CHAPTER – 12

EXPONENTS AND POWERS

EXERCISE – 12.1

Question – 1 Evaluate.

(i) 3^{-2}

(ii) $(4)^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-5}$

Answer:

(i) Formula:

$$(a)^{-m} = \frac{1}{a^m}$$

By using above formula in question:

$$(3)^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

(ii) Formula:

$$(a)^{-m} = \frac{1}{a^m}$$

By using above formula in question:

$$= (-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$$

(iii) Formula:

$$(a)^{-m} = \frac{1}{a^m}$$

By using above formula in question:

$$\left(\frac{1}{2}\right)^{-5} = 2^5 = 32$$

Question – 2 Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$

(ii) $\left(\frac{1}{2^3}\right)$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Answer:

(i) Formula: $a^m \div a^n = a^{(m-n)}$

By using above formula in question:

$(-4)^5 \div (-4)^8$

$= (-4)^{5-8}$

$= (-4)^{-3}$

$= 1/ (-4)^3$

$= -1/64$

(ii) Formula: $(a^m)^n = a^{mn}$

By using above formula in question:

$\left(\frac{1}{2^3}\right)^2$

$= \frac{1}{2^6}$

$= \frac{1}{64}$

(iii) Formula: - $a^m \times b^m = (a \times b)^m$

Using this formula:

$(-3)^4 \times \left(\frac{5}{3}\right)^4$

$= \left(-3 \times \frac{5}{3}\right)^4$

$= (-5)^4$

$= -5 \times (-5) \times (-5) \times (-5)$

$= 3125$

(iv) Formula: - $a^m \div a^n = a^{(m-n)}$

$a^m \times a^n = a^{\{m+n\}}$

By using above formula in question:

$(3^{-7} \div 3^{-10}) \times 3^{-5}$

$= (3^{(-7+10)}) \times 3^{-5}$

$= 3^3 \times 3^{-5}$

$= 3^{(3-5)}$

$= 3^{-2}$

$= 1/3^2$

$= 1/9$

(v) $2^{-3} \times (-7)^{-3}$

By using the formula,

$a^m \times b^m = (a \times b)^m$

$= (2 \times (-7))^{-3}$

$= (-14)^{-3}$

$= \frac{1}{(-14)^{-3}}$

$$= \frac{1}{-14\times(-14)\times(-14)} = -\frac{1}{2744}$$

Question – 3 Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Answer:

(i) Formula:

$A^0 = 1$

$(a)^{-m} = \frac{1}{a^m}$

By using above formula in question:

$(3^0 + 4^{-1}) \times 2^2 = (1 + 1/4) \times 2^2$

$\Rightarrow (3^0 + 4^{-1}) \times 2^2 = (5/4) \times 4$

$\Rightarrow (3^0 + 4^{-1}) \times 2^2 = 5$

(ii) Formula:

$(a)^{-m} = \frac{1}{a^m}$

$a^m \times a^n = a^{\{m+n\}}$

By using above formula in question:

$(2^{-1} \times 4^{-1}) \div 2^{-2} = (2^{-1} \times 2^{-2}) \div 2^{-2}$

$\Rightarrow (2^{-1} \times 4^{-1}) \div 2^{-2} = (2^{-1\,-2}) \div 2^{-2}$

$\Rightarrow (2^{-1} \times 4^{-1}) \div 2^{-2} = (2^{-3}) \div 2^{-2}$

$\Rightarrow (2^{-1} \times 4^{-1}) \div 2^{-2} = 2^{-3+2}$

$\Rightarrow (2^{-1} \times 4^{-1}) \div 2^{-2} = 2^{-1} = 1/2$

(iii) Formula:

$(a)^{-m} = \frac{1}{a^m}$

By using above formula in question:

$\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = 2^2 + 3^2 + 4^2$

$\longrightarrow \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = 4 + 9 + 16 = 29$

(iv) Formula:

$(a)^{-m} = \frac{1}{a^m}$

$(a)^0 = 1$

By using above formula in question:

(3-1 + 4-1 + 5-1)

Since anything raise to the power 0 = 1

(v) Formula:

$(a^m)^n = a^{mn}$

By using above formula in question:

$\rightarrow \left\{\left(-\frac{2}{3}\right)^{-2}\right\}^2 = \left(-\frac{3}{2}\right)^4$

$= \frac{-3\times-3\times-3\times-3}{2\times2\times2\times2}$

$= \frac{81}{16}$

Question – 4 Evaluate

(i) $\frac{8^{-1}\times5^{3}}{2^{-4}}$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Answer:

(i) Formula

$a^m/a^n = a^{(m-n)}$

$a^m \times a^n = a^{(m+n)}$

By using above formula in question:

$\frac{8^{-1}\times5^{3}}{2^{-4}}$

$= \frac{2^{-3}\times5^{3}}{2^{-4}}$

$= \frac{5^3}{2^{-1}} \ \frac{8^{-1}\times5^{3}}{2^{-4}}$

$= \frac{1}{8} \times \frac{1}{\frac{1}{16}} \times 5^3$

$= \frac{16}{8} \times 5^3$

$= 5^3 \times 2$

$= 125 \times 2$

$= 250$

(ii) Formula:

$(a)^{-m} = \frac{1}{a^m}$

$(a^m)^n = a^{mn}$

By using above formula in question:

$(5^{-1} \times 2^{-1}) \times 6^{-1} = \frac{1}{5} \times \frac{1}{2} \times \frac{1}{6}$

$\longrightarrow (5^{-1} \times 2^{-1}) \times 6^{-1} = \frac{1}{60}$

Question – 5 Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Answer:

Formula:

$\frac{a^m}{a^n} = a^{(m-n)}$

$a^m \times a^n = a^{(m+n)}$

$a^n = a^m$, then n = m

By using above formula in question:

$5^m \div 5^{-3} = 5^5$

$\frac{5^m}{5^{-3}} = 5^5$

$\Rightarrow 5^{m-(-3)} = 5^5$

$\Rightarrow 5^{m+3} = 5^5$

$\Rightarrow m + 3 = 5$

$\Rightarrow m = 2$

Question – 6 Evaluate

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Answer:

(i) Formula:

$(a)^{-m} = \frac{1}{a^m}$

By using above formula in question:

$\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \{3^1 - 4^1\}^{-1}$

$\rightarrow \left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \{-1\}^{-1} = -1$

(ii) Formula:

$(a)^{-m} = \frac{1}{a^m}$

$a^m \times a^n = a^{(m+n)}$

By using above formula in question:

$\left\{\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-7}\right\} = \left\{\left(\frac{5}{8}\right)^{-7} \times \left(\frac{5}{8}\right)^{4}\right\}$

$\rightarrow \left\{\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-7}\right\} = \left\{\left(\frac{5}{8}\right)^{-7+4}\right\}$

$\rightarrow \left\{\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-7}\right\} = \left\{\left(\frac{5}{8}\right)^{-3}\right\}$

$\rightarrow \left\{\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-7}\right\} = \left\{\left(\frac{8}{5}\right)^{3}\right\} = \frac{512}{125}$

Question - 7

Simplify

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$

(ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Answer:

(i) Formula:

$$\frac{a^m}{a^n} = a^{(m-n)}$$

$$(a)^{-m} = \frac{1}{a^m}$$

$$a^m \times a^n = a^{(m+n)}$$

By using above formula in question:

$$= \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \; (t \neq 0)$$

$$= \frac{5^2 \times t^{-4}}{5^{-3} \times 2 \times 5 \times t^{-8}}$$

$$= \frac{5^{2+3-1} \times t^{-4+8}}{2}$$

$$= \frac{5^4 \times t^4}{2} = \frac{625}{2} \; t^4$$

(ii) Formula:

$$\frac{a^m}{a^n} = a^{(m-n)}$$

$$(a)^{-m} = \frac{1}{a^m}$$

$$a^m \times a^n = a^{(m+n)}$$

By using above formula in question:

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

$$= \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5+5} \times (5)^{5+7+3}}{(2)^0}$$

$$= 5^5$$

$$= 3125$$

EXERCISE – 12.2

Question – 1 Express the following numbers in standard form.

(i) 0.0000000000085

(ii) 0.00000000000942

(iii) 6020000000000000

(iv) 0.00000000837

(v) 31860000000

Answer:

(i) $0.0000000000085 = 85 \times 10^{-13} = 8.5 \times 10^{-12}$

(ii) $0.00000000000942 = 942 \times 10^{-14} = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 602 \times 10^{13} = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 837 \times 10^{-11} = 8.37 \times 10^{-9}$

(v) $31860000000 = 3186 \times 10^{7} = 3.186 \times 10^{10}$

Question – 2 Express the following numbers in usual form.

(i) 3.02×10^{-6}

(ii) 4.5×10^{4}

(iii) 3×10^{-8}

(iv) 1.0001×10^{9}

(v) 5.8×10^{12}

(vi) 3.61492×10^{6}

Answer:

We know by simple indices that:

a^m = a x a x a x a.....m times

So, $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

And also that

$(a)^{-m} = \frac{1}{a \times a \times a \ldots\ldots m \text{ times}}$

(i)

$3.02 \times 10^{-6} = 3.02 \times \frac{1}{1000000}$

Place the decimal behind the digit of 3.02 up to how many zeroes are there in denominator

$3.02 \times 10^{-6} = 0.00000302$

(ii)

$4.5 \times 10^{4} = 4.5 \times 10000$

Now place the decimal after the digits up to the number of zeroes

$4.5 \times 10^{4} = 45000$

(iii) $3 \times 10^{-8} = 3/100000000 = .00000003$

(iv) $1.001 \times 10^{9} = 1.0001 \times 1000000000 = 1000100000$

(v) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$

(vi) $3.61492 \times 10^{6} = 3.61492 \times 1000000 = 3614920$

Question – 3 Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{10000000} m$

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

Answer:

(i) 1 Micron = 10^{-6} m

(ii) 0.000,000,000,000,000,000,16 Coulomb

= 16/1000000000000000000

= 16 × 10^{-18} = 1.6 × 10^{-19} Coulomb

(iii) 0.0000005 m
= 5/10000000 = 5 ×10^{-7} m

(iv)0.00001275 m
= 1275 /100000000 = 1275 × 10^{-8}
=1.275 × 10^{-5}

(v) 0.07 mm
= 7/100 mm = 7 ×10^{-2} mm

Question – 4 In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Answer:

Thickness of 1 book = 20 mm
Thickness of 5 books = 5 × 20 mm = 100 mm

Thickness of 1 page = 0.016 mm

Thickness of 5 pages = 5 × 0.016 = 0.08 mm

Total Thickness = Thickness of 5 books + Thickness of 5 pages

Total Thickness = 100 + 0.08

Total Thickness = 100.08 mm