Arithmetic Progression

Exercise – 5.1

Question 1:

Given a and d for the following A.P., find the following A.P. :

1. a = 3, d = 2
2. a = -3, d = -2
3. a = 100, d = -7
4. a = -100, d = 7
5. a = 1000, d = -100

Question 1(1):

Solution :

For the given A.P. the first term a = 3 and the common difference d = 2
$\therefore T_1 = a = 3$,
$T_2 = T_1 + d = 3 + 2 = 5$,
$T_3 = T_2 + d = 5 + 2 = 7$,
$T_4 = T_3 + d = 7 + 2 = 9$ and
$T_n = a + (n - 1)d$
$= 3 + (n - 1)(2)$
$T_n = 2n + 1$

Thus, the required A.P. is 3, 5, 7, 9, …..

Question 1(2):

Solution :

Here the first term a = -3 and the common difference d = -2

∴ T_1 = a = -3,

$T_2 = T_1 + d = -3 + (-2) = -5$,

$T_3 = T_2 + d = -5 + (-2) = -7$,

$T_4 = T_3 + d = -7 + (-2) = -9$ and

$T_n = a + (n – 1)d$

$= -3 + (n – 1) (-2)$

$= -3 – 2n + 2$

$T_n = -2n – 1$

Thus, the required A.P is -3, -5, -7, -9, …..

Question 1(3):

Solution :

Here the first term a = 100 and the common difference d = -7

∴ T_1 = a = 100,

$T_2 = T_1 + d = 100 + (-7) = 93$,

$T_3 = T_2 + d = 93 + (-7) = 86$,

$T_4 = T_3 + d = 86 + (-7) = 79$ and

$T_n = a + (n – 1)d$

$= 100 + (n – 1)(-7)$

$T_n = -7n + 107$

Thus, the required A.P. is 100, 93, 86, 79, …..

Question 1(4):

Solution :

Here the first term a = -100 and the common difference d = 7

∴ T_1 = a = -100,

$T_2 = T_1 + d = -100 + (7) = -93$,

$T_3 = T_2 + d = -93 + (7) = -86$,

$T_4 = T_3 + d = -86 + (7) = -79$ and

$T_n = a + (n – 1)d$

$= -100 + (n – 1)(7)$

$T_n = 7n – 107$

Thus, the required A.P. is -100, -93, -86, -79, …..

Question 1(5):

Solution :

Here the first term a = 1000 and the common difference d = -100.

∴ T_1 = a = -1000,

$T_2 = T_1 + d = -1000 + (-100) = 900$,

$T_3 = T_2 + d = 900 + (-100) = 800$,

$T_4 = T_3 + d = 800 + (-100) = 700$ and

$T_n = a + (n – 1)d$

$= -1000 + (n – 1)(-100)$

$T_n = -100n + 1100$

Thus, the required A.P. is 1000, 900, 800, 700, …..

Question 2:

Determine if the following sequences represent an A.P., assuming that the pattern continues. If it is an A.P., find the nth term :

1. 5, -5, 5, -5,……
2. 2, 2, 2, 2,……
3. 1, 11, 111, 1111,….
4. 5, 15, 25, 35, 45,…
5. 17, 22, 27, 32,…
6. 101, 99, 97, 95,…
7. 201, 198, 195, 192,…
8. Natural numbers which are consecutive multiples of 5 in increasing order.
9. Natural numbers which are multiples of 3 or 5 in increasing order.

Question 2(1):

Solution :

For the sequence 5, -5, 5, -5, ….

$T_2 – T_1 = (-5) – 5 = -10$

$T_3 – T_2 = 5 – (-5) = 10$

But, $T_2 – T_1 \neq T_3 – T_2$

Hence, the given sequence is not an A.P.

Question 2(2):

Solution :

For the sequence 2, 2, 2, 2, …..

$T_2 – T_1 = 0$

$T_3 – T_2 = 0$ ……

But, for an A.P, the common difference must be a non-zero constant.

Hence, the given sequence is not an A.P.

Question 2(3):

Solution :

For the sequence 1, 11, 111, 1111, ….

$T_2 – T_1 = 11 – 1 = 10$ and

$T_3 – T_2 = 111 – 11 = 100$

But, $T_2 – T_1 \neq T_3 – T_2$

Hence, the given sequence is not an A.P.

Question 2(4):

Solution :

For the sequence 5, 15, 25, 35, 45, …..

$T_2 – T_1 = 15 – 5 = 10$

$T_3 – T_2 = 25 – 15 = 10$

$T_4 – T_3 = 35 – 25 = 10$

$T_5 – T_4 = 45 – 35 = 10$

So,

$T_2 – T_1 = T_3 – T_2 = T_4 – T_3 = T_5 – T_{4\ ...} = 10$

Assuming that the pattern continues, the given sequence is an A.P.

Here, first term a = 5 and common difference d = 10.

Then,

$T_n = a + (n – 1)d$

$\therefore T_n = 5 + (n – 1)10$

$\therefore T_n = 10n – 5$

Question 2(5):

Solution :

For the sequence 17, 22, 27, 32, ……

$T_2 – T_1 = 22 – 17 = 5$

$T_3 – T_2 = 27 – 22 = 5$

$T_4 – T_3 = 32 – 27 = 5$

So, $T_2 – T_1 = T_3 – T_2 = T_4 – T_3 = 5$

Assuming that the pattern continues, the given sequence is an A.P.

Here the first term a = 17 and common difference d = 5.

Then, $T_n = a + (n – 1)d$

$\therefore T_n = 17 + (n – 1)5$

$\therefore T_n = 5n + 12$

Question 2(6):

Solution :

For the sequence 101, 99, 97, 95, ……

$T_2 – T_1 = 99 – 101 = -2$

$T_3 – T_2 = 97 – 99 = -2$

$T_4 – T_3 = 95 – 97 = -2$

So, $T_2 – T_1 = T_3 – T_2 = T_4 – T_3 \ldots = -2$

Assuming that the pattern continues, the given sequence is an A.P.

Here the first term a =101 and common difference d = -2.

Now, $T_n = a + (n – 1)d$

$\therefore T_n = 101 + (n – 1)(-2)$

$\therefore T_n = -2n + 103$

Question 2(7):

Solution :

For the sequence 201, 198, 195, 192, …..

$T_2 - T_1 = 198 - 201 = -3$

$T_3 - T_2 = 195 - 198 = -3$

$T_4 - T_3 = 192 - 195 = -3$

Then,

$T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = -3$

Assuming that the pattern continues, the given sequence is an A.P.

Here the first term a = 201 and common difference d = -3.

Now, $T_n = a + (n - 1)d$

$\therefore T_n = 201 + (n - 1)(-3)$

$\therefore T_n = -3n + 204$

Question 2(8):

Solution :

The given sequence is 5, 10, 15, 20, …….

Here,

$T_1 = 10 - 5 = 5$

$T_3 - T_2 = 15 - 10 = 5$

$T_4 - T_3 = 20 - 15 = 5$

So, $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = 5$

Assuming that the pattern continues, the given sequence is an A.P.

Here the first term a = 5 and common difference d = 5.

Then,

$T_n = a + (n - 1)d$

$\therefore T_n = 5 + (n - 1)(5)$

$\therefore T_n = 5n$

Question 2(9):

Solution :

The given sequence is 3, 5, 6, 9, 10, 12, 15, ……

Here, $T_1 = a = 3$

$T_2 - T_1 = 5 - 3 = 2$

$T_3 - T_2 = 6 - 5 = 1$

As $T_2 - T_1 \neq T_3 - T_2$

The given sequence is not an A.P.

Question 3:

Find the nth term of the following A.P.'s .

1. 2, 7, 12, 17,…
2. 200, 195, 190, 185,….
3. 1000, 900, 800,…
4. 50, 100, 150, 200,..
5. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$,…..
6. 1.1, 2.1, 3.1, 4.1,……
7. 1.2, 2.3, 3.4, 4.5,…

8. $\frac{5}{3}, \frac{7}{3}, 3, \frac{11}{3}, \frac{13}{3}, 5, \ldots\ldots\ldots$

Solution :

1. For the given A.P.,

First term = a = 2 and

Common difference = d = 7 – 2 = 5

Now n^{th} term,

$T_n = a + (n - 1)d$

$= 2 + (n - 1)(5)$

$\therefore T_n = 5n - 3$

2. For the given A.P.,

First term = a = 200

Common difference = d = 195 – 200 = -5

Now n^{th} term,

$T_n = a + (n - 1)d$

$= 200 + (n - 1)(-5)$

$\therefore T_n = 205 - 5n$

3. For the given A.P.,

First term = a = 1000

Common difference = d = 900 – 1000 = -100

Now n^{th} term,

$T_n = a + (n - 1)d$

$= 1000 + (n - 1)(-100)$

$\therefore T_n = 1100 - 100n$

4. For the given A.P.,

First term = a = 50 Common difference = d = 100 – 50 = 50

Now n^{th} term

$T_n = a + (n - 1)d$

$= 50 + (n - 1)(50)$

$\therefore T_n = 50n$

5.

For the given A.P.,

First term $= a = \frac{1}{2}$

Common difference $= d = \frac{3}{2} - \frac{1}{2} = 1.$

Now n^{th} term,

$T_n = a + (n-1)d$

$= \frac{1}{2} + (n-1)(1)$

$T_n = n - \frac{1}{2}$

For the given A.P.,

6. First term = a = 1.1

Common difference = d = 2.1 – 1.1= 1

Now nth term $T_n = a + (n - 1)d$

$= 1.1 + (n - 1)(1)$

$\therefore T_n = n + 0.1$

7. For the given A.P.,

First term = a = 1.2

Common difference = d = 2.3 – 1.2 = 1.1

Now nth term

$T_n = a + (n – 1)d$

$= 1.2 + (n – 1)(1.1)$

$\therefore T_n = 1.1n + 0.1$

8.

For the given A.P.,

First term $= a = \frac{5}{3}$

Common difference $= d = \frac{7}{3} - \frac{5}{3} = \frac{2}{3}$

Now n^{th} term,

$Tn = a + (n-1)d$

$= \frac{5}{3} + (n-1)\left(\frac{2}{3}\right)$

$\therefore T_n = \frac{2}{3}n + 1$

Question 4:

Find A.P. if T_n, T_m are as given below :

1. T_7, = 12, $T_{12} = 72$
2. $T_2 = 1$, $T_{12} = -9$

Question 4(1):

Solution :

Here $T_n = T_7 = 12$ and $T_m = T_{12} = 72$

We know that,

$d = \frac{T_m - T_n}{m - n} = \frac{72 - 12}{12 - 7} = \frac{60}{5} = 12$

$\therefore d = 12$

Now, $T_7 = a + 6d \quad [T_n = a + (n-1)d]$

$\therefore 12 = a + 6(12)$

$\therefore 12 = a + 72$

$\therefore a = -60$

Thus, the required A.P. is $-60, -48, -36, -24, \ldots\ldots$

$T_n = a + (n-1)d = -60 + (n-1)(12)$

$\therefore T_n = 12n - 72$

Question 4(2):

Solution :

Here, $T_n = T_2 = 1$ and $T_m = T_{12} = -9$

We know that,

$$d = \frac{T_m - T_n}{m - n} = \frac{-9 - 1}{12 - 2} = \frac{-10}{10} = -1$$

$\therefore d = -1$

Now, $T_2 = a + d \quad [T_n = a + (n - 1)d]$

$\therefore 1 = a + (-1)$

$\therefore a = 2$

Thus the required A.P. is 2, 1, 0, −1, ...

$T_n = a + (n - 1)d$

$= 2 + (n - 1)(-1)$

$\therefore T_n = 3 - n$

Question 5:

1. In an A.P., $T_3 = 8$, $T_{10} = T_6 + 20$. Find the A.P.
2. In an A.P. 5th term is 17 and 9th term exceeds 2nd term by 35. Find the A.P.

Question 5(1):

Solution :

Here it is given that,

$T_{10} = T_6 + 20$

$\therefore T_{10} - T_6 = 20$

Next,

$$d = \frac{T_m - T_n}{m - n}$$

$$\therefore d = \frac{T_{10} - T_6}{10 - 6} = \frac{20}{4} = 5$$

Now $T_3 = 8 \quad$ (given)

$\therefore a + 2d = 8 \quad [T_n = a + (n - 1)d]$

$\therefore a + 2(5) = 8$

$\therefore a = -2$

Thus, the required A.P. is − 2, 3, 8, 13,...

$T_n = a + (n - 1)d$

$= -2 + (n - 1)(5)$

$\therefore T_n = 5n - 7$

Question 5(2):

Solution :

Here $T_5 = 17$ and $T_9 = T_2 + 35$

$\therefore T_9 - T_2 = 35$

We know that,

$$d = \frac{T_m - T_n}{m - n}$$

$$\therefore d = \frac{T_9 - T_2}{9 - 2} = \frac{35}{7} = 5$$

Now, $T_5 = 17$

$\therefore a + 4d = 17 \quad [T_n = a + (n-1)d]$

$\therefore a + 4(5) = 17$

$\therefore a = -3$

Thus, the required A.P. is $-3, 2, 7, 12, \ldots$

Also,

$T_n = a + (n-1)d$

$= -3 + (n-1)(5)$

$\therefore T_n = 5n - 8$

Question 6:

Can any term of A.P., 12, 17, 22, 27,… be zero ? Why ?

Solution :

For the given A.P., 12, 17, 22, 27,....

$a = 12$ and $d = 17 - 12 = 5$

Assume that $Tn = 0$; for some $n \in N$

$Tn = a + (n-1)d$

$\therefore 0 = 12 + (n-1)(5)$

$\therefore 0 = 7 + 5n$

$$\therefore n = -\frac{7}{5} \in N$$

Thus, any term of the given A.P. cannot be zero, because for any n^{th} term to be zero, the value of n is found out to be a negative fraction which is contradiction as $n \in N$.

Question 7:

Can any term of A.P., 201, 197, 193,… be 5 ? Why ?

Solution :

For the given A.P., 201, 197, 193. ……

a = 201 and d = 197 – 201 = -4

Assume that $T_n = 5$

$5 = a + (n - 1)d$

$\therefore 5 = 201 + (n - 1)(-4)$

$\therefore 5 = 205 - 4n$

$\therefore 4n = 200$

$\therefore n = 50 \in N$

Thus, the 50^{th} term of the given A.P. is 5.

Question 8:

Which term of A.P., 8, 11, 14, 17,.. is 272 ?

Solution :

For the given A.P. 8, 11, 14, 17, ……….

a = 8 and d = 11 – 8 = 3

Let the n^{th} term of the A.P. be 272.

T_n= a + (n – 1)d

∴ 272 = 8 + (n – 1)(3)

∴ 272 = 3n + 5

∴ 3n = 267

∴ n = 89

Hence the 89^{th} term of the given A.P. is 272.

Question 9:

Find the 10th term from end for A.P., 3, 6, 9, 12,… 300.

Solution :

For the given A.P., 3, 6, 9, 12, ………, 300

a = 3 and d = 6 – 3 = 3

Here T_n = 300, then

300 = 3 + (n – 1)(3) [∵T_n= a + (n – 1)d]

∴ 3(n – 1) = 297

∴ n – 1 = 99

∴ n = 100

Now, the 10^{th} term from the end is 91^{st} term of the A.P.

[∵ 91^{st} term = 100 – 10 + 1]

T_{91} = a + (91 – 1)d

= 3 + 90 × 3

= 273

Thus, the 10^{th} term from the end of the A.P. is 273.

Question 10:

Find the 15th term from end for A.P., 10, 15, 20, 25, 30,…,1000.

Solution :

For the given A.P., 10, 15, 20, 25, 30, …….. ,1000

a = 10 and d = 15 – 10 = 5,

Here T_n = 1000

1000 = 10 + (n – 1)5 [∵ T_n= a + (n – 1)d]

∴ 990 = 5(n – 1)

∴ 198 = n – 1

∴ n = 199

Now, the 15^{th} term from the end is 185^{th} term of the A.P. (∵ 185^{th} term = 199 – 15 + 1)

T_{185} = 10 + (185 – 1)(5)

= 10 + 184 × 5

= 10 + 920

∴ T_{185} = 930

Thus, the 15th term from the end of the A.P. is 930.

Question 11:

If in an AP., $T_7 = 18$, $T_{18} = 7$, find T_{101}.

Solution :

Here, $T_m = T_7 = 18$ and $T_n = T_{18} = 7$

We know that,

$$d = \frac{T_m - T_n}{m - n}$$

$$\therefore d = \frac{T_{18} - T_7}{18 - 7} = \frac{7 - 18}{11} = -1$$

$T_7 = 18$

$\therefore a + 6d = 18 \quad [\because T_n = a + (n-1)d]$

$\therefore a + 6(-1) = 18$

$\therefore a = 24$

Now,

$T_{101} = a + 100d \quad [\because T_n = a + (n-1)d]$

$= 24 + 100(-1)$

$= 24 - 100$

$\therefore T_{101} = -76$

Thus, in the given A.P. $T_{101} = -76$

Question 12:

If in an A.P., $T_m = n$, $T_n = m$, prove $d = -1$.

Solution :

For the given A.P. take $T_m = n$ and $T_n = m$.

We know that,

$$d = \frac{T_m - T_n}{m - n}$$

$$\therefore d = \frac{n - m}{m - n}$$

$$\therefore d = \frac{-(m - n)}{m - n}$$

$\therefore d = -1$

Thus, for an A.P. in which $T_m = n$ and $T_n = m$, we get $d = -1$.

Exercise – 5.2

Question 1:

Find the sum of the first n terms of the A.P. as asked for :

1. 2, 6, 10, 14,… upto 20 terms
2. 5, 7, 9, 11,…… upto 30 terms
3. -10, -12, -14, -16,… upto 15 terms
4. 1, 1.5, 2, 2.5, 3,……

5. $\frac{1}{3}, \frac{4}{3}, \frac{7}{3}, \frac{10}{3}$,..... upto 18 terms

Question 1(1):

Solution :

For the given A.P., 2, 6, 10, 14,...

$a = 2,\ d = 6 - 2 = 4$ and $n = 20$

Now,

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$\therefore S_{20} = \frac{1}{2} \times 20\left[2 \times 2 + (20-1)(4)\right]$$

$$= 10\left[4 + 76\right]$$

$$\therefore S_{20} = 800$$

Question 1(2):

Solution :

For the given A.P. 5, 7, 9, 11, ...

$a = 5,\ d = 7 - 5 = 2$ and $n = 15$

Now,

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$\therefore S_{30} = \frac{1}{2} \times 30\left[2 \times 5 + (30-1)(2)\right]$$

$$= 15 \times 68$$

$$\therefore S_{30} = 1020$$

Question 1(3):

Solution :

For the given A.P. − 10, − 12, − 14, − 16 ...

$a = -10,\ d = -12 - (-10) = -2$ and $n = 15$

Now,

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$\therefore S_{15} = \frac{1}{2} \times 15\left[2 \times (-10) + (15-1)(-2)\right]$$

$$= \frac{1}{2} \times 15 \times (-48)$$

$$\therefore S_{15} = -360$$

Question 1(4):

Solution :

For the given A.P., 1, 1.5, 2, 2.5, 3, ...

$a = 1$, $d = 1.5 - 1 = 0.5$ and $n = 16$

Now,

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$\therefore S_{16} = \frac{1}{2} \times 16\left[2 \times 1 + (16-1)(0.5)\right]$$

$$= 8 \times 9.5$$

$$\therefore S_{16} = 76$$

Question 1(5):

Solution :

For the given A.P. $\frac{1}{3}, \frac{4}{3}, \frac{7}{3}, \frac{10}{3}, \dots$

$a = \frac{1}{3}$, $d = \frac{4}{3} - \frac{1}{3} = 1$ and $n = 18$

Now,

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$\therefore S_{18} = \frac{1}{2} \times 18\left[2 \times \frac{1}{3} + (18-1)(1)\right]$$

$$= 9 \times \left[\frac{2}{3} + 17\right]$$

$$= 9 \times \frac{53}{3}$$

$$= 3 \times 53$$

$$\therefore S_{18} = 159$$

Question 2:

Find the sums indicated below :

1. 3 + 6 + 9 + … + 300
2. 5 + 10 + 15 + ….. + 100
3. 7 + 12 + 17 + 22 + ….. + 102
4. (-100) + (-92) + (-84) + …… + 92
5. 25 + 21 + 17 + 13 + ….. + (-51)

Question 2(1):

Solution :

For the given A.P. $a = 3$, $d = 6 - 3 = 3$ and $T_n = 300$.

$T_n = a + (n-1)d$

$\therefore 300 = 3 + (n-1)(3)$

$\therefore 297 = 3(n-1)$

$\therefore 99 = n - 1$

$\therefore n = 100$

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$\therefore S_{18} = \frac{1}{2} \times 100\left[2 \times 3 + (100-1)3\right]$

$= 50 \times 303$

$\therefore S_{18} = 15150$

Question 2(2):

Solution :

For the given A.P. $a = 5$, $d = 10 - 5 = 5$ and $T_n = 100$.

$T_n = a + (n-1)d$

$\therefore 100 = 5 + (n-1)(5)$

$\therefore 95 = 5(n-1)$

$\therefore 19 = n - 1$

$\therefore n = 20$

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$\therefore S_{20} = \frac{1}{2} \times 20\left[2 \times 5 + (20-1)5\right]$

$= 10\left[10 + 95\right]$

$= 10 \times 105$

$\therefore S_{20} = 1050$

Question 2(3):

Solution :

For the given A.P. $a = 7$, $d = 12 - 7 = 5$ and $T_n = 102$.

$T_n = a + (n-1)d$

$\therefore 102 = 7 + (n-1)(5)$

$\therefore 95 = 5(n-1)$

$\therefore 19 = n - 1$

$\therefore n = 20$

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$\therefore S_{20} = \frac{1}{2} \times 20\left[2 \times 7 + (20-1)5\right]$

$= 10 \times 109$

$\therefore S_{20} = 1090$

Question 2(4):

Solution :

For the given A.P. $a = -100$, $d = -92 - (-100) = 8$ and $T_n = 92$.

$$T_n = a + (n-1)d$$
$$\therefore 92 = -100 + (n-1)(8)$$
$$\therefore 192 = 8(n-1)$$
$$\therefore 24 = n - 1$$
$$\therefore n = 25$$
$$S_n = \frac{1}{2}n[2a + (n-1)d]$$
$$\therefore S_{25} = \frac{1}{2} \times 25[2 \times (-100) + (25-1)(8)]$$
$$= \frac{1}{2} \times 25 \times (-8)$$
$$\therefore S_{25} = -100$$

Question 2(5):

Solution :

For the given A.P. $a = 25$, $d = 21 - 25 = -4$ and $T_n = -51$.

$$T_n = a + (n-1)d$$
$$\therefore -51 = 25 + (n-1)(-4)$$
$$\therefore -76 = -4(n-1)$$
$$\therefore 19 = n - 1$$
$$\therefore n = 20$$
$$S_n = \frac{1}{2}n[2a + (n-1)d]$$
$$\therefore S_{20} = \frac{1}{2} \times 20[2 \times 25 + (20-1)(-4)]$$
$$= 10 \times (-26)$$
$$\therefore S_{20} = -260$$

Question 3:

For a given A.P. with

1. $a = 1$, $d = 2$, find S_{10}.
2. $a = 2$, $d = 3$, find S_{30}.
3. $S_3 = 9$, $S_7 = 49$, find Sn and S_{10}.
4. $T_{10} = 41$, $S_{10} = 320$, find T_n, S_n.
5. $S_{10} = 50$, $a = 0.5$, find d.
6. $S_{20} = 100$, $d = -2$, find a.

Question 3(1):

Solution :

For the given A.P. $a = 1$, $d = 2$ and $n = 10$.

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$
$$\therefore S_{10} = \frac{1}{2} \times 10[2 \times 1 + (10-1)(2)]$$
$$= 5 \times 20$$
$$\therefore S_{10} = 100$$

Question 3(2):

Solution :

For the given A.P. $a = 2,\ d = 3$ and $n = 30$.

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$\therefore S_{30} = \frac{1}{2} \times 30\left[2 \times 2 + (30-1)(3)\right]$

$= 15 \times 91$

$\therefore S_{30} = 1365$

Question 3(3):

Solution :

Here $S_3 = 9$ and $S_7 = 49$

We know that,

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$\therefore S_3 = \frac{1}{2} \times 3\left[2a + (3-1)d\right]$

$\therefore 9 = \frac{1}{2} \times 3\left[2a + 2d\right]$

$\therefore 9 = 3[a + d]$

$\therefore a + d = 3 \qquad ....(1)$

$\therefore S_7 = \frac{1}{2} \times 7\left[2a + (7-1)d\right]$

$\therefore 49 = \frac{1}{2} \times 7\left[2a + 6d\right]$

$\therefore 49 = 7\left[a + 3d\right]$

$\therefore a + 3d = 7 \qquad ...(2)$

Solving equations (1) and (2), we get $d = 2$ and $a = 1$

Now, $S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$= \frac{1}{2} \times n\left[2 \times 1 + (n-1)(2)\right]$

$= \frac{1}{2} \times n\left[2n\right]$

$\therefore S_n = n^2$

Taking $n = 10$, we get $S_{10} = 10^2 = 100$

$\therefore S_n = n^2$ and $S_{10} = 100$

Question 3(4):

Solution :

Here $T_{10} = 41$

$\therefore a + 9d = 41 \quad [\because T_n = a + (n-1)d] \quad \ldots(1)$

Also $S_{10} = 320$

$\therefore \frac{1}{2} \times 10[2a + (10-1)d] = 320 \qquad \left[\because S_n = \frac{1}{2}n[2a + (n-1)d]\right]$

$\therefore 5[2a + 9d] = 320$

$\therefore 2a + 9d = 64$

$\therefore a + a + 9d = 64$

$\therefore a + 41 = 64 \qquad [\because \text{using } (1)]$

$\therefore a = 23$

Substituting in (1), we get $d = 2$

Now,

$T_n = a + (n-1)d$

$= 23 + (n-1)(2)$

$= 23 + 2n - 2$

$\therefore T_n = 2n + 21$

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$= \frac{1}{2} \times n \times [46 + (n-1)(2)]$

$= \frac{1}{2} \times n \times (44 + 2n)$

$= n(22 + n)$

$\therefore S_n = n^2 + 22n$

Question 3(5):

Solution :

We know that,

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$\therefore S_{10} = \frac{1}{2} \times 10[2a + (10-1)d]$

$\therefore 50 = 5[2(0.5) + 9d]$

$\therefore 10 = 1 + 9d$

$\therefore 9 = 9d$

$\therefore d = 1$

Question 3(6):

Solution :

We know that,

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$\therefore S_{20} = \frac{1}{2} \times 20[2a + (20-1)d]$

$\therefore 100 = 10[2a + 19(-2)]$

$\therefore 10 = 2a - 38$

$\therefore 48 = 2a$

$\therefore a = 24$

Question 4:

How many terms of A.P., 2, 7, 12, 17,… add upto 990 ?

Solution :

For the given A.P.

$a = 2, \ d = 7 - 2 = 5$ and $S_n = 990$.

We know that,

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$

$$\therefore 990 = \frac{1}{2}n[4 + (n-1)5]$$

$$\therefore 1980 = n[5n - 1]$$

$$\therefore 1980 = 5n^2 - n$$

$$\therefore 5n^2 - n - 1980 = 0$$

$$\therefore (5n + 99)(n - 20) = 0$$

$$\therefore n = -\frac{99}{5} \text{ or } n = 20$$

Since $n \in N$, $n = -\frac{99}{5}$ is not possible.

$$\therefore n = 20$$

Thus, 20 terms of the given A.P. add upto 990.

Question 5:

The first term of finite A.P. is 5, the last term is 45 and the sum is 500. Find the number of terms.

Solution :

For the given finite A.P. $a = 5$, $l = T_n = 45$ and $S_n = 500$.

$$S_n = \frac{1}{2}n[a + l]$$

$$\therefore 500 = \frac{1}{2}n[5 + 45]$$

$$\therefore 500 = \frac{1}{2}n \times 50$$

$$\therefore 500 = 25n$$

$$\therefore n = 20$$

Thus, the number of terms of the given finite A.P. is 20.

Question 6:

If the first term and the last term of a finite A.P. are 5 and 95 respectively and d = 5, find n and S_n.

Solution :

For the given finite A.P.,

$a = 5, \; l = T_n = 95$ and $d = 5.$

Then,

$T_n = a + (n-1)d$

$\therefore 95 = 5 + (n-1)(5)$

$\therefore 90 = 5(n-1)$

$\therefore 18 = n - 1$

$\therefore n = 19$

Next,

$S_n = \frac{1}{2}n[a + l]$ (finite A.P.)

$= \frac{1}{2} \times 19[5 + 95]$

$= \frac{1}{2} \times 19 \times 100$

$\therefore S_n = 950$

Question 7:

The sum of first n terms of an A.P. is $5n - 2n^2$. Find the A.P. i.e. a and d.

Solution :

Here $S_n = 5n - 2n^2$

$\therefore S_{n-1} = 5(n-1) - 2(n-1)^2$

$= 5n - 5 - 2n^2 + 4n - 2$

$= 9n - 2n^2 - 7$

Now, $T_n = S_n - S_{n-1}$

$= (5n - 2n^2) - (9n - 2n^2 - 7)$

$\therefore T_n = 7 - 4n$, where $n > 1$ …..(i)

Also, $a = T_1 = 7 - 4(1) = 3$

$\therefore a = 3$

Now, $T_2 = 7 - 4(2) = 7 - 8 = (-1)$

$d = T_2 - T_1 = (-1) - 3$

$\therefore d = -4$

Question 8:

Find the sum of all three digit numbers divisible by 3.

Solution :

Amongst three digit numbers divisible by 3, smallest number is 102 and greatest number is 999.
Arranging them in the increasing order, we get a finite A.P. as follows:

102, 105 ,1008, ..., 996, 999.
Here, $a = 102$ $d = 105 - 102 - 3$ and $l = T_n = 999$

$T_n = a + (n-1)d$

$\therefore 999 = 102 + (n-1)(3)$

$\therefore 879 = 3(n-1)$

$\therefore 299 = n-1$

$\therefore n = 300$

Thus, there are 300 terms in this finite A.P.

$S_n = \frac{1}{2}n[a + l]$

$\therefore S_{300} = \frac{1}{2} \times 300[102 + 999]$

$= 150 \times 1101$

$\therefore S_{300} = 165150$

Thus, the sum of all three digit numbers divisible by 3 is 1,65,150.

Question 9:

Find the sum of all odd numbers from 5 to 205.

Solution :

Odd numbers from 5 to 205 are 5, 7, 9, 11, ... , 205.
Arranging all the odd numbers from 5 to 205 in the ascending order, we get the following finite A.P.
5, 7, 9, 11, ... ,203, 205.
Here, $a = 5$, $d = 7 - 5 = 2$, $l = T_n = 205$.

$T_n = a + (n-1)d$

$\therefore 205 = 5 + (n-1)(2)$

$\therefore 200 = 2(n-1)$

$\therefore 100 = n-1$

$\therefore n = 101$

Now, for a finite A.P.

$S_n = \frac{1}{2}n[a + l]$

$S_{101} = \frac{1}{2} \times 101[5 + 205]$

$= \frac{1}{2} \times 101 \times 210$

$= 101 \times 105$

$S_{101} = 10605$

Thus, the sum of all the odd numbers from 5 to 205 is 10605.

Question 10:

Which term of A.P. 121, 117, 113,….. is its first negative term? If it is the nth term, find S_n.

Solution :

For the given A.P., 121, 117, 113, ...

$a = 121$ and $d = 117 - 121 = -4$

Let the n^{th} term of the given A.P. be its first negative term.

$T_n < 0$

$\therefore a + (n-1)d < 0$

$\therefore 121 + (n-1)(-4) < 0$

$\therefore 121 < 4(n-1)$

$\therefore \frac{121}{4} < n - 1$

$\therefore 30\frac{1}{4} + 1 < n$

$\therefore n > 31\frac{1}{4}$

The smallest integer greater than $31\frac{1}{4}$ is 32.

$\therefore n = 32$

Thus, the 32^{nd} term of the given A.P. is its first negative term.

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$

$$\therefore S_{32} = \frac{1}{2} \times 32[242 + (32-1)(-4)]$$

$$= 16[242 - 124]$$

$$= 16 \times 118$$

$$\therefore S_{32} = 1888$$

Exercise – 5

Question 1:

If $T_n = 6n + 5$, find S_n.

Solution :

Here $T_n = 6n + 5$

So,

$T_1 = 6(1) + 5 = 11,$

$T_2 = 6(2) + 5 = 17$

$T_3 = 6(3) + 5 = 23$

Thus, the given A.P. is 11, 17, 23, ...

Here, $a = 11$ and $d = 17 - 11 = 6$

Now,

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$

$$= \frac{1}{2} \times n[2(11) + (n-1)(6)]$$

$$= \frac{1}{2}n[6n + 16]$$

$$= n[3n + 8]$$

$$\therefore S_n = 3n^2 + 8n$$

Question 2:

If $S_n = n^2 + 2B$, find T_n.

Solution :

Here $S_n = n^2 + 2n$

$\therefore S_{n-1}$

$= (n - 1)^2 + 2(n - 1)$

$= n^2 - 2n + 1 + 2n - 2$

$= n^2 - 1$

Now, $T_1 = S_1 = (1)^2 + 2(1) = 3$

And, T_n

$= S_n - S_{n-1}$, where $n > 1$

$= (n^2 + 2n) - (n^2 - 1)$

$\therefore T_n = 2_n + 1$

Question 3:

If the gum of firgt n terms of A.P, 30, 27, 24, 21………… is 120, find number of terms and the last term.

Solution :

Here the given A.P. is 30, 27, 24, 21, ……

$a = 30,\ d = 27 - 30 = -3$ and $S_n = 120.$

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$120 = \frac{1}{2}n\left[2(30) + (n-1)(-3)\right]$$

$\therefore 240 = n[63 - 3n]$

$\therefore 240 = 63n - 3n^2$

$\therefore 3n^2 - 63n + 240 = 0$

$\therefore n^2 - 21n + 80 = 0$

$\therefore (n - 5)(n - 16) = 0$

$\therefore n = 5$ or $n = 16$

Thus, the number of terms is either 5 or 16.

If $n = 5$, then last term

$T_5 = a + 4d = 30 + 4(-3) = 18$

If $n = 16$, then last term

$T_{16} = a + 15d = 30 + 15(-3) = -15$

Thus, the last term is $T_5 = 18$ or $T_{16} = -15.$

Question 4:

Which term of A.P., 100, 97, 94, 91… will be its first -ve term ?

Solution :

Here for the given A.P., 100, 97, 94, 91, ...
$a = 100$ and $d = 97 - 100 = -3$.
Let the n^{th} term of the A.P. be its first negative term.
$\therefore T_n < 0$
$\therefore a + (n-1)d < 0$
$\therefore 100 + (n-1)(-3) < 0$
$\therefore 100 < 3(n-1)$
$\frac{100}{3} < n - 1$
$33\frac{1}{3} + 1 < n$
$n > 34\frac{1}{3}$
So, for $n \in N$, the smallest positive integer greater than $34\frac{1}{3}$ is 35.
Hence, the 35^{th} term of the A.P. is its first negative term.

Question 5:

Find the sum of all 3 digit natural multiples of 6.

Solution :

Smallest 3 digit natural number = 100
Greatest 3 digit natural number = 999
$\therefore 100 = 16 \times 6 + 4$ and
$999 = 166 \times 6 + 3$.

$\therefore$ The first 3 digit natural numbers which is a multiple of 6 is 102.
Second number is 108 and so on.
Thus the 3 digit natural numbers which are multiples of 6
form the finite A.P., 102, 108, 114,..., 996.
For the above A.P., $a = 102$, $d = 108 - 102 = 6$ and $l = T_n = 996$.
Now, $T_n = a + (n-1)d$
$\therefore 996 = 102 + (n-1)(6)$
$\therefore 996 = 96 + 6n$
$\therefore 900 = 6n$
$\therefore n = 150$
Now,
$S_n = \frac{1}{2}n(a + l)$
$\therefore S_{150} = \frac{1}{2} \times 150(102 + 996)$
$= 75 \times 1098$
$= 82{,}350$
Thus, the required sum is 82,350.

Question 6:

The ratio of the sum to m terms to sum to n terms of an A.P. is $\frac{m^2}{n^2}$. Find the ratio of its mth Term tor its nth term.

Solution :

The ratio of the sum of first m terms of an A.P. and the sum of first n terms of the A.P. is $\frac{m^2}{n^2}$

$\therefore \frac{m^2}{n^2} = k$

Let the sum of the first m terms be km^2 $(k \neq 0)$.

Then, the sum of first n terms is kn^2.

Now, $S_m = km^2$

$\therefore S_{m-1} = k(m-1)^2 = k(m^2 - 2m + 1)$

$T_m = S_m - S_{m-1}$

$= km^2 - k(m^2 - 2m + 1)$

$= km^2 - km^2 + 2km - k = 2km - k$

$\therefore T_m = k(2m - 1)$

Similarly we can show that $S_n = kn^2$

$\therefore S_{n-1} = k(n-1)^2$

$T_n = S_n - S_{n-1}$

$= kn^2 - k(n-1)^2$

$= k[n^2 - (n-1)^2]$

$= k[n^2 - n^2 + 2n - 1]$

$\therefore T_n = k(2_n - 1)$

Now, $\frac{T_m}{T_n} = \frac{k(2m-1)}{k(2n-1)} = \frac{2m-1}{2n-1}$

Thus, the ratio of the mth term of the A.P. and the n^{th} term of the A.P. is $\frac{2m-1}{2n-1}$.

Question 7:

Sum to first l, m, n terms of A.P. are p, q, r. Prove that $\frac{p}{l}$ (m – n) + $\frac{q}{m}$ (n – l) + $\frac{r}{n}$ (l – m) = 0

Solution :

Here it is given that $S_l = p$, $S_m = q$ and $S_n = r$.

We know that,

$$S_l = \frac{1}{2} l\left[2a + (l-1)d\right] = p$$

Similarly,

$$\frac{1}{2} m\left[2a + (m-1)d\right] = q \text{ and } \frac{1}{2} n\left[2a + (n-1)d\right] = r$$

Then,

$$\frac{2p}{l} = 2a + (l-1)d \qquad \ldots(1)$$

$$\frac{2q}{m} = 2a + (m-1)d \qquad \ldots(2)$$

$$\frac{2r}{n} = 2a + (n-1)d \qquad \ldots(3)$$

Multiplying equations (1), (2) and (3) by $(m-n)$, $(n-l)$ and $(l-m)$ respectively and then adding, we get

$$\frac{2p}{l}(m-n) + \frac{2q}{m}(n-l) + \frac{2r}{n}(l-m)$$

$$= [2a + (l-1)d](m-n) + [2a + (m-1)d](n-l) + [2a + (n-1)d](l-m)$$

$$= 2a(m-n) + d(l-1)(m-n) + 2a(n-l) + d(m-1)(n-l) + 2a(l-m) + d(n-1)(l-m)$$

$$= 2a[m-n+n-l+l-m] + d[lm-ln-m+n+mn-ml-n+l+nl-nm-l+m]$$

$$= 2a \times 0 + d \times 0$$

$$\therefore \frac{p}{l}(m-n) + \frac{q}{m}(n-l) + \frac{r}{m}(l-m) = 0 \qquad (\because \text{dividing by } 2)$$

Question 8:

The ratio of sum to n terms of two A.P.'s is $\frac{8n+1}{7n+3}$ for every n ∈ N. Find the ratio of their 7th terms and mth terms.

Solution :

Let the two A.P.'s be a, a + d, a + 2d, ……, a + (n − 1)d
and A, A + D, A + 2D, ….., A + (n − 1)D
We denote the sum of n terms of the first A.P. by S_n
and its m^{th} term by T_m.
We denote the sum of n terms of the second A.P. by S_N
and its m^{th} term by T'_m.
Then, according to the data,

$$\frac{S_n}{S_N} = \frac{8n+1}{7n+3}$$

$$\therefore \frac{\frac{1}{2}n\left[2a+(n-1)d\right]}{\frac{1}{2}n\left[2A+(n-1)D\right]} = \frac{8n+1}{7n+3}$$

$$\therefore \frac{2a+(n-1)d}{2A+(n-1)D} = \frac{8n+1}{7n+3}$$

Now, to get the 7^{th} term ratios, we take n = 13 (i.e. 7 × 2 − 1)

$$\frac{2a+(13-1)d}{2A+(13-1)A} = \frac{8(13)+1}{7(13)+3}$$

$$\therefore \frac{2(a+6d)}{2(A+6D)} = \frac{105}{94}$$

$$\therefore \frac{a+6d}{A+6D} = \frac{105}{94}$$

$$\frac{T_7}{T'_7} = \frac{105}{94}$$

Thus, the ratio of the 7^{th} terms of the given A.P.'s is $\frac{105}{94}$.

Next, taking n = 2m − 1,

$$\frac{2a+(2m-1-1)d}{2A+(2m-1-1)D} = \frac{8(2m-1)+1}{7(2m-1)+3}$$

$$\therefore \frac{2\left[a+(m-1)d\right]}{2\left[A+(m-1)D\right]} = \frac{16m-7}{14m-4} \qquad [\because \text{taking 2 common}]$$

$$\therefore \frac{T_m}{T'_m} = \frac{16m-7}{14m-4}$$

Thus, the ratio m^{th} terms of the A.P.'s is $\frac{16m-7}{14m-4}$.

Question 9:

Three numbers in A.P. have the sum 18 and the sum Of their squares is 180. Find the numbers in the increasing order.

Solution :

Suppose the three numbers in A.P. are
a – d, a and a + d.
According to the first condition.
(a – d) + a + (a + d) = 18
∴ 3a = 18
∴ a = 6
According to the second condition,
$(a-d)^2 + a^2 + (a+d)^2 = 180$
$\therefore a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 180$
$\therefore 3a^2 + 21d^2 = 180$

∴ $108 + 2d^2 = 180$
∴ $2d^2 = 72$
∴ $d^2 = 36$
∴ d = 6 or d = -6
Taking a = 6 and d = 6,
First term = a – d = 6 – 6 = 0
Second term = a = 6 and
Third term = a + d = 6 + 6 = 12
Taking a = 6 and d = -6
First term = a – d = 6 – (-6) = 12
Second term = a = 6 and
Third term a + d = 6 + (-6) = 0
Thus, the required numbers are 0, 6 and 12 or 12, 6 and 0.
Arranging the numbers in the increasing order-
0, 6 and 12.

Question 10:

In potato race bucket is placed at the starting point. It 5 m away the from the first potato. The rest of the potatoes are placed in a straight line each 3 m away from the other. Each competitor starts from the bucket. Picks up the nearest potato and runs back and drops it in the bucket and continues till all potatoes are placed in the bucket. What is total distance covered if 15 potatoes
are placed in the race ?

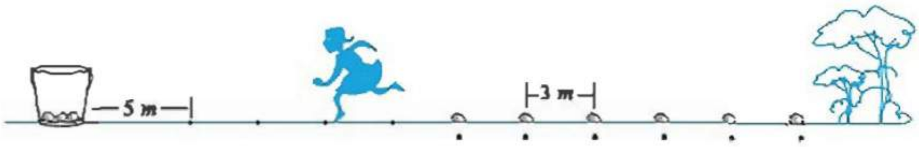

Figure 5.14

If the distance covered is 1340 m, find the number of potatoes?

Solution :

From the given data we calculate the distance covered for each potato.
First potato = 2 × 5 = 10
Second potato = 10 + 2 × 3 = 16 m
Third potato = 16 + 2 × 3 = 22 m, ...
Thus, the distances to be covered form an A.P.
10 m, 16 m, 22 m, ...
The total distance to be covered for 15 potatoes is given by S_{15}.

Here, a = 10 and d = 6 for the said A.P.

$$Sn = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$\therefore S_{15} = \frac{1}{2} \times 15\left[2(10) + (15-1)(6)\right]$$

$$= \frac{1}{2} \times 15 \times 104$$

$$= 15 \times 42$$

$$\therefore S_{15} = 780$$

Thus, if 15 potatoes are placed in the race,
total distance to be covered is 780 m.

Next given is, total distance to be covered is 1340m and we need to find the number of potatoes.
Let the number of potatoes be n, then we take

$S_n = 1340$

$\therefore 1340 = \frac{1}{2}n[2(10) + (n-1)(6)]$

$\therefore 1340 = \frac{1}{2}n[14 + 6n]$

$\therefore 1340 = n[7 + 3n]$

$\therefore 3n^2 + 7n - 1340 = 0$

$n = -\frac{67}{3}$ or $n = 20$

Since $-\frac{67}{3} \in N$, $n = -\frac{67}{3}$ is not possible.

$\therefore n = 20$

Thus, if the total distance to be covered is 1340m, the number of potatoes placed in the race is 20.

Question 11:

A ladder has rungs 25 cm apart. The rungs decrease uniformly from 60 cm at bottom to 40 cm at top. If the distance between the top rung and the bottom rung is 2.5 m, find length of the wood required.

Figure 5.15

Solution :

Here, the distance between two consecutive rungs is 25 cm and the distance between the top rung and the bottom rung is 2.5 m = 250 cm.

$\therefore$ No. of rungs = $\frac{250}{25} + 1 = 11$

The length of the bottom rung is 60 cm and going upwards the length of the rung decreases uniformly.

Length of the last rung is 40 cm.

So, the length of rung will from a finite A.P. in which the first term $a = T_1 = 60$ and the 11^{th} term $T_{11} = 40$.

Now,

$$d = \frac{T_m - T_n}{m - n}$$

$$= \frac{T_{11} - T_1}{11 - 1}$$

$$= \frac{40 - 60}{10}$$

$$d = -2$$

The length of wood required is given by S_{11}.

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$\therefore S_{11} = \frac{1}{2} \times 11\left[2(60) + (11-1)(-2)\right]$$

$$= \frac{1}{2} \times 11[120 - 20]$$

$$= \frac{1}{2} \times 11 \times 100$$

$$= 550$$

Thus, the length of wood required is 550 cm = 5.5 m.

Question 12:

A man purchased LCD TV for ₹ 32.500. He paid ₹ 200 initially and increasing the payment by ₹ 150 every month. How many months did he take to make the complete payment ?

Solution :

Here, amount paid as down payment = Rs. 200
Amount paid in 1st installment = Rs. 200 + Rs. 150 = Rs. 350
Amount paid in the 2nd installment = Rs. 350 + Rs. 150 = Rs. 500, and so on
The amount paid every month increases every month and
forms a finite A.P. 200, 350, 500, ..., T_n
Total sum paid S_n = Rs. 32,500

$$S_n = \frac{1}{2}n[2a + (n-1)d]$$
$$\therefore 32500 = \frac{1}{2}n[2(200) + (n-1)150]$$
$$\therefore 32500 = \frac{1}{2}n[250 + 150n]$$
$$\therefore 32500 = n[125 + 75n]$$
$$\therefore 32500 = 125n + 75n^2$$
$$\therefore 75n^2 + 125n + 32500 = 0$$
$$\therefore 3n^2 + 5n - 1300 = 0 \qquad \text{(Dividing by 25)}$$
$$\therefore (3n + 65)(n - 20) = 0$$
$$n = -\frac{65}{3} \text{ or } n = 20$$

But $-\frac{65}{3} \notin N$, hence, $n = -\frac{65}{3}$ is not possible.

$\therefore n = 20$

Thus there are 20 terms in this finite A.P.
Among these 20 terms, the first term is the down payment, hence the man took 20 − 1 = 19 months to make the complete payment.

Question 13:

In an A.P., $T_1 = 22$, $T_n = -11$, $S_n = 66$. find n.

Solution :

Considering $l = T_n$ as the last term in the given A.P.
$T_1 = a = 22$,
$l = T_n = -11$ and $S_n = 66$
Now,

$$S_n = \frac{n}{2}(a + l)$$
$$\therefore 66 = \frac{n}{2}(22 - 11)$$
$$\therefore 66 = \frac{n}{2} \times 11$$
$$\therefore n = 12$$

Question 14:

In an A.P. $a = 8$, $T_n = 33$, $S_n = 123$, find d and n.

Solution :

Considering $l = T_n$ as the last term in the given A.P., we have
$a = T_1 = 8$, $l = T_n = 33$ and $S_n = 123$.
Now,

$$S_n = \frac{n}{2}(a + l)$$
$$\therefore 123 = \frac{n}{2}(8 + 33)$$
$$\therefore 123 = \frac{41}{2}n$$
$$\therefore 123 \times \frac{2}{41} = n$$
$$\therefore n = 6$$

Also,

$$T_n = a + (n - 1)d$$
$$\therefore T_6 = a + 5d$$
$$\therefore 33 = 8 + 5d$$
$$\therefore 25 = 5d$$
$$\therefore d = 5$$

Question 15:

Select a proper option (a), (b), (c) or (d) from given option :

Question 15(1):

If $T_3 = 8$, $T_7 = 24$, then $T_{10} = \ldots\ldots$

Solution :

d. 36

We need to first find d.

$$d = \frac{T_7 - T_3}{7 - 3} = \frac{24 - 8}{4} = \frac{16}{4} = 4$$

Now, $T_3 = 8$

$$\therefore a + 2d = 8$$
$$\therefore a + 2(4) = 8$$
$$\therefore a + 8 = 8$$
$$\therefore a = 0$$
$$T_{10} = a + 9d = 0 + 9(4) = 36$$

Question 15(2):

If $S_n = 2n^2 + 3n$, then d =

Solution :

b. 4

Here, $a = T_1 = S_1 = 2(1)^2 + 3(1) = 5$

and $a + (a + d) = S_2 = 2(2)^2 + 3(2)$

$\therefore 2a + d = 14$

$\therefore 2(5) + d = 14$

$\therefore d = 4$

Question 15(3):

If the sum of the three consecutive terms of A.P. is 48 and the product of the first and the last is 252, then d = ………

Solution :

a. 2
Let the three consecutive terms of the A.P. be
a – d, a, a + d.
∴ (a – d) + a + (a + d) = 48 (first condition)
∴ 3a = 48
∴ a = 16
(a – d)(a + d) = 252 (second condition)
∴ $a^2 - d^2 = 252$
∴ $(16)^2 - d^2 = 252$
∴ $256 - 252 = d^2$
∴ $d^2 = 4$
∴ d = 2 or d = -2
But (-2) is not there in the given options.
∴ d = 2

Question 15(4):

If a = 2 and d = 4, then S_{20} = …..

Solution :

b. 800

For a = 2, d = 4

$$Sn = \frac{1}{2}n[2a + (n-1)d]$$

$$\therefore S_{20} = \frac{1}{2} \times 20[4 + (20-1)(4)]$$

$$\therefore S_{20} = 10 \times 80$$

$$\therefore S_{20} = 800$$

Question 15(5):

If 3 + 5 + 7 + 9 +….. upto n terms = 288, then n = ……

Solution :

c. 16
3 + 5 + 7 + 9 + ….. upto n terms = 288
∴ 1 + (3 + 5 + 7 + 9 + …. upto n terms) = 288 + 1
∴ 1 + 3 + 5 + …… upto (n + 1) terms = 289
∴ $(n + 1)^2 = (17)^2$ [∵1 + 3 + 5+ ….. n terms = n^2]
∴ n + 1 = 17
∴ n = 16

Question 15(6):

Four numbers are in A.P. and their sum is 72 and the largest of them is twice the smallest. Then the numbers are ..

Solution :

b. 12, 16, 20, 24
Let the four numbers in A.P. be
a – 3d, a – d, a + d and a + 3d.
∴ (a – 3d) + (a – d) + (a + d) + (a + 3d) = 72
∴ 4a = 72
∴ a = 18
Also,
a + 3d = 2(a – 3d)
∴ a + 3d = 2a – 6d
∴ 9d = a
∴ 9d = 18
∴ d = 2
∴ First number = 18 – 3(2) = 12,
Second number = 18 – 2 = 16,
Third number = 18 + 2 = 20,
Fourth number = 18 + 3(2) = 24

Question 15(7):

If $S_1 = 2 + 4 + \ldots + 2n$ and $S_2 = 1 + 3 + \ldots + (2n - 1)$, then $S_1 : S_2 = \ldots\ldots$

Solution :

a. $\frac{n+1}{n}$

$S_1 = 2 + 4 + \ldots\ldots + 2n$
$= 2(1 + 2 + \ldots\ldots + n)$
$= 2\frac{n(n+1)}{n}$
$\therefore S_1 = n(n+1)$
Next, $S_2 = 1 + 3 + \ldots\ldots + (2n - 1)$
= Sum of first n odd natural numbers
$\therefore S_2 = n^2$

Now,
$$S_1 : S_2 = \frac{S_1}{S_2} = \frac{n(n+1)}{n^2} = \frac{n+1}{n}$$

Question 15(8):

For A.P., $S_n - 2S_n - 1 + S_n - 2 = \ldots\ldots$ (n > 2)

Solution :

b. d

For $S_n - 2S_{n-1} + S_{n-2}$

$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$

$= T_n - T_{n-1}$

$= d$

Question 15(9):

If $S_m = n$ and $S_n = m$ then $S_{m+n} = \ldots\ldots$

Solution :

a. $-(m+n)$

Here $S_m = n$ and $S_n = m$

$\therefore \frac{1}{2}m[2a+(m-1)d] = n$ and $\frac{1}{2}n[2a+(n-1)d] = m$

$2ma + (m^2 - m)d = 2n$ and $2na + (n^2 - n)d = 2m$

Taking the difference of above equations,

$2a(m-n) + (m^2 - m - n^2 + n)d = 2n - 2m$

$\therefore 2a(m-n) + (m-n)(m+n-1)d = -2(m-n)$

$\therefore 2a + (m+n-1)d = -2 \quad (\because m \neq n)$

Now,

$S_{m+n} = \frac{1}{2}(m+n)[2a + (m+n-1)d]$

$= \frac{1}{2}(m+n)(-2)$

$\therefore S_{m+n} = -(m+n)$

Question 15(10):

If $T_4 = 7$ and $T_7 = 4$, then $T_{10} = \ldots\ldots$

Solution :

d. 1

$d = \frac{T_7 - T_4}{7-4} = \frac{4-7}{3} = \frac{-3}{3} = (-1)$

$T_4 = 7$

$a + 3d = 7$

$a + 3(-1) = 7$

$a = 7 + 3$

$a = 10$

$T_{10} = a + 9d$

$= 10 + 9(-1)$

$= 1$

Question 15(11):

If $2k + 1, 13, 5k - 3$ are three consecutive terms of A.P., then $k = \ldots\ldots$

Solution :

c. 4

2k + 1, 13, 5k – 3 are three consecutive terms of the A.P.

$\therefore$ 13 – (2k + 1) = (5k – 3) – 13

$\therefore$ 13 + 13 = 5k – 3 + 2k + 1

$\therefore$ 26 = 7k – 2

$\therefore$ 28 = 7k

$\therefore$ k = 4

Question 15(12):

(1) + (1 + 1) + (1 + 1 + 1) +…+ (1 + 1 + 1 +…n – 1 times) = ……..

Solution :

a. $\frac{(n-1)n}{2}$

$$(1) + (1 + 1) + (1 + 1 + 1) + \ldots + (1 + 1 + 1 + \ldots\ldots n-1 \text{ times})$$
$$= 1 + 2 + 3 + \ldots + (n-1)$$
$$= \frac{(n-1)(n-1+1)}{2} \quad (\text{formula for sum of first } (n-1) \text{ natural numbers})$$
$$= \frac{(n-1)n}{2}$$

Question 15(13):

In the A.P., 5, 7, 9, 11, 13, 15,… the sixth term which is prime is …….

Solution :

b. 19

The given A.P. is 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, ……….

Amongst them, primes are 5, 7, 11, 13, 17, 19, 23, ……. Then, the sixth prime is 19.

Question 15(14):

For A.P. $T_{18} - T_8$ = …..

Solution :

b. 10d

For any A.P.

$$d = \frac{T_m - T_n}{m - n}$$
$$\therefore d = \frac{T_{18} - T_8}{18 - 8}$$
$$\therefore d = \frac{T_{18} - T_8}{10}$$
$$T_{18} - T_8 = 10d$$

Question 15(15):

If for A.P., $T_{25} - T_{20} = 15$ then d = ……

Solution :

a. 3

For any A.P.

$$d = \frac{T_m - T_n}{m - n}$$

$$\therefore d = \frac{T_{25} - T_{20}}{25 - 20}$$

$$= \frac{15}{5} = 3$$