Chapter 5

Quadratic Equations

Solutions (Set-1)

Very Short Answer Type Questions :

1. $(x - 3)(x + 2) = 0$ then find the value of x.

Sol. $(x - 3)(x + 2) = 0$

$x - 3 = 0$ or $x + 2 = 0$

$x = 3$ $\quad$ $x = -2$

$x = 3, -2$

2. If α, β are the roots of the equation $x^2 + 5x - 7 = 0$ then find the value of $\frac{\alpha+\beta}{\alpha\beta}$.

Sol. $x^2 + 5x - 7 = 0$

$\alpha + \beta = -5$

$\alpha\beta = -7$

$\frac{\alpha+\beta}{\alpha\beta} = \frac{5}{7}$

3. Write the number of real roots of the equation $(x + 2)^2 + (x - 3)^2 + (x - 4)^2 = 0$.

Sol. $(x+2)^2 + (x-3)^2 + (x-4)^2 = 0$

$x + 2 = 0$ and $x - 3 = 0$ and $x - 4 = 0$

$x = -2$ $\quad$ $x = 3$ $\quad$ $x = 4$

$\Rightarrow$ No solution.

4. If $2+\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, then write the value of p and q.

Sol. $p = -\left[2+\sqrt{3}+2-\sqrt{3}\right] = -4$

$q = \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) = 1$

5. If a and b are roots of the equation $x^2 - x + 1 = 0$, then write the value of $a^2 + b^2$.

Sol. $(a^2 + b^2) = (a+b)^2 - 2ab = 1 - 2(1) = -1$

Short Answer Type Questions :

6. Solve the quadratic equation $25x^2 - 30x + 11 = 0$.

Sol. $25x^2 - 30x + 11 = 0$

Here, $b^2 - 4ac = (-30)^2 - 4(25)(11)$

$= 900 - 1100$

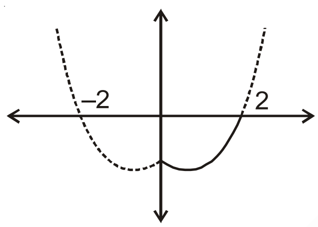
$= -200$

$\therefore$ The solutions of the equation $25x^2 - 30x + 11 = 0$ are given by

$$-\frac{(-30) \pm \sqrt{-200}}{50} = \frac{30 \pm \sqrt{200}i}{50} = \frac{30 \pm 10\sqrt{2}i}{50} = \frac{3}{5} \pm \frac{\sqrt{2}}{5}i$$

7. For the equation $|x|^2 + |x| - 6 = 0$, find the sum of real roots.

Sol. Sum of roots = 2 – 2 = 0



8. Find the values of k for which the quadratic equation $x^2 - kx + k + 2 = 0$ has equal roots.

Sol. $D = 0$

$k^2 - 4(k+2) = 0$

$k = 2 \pm \sqrt{12}$

9. Find the total number of solutions of $x^2 + |x - 1| = 1$.

Sol. 0

Long Answer Type Questions :

10. Solve the quadratic equation $x^2 - \left(3\sqrt{2} - 2i\right)x - 6\sqrt{2}i = 0$, $x \in C$.

Sol. $x^2 - \left(3\sqrt{2} - 2i\right)x - 6\sqrt{2}i = 0$

Here, $b^2 - 4ac = \left(3\sqrt{2} - 2i\right)^2 - 4(1)(-6\sqrt{2}i)$

$= 18 + 4i^2 - 12\sqrt{2}i + 24\sqrt{2}i$

$= 18 + 14i^2 + 12\sqrt{2}i$

$= \left(3\sqrt{2}\right)^2 + (2i)^2 + 2\left(3\sqrt{2}\right)(2i)$

$= \left(3\sqrt{2} + 2i\right)^2$

$\therefore$ The solution of the equation $x^2-\left(3\sqrt{2}-2i\right)x-6\sqrt{2}i=0$ are

$$\frac{\left(3\sqrt{2}-2i\right)+\sqrt{\left(3\sqrt{2}+2i\right)^2}}{2} \text{ and } \frac{\left(3\sqrt{2}-2i\right)-\sqrt{\left(3\sqrt{2}+2i\right)^2}}{2}$$

$$=\frac{\left(3\sqrt{2}-2i\right)+\left(3\sqrt{2}+2i\right)}{2} \text{ and } \frac{\left(3\sqrt{2}-2i\right)-\left(3\sqrt{2}+2i\right)}{2}$$

$= 3\sqrt{2}$ and $-2i$

11. Solve the quadratic equation $2x^2 - (3 + 7i)x - (3 - 9i) = 0$, $x \in C$.

Sol. $2x^2 - (3 + 7i)x - (3 - 9i) = 0$

Here, $b^2 - 4ac = (3 + 7i)^2 + 4(2)(3 - 9i)$

$= 9 + 49i^2 + 42i + 24 - 72i$

$= 9 - 49 + 24 + 42i - 72i$

$= -16 - 30i$

$\therefore$ The solutions of the equation are given by

$$\alpha=\frac{(3+7i)+\sqrt{-16-30i}}{4} \text{ and } \beta=\frac{(3+7i)-\sqrt{-16-30i}}{4}$$

Firstly, we find $\sqrt{-16-30i}$

Let $x + iy = \sqrt{-16-30i}$

$\Rightarrow$ $(x + iy)^2 = -16 - 30i$

$\Rightarrow$ $(x^2 - y^2) + 2ixy = -16 -30i$

$\Rightarrow$ $x^2 - y^2 = -16$ ...(i)

and $2xy = -30$ ...(ii)

Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$= (-16)^2 + (-30)^2 = 256 + 900 = 1156$

$\therefore$ $x^2 + y^2 = \sqrt{1156}$

$x^2 + y^2 = 34$ ...(iii)

From (i) and (ii), we get

$x = \pm 3$

from (ii),

When $x = 3$, $y = -5$ and when $x = -3$, $y = 5$

$\Rightarrow$ $\sqrt{-16-30i}=3-5i$ or $-3 + 5i$

$\Rightarrow \quad \alpha = \dfrac{(3+7i)+(3-5i)}{4}$ and $\beta = \dfrac{(3+7i)-(3-5i)}{4}$

$\Rightarrow \quad \alpha = \dfrac{6+2i}{4}$ and $\beta = \dfrac{12i}{4}$

$\Rightarrow \quad \alpha = \dfrac{3}{2}+\dfrac{i}{2}$ and $\beta = 3i$

Hence the roots of the given equation are $\dfrac{3}{2}+\dfrac{i}{2}$ and $3i$

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Chapter 5

Quadratic Equations

Solutions (Set-2)

1. If the difference of the roots of the equation $x^2 - px + q = 0$ is unity,

(1) $p^2 + 4q = 1$

(2) $p^2 - 4q = 1$

(3) $p^2 - 4q^2 = (1 + 2q)^2$

(4) $4p^2 + q^2 = (1 + 2p)^2$

Sol. Answer (2)

$x^2 - px + q = 0$

Let α, β be roots

$(\alpha - \beta) = 1$

$(\alpha - \beta)^2 = 1$

$(\alpha + \beta)^2 - 4\alpha\beta = 1$

$p^2 - 4q = 1$

2. If α and β are the roots of the equation $x^2 - px + 16 = 0$, such that $\alpha^2 + \beta^2 = 9$, then the value of p is

(1) $\pm\sqrt{6}$

(2) $\pm\sqrt{41}$

(3) ± 8

(4) ± 7

Sol. Answer (2)

$x^2 - px + 16 = 0$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$9 = p^2 - 32$

$p^2 = 41$

$p = \pm\sqrt{41}$

3. Let α and β are the roots of the equation $x^2 + x + 1 = 0$ then

(1) $\alpha^2 + \beta^2 = 4$

(2) $(\alpha - \beta)^2 = 3$

(3) $\alpha^3 + \beta^3 = 2$

(4) $\alpha^4 + \beta^4 = 1$

Sol. Answer (3)

$x^2 + x + 1 = 0$

$\alpha + \beta = -1$

$\alpha\beta = -1$

(1) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2(1)$

$= 1 - 2 = -1$

(2) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= (-1)^2 - 4 \times 1 = -3$

(3) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (\alpha+\beta)((\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta)$

$= (\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta)$

$= (-1)((-1)^2 - 3\alpha 1)$

$= (-1)\ (1 - 3) = 2$

Alternative

$x^2 + x + 1 = 0$

$x = \omega, \omega^2$ (complex root of unity)

$\therefore\ \omega^3 + (\omega^2)^3 = 2$

(4) $\alpha^4 + \beta^2 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= (-1)^2 - 2\ \ 1$

$= 1 - 2$

$= -1$

4. If the ratio of the roots of $lx^2 - nx + n = 0$ is $p : q$, then

(1) $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$ (2) $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} - \sqrt{\frac{n}{l}} = 0$ (3) $\sqrt{\frac{q}{p}} + \sqrt{\frac{p}{q}} + \sqrt{\frac{l}{n}} = 1$ (4) $\sqrt{\frac{q}{p}} + \sqrt{\frac{p}{q}} + \sqrt{\frac{l}{n}} = 0$

Sol. Answer (2)

$\frac{p}{q} = \frac{\alpha}{\beta},\ \alpha + \beta = +\frac{n}{l},\ \alpha\beta = \frac{n}{l}$

Now, $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}} = \frac{+\frac{n}{l}}{\sqrt{\frac{n}{l}}} = +\sqrt{\frac{n}{l}}$

$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = +\sqrt{\frac{n}{l}} \Rightarrow \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} - \sqrt{\frac{n}{l}} = 0$

5. For the equation $|x^2| + |x| - 6 = 0$, the roots are

(1) Real and equal (2) Real with sum 0 (3) Real with sum 1 (4) Real with product 0

Sol. Answer (2)

$$\left|x^2\right| + |x| - 6 = 0 \Rightarrow |x|^2 + |x| - 6 = 0$$

$$\Rightarrow (|x|+3)(|x|-2) = 0 \Rightarrow |x| = -3, |x| = 2$$

$\Rightarrow x = \pm 2$

Two roots are real, with sum 0.

6. If the minimum value of $x^2 + 2x + 3$ is m and maximum value of $-x^2 + 4x + 6$ is M then the value of $m + M$ is

(1) 10 (2) 11 (3) 12 (4) 13

Sol. Answer (3)

$x^2 + 2x + 3 = (x + 1)^2 + 2 \Rightarrow m = 2$

$$\begin{aligned}-x^2 + 4x + 6 &= -x^2 + 4x + 4 - 4 + 6 \\ &= 6 - (x^2 - 4x + 4) + 4 \\ &= 10 - (x^2 - 4x + 4) \\ &= 10 - (x - 2)^2\end{aligned}$$

$\Rightarrow M = 10$

$m + M = 2 + 10 = 12$

7. The values of a, for which the quadratic equation $3x^2 + 2(a^2 + 1)x + (a^2 - 3a + 2) = 0$ possesses roots of opposite sign, are

(1) $1 < a < 2$ (2) $a \in (2, \infty)$ (3) $1 < a < 3$ (4) $-1 < a < 0$

Sol. Answer (1)

For roots of opposite sign, product < 0

$$\Rightarrow \frac{a^2 - 3a + 2}{3} < 0 \Rightarrow (a-2)(a-1) < 0$$

$\Rightarrow 1 < a < 2$

8. If 1, 2, 3 are the roots of the equation $x^3 + ax^2 + bx + c = 0$, then

(1) $a = 1, b = 2, c = 3$ (2) $a = -6, b = 11, c = -6$

(3) $a = 6, b = 11, c = 6$ (4) $a = 6, b = 6, c = 6$

Sol. Answer (2)

If 1, 2, 3 are roots of equation then

$x^3 + ax^2 + bx + c = 0$

$\Rightarrow 1 + 2 + 3 = -a \Rightarrow a = -6$

$1\cdot2 + 2\cdot3 + 1\cdot3 = b \Rightarrow b = 11$

$1\cdot2\cdot3 = -c \Rightarrow c = -6$

9. If $x^2 + px + q$ is an integer for every integral x, then

(1) p is always an integer but q need not be an integer

(2) q is always an integer but p need not be an integer

(3) $(p + q)$ are always integers

(4) p and q) are always integers

Sol. Answer (4)

Let $f(x) = x^2 + px + q$; x $f(x) \in$ Integers

As $f(0) = q \Rightarrow q \in I$

Also $f(1) = (1 + p + q)$ is an integer

$\Rightarrow$ $p \in I$ as $q \in I$

Therefore $p, q \in I$

10. If $a + b + c = 0$ and a, b, c are rational, then the roots of the equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are

(1) Rational (2) Irrational (3) Imaginary (4) Equal

Sol. Answer (1)

$(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$

Put $x = 1$, $b + c - a + c + a - b + a + b - c = a + b + c = 0$

$\therefore$ 1 is the root of the equation.

$\therefore$ Roots are rational.

11. If the equation $x^2 + px + p = 0$, $p \in I$ has both the roots integers, then $(p^2 - 4p)$ can attain

(1) No integral value (2) One integral value (3) Two integral values (4) Three integral values

Sol. Answer (2)

Let $p^2 - 4p = m^2$ as $x = \dfrac{-p \pm \sqrt{p^2 - 4p}}{2}$ is an integer

$\Rightarrow$ $p^2 - 4p - m^2 = 0$

$\Rightarrow$ $p = 2 \pm \sqrt{4 + m^2}$

$\Rightarrow$ $\sqrt{4 + m^2}$ is an integer

Let $4 + m^2 = \lambda^2$ where $\lambda \in I$

$\Rightarrow$ $\lambda^2 - m^2 = 4$ $\Rightarrow$ $\lambda = \pm 2, m = 0$

It follows that $p^2 - 4p = 0$ $\Rightarrow$ $p(p - 4) = 0$ for $p = 0, 4$

12. The number of positive integers n for which $n^2 + 96$ is a perfect square, is

(1) One (2) Two (3) Four (4) Infinite

Sol. Answer (3)

Let $n^2 + 96 = \lambda^2$, $\lambda \in I^+$

$\Rightarrow \quad \lambda^2 - n^2 = 96 \;\Rightarrow\; (\lambda+n)(\lambda-n) = 96$

$\lambda + n$ and $\lambda - n$ must both be even.

Now, 96 = 2 × 48, 4 × 24, 6 × 16, 8 × 12

So, the number of solutions = 4

13. The set of value of λ for which the equation $x^3 - 3x + \lambda = 0$ has three distinct real roots, is

(1) R (2) (–2, 2) (3) (–1, 2) (4) (–3, 1)

Sol. Answer (2)

Let $f(x) = x^3 - 3x + \lambda$, $f'(x) = 3x^2 - 3$, $f'(x) = 0 \Rightarrow x = -1, 1$

Now, $f(-1)\,f(1) < 0 \;\Rightarrow\; (-1+3+\lambda)(1-3+\lambda) < 0$

$\Rightarrow\; (\lambda+2)(\lambda-2) < 0$

$\Rightarrow\; -2 < \lambda < 2$

14. If the equation $y = \lambda x + a\sqrt{1+\lambda^2}$, regarded as a quadratic in λ, will have equal roots, then $x^2 + y^2$ is equal to

(1) $-a^2$ (2) a^2 (3) 0 (4) –1

Sol. Answer (2)

$y = \lambda x + a\sqrt{1+\lambda^2} \;\Rightarrow\; y - \lambda x = a\sqrt{1+\lambda^2}$

$\Rightarrow \quad (y-\lambda x)^2 = a^2(1+\lambda^2) \;\Rightarrow\; \lambda^2(x^2-a^2) - 2\lambda yx + y^2 - a^2 = 0$

For equal roots, $4y^2x^2 - 4(x^2-a^2)(y^2-a^2) = 0$

$\Rightarrow \quad a^2(x^2+y^2) - a^4 = 0 \;\Rightarrow\; x^2 + y^2 = a^2$

15. If $\sec\alpha$, $\tan\alpha$ are roots of $ax^2 + bx + c = 0$, then

(1) $a^4 - b^4 + 4ab^2c = 0$ (2) $a^4 + b^4 - 4ab^2c = 0$ (3) $a^2 - b^2 = 4ac$ (4) $a^2 + b^2 = ac$

Sol. Answer (1)

We know

$\sec^2\alpha - \tan^2\alpha = 1$

$(\sec\alpha - \tan\alpha)(\sec\alpha + \tan\alpha) = 1$

$\left(\dfrac{\sqrt{b^2-4ac}}{a}\right)\left(-\dfrac{b}{a}\right) = -1$

Squaring both side

$(b^2 - 4ac)b^2 = a^4$

$a^4 - b^4 + 4ab^2c = 0$

16. If α, β are roots of $ax^2 + bx + c = 0$, then the equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ has roots

(1) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ (2) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$ (3) $\frac{\alpha}{1+\alpha}, \frac{\beta}{1+\beta}$ (4) $\frac{1+\alpha}{\alpha}, \frac{1+\beta}{\beta}$

Sol. Answer (3)

$ax^2 + bx + c = 0$, Given equation is $an^2 - bx(x-1) + c(x-1)^2 = 0$

$$a\left(\frac{-x}{x-1}\right)^2 + b\left(\frac{-x}{x-1}\right) + 1 = 0$$

Now, Replacing x by $\alpha = -\frac{x}{x-1}$

$$\frac{ax^2}{(x-1)^2} - \frac{bx}{x-1} + c = 0 \Rightarrow ax^2 - bx(x-1) + c(x-1)^2 = 0$$

$\alpha = \frac{-x}{x-1} \Rightarrow x = \frac{\alpha}{1+\alpha}$ is the root of the above equation.

17. The equation $\frac{a(x-b)(x-c)}{(a-b)(a-c)} + \frac{b(x-c)(x-a)}{(b-c)(b-a)} + \frac{c(x-a)(x-b)}{(c-a)(c-b)} = x$ is satisfied by

(1) No value of x (2) Exactly two values of x

(3) Exactly three values of x (4) All values of x

Sol. Answer (4)

$$\frac{a(x-b)(x-c)}{(a-b)(a-c)} + \frac{b(x-c)(x-a)}{(b-c)(b-a)} + \frac{c(x-a)(x-b)}{(c-a)(c-b)} = x$$

is satisfied by $x = a$, $x = b$, $x = c$.

A quadratic equation is satisfied by more than two values of x. So it is an identity. Hence it is satisfied by all values of x.

18. Select the false statement.

(1) If '1' is the root of $8x^8 + 7x^7 + 6x^6 + 5x^5 + 4x^4 + 3x^3 + 2x^2 + x - k = 0$ then the value of k is 36

(2) The roots of the equation $x^2 + kx - 1 = 0$, $k \in R$, are real and distinct

(3) If $b^2 - 4ac$ is not a perfect square then the roots of $ax^2 + bx + c = 0$ are both either irrational or rational

(4) If $a + b + c = 0$ then '1' is the root of $ax^2 + bx + c = 0$

Sol. Answer (3)

(1) Put x = -1

$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 - k = 0$

$k = \frac{8 \times 9}{2} = 36$, True.

(2) $D = b^2 - 4ac = k^2 + 4 > 0$

Hence roots are real. True.

(3) If $b^2 - 4ac$ is negative then roots may be imaginary. Hence statement is false.

(4) f '1' is the root of the equation then

$a(1)^2 + b(1) + c = 0$

$a + b + c = 0$

19. If the equation $(k^2 – 3k + 2)x^2 + (k^2 – 5k + 4)x + (k^2 – 6k + 5) = 0$ is an identity then the value of k is

(1) 1 (2) 2 (3) 3 (4) 4

Sol. Answer (1)

For an identity

$(k^2 – 3k + 2) = 0$

$\Rightarrow (k – 1)(k – 2) = 0$

$\Rightarrow k = 1, k = 2$

$k^2 – 5k + 4 = 0 \Rightarrow (k – 1)(k – 4) = 0$

$k = 1, 4$

$k^2 – 6k + 5 = 0 \Rightarrow (k – 5)(k – 1) = 0$

$k = 1, 5$

Common value of $k = 1$.

20. If $0 < p < q < r$ and the roots α, β of the equation $px^2 + qx + r = 0$ are imaginary, then

(1) $|\alpha| = |\beta|$ (2) $|\alpha| < 1$ (3) $|q| < 1$ (4) $|\alpha| \neq |\beta|$

Sol. Answer (1)

α, β are imaginary roots.

$\therefore$ they are complex conjugate of each other.

$\Rightarrow \bar{\alpha} = \beta \Rightarrow |\bar{\alpha}| = |\beta| \Rightarrow |\alpha| = |\beta|$

21. If $a, b, c \in R$ and the equations $ax^2 + bx + c = 0$ and $x^2 + x + 1 = 0$ have a common root then $a : b : c$ is equal to

(1) 1 : 1 : 1 (2) 1 : 2 : 3 (3) 2 : 3 : 1 (4) 3 : 2 : 1

Sol. Answer (1)

$x^2 + x + 1 = 0$ … (i)

Discriminant $= b^2 – 4ac = 1 – 4 \times 1 \times 1 = –3$

Hence the roots of $x^2 + x + 1 = 0$ and not real.

So roots will be in pair.

Also the roots of $ax^2 + bx + c = 0$ will be non-real.

Clearly both roots of the equations are common.

$$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{1}$$

$\Rightarrow a : b : c = 1 : 1 : 1$

22. Consider the equation $ax^2 + bx + c = 0$, where $a \neq 0$, $a, b, c \in R$ then

(1) If one root is $\alpha + \sqrt{\beta}$, then other root is $\alpha - \sqrt{\beta}$

(2) If $a = 1$ and b and c are integers, then root will be integer

(3) If one root is $\alpha + i\beta$, then other root will be $\alpha – i\beta$

(4) If roots are of opposite sign, then $b \neq 0$

Sol. Answer (3)

(1) Root will be of the form $\alpha \pm \sqrt{\beta}$ of *a, b, c* are rational.

(2) There is no information about $b^2 - 4ac$

Hence statement is false.

(3) As *a, b, c* are real and one root is $\alpha + i\beta$ then other root will be $\alpha - i\beta$.

(4) If roots are of opposite sign then $\alpha\beta < 0 \Rightarrow \frac{c}{a} < 0$

23. The least integral value of k for which the equation $x^2 - 2(k + 2)x + 12 + k^2 = 0$ has two distinct real roots is

(1) 0 (2) 2 (3) 3 (4) 4

Sol. Answer (3)

The given equation is $x^2 - 2(k + 2)x + 12 + k^2 = 0$ has distinct real roots when $D > 0$

$\Rightarrow 4(k + 2)^2 - 4(12 + k^2) > 0$

$\Rightarrow k^2 + 4 + 4k - 12 - k^2 > 0$

$\Rightarrow 4k - 8 > 0$

$\Rightarrow k > 2$

So least integral value of k is 3.

24. For all $x \in R$ if $mx^2 - 9mx + 5m + 1 > 0$, then m lies in the interval

(1) $\left(-\frac{61}{4}, 0\right)$ (2) $\left(\frac{4}{61}, \frac{61}{4}\right)$ (3) $\left[0, \frac{4}{61}\right)$ (4) $\left(\frac{-4}{61}, 0\right)$

Sol. Answer (3)

Let $y = mx^2 - 9m + 5m + 1$

We need $y > 0$

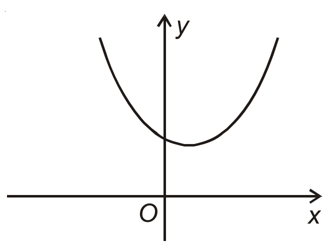
$\Rightarrow$ Upward parabola above x-axis.

$mx^2 - 9mx + 5m + 1 > 0, \ \forall \ x \in R.$

$\Rightarrow \ D < 0, \ a > 0$

i.e., $81m^2 - 4(m)(5m+1) < 0$ and $m > 0$

$\Rightarrow \ m(61m - 4) < 0$ and $m > 0 \ \Rightarrow \ 0 < m < \frac{4}{61}$



Also for $m = 0$,

$0x^2 - 9(0)x + 0 + 1 = 1 > 0, \ \forall x \in R$

$\therefore \ m \in \left[0, \frac{4}{61}\right)$

25. The roots of the equation $x^3 - 2x^2 - x + 2 = 0$ are

(1) 1, 2, 3 (2) –1, 1, 2 (3) –1, 0, 1 (4) –1, –2, 3

Sol. Answer (2)

$x^3 - 2x^2 - x + 2 = 0$

As $x = 1$ is the root of the equation

Hence we may write

$x^3 - 2x^2 - x + 2$

$= x^2 (x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1) (x^2 - x - 2)$

$= (x - 1) (x - 2) (x + 1)$

Roots = 1, –1, 2.

26. The number of roots of the equation $\sqrt{x-3}(x^2 - 7x + 10) = 0$ is

(1) 2 (2) 3 (3) Zero (4) 1

Sol. Answer (1)

$\sqrt{x-3}(x^2 - 7x + 10) = 0$

$\Rightarrow$ $x = 3$ and $x = 2, 5$

But $x \geq 3$

$\therefore$ Only two roots.

27. Consider the following statements

S_1 : The number of real roots of $\cos(e^x) = (2007)^x + (2007)^{-x}$ is zero.

S_2 : The number of real roots of $(x + 1)^{2006} + (x + 2)^{2006} + (x + 3)^{2006} + ... + (x + 2007)^{2006}$ is zero.

S_3 : The number of real roots of $x^4 + x^2 + 1 = 0$ is zero.

Then the true statement(s) is/are

(1) S_1 only (2) S_2 only (3) S_1 & S_2 only (4) S_1, S_2 & S_3

Sol. Answer (4)

S_1 : $\cos x^x \in [-1, 1]$

And $(2007)^x + (2007)^{-x} > 2$

Hence no solution exist.

S_2 : As L.H.S. $\neq$ 0 at any x

Hence no solution exist.

S_3 : As $x^4 + x^2 + 1 > 0$ have no solution exist.

28. If $p + iq$ be one of the roots of the equation $x^3 + ax + b = 0$, then $2p$ is one of the roots of the equation

(1) $x^3 + ax + b = 0$ (2) $x^3 - ax - b = 0$ (3) $x^3 + ax - b = 0$ (4) $x^3 + bx + a = 0$

Sol. Answer (3)

$p + iq$ is one root $\Rightarrow$ $p - iq$ is other root.

Let α be third root.

Now sum =

$\Rightarrow \quad \alpha = -2p$

$\alpha = -2p$ is root of $x^3 + ax + b = 0$

$\therefore \quad 2p$ is root of $(-x)^3 - ax + b = 0$

$\Rightarrow \quad x^3 + ax - b = 0$

29. If the sum of the roots of the quadratic equation $x^2 + bx + c = 0$ (a, b, $c \neq 0$) is equal to sum of squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in

(1) A.P. (2) G.P. (3) H.P. (4) None of these

Sol. Answer (3)

Given, $\alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$

Also, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\alpha +$

$\Rightarrow \quad (\alpha+\beta)(\alpha^2\beta^2) = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta \quad \Rightarrow \quad \left(-\frac{b}{a}\right)\left(\frac{c^2}{a^2}\right) = \frac{b^2}{a^2} - \frac{2c}{a}$

$\Rightarrow \quad -bc^2 = b^2a - 2ca^2 \quad \Rightarrow \quad 2ca^2 = bc^2 + b^2a$

$\Rightarrow \quad 2\frac{a}{b} = \frac{c}{a} + \frac{b}{c} \quad \Rightarrow \quad \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.

30. If a, b are real and unequal, then the roots of the quadratic equation $(a - b)x^2 - 5(a + b)x - 2(a - b) = 0$ are

(1) Real and equal (2) Non-real complex (3) Real and unequal (4) None of these

Sol. Answer (3)

The given quadratic equation is $(a - b)x^2 - 5(a + b)x - 2(a - b) = 0$

The discriminant

$D = (-5(a + b))^2 + 8(a - b)(a - b)$

$= 25(a + b)^2 + 8(a - b)^2$

Hence $D > 0 \ \forall \ a$ & b.

So, roots are real and unequal.

31. The roots x_1 and x_2 of the equation $x^2 + px + 12 = 0$ are such that their difference is 1. Then the positive value of p is

(1) 1 (2) 2 (3) 3 (4) 7

Sol. Answer (4)

$$\alpha - \beta = \frac{\sqrt{D}}{a} = 1$$

$\Rightarrow$ $\sqrt{p^2 - 48} = 1$

$\Rightarrow$ $p = \pm 7$; but p is positive, hence $p = 7$.

32. The number of integral values of x satisfying the equation $2^x (4 - x) = 2x + 4$ is

(1) Zero (2) 1 (3) 2 (4) 3

Sol. Answer (4)

$$2^x = \frac{2x+4}{4-x}$$

Here $2^x > 0 \Rightarrow \frac{2x+4}{4-x} > 0$

$\therefore$ $x \in (-2, 4)$

But only values of $x = 0, 1, 2$, satisfy the equation.

33. Let α and β be the real roots of the equation $x^2 - x(\lambda - 2) + (\lambda^2 + 3\lambda + 5) = 0$. The maximum value of $\alpha^2 + \beta^2$ is

(1) 18 (2) 20 (3) 27 (4) 19

Sol. Answer (1)

$\because$ $\alpha + \beta = \lambda - 2$

$\alpha \cdot \beta = \lambda^2 + 3\lambda + 5$

$\therefore$ $\alpha^2 + \beta^2 = (\lambda^2 - 4\lambda + 4(-2(\lambda^2 + 3\lambda + 5) = 19 - (\lambda + 5)^2$

$\therefore$ Maximum value of $\alpha^2 + \beta^2 = 18$

$\because$ Roots are real, $\therefore$ $D \geq 0$

$\therefore$ $\lambda \in \left[-4, -\frac{4}{3}\right]$

34. If a and b are the non-zero roots of equation $x^2 + ax + b = 0$, then $(a + b)$ is equal to

(1) –1 (2) 2 (3) 1 (4) –2

Sol. Answer (1)

$\because$ a, b are non-zero roots of $x^2 + ax + b = 0$

$\therefore$ $a + b = -a$

and $ab = b$

$\therefore$ Either $a = b = 0$

or $a = 1$ and $b = -2$

$\therefore$ $a + b = -1$

35. (α_1, α_2), (α_2, α_3) and (α_3, α_1) are respectively the roots of $x^2 - 2ax + 2 = 0$, $x^2 - 2bx + 3 = 0$ and $x^2 - 2cx + 6 = 0$. If $\alpha_1, \alpha_2, \alpha_3 \in R^+$, then the value of $a + b + c$ is equal to

(1) 2 (2) 3 (3) 6 (4) 12

Sol. Answer (3)

$\alpha_1\alpha_2 = 2,\ \alpha_2\alpha_3 = 3,\ \alpha_3\alpha_1 = 6$

Thus $\alpha_1^2\alpha_2^2\alpha_3^2 = 36 \Rightarrow \alpha_1 = 2, \alpha_2 = 1, \alpha_3 = 3$

$2a = 3$, $2b = 4$, $2c = 5$

$\Rightarrow a + b + c = 6$

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