011)***



(a) Restoring torque in case A = Restoring torque in case B
(b) restoring torque in case A < Restoring torque in case B
(c) Angular frequency for case A > angular frequency for case B.
(d) Angular frequency for case A < Angular frequency for case B.

10. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies $\omega_1$ and $\omega_2$ and have total energies $E_1$ and $E_2$, respectively. The variations of their momenta $p$ with positions $x$ are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are) ***(JEE Adv. 2015)***




(a) $E_1\omega_1 = E_2\omega_2$
(b) $\frac{\omega_2}{\omega_1} = n^2$
(c) $\omega_1\omega_2 = n^2$
(d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

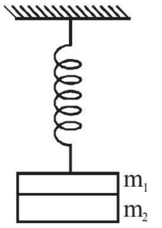
11. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position $x_0$. Consider two cases: (i) when the block is at $x_0$; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m(<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M? ***(JEE Adv. 2016)***

(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged.
(b) The final time period of oscillation in both the cases is same.
(c) The total energy decreases in both the cases.
(d) The instantaneous speed at $x_0$ of the combined masses decreases in both the cases

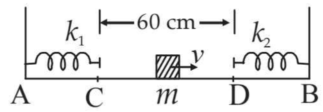
## E Subjective Problems

1. A mass $M$ attached to a spring, oscillates with a period of 2sec. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass $M$ assuming that Hook's Law is obeyed. ***(1979)***

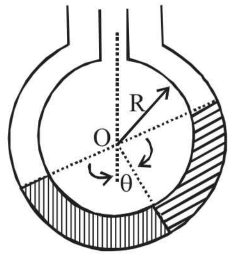
2. Two masses $m_1$ and $m_2$ are suspended together by a massless spring of spring constant $k$. When the masses are in equilibrium, $m_1$ is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of $m_2$. ***(1981 - 3 marks)***




3. Two light springs of force constants $k_1$ and $k_2$ and a block of mass $m$ are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the springs is 60 cms. If the block moves along AB with a velocity 120 cm/sec in between the springs, calculate the period of oscillation of the block ($k_1 = 1.8$ N/m, $k_2 = 3.2$ N/m, $m = 200$ gm) ***(1985 - 6 Marks)***



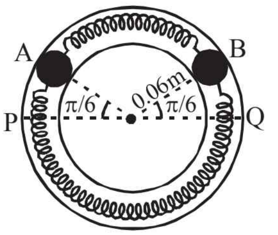

4. Two non-viscous, incompressible and immiscible liquids of densities $\rho$ and $1.5\rho$ are poured into the two limbs of a circular tube of radius $R$ and small cross section kept fixed in a vertical plane as shown in fig. Each liquid occupies one fourth the circumference of the tube. ***(1991 - 4 + 4 marks)***




(a) Find the angle $\theta$ that the radius to the interface makes with the vertical in equilibrium position.
(b) If the whole is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

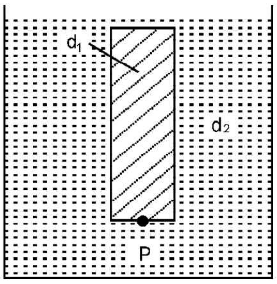
5. Two identical balls $A$ and $B$ each of mass 0.1 kg, are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in Fig. The pipe is fixed in a horizontal plane.

The centres of the balls can move in a circle of radius 0.06 $\pi$ meter. Each spring has a natural length zof $0.06\pi$ meter and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter $PQ$ of the circle (as shown in Fig.) and released from rest. ***(1993 - 6 marks)***

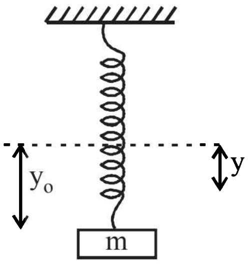



(i) Calculate the frequency of oscillation of ball $B$.
(ii) Find the speed of ball $A$ when $A$ and $B$ are at the two ends of the diameter $PQ$.
(iii) What is the total energy of the system

**6.** A thin rod of length $L$ and area of cross-section $S$ is pivoted at its lowest point $P$ inside a stationary, homogeneous and non-viscous liquid . The rod is free to rotate in a vertical plane about a horizontal axis passing through $P$. The density $d_1$ of the material of the rod is smaller than the density $d_2$ of the liquid. The rod is displaced by a small angle $\theta$ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. ***(1996 - 5 Marks)***



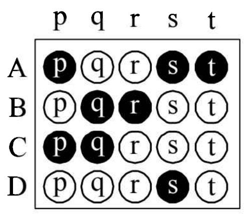

**7.** A small body attached to one end of a vertically hanging spring is performing SHM about its mean position with angular frequency $\omega$ and amplitude a. If at a height $y^*$ from the mean position, the body gets detached from the spring, calculate the value of $y^*$ so that the height H attained by the mass is maximum. The body does not interact with the spring during its subsequent motion after detachment. ($a\omega^2 > g$) ***(2005 - 4 Marks)***




## F Match the Following

**DIRECTIONS (Q. No. 1-2) :** *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

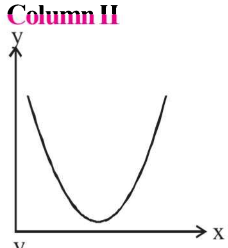
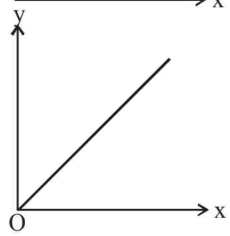
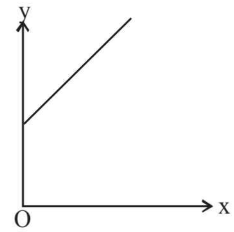
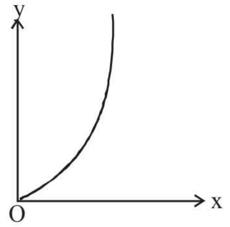
*If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.*




**1.** **Column I** describes some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situations in **Column I** with the characteristics in **Column II** and indicate your answer by darkening appropriate bubbles in 4 × 4 matrix given in the *ORS*. ***(2007)***

| **Column I** | **Column II** |
|---|---|
| (A) The object moves on the $x$ -axis under a conservative force in such a way that its "speed " and position" satisfy $v = c_1\sqrt{c_2 - x^2}$ where $c_1$ and $c_2$ are positive constants. | (p) The object executes a simple harmonic motion. |
| (B) The object moves on the $x$- axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where $k$ is a positive constant. | (q) The object does not change its direction. |
| (C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration. | (r) The kinetic energy of the object keeps on decreasing. |
| (D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e / R_e}$, where, $M_e$ is the mass of the earth and $R_e$ is the radius of the earth, Neglect forces from objects other than the earth. | (s) The object can change its direction only once. |

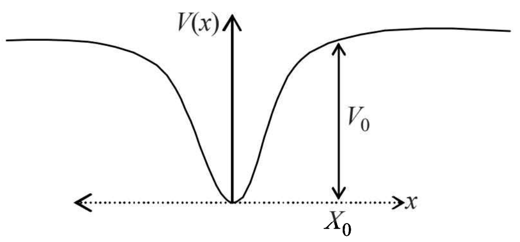
**2.** Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graph given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. ***(2008)***

| Column I | | Column II |
|---|---|---|
| (A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis) | (p) |  |
| (B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction. | (q) |  |
| (C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle. | (r) |  |
| (D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis) | (s) |  |

## G Comprehension Based Questions

### PASSAGE - 1

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$ ,as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from $kx^2$ and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to $V_0$ for $|x| \geq X_0$ (see figure). *(2010)*




**1.** If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$ (b) $E > 0$
(c) $V_0 > E > 0$ (d) $E > V_0$

**2.** For periodic motion of small amplitude A, the time period T of this particle is proportional to

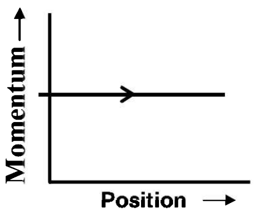
(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

**3.** The acceleration of this particle for $|x| > X_0$ is

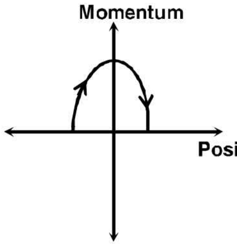
(a) proportional to $V_0$ (b) proportional to $\frac{V_0}{mX_0}$
(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (d) zero

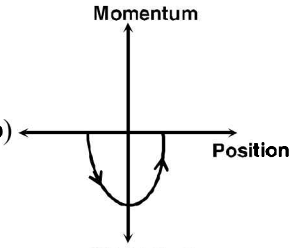
### PASSAGE - 2

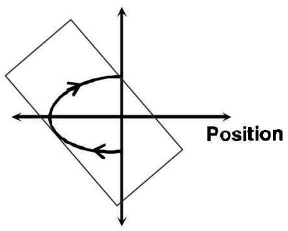
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momenum are changed. Here we consider some simple dynamical systems in one dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which positon or momentum upwards (or to right) is positive and downwards (or to left) is negative. *(2011)*

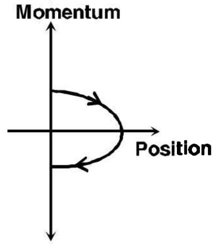


4. The phase space diagram for a ball thrown vertically up from ground is

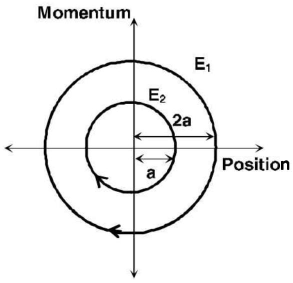
(a) 


(b) 


(c) 


(d) 


5. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and $E_1$ and $E_2$ are the total mechanical energies respectively. Then



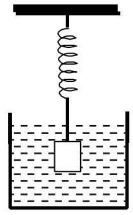

(a) $E_1 = \sqrt{2}E_2$

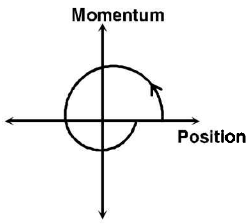
(b) $E_1 = 2E_2$

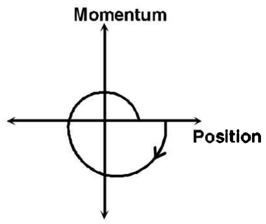
(c) $E_1 = 4E_2$

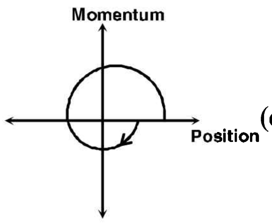
(d) $E_1 = 16E_2$

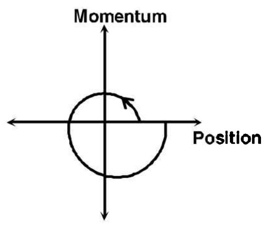
6. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(a) 


(b) 


(c) 


(d) 


## Section-B JEE Main / AIEEE

1. In a simple harmonic oscillator, at the mean position **[2002]**
(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum.

2. If a spring has time period T, and is cut into $n$ equal parts, then the time period of each part will be **[2002]**
(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$
(c) $nT$ (d) $T$

3. A child swinging on a swing in sitting position, stands up, then the time period of the swing will **[2002]**
(a) increase
(b) decrease
(c) remains same
(d) increases of the child is long and decreases if the child is short

4. A mass $M$ is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period $T$. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the ratio of $\frac{m}{M}$ is **[2003]**
(a) $\frac{3}{5}$ (b) $\frac{25}{9}$ (c) $\frac{16}{9}$ (d) $\frac{5}{3}$

5. Two particles $A$ and $B$ of equal masses are suspended from two massless springs of spring constant $k_1$ and $k_2$, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of $A$ and $B$ is **[2003]**
(a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_2}{k_1}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1}{k_2}$

6. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is **[2003]**
(a) 11% (b) 21% (c) 42% (d) 10%

7. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is **[2003]**
(a) $-4$ (b) 4 (c) $4\sqrt{2}$ (d) 8

8. A body executes simple harmonic motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement $x$. Which of the following statements is true ? **[2003]**
(a) K.E. is maximum when $x = 0$
(b) T.E is zero when $x = 0$
(c) K.E is maximum when $x$ is maximum
(d) P.E is maximum when $x = 0$

**9.** The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is $t_0$ in air. Neglecting frictional force of water and given that the density of the bob is $(4/3)\times 1000\ \text{kg/m}^3$. What relationship between $t$ and $t_0$ is true **[2004]**

(a) $t = 2t_0$ (b) $t = t_0/2$
(c) $t = t_0$ (d) $t = 4t_0$

**10.** A particle at the end of a spring executes S.H.M with a period $t_1$. while the corresponding period for another spring is $t_2$. If the period of oscillation with the two springs in series is T then **[2004]**

(a) $T^{-1} = t_1^{-1} + t_2^{-1}$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T = t_1 + t_2$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

**11.** The total energy of a particle, executing simple harmonic motion is **[2004]**

(a) independent of $x$ (b) $\propto x^2$
(c) $\propto x$ (d) $\propto x^{1/2}$
where $x$ is the displacement from the mean position, hence total energy is independent of $x$.

**12.** A particle of mass m is attached to a spring (of spring constant $k$) and has a natural angular frequency $\omega_0$. An external force $F(t)$ proportional to $\cos\omega t(\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to **[2004]**

(a) $\dfrac{1}{m(\omega_0^2+\omega^2)}$ (b) $\dfrac{1}{m(\omega_0^2-\omega^2)}$
(c) $\dfrac{m}{\omega_0^2-\omega^2}$ (d) $\dfrac{m}{(\omega_0^2+\omega^2)}$

**13.** In forced oscillation of a particle the amplitude is maximum for a frequency $\omega_1$ of the force while the energy is maximum for a frequency $\omega_2$ of the force; then **[2004]**

(a) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(b) $\omega_1 > \omega_2$
(c) $\omega_1 = \omega_2$
(d) $\omega_1 < \omega_2$

**14.** Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \dfrac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is **[2005]**

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{-\pi}{6}$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{-\pi}{3}$

**15.** The function $\sin^2(\omega t)$ represents **[2005]**

(a) a periodic, but not SHM with a period $\dfrac{\pi}{\omega}$
(b) a periodic, but not SHM with a period $\dfrac{2\pi}{\omega}$
(c) a SHM with a period $\dfrac{\pi}{\omega}$
(d) a SHM with a period $\dfrac{2\pi}{\omega}$

**16.** The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would **[2005]**

(a) first decrease and then increase to the original value
(b) first increase and then decrease to the original value
(c) increase towards a saturation value
(d) remain unchanged

**17.** If a simple harmonic motion is represented by $\dfrac{d^2x}{dt^2} + \alpha x = 0$, its time period is **[2005]**

(a) $\dfrac{2\pi}{\sqrt{\alpha}}$ (b) $\dfrac{2\pi}{\alpha}$ (c) $2\pi\sqrt{\alpha}$ (d) $2\pi\alpha$

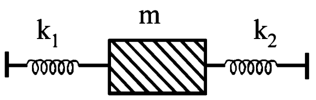
**18.** The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is **[2006]**

(a) 0.01 s (b) 10 s (c) 0.1 s (d) 100 s

**19.** Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? **[2006]**

(a) $\dfrac{1}{6}$s (b) $\dfrac{1}{4}$s (c) $\dfrac{1}{3}$s (d) $\dfrac{1}{12}$s

**20.** Two springs, of force constants $k_1$ and $k_2$ are connected to a mass $m$ as shown. The frequency of oscillation of the mass is $f$. If both $k_1$ and $k_2$ are made four times their original values, the frequency of oscillation becomes **[2007]**




(a) $2f$ (b) $f/2$ (c) $f/4$ (d) $4f$

**21.** A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is **[2007]**

(a) $2\pi^2 ma^2\nu^2$ (b) $\pi^2 ma^2\nu^2$
(c) $\dfrac{1}{4}ma^2\nu^2$ (d) $4\pi^2 ma^2\nu^2$

**22.** The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2\times 10^{-2}\cos\pi t$ metre. The time at which the maximum speed first occurs is **[2007]**

(a) 0.25 s (b) 0.5 s
(c) 0.75 s (d) 0.125 s

**23.** A point mass oscillates along the x-axis according to the law $x = x_0\cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then **[2007]**

(a) $A = x_0\omega^2,\ \delta = 3\pi/4$ (b) $A = x_0,\ \delta = -\pi/4$
(c) $A = x_0\omega^2,\ \delta = \pi/4$ (d) $A = x_0\omega^2,\ \delta = -\pi/4$

**24.** If $x$, $v$ and $a$ denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period $T$, then, which of the following does not change with time? **[2009]**

(a) $aT/x$ (b) $aT + 2\pi v$
(c) $aT/v$ (d) $a^2T^2 + 4\pi^2v^2$

**25.** Two particles are executing simple harmonic motion of the same amplitude $A$ and frequency $\omega$ along the $x$-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is: **[2011]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

**26.** A mass $M$, attached to a horizontal spring, executes S.H.M. with amplitude $A_1$. When the mass $M$ passes through its mean position then a smaller mass $m$ is placed over it and both of them move together with amplitude $A_2$. The ratio of $\left(\frac{A_1}{A_2}\right)$ is: **[2011]**

(a) $\frac{M+m}{M}$ (b) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$

(c) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$ (d) $\frac{M}{M+m}$

**27.** If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between $t = 0$s to $t = \tau$ s, then $\tau$ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with $b$ as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds : **[2012]**

(a) $\frac{0.693}{b}$ (b) b (c) $\frac{1}{b}$ (d) $\frac{2}{b}$

**28.** The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to $\alpha$ times its original magnitude, where $\alpha$ equals

(a) 0.7 (b) 0.81 **|JEE Main 2013|**
(c) 0.729 (d) 0.6

**29.** An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is $V_0$ and its pressure is $P_0$. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **|JEE Main 2013|**

(a) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

**30.** A particle moves with simple harmonic motion in a straight line. In first $\tau$ s, after starting from rest it travels a distance a, and in next $\tau$ s it travels 2a, in same direction, then: **|JEE Main 2014|**

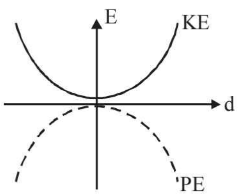
(a) amplitude of motion is 3a
(b) time period of oscillations is $8\tau$
(c) amplitude of motion is 4a
(d) time period of oscillations is $6\tau$

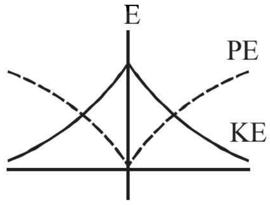
**31.** A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to $T_M$. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :
(*g = gravitational acceleration*) **[JEE Main 2015]**

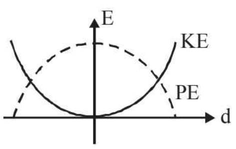
(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

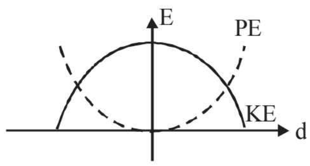
(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

**32.** For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*) **|JEE Main 2015|**

(a)



(b)



(c)



(d)



**33.** A particle performs simple harmonic mition with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is : **[JEE Main 2016[**

(a) $A\sqrt{3}$ (b) $\frac{7A}{3}$

(c) $\frac{A}{3}\sqrt{41}$ (d) 3A

# 10 Simple Harmonic Motion (Oscillations)

## Section-A : JEE Advanced/ IIT-JEE

**A** **1.** 0.06

**C** **1.** (d) **2.** (d) **3.** (a) **4.** (a) **5.** (a) **6.** (b) **7.** (d) **8.** (c) **9.** (d)
**10.** (b) **11.** (a)

**D** **1.** (c) **2.** (b, c) **3.** (b) **4.** (d) **5.** (b) **6.** (a) **7.** (a, c) **8.** (a, b, c) **9.** (a, d)
**10.** (b, d) **11.** (a,b,c)

**E** **1.** 1.6 kg **2.** $\sqrt{\frac{k}{m_2}}, \frac{m_1 g}{k}$ **3.** 2.83 sec. **4. (a)** $\theta = \tan^{-1}\frac{1}{5}$ **(b)** $\frac{2\pi\sqrt{1.803R}}{\sqrt{g}}$

**5.** **(i)** $\frac{1}{\pi}s^{-1}$ **(ii)** $0.02\pi$ m/s **(iii)** $3.95 \times 10^{-4}$ J **6.** $\sqrt{\frac{3(d_2-d_1)g}{2d_1L}}$ **8.** $y^* = \frac{g}{\omega^2}$

**F** **1.** (A) → p; (B) → q, r; (C) → p; (D) → q, r **2.** (A) → p; (B) → q, r, s; (C) → s; (D) → q

**G** **1.** (c) **2.** (b) **3.** (d) **4.** (d) **5.** (c) **6.** (b)

## Section-B : JEE Main/ AIEEE

**1.** (c) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (d) **7.** (c) **8.** (a) **9.** (a) **10.** (b) **11.** (a) **12.** (b)
**13.** (c) **14.** (b) **15.** (c) **16.** (b) **17.** (a) **18.** (a) **19.** (a) **20.** (a) **21.** (b) **22.** (b) **23.** (a) **24.** (a)
**25.** (d) **26.** (c) **27.** (d) **28.** (c) **29.** (c) **30.** (d) **31.** (c) **32.** (d) **33.** (d)

## Section-A JEE Advanced/ IIT-JEE

### A. Fill in the Blanks

**1.** $x = 0.04$ m, K.E. = 0.5 J and P.E. = 0.4 J
T.E. = (0.5 + 0.4) J = 0.9 J

Now, T.E. $= \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}m \times 4\pi^2 v^2 a^2$

$\Rightarrow 0.9 = \frac{1}{2} \times 0.2 \times 4\pi^2 \times \frac{25}{\pi} \times \frac{25}{\pi} \times a^2 \Rightarrow a = 0.06\text{m}$

### C. MCQs with ONE Correct Answer

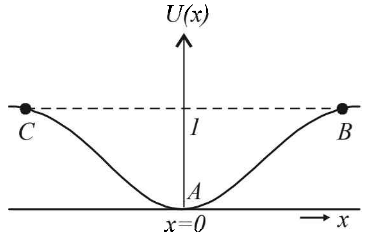
**1.** **(d)** Both the bodies oscillate in simple harmonic motion, for which the maximum velocities will be
Given that $v_1 = v_2 \Rightarrow a_1\omega_1 = a_2\omega_2$

$\therefore \quad a_1 \times \frac{2\pi}{T_1} = a_2 \times \frac{2\pi}{T_2}$

$$\Rightarrow \frac{a_1}{a_2} = \frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{m}{k_1}}}{2\pi\sqrt{\frac{m}{k_2}}} = \sqrt{\frac{k_2}{k_1}}$$

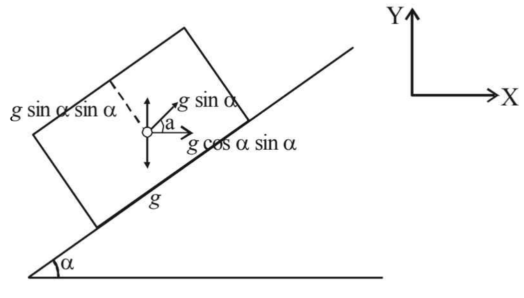
**2.** **(d)** Let us plot the graph of the mathematical equation

$$U(x) = K\left[1 - e^{-x^2}\right], \quad F = -\frac{dU}{dx} = 2kxe^{-x^2}$$




From the graph it is clear that the potential energy is minimum at $x = 0$. Therefore, $x = 0$ is the state of stable equilibrium. Now if we displace the particle from $x = 0$ then for displacements the particle tends to regain the position $x = 0$ with a force $F = \frac{2kx}{e^{x^2}}$. Therefore for small values of $x$ we have $F \propto x$.

**3.** **(a)** As shown in the figure, $g\sin\alpha$ is the pseudo acceleration applied by the observer in the accelerated frame




$a_x = g\sin\alpha\cos\alpha$

$a_y = g - g\sin^2\alpha = g(1 - \sin^2\alpha) = g\cos^2\alpha$

$a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{g^2\sin^2\alpha\cos^2\alpha + g^2\cos^4\alpha}$

$= g\cos\alpha\sqrt{\sin^2\alpha + \cos^2\alpha} = g\cos\alpha$

$\therefore \quad T = 2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$

**NOTE :** Whenever point of suspension is accelerating use $T = 2\pi\sqrt{\dfrac{L}{g_{eff}}}$

**4. (a)** The velocity of a body executing S.H.M. is maximum at its centre and decreases as the body proceeds to the extremes. Therefore if the time taken for the body to go from $O$ to $A/2$ is $T_1$ and to go from $A/2$ to $A$ is $T_2$ then obviously
$T_1 < T_2$

**5. (a)** NOTE : In S.H.M., at extreme position, $P.E.$ is maximum when
$t = 0, x = A$.
i.e., at time $t = 0$, the particle executing S.H.M. is at its extreme position.
Therefore P.E. is max. The graph I and III represent the above characteristics.

**6. (b)** $y = kt^2$

$\therefore \quad \dfrac{dy}{dt} = 2kt \quad \text{or} \quad \dfrac{d^2y}{dt^2} = 2k$

or $a_g = 2\,\text{m/s}^2 \quad (\because k = 1\ \text{m/s}^2 \text{ given})$

We know that $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

$\therefore \quad \dfrac{T_1^2}{T_2^2} = \dfrac{g_2}{g_1} \Rightarrow \dfrac{T_1^2}{T_2^2} = \dfrac{12}{10} = \dfrac{6}{5}$

$[\because g_1 = 10\ \text{m/s}^2 \text{ and } g_2 = g + 2 = 12\ \text{m/s}^2]$

**7. (d)** From the graph it is clear that the amplitude is 1 cm and the time period is 8 second. Therefore the equation for the S.H.M. is

$$x = a\sin\left(\frac{2\pi}{T}\right)\times t = 1\sin\left(\frac{2\pi}{8}\right)t = \sin\frac{\pi}{4}t$$

The velocity ($v$) of the particle at any instant of time '$t$' is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[\sin\left(\frac{\pi}{4}\right)t\right] = \frac{\pi}{4}\cos\left(\frac{\pi}{4}\right)t$$

The acceleration of the particle is

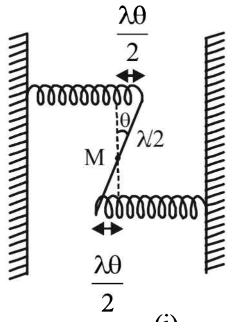
$$\frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2\sin\left(\frac{\pi}{4}\right)t$$

At $t = \dfrac{4}{3}s$ we get

$$\frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2\sin\frac{\pi}{4}\times\frac{4}{3} = \frac{-\pi^2}{16}\sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{3}\pi^2}{32}\,\text{cm/s}^2$$

**8. (c)** Figure shows the rod at an angle θ with respect to its equilibrium position. Both the springs are stretched by length $\dfrac{\ell\theta}{2}$.




The restoring torque due to the springs
τ = – 2 (Restoring force) × perpendicular distance

$$\tau = -2k\left(\frac{\ell\theta}{2}\right)\times\frac{\ell}{2} = -k\frac{\ell^2}{2}\theta \quad \text{... (i)}$$

If $I$ is the moment of inertia of the rod about $M$ then

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \quad \text{... (ii)}$$

From (i) & (ii) we get

$$I\frac{d^2\theta}{dt^2} = -k\frac{\ell^2\theta}{2} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{k}{I}\frac{\ell^2}{2}\theta = \frac{-k}{M\ell^2/12}\frac{\ell^2}{2}\theta$$

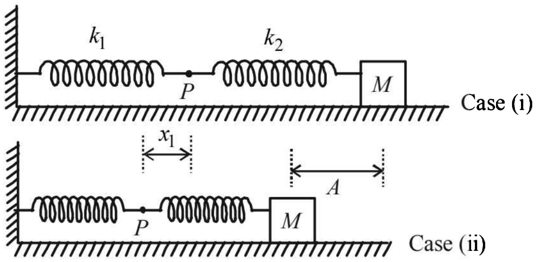
$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{6k}{M}\theta$$

Comparing it with the standard equation of rotational SHM we get

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \Rightarrow \omega^2 = \frac{6k}{M} \Rightarrow \omega = \sqrt{\frac{6k}{M}}$$

$$\Rightarrow 2\pi\nu = \sqrt{\frac{6k}{M}} \Rightarrow \nu = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

**9. (d)**




In case (ii), the springs are shown in the maximum compressed position. If the spring of spring constant $k_1$ is compressed by $x_1$ and that of spring constant $k_2$ is compressed by $x_2$ then

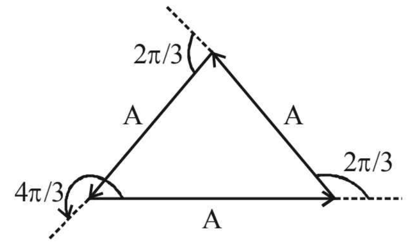
$$x_1 + x_2 = A \quad \text{... (i)}$$

and $k_1x_1 = k_2x_2 \Rightarrow x_2 = \dfrac{k_1x_1}{k_2}$ ...(ii)

From (i) & (ii)

$$x_1 + \frac{k_1x_1}{k_2} = A \Rightarrow x_1 = \frac{k_2A}{k_2 + k_1}$$

**10. (b)** Two sinusoidal displacements have amplitude A each, with a phase difference of $2\dfrac{\pi}{3}$. It is given that sinusoidal displacement $x_3(t)$ brings the mass to a complete rest. This is possible when the amplitude of third is A and is having a phase difference of $4\dfrac{\pi}{3}$ with respect to $x_1$ (t) as shown in the figure.




11. (a) $T=\dfrac{2V\sin\theta}{g} \quad \therefore \quad 1=\dfrac{2V\sin 45^\circ}{g} \quad \therefore v=\sqrt{50}\ \text{ms}^{-1}$

## D. MCQs with ONE or MORE THAN ONE Correct

1. (c) **NOTE :** During one complete oscillation, the kinetic energy will become maximum twice.
Therefore the frequency of kinetic energy will be $2f$.

2. (b, c) The total energy of the oscillator

$$=\frac{1}{2}kA^2 = \text{Max. K.E.}$$

$$=\frac{1}{2}\times 2\times 10^6 \times (0.01)^2 = 100\,\text{J}$$

As total mechanical energy = 160 J
The P.E. at equilibrium position is not zero.
P.E. at mean position = (160 – 100) J = 60 J
∴ Max P.E. = (100 + 60 ) J = 160 J.

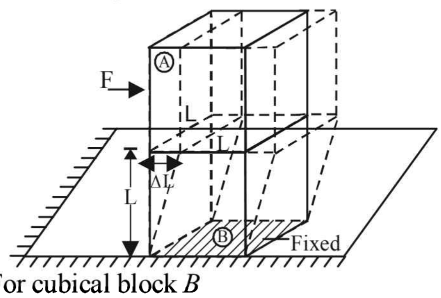
| Extreme position | Mean position |
|---|---|
| K.E. = 0 | K.E. = 100J |
| P. E. = 160J | P. E. = 60J |

3. (b) If $x$ is the displacement then,

$$\therefore \quad M\omega^2 x = [\rho A g + k]x$$

$$\Rightarrow \omega = \left[\frac{\rho A g + k}{M}\right]^{1/2} \quad \Rightarrow v = \frac{1}{2\pi}\left[\frac{\rho A g + k}{M}\right]^{1/2}$$

4. (d) **NOTE :** When a force is applied on cubical block $A$ in the horizontal direction then the lower block $B$ will get distorted as shown by dotted lines and $A$ will attain a new position (without distortion as $A$ is a rigid body) as shown by dotted lines.




For cubical block $B$

$$\eta = \frac{F/A}{\Delta L/L} = \frac{F}{A}\times\frac{L}{\Delta L} = \frac{F}{L^2}\times\frac{L}{\Delta L} = \frac{F}{L\times\Delta L}$$

⇒ $F = \eta L \Delta L$
$\eta L$ is a constant
⇒ Force $F \propto \Delta L$ and directed towards the mean position, oscillation will be simple harmonic in nature.
Here, $M\omega^2 = \eta L$

$$\Rightarrow \omega = \sqrt{\frac{\eta L}{M}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{\eta L}{M}} \Rightarrow T = 2\pi\sqrt{\frac{M}{\eta L}}$$

5. (b) Let us consider the wire also as a spring. Then the case becomes that of two spring attached in series. The equivalent spring constant is

$$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{k'}$$

where $k'$ is the spring constant of the wire

Now, $Y = \dfrac{F/A}{\Delta L/L} = \dfrac{F}{A}\times\dfrac{L}{\Delta L}$

or $\dfrac{F}{\Delta L} = \dfrac{YA}{L}$ or, $k' = \dfrac{YA}{L}$

We know that time period of the system

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{m\left[\frac{1}{k}+\frac{1}{k'}\right]}$$

$$\Rightarrow T = 0\,2\pi\sqrt{m\left[\frac{1}{k}+\frac{L}{YA}\right]} = 2\pi\sqrt{\frac{m\,(YA+kL)}{kYA}}$$

6. (a) $U(x) = k|x|^3$

$$\therefore \quad [k] = \frac{[U]}{[x^3]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

Now time period may depend on $T \propto (\text{mass})^x (\text{amplitude})^y (k)^z$

∴ $[M^0L^0T] = [M]^x[L]^y[ML^{-1}T^{-2}]^z$
$= [M^{x+z}L^{y-z}T^{-2z}]$

Equating the powers, we get
$-2z = 1$ or $z = -1/2$
$y - z = 0$ or $y = z = -1/2$
Hence $T \propto (\text{amplitude})^{-1/2} \propto a^{-1/2}$

7. (a, c) From superposition principle

$y = y_1 + y_2 + y_3$
$= a\sin\omega t + a\sin(\omega t + 45^\circ) + a\sin(\omega t + 90^\circ)$
$= a[\sin\omega t + \sin(\omega t + 90^\circ)] + a\sin(\omega t + 45^\circ)$
$= 2a\sin(\omega t + 45^\circ)\cos 45^\circ + a\sin(\omega t + 45^\circ)$
$= (\sqrt{2}+1)\,a\sin(\omega t + 45^\circ)$
$= A\sin(\omega t + 45^\circ)$

Therefore resultant motion is simple harmonic of amplitude $A = (\sqrt{2}+1)\,a$
and which differ in phase by 45° relative to the first.

Energy in SHM ∝ (amplitude)$^2$ $\left[\because E = \dfrac{1}{2}mA^2\omega^2\right]$

$$\therefore \quad \frac{E_{resultant}}{E_{single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$$\therefore \quad E_{resultant} = (3+2\sqrt{2})\,E_{single}$$

8. (a, b, c) The given equation is
$x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$

**NOTE THIS STEP**

Rearranging the equation in a meaningful form (for interpretation of SHM)

$$x = \frac{A}{2}(2\sin^2\omega t) + \frac{B}{2}(2\cos^2\omega t) + \frac{C}{2}(2\sin\omega t\cos\omega t)$$

$$= \frac{A}{2}[1-\cos 2\omega t] + \frac{B}{2}[1+\cos 2\omega t] + \frac{C}{2}[\sin 2\omega t]$$

(a) For $A=0$ and $B=0, x=\dfrac{C}{2}\sin(2\omega t)$

The above equation is that of SHM with amplitude $\dfrac{C}{2}$ and angular frequency $2\omega$. Thus option (a) is correct.

(b) If $A=B$ and $C=2B$ then $x=B+B\sin 2\omega t$

This is equation of SHM. The mean position of the particle executing SHM is not at the origin.

Option (b) is correct.

(c) $A=-B, C=2B$; Therefore

$x=B\cos 2\omega t+B\sin 2\omega t$

Let $B=X\cos\phi=X\sin\phi$ then

$x=X\sin 2\omega t\cos\phi+X\cos 2\omega t\sin\phi$

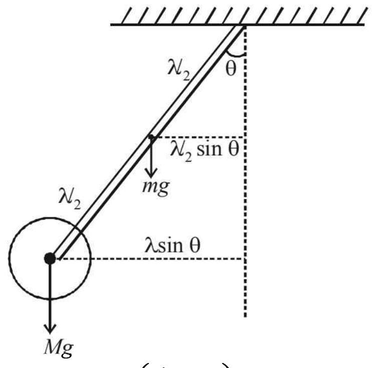
This represents equation of SHM.

(d) $A=B, C=0$ and $x=A$. This equation does not represents SHM.

**9. (a,d)**

Applying $\tau=I\alpha$

**For case A:** $mg\left(\dfrac{\ell}{2}\sin\theta\right)+Mg(\ell\sin\theta)=I_A\alpha_A$




**For case B:** $mg\left(\dfrac{\ell}{2}\sin\theta\right)+Mg(\ell\sin\theta)=I_B\alpha_B$

The restoring torque in both the cases is same.

Also $I_A>I_B \quad \therefore \alpha_A<\alpha_B$

$\therefore \omega_A<\omega_B$

(a) and (d) are correct options.

**10. (b,d)**

Maximum linear momentum in case 1 is $(p_1)_{max}=mv_{max}$

$b=m[aw_1]$ ...(i)

Maximum linear momentum in case 2 is $(p_2)_{max}=mv_{max}$

$R=m[R\omega_2]$

$\therefore 1=m\omega_2$ ...(ii)

Dividing (i) & (ii) $\quad \dfrac{b}{1}=\dfrac{a\omega_1}{\omega_2}$

$\therefore \dfrac{\omega_1}{\omega_2}=\dfrac{b}{a}=\dfrac{1}{n^2}$ $\quad\therefore$ B is a correct option.

Also $E_1=\dfrac{1}{2}m\omega_1^2a^2$

$E_2=\dfrac{1}{2}m\omega_2^2R^2$

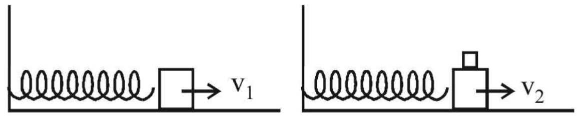
$\therefore \dfrac{E_1}{E_2}=\dfrac{\omega_1^2}{\omega_2^2}\times\dfrac{a^2}{R^2}=\dfrac{\omega_1^2}{\omega_2^2}\times\dfrac{1}{n}=\dfrac{\omega_1^2}{\omega_2^2}\times\dfrac{\omega_2}{\omega_1}=\dfrac{\omega_1}{\omega_2}$

$\therefore \dfrac{E_1}{\omega_1}=\dfrac{E_2}{\omega_2}$ D is the correct option

**11. (a, b, d)**

**Case (i) :** Applying conservation of linear momentum.

$MV_1=(M+m)V_2 \qquad -(1) \therefore V_2=\left(\dfrac{M}{M+m}\right)V_1$




From (1)

$M(A_1\times\omega_1)=(M+m)(A_2\times\omega_2)$

$\therefore MA_1\times\sqrt{\dfrac{K}{M}}=(M+m)A_2\times\sqrt{\dfrac{K}{M+m}}$

$\therefore A_2=\sqrt{\dfrac{M}{M+m}}=A_1$

Also $E_1=\dfrac{1}{2}MV_1^2$

and $\quad E_2=\dfrac{1}{2}(M+m)V_2^2=\dfrac{1}{2}(M+m)$

$\times\dfrac{M^2V_1^2}{(M+m)^2}=\dfrac{1}{2}\left(\dfrac{M}{M+m}\right)^2V_1^2$

Clearly $E_2<E_1$

The new time Period $T_2=2\sqrt{\dfrac{m+M}{K}}$

**Case (ii) :** The new time Period $T_2=2\sqrt{\dfrac{m+M}{K}}$

Also $A_2=A_1$

Here $E_2=E_1$

The instantaneous value of speed at $X_o$ of the combined masses decreases in both the cases.

## E. Subjective Problems

**1. Key Concept:**

The time period $T$ of the spring is $T=2\pi\sqrt{\dfrac{M}{k}}$

or, $2=2\pi\sqrt{\dfrac{M}{k}}$ .... (i)

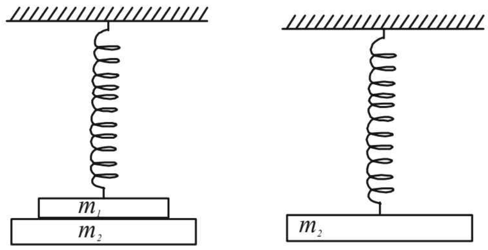
In the second case

$3=2\pi\sqrt{\dfrac{M+2}{k}}$ ... (ii)

From (i) and (ii)

$\dfrac{2}{3}=\sqrt{\dfrac{M}{M+2}} \Rightarrow \dfrac{4}{9}=\dfrac{M}{M+2} \Rightarrow M=1.6$ kg.

**2.**



**NOTE**

When mass $m_1$ is removed then the equilibrium will get disturbed. There will be a restoring force in the upward direction. The body will undergo S.H.M. now.

Let $x_1$ be the extension of the spring when $(m_1+m_2)$ are suspended and $x_2$ be the extension of spring when $m_1$ is removed.

$\therefore \quad kx_1 = (m_1+m_2)g \quad$ or $\quad x_1 = \dfrac{(m_1+m_2)g}{k}$

and, $kx_2 = m_2 g \quad$ or $\quad x_2 = \dfrac{m_2 g}{k}$

Amplitude of oscillation $= x_1 - x_2$

or $\quad A = \dfrac{(m_1+m_2)g - m_2 g}{k} = \dfrac{m_1 g}{k}$

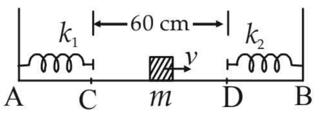
Let at any instant the mass $m_2$ be having a displacement $x$ from the mean position. Restoring force will be

$$F = -kx \text{ or } m_2 a = -kx \Rightarrow a = -\frac{k}{m_2}x$$

Comparing this with $a = -\omega^2 x$,

we get $\omega^2 = \dfrac{k}{m_2} \Rightarrow \omega = \sqrt{\dfrac{k}{m_2}}$

**3.** The mass will strike the right spring, compress it. The K.E. of the mass will convert into P.E. of the spring. Again the spring will return to its natural size thereby converting its P.E. to K.E. of the block. The time taken for this process will be $\dfrac{T}{2}$, where $T = 2\pi\sqrt{\dfrac{m}{k}}$.




$\therefore \quad t_1 = \dfrac{T}{2} = \pi\sqrt{\dfrac{m}{k_2}} = \pi\sqrt{\dfrac{0.2}{3.2}} = 0.785$ sec

The block will move from A to B without any acceleration. The time taken will be

$$t_2 = \frac{60}{120} = 0.5$$

Now the block will compress the left spring and then the spring again attains its natural length. The time taken will be

$$t_3 = \pi\sqrt{\frac{m}{k_1}} = \pi\sqrt{\frac{0.2}{1.8}} = 1.05 \text{ sec.}$$

Again the block moves from B to A, completing one oscillation. The time taken for doing so

$$t_4 = \frac{60}{120} = 0.5$$

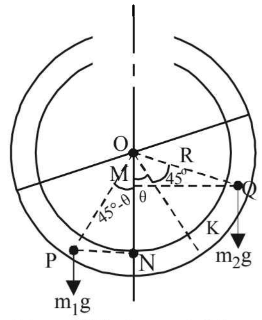
$\therefore$ The complete time of oscillation will be

$= t_1 + t_2 + t_3 + t_4$

$= 0.785 + 0.5 + 1.05 + 0.5$

$= 2.83$ (app.)

**4.**




At equilibrium, taking torque of liquids about O

$$\begin{pmatrix}\text{Torque due to}\\ \text{liquid of density } \rho\end{pmatrix} = \begin{pmatrix}\text{Torque due to}\\ \text{liquid of density } 1.5\rho\end{pmatrix}$$

$m_2 g \times QM = m_1 g \times PN$

$\therefore \quad m_2 g R \sin(45° + \theta) = m_1 g R \sin(45° - \theta)$

$V\rho g R \sin(45° + \theta) = 1.5 V\rho g R \sin(45° - \theta)$... (i)

$\Rightarrow \dfrac{\sin(45+\theta)}{\sin(45-\theta)} = 1.5$

$\Rightarrow \dfrac{\sin 45° \cos\theta + \cos 45° \sin\theta}{\sin 45° \cos\theta - \cos 45° \sin\theta} = \dfrac{3}{2}$

$\Rightarrow \tan\theta = \dfrac{1}{5}$

**NOTE THIS STEP :** Let us now displace the liquids in anticlockwise direction along the circumference of tube through an angle $\alpha$.

The net torque

$\tau = m_2 g R \sin(45° + \theta + \alpha) - m_1 g R \sin(45° - \theta - \alpha)$

$= V\rho g R \sin(45° + \theta + \alpha) - 1.5 V\rho g R (45° - \theta - \alpha)$

$= V\rho g R \sin(\theta + 45°)\cos\alpha + V\rho g R \cos(45° + \theta)\sin\alpha - 1.5 V\rho g R \sin(45° - \theta)\cos\alpha + 1.5 V\rho g R \cos(45° - \theta)\sin\alpha$

Using eq. (i) we get

$\tau = V\rho g R[\cos(45° + \theta)\sin\alpha + 1.5\cos(45° - \theta)\sin\alpha]$

$\tau = V\rho g R[\cos(45° + \theta) + 1.5\cos(45° - \theta)]\sin\alpha$

when $\alpha$ is small (given)

$\therefore \quad \sin\alpha \approx \alpha$

$\tau = V\rho g R[\cos(45° + \theta) + 1.5\cos(45° - \theta)]\alpha$

Since, $\tau$ and $\alpha$ are proportional and directed towards mean position.

$\therefore$ The motion is simple harmonic.

Moment of inertia about $O$ is

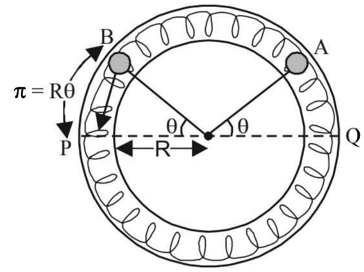
$I = V\rho R^2 + 1.5 V\rho R^2$

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$= 2\pi\frac{\sqrt{(V\rho \times 2.5R^2)}}{\sqrt{[\cos(45+\theta) + 1.5\cos(45-\theta)]V\rho g R}}$$

$= 2\pi\dfrac{\sqrt{1.803R}}{\sqrt{g}}$ (using the value $\tan\theta = \dfrac{1}{5}$)

**5.** (i) As both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter $PQ$ of the circle and released from rest. It results into compression of spring in upper segment and an equal elongation of spring in lower segment. Let it be $x$. $PB$ and $QA$ denote $x$ in the figure.




Compression = $R\theta$ = elongation = $x$

∴ Force exerted by each spring on each ball = 2 $kx$

∴ Total force on each ball due to two springs = 4 $kx$

∴ Restoring torque about origin $O = -(4kx)R$

∴ $\tau = -4k(R\theta)R$, where $\theta$ = Angular displacement

or $\tau = -4kR^2\theta$

Since torque ($\tau$) is proportional to $\theta$, each ball executes angular SHM about the centre $O$.

Again, $\tau = -4kR^2\theta$

or $I\alpha = -4kR^2\theta$ where $\alpha$ = angular acceleration

or $(mR^2)\alpha = -4kR^2\theta$ or $\alpha = -\left(\frac{4k}{m}\right)\theta$

∴ Frequency $f = \frac{1}{2\pi}\sqrt{\frac{\alpha}{\theta}}$

∴ Frequency of each ball $= \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$

$$= \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi}\text{ sec}^{-1} \quad ..(ii)$$

(ii) Let velocity at the mean position be $v_{max}$.

Loss in elastic potential energy = Gain in kinetic energy

$$2\left[\frac{1}{2}K\left(2R\frac{\pi}{6}\right)^2\right] = 2\times\left[\frac{1}{2}mv_{max}^2\right]$$

$$\therefore \quad v_{max} = \sqrt{\frac{K}{m}}\times\frac{R\pi}{3} = 0.02\ \pi \text{ m/s}$$

(iii) Total energy $= 2\left[\frac{1}{2}mv_{max}^2\right]$

$$= 2\left[\frac{1}{2}\times 0.1(0.02\pi)^2\right]$$

$$= 3.95\times 10^{-4}\text{ J}$$

**6.** Let $\theta$ be the angle made by the rod at any instant $t$.

The volume of the rod = $LS$

Weight of the rod = $LS\,d_1 g$

Upthrust acting on rod = $LS\,d_2 g$

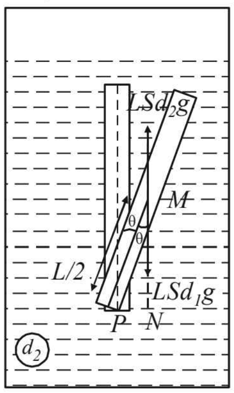
Since, $d_2 > d_1$ (given). Therefore net force acting at the centre of mass of the rod at tilted position is

$(LS\,d_2 g - LSd_1 g)$

Taking moment of force about $P$

$\tau = (LSd_2 g - LSd_1 g)\times PN$

where $PN$ = perpendicular distance of line of action of force from $P$




$$\therefore \quad \tau = LSg\,(d_2 - d_1)\times\frac{L}{2}\sin\theta$$

when $\theta$ is small, $\sin\theta \approx \theta$

$$\therefore \quad \tau = \frac{L^2 Sg}{2}(d_2 - d_1)\theta. \text{ Since, } \tau \propto \theta$$

∴ Motion is simple harmonic.

On comparing it with $\tau = C\theta$

We get $C = \frac{L^2 Sg}{2}(d_2 - d_1)$

$$\Rightarrow \quad I\omega^2 = \frac{L^2 Sg}{2}(d_2 - d_1) \quad ...(i)$$

The moment of inertia $I$ of the rod about $P$

$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$

$$I = \frac{1}{3}ML^2 = \frac{1}{3}LSd_1L^2 \quad ...(ii)$$

From (i) and (ii)

$$\omega^2\times\frac{L^3}{3}Sd_1 = \frac{L^2 Sg}{2}(d_2 - d_1)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3Sg(d_2 - d_1)}{2LSd_1}} \Rightarrow \omega = \sqrt{\frac{3(d_2 - d_1)g}{2d_1 L}}$$

**7.** If a small mass is attached to one end of a vertically hanging spring then it performs SHM.

Angular frequency = $\omega$, Amplitude = $a$

Under SHM, velocity $v = \omega\sqrt{a^2 - y^2}$

After detaching from spring, net downward acceleration of the block = $g$.

∴ Height attained by the block = $h$

$$\therefore \quad h = y + \frac{v^2}{2g} \text{ or } h = y + \frac{\omega^2(a^2 - y^2)}{2g}$$

For $h$ to be maximum, $\frac{dh}{dy} = 0, y = y^*$.

$$\therefore \quad \frac{dh}{dy} = 1 + \frac{\omega^2}{2g}(-2y^*) \text{ or } 0 = 1 - \frac{2\omega^2 y^*}{2g}$$

$$\text{or} \quad \frac{\omega^2 y^*}{g} = 1 \text{ or } y^* = \frac{g}{\omega^2}$$

Since $a\omega^2 > g$ (given)

$\therefore\ a > \frac{g}{\omega^2}\ \therefore a > y^*. y^*$ from mean position $< a$.

Hence $y^* = \frac{g}{\omega^2}$.

## F. Match the Following

**1.** **A→p**

**Reason :** For a simple harmonic motion $v = a\sqrt{\omega^2 - x^2}$. On comparing it with $v = c_1\sqrt{c_2 - x^2}$ we find the two comparable.

**B → q, r**

**Reason :** $v = -kx$

when $x$ is positive; $v$ is – ve, and as $x$ decreases, $v$ decreases. Therefore kinetic energy will decreases. When $x = 0$, $v = 0$. Therefore the object does not change its direction.

When $x$ is negative, $v$ is positive. But as $x$ decreases in magnitude, $v$ also decreases. Therefore kinetic energy decreases. When $x = 0$, $v = 0$. Therefore the object does not change its direction.

**C → p**

**Reason :** When $a = 0$, let the spring have an extension $x$. Then $kx = mg$.

When the elevator starts going upwards with a constant acceleration, as seen by the observer in the elevator, the object is at rest.

$\therefore \quad ma + mg = kx'$

$\Rightarrow \quad ma = k(x' - x)$ (Since $a$ is constant)

**D → q, r**

The speed is $\sqrt{2}$ times the escape speed. Therefore the object will leave the earth. It will therefore not change the direction and its kinetic energy will keep on decreasing.

**2.** **A → p**

When simple pendulum is displaced from the mean position towards any of the extreme position, its potential energy increases. In case of a S.H.M. we get a parabola for potential energy versus displacement

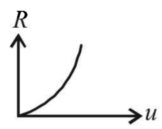
**B → q, r, s**

S = ut for zero acceleration. Therefore we get a straight line passing through the origin. Therefore option (q) is correct.

$S = ut + \frac{1}{2}at^2$ for constant positive acceleration. In this case we get a part of parabola as a graph line between $s$ versus $t$ as shown by graph (s).

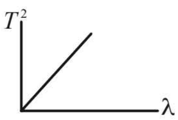
(p) is ruled out because if $a$ is –ve and $v$ is positive.

**C → s**




$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R \propto u^2$ (for consant $\theta$ and $g$)

**D → q** $T = 2\pi\sqrt{\frac{\ell}{g}} \quad \therefore T^2 \propto \ell$




## G. Comprehension Based Questions

**1.** **(c)** If the energy is zero, the particle will not perform oscillations. Therefore E should be greater than zero. Further if $E = V_0$, the potential energy will become constant as depicted in the graph given. In this case also the particle will not oscillate.

Therefore $E$ should be greater than zero but less than $V_0$.

**2.** **(b)** We can get the answer of this question with the help of dimensional analysis.

Given potential energy = $\alpha x^4$

$$\alpha = \frac{\text{Potential energy}}{x^4} = \frac{ML^2T^{-2}}{L^4} = [ML^{-2}T^{-2}]$$

$$\text{Now } \frac{1}{A}\sqrt{\frac{m}{\alpha}} = \frac{1}{L}\sqrt{\frac{M}{ML^{-2}T^{-2}}} = T$$

Therefore option (b) is correct.

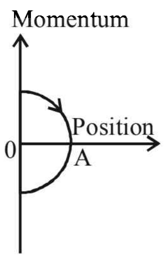
**3.** **(d)** $F = \frac{-dV(x)}{dx}$

As $V(x)$ = constant for $x > X_0$

$\therefore F = 0$ for $x > X_0$

Since $F = 0$, $a = 0$

**4.** **(d)** When the ball is thrown upwards, at the point of throw (O) the linear momentum is in upwards direction (and has a maximum value) and the position is zero. As the time passes, the ball moves upwards and its momentum goes on decreasing and the position becomes positive. The momentum becomes zero at the topmost point (A).




As the time increases, the ball starts moving down with an increasing linear momentum in the downward direction (negative) and reaches back to its original position.

These characteristics are represented by graph (d).

**5.** **(c)** We know that the

total mechanical energy $\propto$ (Amplitude)$^2$

$\therefore \quad E_1 \propto (2a)^2$

& $\quad E_2 \propto a^2$

$\therefore \quad \frac{E_1}{E_2} = 4$

**6.** **(b)** As the mass is osicillating in water its amplitude will go on decreasing and the amplitude will decrease with time. Options (c) and (d) cannot be true.

When the position of the mass is at one extreme end in the positive side (the topmost point), the momentum is zero. As the mass moves towards the mean position the momentum increases in the negative direction. These characteristics are depicted in option (b).

## Section-B **JEE Main/ AIEEE**

**1.** **(c)** $K.E = \frac{1}{2}k(A^2 - x^2)$; $U = \frac{1}{2}kx^2$

**At the mean position** $x = 0$

$\therefore$ K.E. $= \frac{1}{2}kA^2 =$ Maximum and $U = 0$

**2.** **(b)** Let the spring constant of the original spring be $k$.

Then its time period $T = 2\pi\sqrt{\frac{m}{k}}$ where m is the mass of oscillating body.

When the spring is cut into $n$ equal parts, the spring constant of one part becomes $nk$. Therefore the new time period,

$$T' = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}$$

**3.** **(b)** **KEY CONCEPT :** The time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ where

$\ell$ = distance between the point of suspension and the centre of mass of the child. This distance decreases when the child stands

$\therefore T' < T$ i.e., the period decreases.

**4.** **(c)** $T = 2\pi\sqrt{\frac{M}{k}}$

$$T' = 2\pi\sqrt{\frac{M+m}{k}} = \frac{5T}{3}$$

$$\therefore 2\pi\sqrt{\frac{M+m}{k}} = \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}} \Rightarrow \frac{m}{M} = \frac{16}{9}$$

**5.** **(c)** Maximum velocity during SHM $= A\omega = A\sqrt{\frac{k}{m}}$

$$\left[\because \omega = \sqrt{\frac{k}{m}}\right]$$

Here the maximum velocity is same and m is also same

$$\therefore A_1\sqrt{k_1} = A_2\sqrt{k_2} \qquad \therefore \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

**6.** **(d)** $T = 2\pi\sqrt{\frac{\ell}{g}}$ and $T' = 2\pi\sqrt{\frac{1.21\ell}{g}}$

$(\because \ell' = \ell + 21\% \text{ of } \ell)$

$$\% \text{ increase} = \frac{T'-T}{T} \times 100 = \frac{\sqrt{1.21\ell} - \sqrt{\ell}}{\sqrt{\ell}} \times 100 = 10\%$$

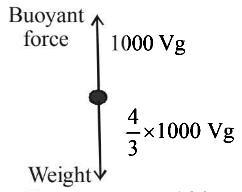
**7.** **(c)** $x = 4(\cos \pi t + \sin \pi t) = \sqrt{2} \times 4\left(\frac{\sin \pi t}{\sqrt{2}} + \frac{\cos \pi t}{\sqrt{2}}\right)$

$$x = 4\sqrt{2}\sin(\pi t + 45°)$$

**8.** **(a)** K.E. $= \frac{1}{2}m\omega^2(a^2 - x^2)$

When $x = 0$, K.E is maximum and is equal to $\frac{1}{2}m\omega^2 a^2$.

**9.** **(a)** $t = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$ ; $t_0 = 2\pi\sqrt{\frac{\ell}{g}}$




$mg_{eff} = mg - B = my - V \times 100 \times g$

$$\therefore g_{eff} = g - \frac{100}{(m/v)}g = g - \frac{1000}{\frac{4}{3}\times 1000}g = \frac{g}{4}$$

$$\therefore t = 2\pi\sqrt{\frac{\ell}{g/4}} \qquad t = 2t_0$$

**10.** **(b)** For first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$,

For second spring, $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

when springs are in series then, $k_{eff} = \frac{k_1 k_2}{k_l + k_2}$

$$\therefore T = 2\pi\sqrt{\frac{m(k_l + k_2)}{k_1 k_2}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{k_2} + \frac{m}{k_1}} = 2\pi\sqrt{\frac{t_2^2}{(2\pi)^2} + \frac{t_1^2}{(2\pi)^2}}$$

$$\Rightarrow T^2 = t_1^2 + t_2^2$$

where $x$ is the displacement from the mean position

**11.** **(a)** At any instant the total energy is

$\frac{1}{2}kA^2$ = constant, where $A$ = amplitude

hence total energy is independent of $x$.

**12.** **(b)** Equation of displacement is given by

$x = A\sin(\omega t + \phi)$

where $A = \dfrac{F_0}{m(\omega_0{}^2 - \omega^2)}$

**13.** **(c)** The maximum of amplitude and energy is obtained when the frequency is equal to the natural frequency (resonance condition)

$\therefore \omega_1 = \omega_2$

**14.** **(b)** $v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

$$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin \pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$$

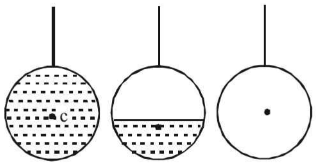
$$\therefore \text{Phase diff.} = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$$

**15.** **(c)** $y = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$

**16. (b)** Centre of mass of combination of liquid and hollow portion (at position $\ell$), first goes down (to $\ell + \Delta\ell$) and when total water is drained out, centre of mass regain its original position (to $\ell$),

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\therefore$ '$T$' first increases and then decreases to original value.



**17. (a)** $\frac{d^2x}{dt^2} = -\alpha x = -\omega^2 x$

$\Rightarrow \omega = \sqrt{\alpha}$ or $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$

**18. (a)** Maximum velocity,

$v_{max} = a\omega, \quad v_{max} = a \times \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01\ s$

**19. (a)** K.E. of a body undergoing SHM is given by,

$K.E. = \frac{1}{2}ma^2\omega^2\cos^2\omega t, \quad T.E. = \frac{1}{2}ma^2\omega^2$

Given K.E. = 0.75 T.E.

$\Rightarrow 0.75 = \cos^2\omega t \Rightarrow \omega t = \frac{\pi}{6}$

$\Rightarrow t = \frac{\pi}{6 \times \omega} \Rightarrow t = \frac{\pi \times 2}{6 \times 2\pi} \Rightarrow t = \frac{1}{6}\ s$

**20. (a)** The two springs are in parallel.

$f = \frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}}$ ....(i)

$f' = \frac{1}{2\pi}\sqrt{\frac{4K_1 + 4K_2}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{4(K_1 + 4K_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}}\right)$

$= 2f$ from eqn. (i)

**21. (b)** **KEY CONCEPT :** The instantaneous kinetic energy of a particle executing S.H.M. is given by

$K = \frac{1}{2}ma^2\omega^2\sin^2\omega t$

$\therefore$ average K.E. $= \langle K \rangle = \langle \frac{1}{2}m\omega^2 a^2 \sin^2\omega t \rangle$

$= \frac{1}{2}m\omega^2 a^2 \langle \sin^2\omega t \rangle$

$= \frac{1}{2}m\omega^2 a^2\left(\frac{1}{2}\right) \quad \left(\because \langle \sin^2\theta \rangle = \frac{1}{2}\right)$

$= \frac{1}{4}m\omega^2 a^2 = \frac{1}{4}ma^2(2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$

or, $\langle K \rangle = \pi^2 ma^2\nu^2$

**22. (b)** Here, $x = 2 \times 10^{-2}\cos\pi t$

$\therefore v = \frac{dx}{dt} = 2 \times 10^{-2}\pi\sin\pi t$

For the first time, the speed to be maximum,

$\sin\pi t = 1$ or, $\sin\pi t = \sin\frac{\pi}{2}$

$\Rightarrow \pi t = \frac{\pi}{2}$ or, $t = \frac{1}{2} = 0.5$ sec.

**23. (a)** Here,

$x = x_0\cos(\omega t - \pi/4)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -x_0\omega\sin\left(\omega t - \frac{\pi}{4}\right)$

Acceleration,

$a = \frac{dv}{dt} = -x_0\omega^2\cos\left(\omega t - \frac{\pi}{4}\right)$

$= x_0\omega^2\cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$

$= x_0\omega^2\cos\left(\omega t + \frac{3\pi}{4}\right)$ ...(1)

Acceleration, $a = A\cos(\omega t + \delta)$ ...(2)

Comparing the two equations, we get

$A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$.

**24. (a)** For an SHM, the acceleration $a = -\omega^2 x$ where $\omega^2$ is a constant. Therefore $\frac{a}{x}$ is a constant. The time period $T$ is also constant. Therefore $\frac{aT}{x}$ is a constant.

**25. (d)** For $X_0 + A$ to be the maximum seperation y one body is at the mean position, the other should be at the extreme.

**26. (c)** The net force becomes zero at the mean point. Therefore, linear momentum must be conserved.

$\therefore \quad Mv_1 = (M + m)v_2$

$MA_1\sqrt{\frac{k}{M}} = (M + m)A_2\sqrt{\frac{k}{m + M}} \therefore \quad \left(V = A\sqrt{\frac{k}{M}}\right)$

$A_1\sqrt{M} = A_2\sqrt{M + m} \quad \therefore \quad \frac{A_1}{A_2} = \sqrt{\frac{m + M}{M}}$

**27. (d)** The equation of motion for the pendulum, suffering retardation

$I\alpha = -mg(\ell\sin\theta) - mbv(\ell)$ where $I = m\ell^2$

and $\alpha = d^2\theta/dt^2$

$\therefore \frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\tan\theta + \frac{bv}{\ell}$

On solving we get $\theta = \theta_0\, e^{-\frac{bt}{2}}\sin(\omega t + \phi)$

According to questions $\frac{\theta_0}{e} = \theta_0 e^{\frac{-b\tau}{2}}$

$\therefore \quad \tau = \frac{2}{b}$

**28. (c)** $\because A = A_0 e^{-\frac{bt}{2m}}$ (where, $A_0$ = maximum amplitude)

According to the questions, after 5 second,

$$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} \quad \ldots(i)$$

After 10 more second,

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \ldots(ii)$$

From eq$^n$s (i) and (ii)

$A = 0.729\, A_0 \;\therefore\; \alpha = 0.729$

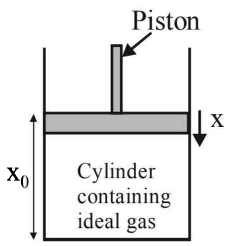
**29. (c)** $\frac{Mg}{A} = P_0$ $\qquad P_0 V_0^{\gamma} = PV^{\gamma}$

$Mg = P_0 A \quad \ldots(1)$ $\qquad P_0 A x_0^{\gamma} = PA(x_0 - x)^{\gamma}$

$$P = \frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}$$

Let piston is displaced by distance x

$$Mg - \left(\frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) A = F_{restoring}$$




$$P_0 A\left(1 - \frac{x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) = F_{restoring} \quad [x_0 - x \approx x_0]$$

$$F = -\frac{\gamma P_0 A x}{x_0}$$

$\therefore$ Frequency with which piston executes SHM.

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A}{x_0 M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{M V_0}}$$

**30. (d)** In simple harmonic motion, starting from rest,

At $t = 0$, $x = A$

$x = A\cos\omega t \quad \ldots..(i)$

When $t = \tau$, $x = A - a$

When $t = 2\tau$, $x = A - 3a$

From equation (i)

$A - a = A\cos\omega\tau \quad \ldots...(ii)$

$A - 3a = A\cos 2\omega\tau \quad \ldots...(iii)$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1 \ldots(iv)$

From equation (ii), (iii) and (iv)

$$\frac{A - 3a}{A} = 2\left(\frac{A - a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A - 3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$

$\Rightarrow 2a^2 = aA \Rightarrow A = 2a$

$\Rightarrow \frac{a}{A} = \frac{1}{2}$

Now, $A - a = A\cos\omega\tau$

$\Rightarrow \cos\omega\tau = \frac{A - a}{A} \Rightarrow \cos\omega\tau = \frac{1}{2}$

or, $\frac{2\pi}{T}\tau = \frac{\pi}{3} \Rightarrow T = 6\tau$

**31. (c)** As we know, time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added then

$$T_M = 2\pi\sqrt{\frac{\ell + \Delta\ell}{g}}$$

$$\frac{T_M}{T} = \sqrt{\frac{\ell + \Delta\ell}{\ell}} \text{ or } \left(\frac{T_M}{T}\right)^2 = \frac{\ell + \Delta\ell}{\ell}$$

or, $\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{Ay}$ $\qquad \left[\because \Delta\ell = \frac{Mg\ell}{Ay}\right]$

$$\therefore \frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

**32. (d)** $K.E = \frac{1}{2}k(A^2 - d^2)$

and $P.E. = \frac{1}{2}kd^2$

At mean position $d = 0$. At extremes positions $d = A$

**33. (b)** We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $\quad V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $\quad 3v = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

Where A' = final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get $\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$

$$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9} \qquad \therefore A' = \frac{7A}{3}$$