Alternating Current

Alternating Current:

An alternating current (a.c.) is a current which continuously, changes in magnitude and periodically reverse in direction'.

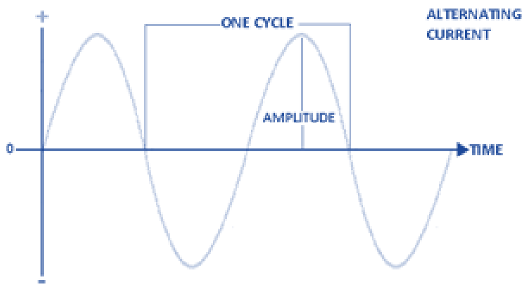
$i = I_0 \sin \omega t = I_0 \sin (2\pi/T)\, t$

Here I_0 is the peak value of *a.c*.

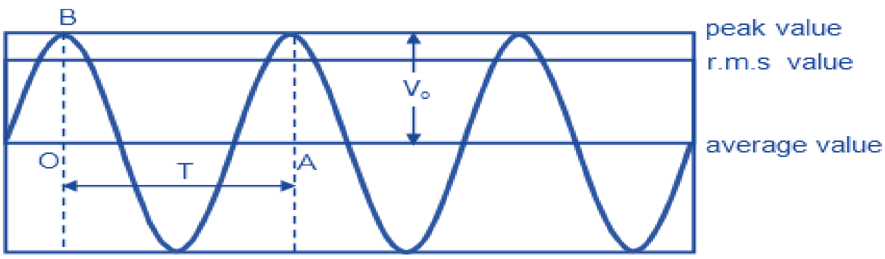
(a) Current, $I = I_0 \sin \omega t$

(b) Angular frequency, $\omega = 2\pi n$ (n is the frequency of a.c.)

(c) $I = I_0 \sin 2\pi nt$



- **Mean value of A.C or D.C. value of A.C.:-**
  Mean value of *a.c.* is that value of steady current which sends the same amount of charge, through a circuit, in same time as is done by *a.c.* in one half-cycle.



$(I_{av})_{\text{half cycle}} = (2/\pi) I_0$

Thus, mean value of alternating current is $2/\pi$ times (0.637 times) its peak value.

$(V_{av})_{\text{half cycle}} = (2/\pi)\, V_0$

- **Average value of A.C. over a complete cycle:-**

  $I_{av} = 0$

  The average value of *a.c.* taken over the complete cycle of *a.c.* is zero.

- **Root mean square value of a.c. or virtual value of a.c.:-**

  Root mean square value of alternating current is defined as that value of steady current which produces same heating effect, in a resistance, in a certain time as is produced by the alternating current in same resistance in same time. The *r.m.s* value of *a.c.* is also called its virtual value.

  $I_{rms} = I_0/\sqrt{2}$

Root mean square value of alternating current is $I/\sqrt{2}$ times (or 0.707 times) the peak value of current.

Similarly, $V_{rms} = V_0/\sqrt{2}$

Here V_0 is the peak value of *e.m.f.*

- **Form Factor:-**

Form Factor = rms value/average value = $(V_0/\sqrt{2})/(2V_0/\pi) = \pi/2\sqrt{2}$

- **Current elements:-**

(a) Inductive reactance:- $X_L = \omega L$

Here, $\omega = 2\pi n$, n being frequency of *a.c.*
L is the coefficient of self-inductance of coil.

(b) Capacitative reactance:- $X_c = 1/\omega C$

Here C is the capacity of the condenser

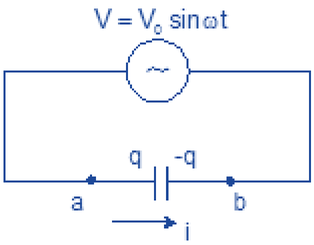
- **Capacitor in AC circuit:-**

$q = CV_0 \sin\omega t$

$I = I_0 \sin(\omega t + \pi/2)$

$V_0 = I_0/\omega C$

$X_c = 1/\omega C$



- **Inductor in AC circuit:-**

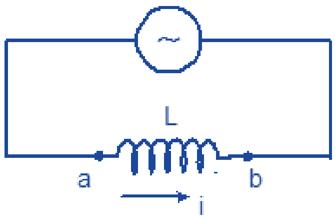
$V_L = L(dI/dt) = LI_0\omega \cos\omega t$

$I = (V_0/\omega L) \sin\omega t$

Here, $I_0 = V_0/\omega L$

$X_L = \omega L$

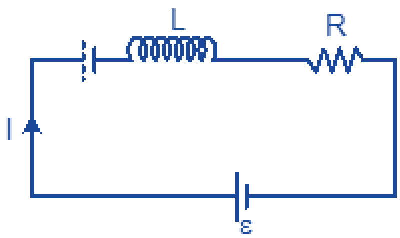
And the maximum current, $I_0 = V_0/XL$



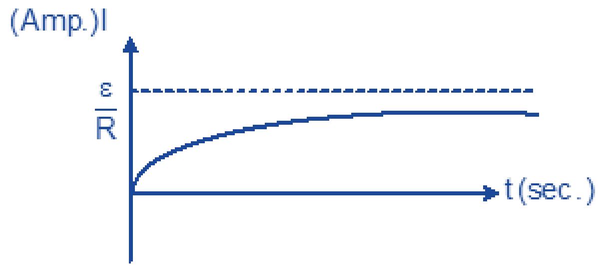
- **R-L circuit:-**

?$I = \varepsilon/R\,[1 - e^{-Rt/L}]$

$V = \varepsilon\, e^{-Rt/L}$



- **Graph between I (amp) and t (sec):-**



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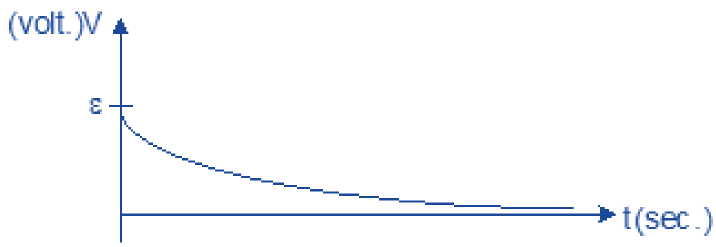
- **Graph between potential difference across inductor and time:-**



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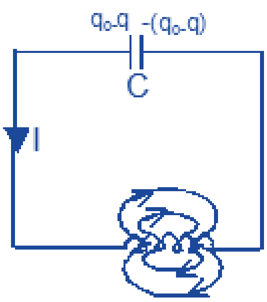
- **L-C Circuit:-**



?f = 1/2π√*LC*

q = q_0 sin (*ωt*+?)

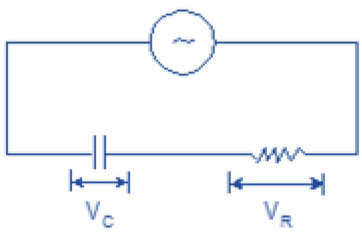
I = $q_0 \omega$sin (*ωt*+?)

ω = 1/√*LC*

- **The total energy of the system remains conserved,**

½ CV^2 + ½ Li^2 = constant = ½ CV_0^2 = ½ Li_0^2

- **Series in C-R circuit:-**



V = IZ

The modulus of impedance, |*Z* |= √R^2+(1/*ωC*)^2

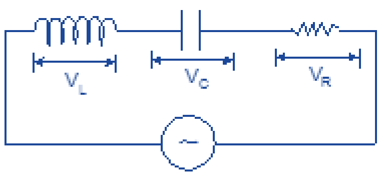
The potential difference lags the current by an angle, ? = $\tan^{-1}$(1/*ωCR*)

- **Series in L-C-R Circuit:**

V = IZ

The modulus of impedance, |*Z* |= √[R^2+(*ωL*-1/*ωC*)^2]

The potential difference lags the current by an angle, ? = $\tan^{-1}$[*ωL* -1/*ωC*)/*R*]



- **Circuit elements with A.C:-**

Circuit elements	Amplitude relation	Circuit quantity	Phase of V
Resistor	$V_0 = i_0R$	R	In phase with i
Capacitor	$V_0 = i_0X_C$	C	Lags i by 90°
Inductor	$V_0 = i_0X_L$	$X_L = wL$	Leads i by 90°

- **Resonance:-**

(a) Resonance frequency:- $f_r = 1/2\pi\sqrt{LC}$

(b) At resonance, $X_L = X_C$, ? = 0, Z = R(minimum), cos? = 1, sin? = 0 and current is maximum $(=E_0/R)$

- **Half power frequencies:-**

(a) Lower, $f_1 = f_r - R/4\pi L$ or $\omega_1 = \omega_r - R/2L$

(b) Upper, $f_2 = f_r + R/4\pi L$ or $\omega_2 = \omega_r + R/2L$

- **Band width:-** $\Delta f = R/2\pi L$ or $\Delta f = R/L$
- **Quality Factor:-**

(a) $Q = \omega_r/\Delta\omega = \omega_r L/R$

(b) As $\omega = 1/\sqrt{LC}$, So $Q \propto \sqrt{L}$, $Q \propto 1/R$ and $Q \propto 1/\sqrt{C}$

(c) $Q = 1/\omega_r CR$

(d) $Q = X_L/R$ or $Q = X_C/R$

(e) $Q = f_r/\Delta f$

- **At resonance, peak voltages are:-**

(a) $(V_L)_{res} = e_0Q$

(b) $(V_C)_{res} = e_0Q$

(c) $(V_R)_{res} = e_0$

- **Conductance, susceptance and admittance:-**

(a) Conductance, $G = 1/R$

(b) Susceptance, $S = 1/X$

(c) $S_L = 1/X_L$ and $S_C = 1/X_C = \omega C$

(d) Admittance, $Y = 1/Z$

(e) Impedance add in series while add in parallel

- **Power in AC circuits:-**

Circuit containing pure resistance:- $P_{av} = (E_0/\surd 2)\times(I_0/\surd 2) = E_v \times I_v$

Here E_v and I_v are the virtual values of *e.m.f* and the current respectively.

Circuit containing impedance (a combination of *R*,*L* and *C*):-

$P_{av} = (E_0/\surd 2)\times(I_0/\surd 2)\cos ? = (E_v \times I_v)\cos ?$

Here cos*?* is the power factor.

(a) Circuit containing pure resistance, $P_{av} = E_v I_v$

(b) Circuit containing pure inductance, $P_{av} = 0$

(c) Circuit containing pure capacitance, $P_{av} = 0$

(d) Circuit containing resistance and inductance,

$Z = \surd R^2 + (\omega L)$

$\cos ? == R/Z = R/[\surd\{R^2 + (\omega L)^2\}]$

(e) Circuit containing resistance and capacitance:-

$Z = \surd R^2 + (1/\omega C)^2$

$\cos ? == R/Z = R/[\surd\{R^2 + (1/\omega C)^2\}]$

(f) Power factor, cos*?* = Real power/Virtual power = $P_{av}/E_{rms} I_{rms}$

- **Transformer:-**

(a) $C_p = N_p\ (d?/dt)$ and $e_s = N_s\ (d?/dt)$

(b) $e_p/e_s = N_p/N_s$

(c) As, $e_p I_p = e_s I_s$, Thus, $I_s/I_p = e_p/e_s = N_p/N_s$

(d) Step down:- $e_s < e_p$, $N_s < N_p$ and $I_s > I_p$

(e) Step up:- $e_s > e_p$, $N_s > N_p$ and $I_s < I_p$

(f) Efficiency, $\eta = e_s I_s / e_p I_p$

- **AC Generator:-**

$e = e_0 \sin(2\pi f t)$

Here, $e_0 = NBA\omega$