JEE-MAIN 2025
Physics
Practice Test - 5

Date :	Time : 75 Min.	Max. Marks : 80

Marking Scheme :

SCQ - 1 - 3, (4, –1)
MCQ = 4 - 9, (4, –1)
Comprehension = 10 - 18, (4, –1)
Integer = 19 - 20, (4, –1)

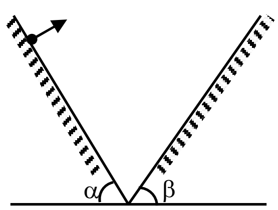
SECTION-1 : (Only One option correct type)

This section contains **3 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Consider two identical homogeneous balls, A and B, with the same initial temperatures. One of them is at rest on a horizontal plane, while the second one hangs on a thread as shown in figure. The same quantities of heat Q have been supplied to both balls. Rise of temperature of ball A is ΔT_A and of ball B is ΔT_B. Mass of one ball is m, radius is r, specific heat is c and coefficient of linear expansion of material of ball is α . Value of $\Delta T_A - \Delta T_B$ will be (All kinds of heat losses are negligible.) (Given that *Q* = 50 kcal, J = 4.16 J/cal, g = 10 m/s^2, m = 47 kg, r = 0.1 m, c = 0.031 cal/(g · K), $\alpha = 29 \times 10^{-6}$ K^{-1}.

(A) 1.5×10^{-5} K (B) 3×10^{-5} K (C) 4.5×10^{-5} K (D) 6×10^{-5} K

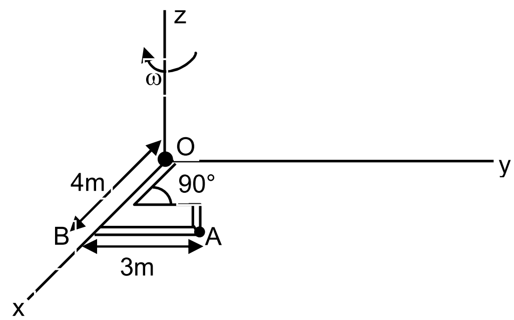
2. Two planes are inclined at angles α and β with the horizontal and a particle is projected at right angle to the one plane from a point at a distance 'a' from the point of intersection of the planes as shown in the figure . If the particle strikes to the other plane at right angle, time of flight is :



(A) $\sqrt{\dfrac{2a\sin^2(\alpha+\beta)}{g(\sin\alpha - \sin\beta\cos(\alpha+\beta))}}$ (B) $\sqrt{\dfrac{2a\sin^2(\alpha-\beta)}{g(\sin\alpha - \sin\beta\cos(\alpha+\beta))}}$

(C) $\sqrt{\dfrac{2a\sin^2(\alpha+\beta)}{g(\sin\alpha - \sin\beta\cos(\alpha-\beta))}}$ (D) $\sqrt{\dfrac{2a\sin^2(\alpha-\beta)}{g(\sin\alpha - \sin\beta\cos(\alpha-\beta))}}$

3. A rod is bent into the "L" shape and hinged at O so that it can be rotated about z-axis in x-y plane as shown in the figure. At the position shown in the figure, the angular velocity is 2rad/sec and angular velocity is decreasing at the rate of 4rad/s^2. Acceleration of end A will be :



(A) $\vec{a} = \left(-4\hat{i} + 28\hat{j}\right)$ (B) $\vec{a} = \left(28\hat{i} - 4\hat{j}\right)$ (C) $\vec{a} = \left(-28\hat{i} - 4\hat{j}\right)$ (D) $\vec{a} = \left(-28\hat{i} + 4\hat{j}\right)$

SECTION-2 : (One or more option correct type)

This section contains **6 multiple choice question.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

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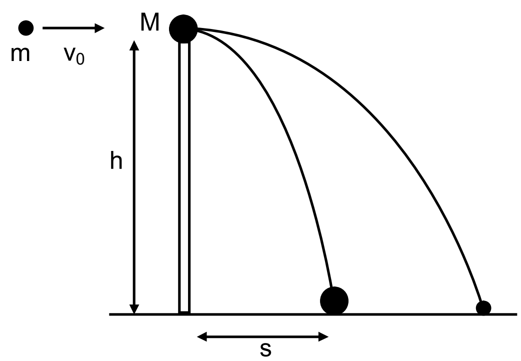
4. A small ball with mass M rests on a vertical column with height h. A bullet with mass m, moving with velocity v_0, passes horizontally through the center of the ball. The ball reaches the ground at a distance s from vertical column. Choose the correct option(s)

Neglect resistance of the air.



(A) The bullet reaches the ground at a distance $d = v_0\sqrt{\dfrac{2h}{g}} - \dfrac{M}{m}s$ from vertical column.

(B) The bullet reaches the ground at a distance $d = v_0\sqrt{\dfrac{2h}{g}} - 2\sqrt{\dfrac{M}{m}}s$ from vertical column.

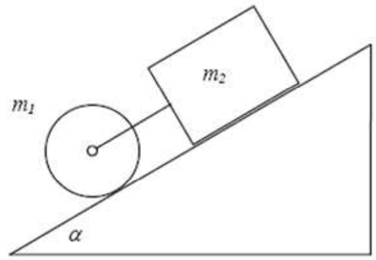
(C) Fraction of kinetic energy of the bullet converted into heat when the bullet passed through the ball is

$$\frac{M}{m}\frac{s^2}{v_0^2}\frac{g}{2h}\left(2\frac{v_0}{s}\sqrt{\frac{2h}{g}} - \frac{M+m}{m}\right)$$

(D) Fraction of kinetic energy of the bullet converted into heat when the bullet passed through the ball is

$$\frac{M}{m}\frac{s^2}{v_0^2}\frac{g}{h}\left(\frac{v_0}{s}\sqrt{\frac{2h}{g}} - \frac{M+m}{m}\right)$$

5. On an inclined plane of 30° a block, mass m_2 = 4 kg, is joined by a light cord to a uniform solid cylinder, mass m_1 = 8 kg, radius r = 5 cm. The coefficient of friction between the block and the inclined plane μ = 0.2. Friction at the bearing is negligible. There is no slipping between cylinder and incline plane. Choose the correct option(s).



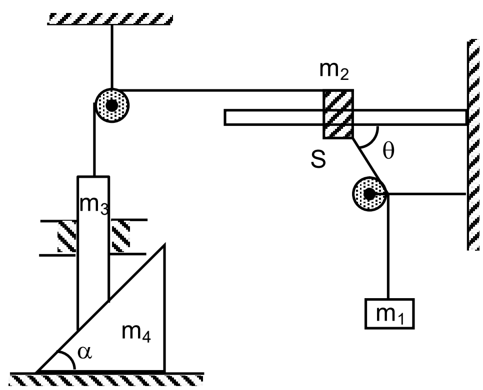
(A) Acceleration of block is 4.25m/s^2

(B) Friction acting between cylinder and incline plane is 13.01N.

(C) Tension in the cord is 0.8 N.

(D) Acceleration of the block is 3.25m/s^2

6. Consider an arrangement shown in the figure. The block of mass m_3 is constrained to move in the vertical direction only. The wedge of mass m_4 moves in the horizontal direction. The slider 'S' of mass m_2 moves on a fixed horizontal rod. The friction between all the contact surfaces is negligible. At a particular instant the string connecting the slider 'S' to the block of mass m_1 is making angle θ with the rod and everything is at rest. Choose the correct option (s) for the given instant. The acceleration due to gravity is g.



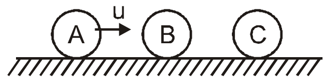
(A) Acceleration of m_1 is $\dfrac{g(m_3 - m_1\cos\theta)\cos\theta}{(m_1\cos^2\theta + m_2 + m_3 + m_4\cot^2\alpha)}$

(B) Acceleration of m_1 is $\dfrac{g(m_3 + m_2 - m_1\cos\theta)\cos\theta}{(m_1\cos^2\theta + m_2 + m_3 + m_4\cot^2\alpha)}$

(C) Acceleration of m_3 is $\dfrac{g(m_3 - m_1\cos\theta)\cot\alpha}{(m_1\cos^2\theta + m_2 + m_3 + m_4\cot^2\alpha)}$

(D) Acceleration of m_3 is $\dfrac{g(m_3 + m_2 - m_1\cos\theta)\cot\alpha}{(m_1\cos^2\theta + m_2 + m_3 + m_4\cot^2\alpha)}$

7. Three balls A, B and C each of mass m and same size, are placed along same line on smooth horizontal surface. A is given a velocity u towards B as shown. If coefficient of restitution for collision between A and B is $\frac{1}{2}$ and between B and C is $\frac{1}{\ }$. Choose the correct option(s).(Assume all the collisions to be headon)



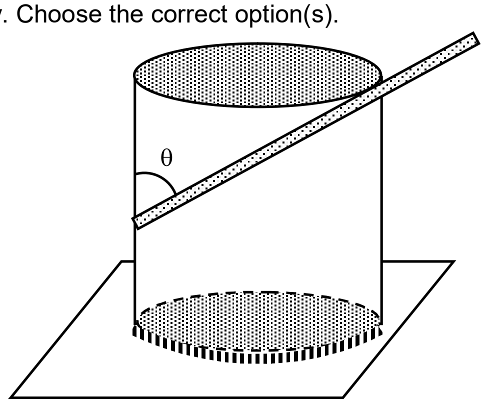
(A) The total energy loss due to all possible collisions will be $\frac{3}{16}$ mu^2

(B) The total energy loss due to all possible collisions will be $\frac{5}{16}$ mu^2

(C) Final velocity of A and B will be same.

(D) Final velocity of C is twice than A.

8. A vertical hollow cylinder is fixed on the ground. A uniform rod can be balanced partly in and partly out of the cylinder with the lower end of the rod resting against the vertical wall of the cylinder, as shown in the figure. The angle made by rod with the vertical in equilibrium is θ. Maximum and minimum value of θ are α and β respectively. Choose the correct option(s).



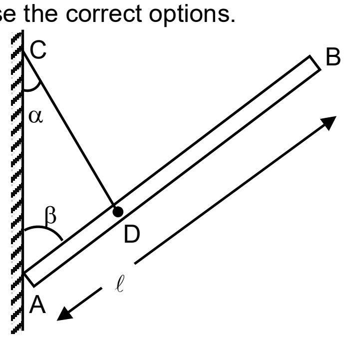
(A) Coefficient of friction between rod and cylinder is $\tan\left[\frac{1}{2}\tan^{-1}\left(\frac{\sin^3\alpha - \sin^3\beta}{\sin^2\alpha\cos\alpha + \sin^2\beta\cos\beta}\right)\right]$

(B) Coefficient of friction between rod and cylinder is $\tan\left[\frac{1}{2}\tan^{-1}\left(\frac{\sin^3\alpha + \sin^3\beta}{\sin^2\alpha\cos\alpha + \sin^2\beta\cos\beta}\right)\right]$

(C) At maximum angle lower end of the rod will have sliding tendency upwards.

(D) At minimum angle lower end of the rod will have sliding tendency upwards.

9. A homogeneous rod AB of mass m and length ℓ resting with its lower end against a vertical wall and kept in inclined position by a string CD as shown in the figure. Given that AD = $\frac{AB}{4}$. Angles formed by the string and the rod with the wall are α and β respectively .Coefficient of friction between end A of the rod and vertical wall is μ. Choose the correct options.



(A) If $\alpha = \beta$ friction between end A and vertical wall will be zero.

(B) If $\beta > \alpha$ friction on end A of the rod will be in downward direction.

(C) If $\beta < \alpha$ friction on end A of the rod will be in vertically upward direction.

(D) If $\beta = 53°$ and $\alpha = 37°$ minimum value of μ required is 7/12.

SECTION – 3 : (Paragraph Type)

This section contains **3 paragraphs** each describing theory, experiment, data etc**. Nine questions** relate to three paragraphs. Each question of a paragraph has **only one correct answer** among the four choices (A), (B), (C) and (D).

Paragraph for Questions 10 and 12

Three cylinders with the same mass, the same length and the same external radius are initially resting on an inclined plane having angle α with horizontal. The coefficient of sliding friction on the inclined μ is known and has the same value for all the cylinders. The first cylinder is empty (tube) , the second is homogeneous filled, and the third has a cavity exactly like the first, but closed with two negligible mass lids and filled with a liquid with the same density like the cylinder's walls. The friction between the liquid and the cylinder wall is considered negligible. The density of the material of the first cylinder is n times greater than that of the second or of the third cylinder. Answer the following three questions .

10. The linear acceleration of the third cylinder in the non-sliding case will be

(A) $a_3 = \dfrac{2g\sin\alpha}{3}$ (B) $a_3 = \dfrac{2g\sin\alpha}{3-\left(1-\frac{1}{n}\right)^2}$

(C) $a_3 = \dfrac{2g\sin\alpha}{3+\left(1-\frac{1}{n}\right)}$ (D) $a_3 = \dfrac{2g\sin\alpha}{3+\left(1+\frac{1}{n}\right)}$

11. Condition for angle α of the inclined plane so that no cylinder slips.

(A) $\tan\alpha < \mu\dfrac{4n-1}{2n-1}$ (B) $\tan\alpha < \mu\dfrac{3n-1}{2n-1}$

(C) $\tan\alpha < \mu\dfrac{4n+1}{2n+1}$ (D) $\tan\alpha < \mu\dfrac{4n-1}{2n+1}$

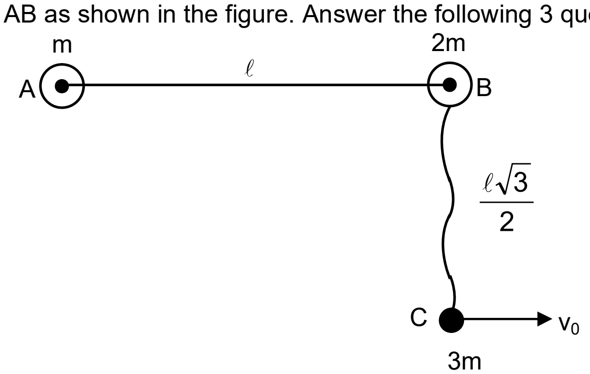
12. The interaction force between the liquid and the walls of the cylinder in the case of slipping of this cylinder, knowing that the liquid mass is m_1, will be

(A) $m_1 g\sin\alpha\sqrt{1+\mu^2}$ (B) $m_1 g\cos\alpha\sqrt{1-\mu^2}$

(C) $m_1 g\sin\alpha\sqrt{1-\mu^2}$ (D) $m_1 g\cos\alpha\sqrt{1+\mu^2}$

Paragraph for Questions 13 and 15

There particles A,B and C of mass m, 2m and 3m respectively lie on a smooth horizontal table. A and B as well as B and C are connected by light inextensible strings each of equal length l. The string connecting A and B is tight. The initial distance between B and C is $\dfrac{l\sqrt{3}}{2}$ and particle C is given a velocity v_0 parallel to AB as shown in the figure. Answer the following 3 questions.



13. Velocity of the particle A, just after its starts moving is :

(A) $\dfrac{11v_0}{38}$ (B) $\dfrac{2v_0}{19}$ (C) $\dfrac{4v_0}{19}$ (D) $\dfrac{9v_0}{38}$

14. Total energy loss due to non-conservative force devolved in strings during the time strings become taut is :

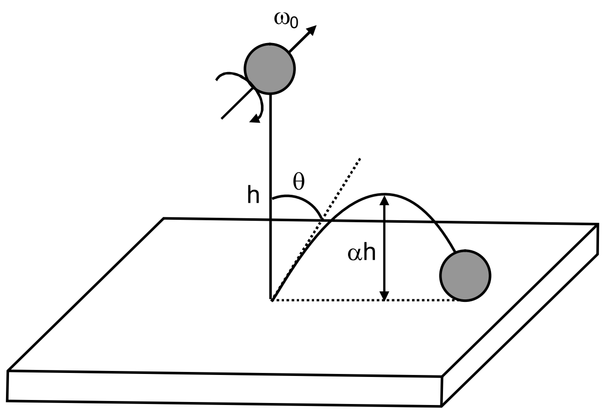
(A) $\frac{2705}{2888}mv_0^2$ (B) $\frac{3705}{1888}mv_0^2$ (C) $\frac{3705}{2888}mv_0^2$ (D) $\frac{2705}{1888}mv_0^2$

15. Impulse on particle B due to the string connecting B and C during the time just before and just after strings become taut is :

(A) $\frac{3mv_0}{19}$ (B) $\frac{9mv_0}{19}$ (C) $\frac{6mv_0}{19}$ (D) $\frac{12mv_0}{19}$

Paragraph for Questions 16 and 18

The figure shows a solid, homogeneous ball radius *R*. Before falling to the floor its center of mass is at rest, but the ball is spinning with angular velocity ω_0 about a horizontal axis through its center. The lowest point of the ball is at a height *h* above the floor.



When released, the ball falls under gravity, and rebounds to a new height such that its lowest point is now αh above the floor. The deformation of the ball and the floor on impact may be considered negligible. Ignore the presence of the air. The impact time, although, is finite. The mass of the ball is *m*, the acceleration due the gravity is *g*, the dynamic coefficient of friction between the ball and the floor is μ_k, and the moment of inertia of the ball about the given axis is:

$$I = \frac{2mR^2}{5}$$

You are required to consider two situations, in the first, the ball slips during the entire impact time, and in the second the slipping stops before the end of the impact time.

Answer the following three questions.

16. If the ball slips during the entire impact time, value of tanθ is (where θ is the rebound angle indicated in the diagram)

(A) $\tan\theta = \mu_k \frac{(1-c)}{c}$, where c = $\sqrt{\alpha}$ (B) $\tan\theta = \mu_k \frac{(1+c)}{c}$, where c = $\sqrt{\alpha}$

(C) $\tan\theta = \mu_k \frac{(1+c^3)}{c^3}$, where c = $\sqrt{\alpha}$ (D) $\tan\theta = \mu_k \frac{(1+c)}{(2-c)}$, where c = $\sqrt{\alpha}$

17. If the ball slips during the entire impact time, minimum value of ω_0 required is :

(A) $\omega_{0\min} = \frac{7\mu_k(1+c)\sqrt{gh}}{R}$, where c = $\sqrt{\alpha}$ (B) $\omega_{0\min} = \frac{7\mu_k(1-c)\sqrt{gh}}{R}$, where c = $\sqrt{\alpha}$

(C) $\omega_{0\min} = \frac{7\mu_k(1+c)\sqrt{2gh}}{R}$, where c = $\sqrt{\alpha}$ (D) $\omega_{0\min} = \frac{7\mu_k(1+c)\sqrt{2gh}}{2R}$, where c = $\sqrt{\alpha}$

18. If the ball stops slipping before the end of the impact time, the horizontal distance traveled in flight before first and second impact is:

(A) $\frac{4}{7}c\sqrt{\frac{2hR}{g}}R\omega_0$, where c = $\sqrt{\alpha}$

(B) $\frac{4}{7}c\sqrt{\frac{hR}{g}}R\omega_0$, where c = $\sqrt{\alpha}$

(C) $\frac{2}{7}c\sqrt{\frac{2hR}{g}}R\omega_0$, where c = $\sqrt{\alpha}$

(D) $\frac{4}{7}c\sqrt{\frac{hR}{2g}}R\omega_0$, where c = $\sqrt{\alpha}$

SECTION-4 : (Integer value correct Type)

This section contains **2 questions**. The answer to each question is a **single digit integer,** ranging from 0 to 9 (both inclusive)

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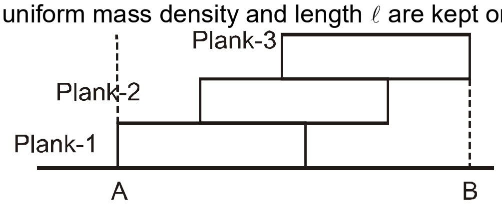
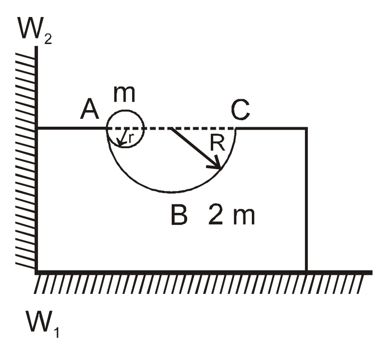
19. Three identical planks of uniform mass density and length ℓ are kept on each other as shown.



maximum length of AB so that all the planks remain in equilibrium is $\frac{\alpha\ell}{}$. Find α (Assume all surfaces to be smooth) :

20. A sphere of mass m and radius r is released from a wedge of mass 2m as show. ABC is hemispherical position of radius R. Impulse imparted to the system consisting wedge and sphere by the vertical wall w_1w_2 till the time sphere reaches at the bottom most position of spherical portion for the first time is $m\sqrt{\frac{10g\ \ (R-r)}{\alpha}}$. Find α. Friction between wedge and horizontal surface is absent and between sphere and wedge friction is sufficient to avoid slipping between them.



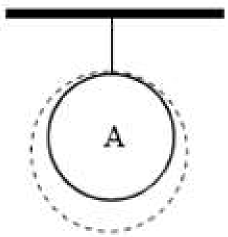
ANSWER KEYS

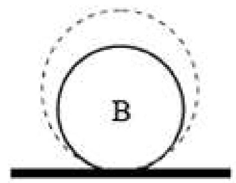
1.	(A)	**2.**	(A)	**3.**	(D)	**4.**	(AC)	**5.**	(BD)
6.	(AC)	**7.**	(BCD)	**8.**	(AC)	**9.**	(ABCD)	**10.**	(B)
11.	(A)	**12.**	(D)	**13.**	(B)	**14.**	(C)	**15.**	(D)
16.	(B)	**17.**	(D)	**18.**	(A)	**19.**	7	**20.**	7
21.	(A)								

SOLUTIONS

Solutions:

Sol.1





For A

$mc\Delta T_A = Q + mgr\alpha\Delta T_A \Rightarrow \qquad \Delta T_A = \dfrac{Q}{m(C - gr\alpha)}$

for B

$mc\Delta T_B = Q - mgr\alpha\Delta T_B \Rightarrow \qquad \Delta T_A = \dfrac{Q}{m(C + gr\alpha)}$

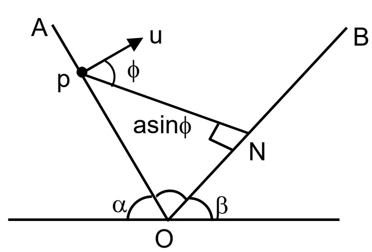
$Q = 208 \times 10^3$ joule
$c = 0.13$ Jg–k
$\alpha = 29 \times 10^{-6}\ K^{-1}$
$r = \dfrac{1}{10}m$
$g = 10m/s^2$

$\Delta T_A - \Delta T_B = \dfrac{Q}{m(c - gr\alpha)} - \dfrac{Q}{m(c + gr\alpha)}$

$= \dfrac{2Qgr\alpha}{mc^2}$

$= 1.5 \times 10^{-5}$ k

Sol.2 PN is perpendicular to OB, for motion parallel to OB



$0 = (u \operatorname{Sin}\phi) - (g \operatorname{Sin}\beta)t$

For motion parallel to PN (perpendicular to OB

$a\sin\phi = (u\cos\phi)\, t - \frac{1}{2}(g\cos\beta)t^2$

Also , $\phi = \pi - (\alpha + \beta)$

From equations (i), (ii) and (iii)

$$u^2 = \frac{2ag\sin\beta}{\sin\alpha - \sin\beta\cos(\alpha+\beta)}$$

Now from equation (i) and (iv)

$$t = \sqrt{\frac{2a\sin^2(\alpha+\beta)}{g(\sin\alpha - \sin\beta\cos(\alpha+\beta))}}$$

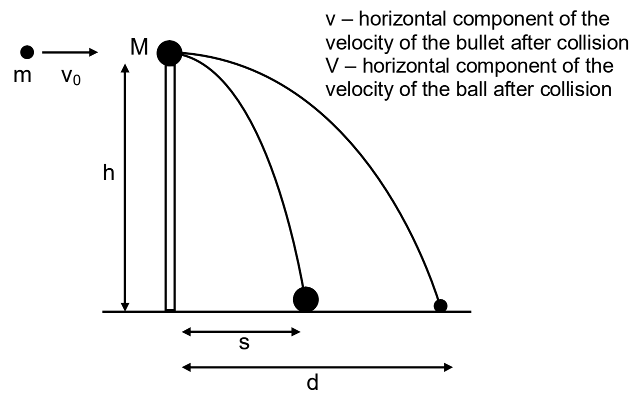
Sol.3 Here , $= \vec{\omega} = -2\hat{k}, \vec{\alpha} = 4\hat{k}$

$\vec{r} = (4\hat{i} + 3\hat{j})$

Now, $\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) + (\vec{\alpha} \times \vec{r})$

$\Rightarrow$ $\vec{a} = (-28\hat{i} + 4\hat{j})$

Sol.4



We will use notation shown in Fig. 2

As no horizontal force acts on the system ball + bullet, the horizontal component of momentum of this system before collision and after collision must be the same:

$$mv_0 = mv + MV.$$

So,

$$v = v_0 - \frac{M}{m}V$$

From conditions described in the text of the problem it follows that

$$v > V$$

After collision both the ball and the bullet continue a free motion in the gravitational field with initial horizontal velocities v and V, respectively. Motion of the ball and motion of the bullet are continued for the same time

$$t = \sqrt{\frac{2h}{g}}$$

It is time of free fall from height h.

The distances passed by the ball and bullet during time t are.

respectively Thus.

$$V = s\sqrt{\frac{g}{2h}}$$

Therefore

$$v = v_0 - \frac{M}{m}s\sqrt{\frac{g}{2h}}$$

Finally

$$d = v_0\sqrt{\frac{2h}{g}} - \sqrt{\frac{M}{m}}s$$

$$E_0 = \frac{mv_0^2}{2}$$

Immediately after the collision the total kinetic energy of the system is equal to the sum of the kinetic energy of the bullet and the ball:

$$E_m = \frac{mv^2}{2}, E_M = \frac{MV^2}{2}$$

Their difference, converted into heat, was

$\Delta E = E_0 - (E_m + E_M)$

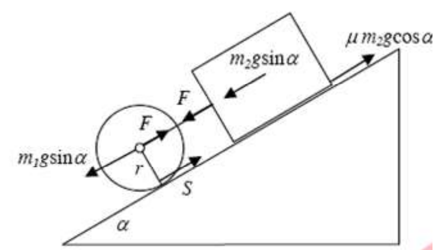
It is the following part of the initial kinetic energy of the bullet:

$$p = \frac{\Delta E}{E_0} = 1 - \frac{E_m + E_M}{E_0}$$

By using expressions for energies and velocities (quoted earlier) we get

$$p = \frac{M}{m}\frac{s^2}{v_0^2}\frac{g}{2h}\left(2\frac{v_0}{s}\sqrt{\frac{2h}{g}} - \frac{M+m}{m}\right)$$

Sol.5



If the cord is stressed the cylinder and the block are moving with the same acceleration *a*. Let *F* be the tension in the cord, *S* the frictional force between the cylinder and the inclined plane (*Fig. 2*). The angular acceleration of the cylinder is *a*/*r*. The net force causing the acceleration of the block:

$$m_2 a = m_2 g\cos\alpha + F,$$

and the net force causing the acceleration of the cylinder:

$$m_1 a = m_1 g\sin\alpha - S - F$$

The equation of motion for the rotation of the cylinder:

$$Sr = \frac{a}{r}.I.$$

(*I* is the moment of inertia of the cylinder, $S \cdot r$ is the torque of the frictional force.) Solving the system of equations we get

$$a = g.\frac{(m_1 + m_2)\sin\alpha - \mu m_2\cos\alpha}{m_1 + m_2 + \frac{I}{r^2}}$$

$$s = \frac{I}{r^2}.g.\frac{(m_1 + m_2)\sin\alpha - \mu m_2\cos\alpha}{m_1 + m_2 + \frac{I}{r^2}}$$

$$F = m_2 g.\frac{\mu\left(m_1 + \frac{I}{r^2}\right)\cos\alpha - \frac{I\sin\alpha}{r^2}}{m_1 + m_2\frac{I}{r^2}}$$

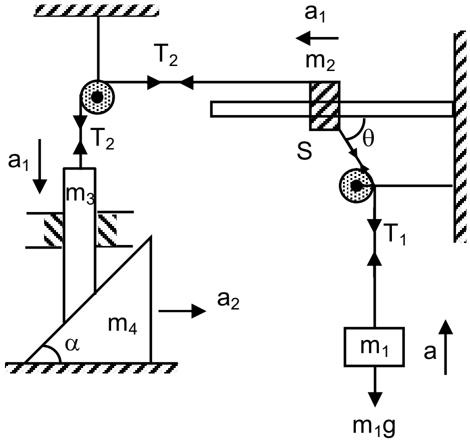
The moment of inertia of a solid cylinder is $I = \frac{m_1 r^2}{2}$. Using the given numerical values :

$$a = g.\frac{(m_1 + m_2)\sin\alpha - \mu m_2\cos\alpha}{1.5m_1 + m_2} = 3.25 m/s^2$$

$$S = \frac{m_1 g}{2}.\frac{(m_1 + m_2)\sin\alpha - \mu m_2\cos\alpha}{1.5m_1 + m_2} = 13.01N$$

$$F = m_2 g.\frac{(1.5\mu\cos\alpha - 0.5\sin\alpha)m_1}{1.5m_1 + m_1} = 0.192\text{N}$$

Sol.6 From the geometry of the figure.



$a_1 = a\sec\theta$ ....(i)

$a_2 = a_1\cot\alpha$ ....(ii)

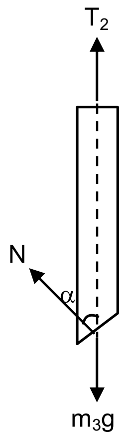
For the block of mass m_1 ....(iii)

$m_1a = T_1 - m_1g$ ....(iv)

For the slider 'S'

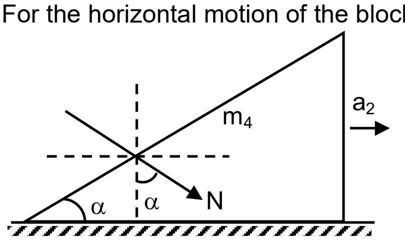
$m_2a_1 = T_2 - T_1\cos\theta$ ....(iv)

For the vertical motion of the block of mass m_3



$m_3a_1 = m_3g - T_1 - N\cos\alpha$ ....(v)

For the horizontal motion of the block of mass m_4.



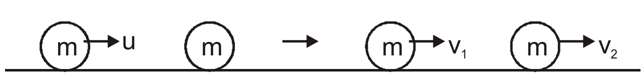
$m_4a_2 = N\sin\alpha$ ....(vi)

After solving these equations, we get

$$\alpha = \frac{g(m_3 - m_1\cos\theta)\cos\theta}{(m_1\cos^2\theta + m_2 + m_3 + m_4\cot^2\alpha)}$$

Just before collision **Just after collision**

Sol.7



applying conservation of linear momentum $mu = mv_2 + mv_1$

js[kh; laosx lj{k.k }kjk $mu = mv_2 + mv_1$

coefficient of restitution $v_2 - v_1 = eu$

izR;koLFkku xq.kkad $v_2 - v_1 = eu$

$\Rightarrow \quad v_2 = \frac{u}{2}(1 + e)$ $\qquad v_1 = \frac{u}{2}(1 - e)$

for collision between A and B

A rFkk B ds e/; VDdj ds fy,

$v_B = \frac{u}{2}\left(1+\frac{1}{2}\right) = \frac{3u}{4}$

$v_A = \frac{u}{2}\left(1-\frac{1}{2}\right) = \frac{u}{4}$

for collision between B and C

B rFkk C ds e/; VDdj ds fy,

$v_C = \frac{1}{2}\left(\frac{3u}{4}\right)\left(1+\frac{1}{3}\right) = \frac{u}{2}$

$v_B' = \frac{1}{2}\left(\frac{3u}{4}\right)\left(1-\frac{1}{3}\right) = \frac{u}{4}$

Total number of collisions = 2

dqy VDdjksa dh la[;k = 2

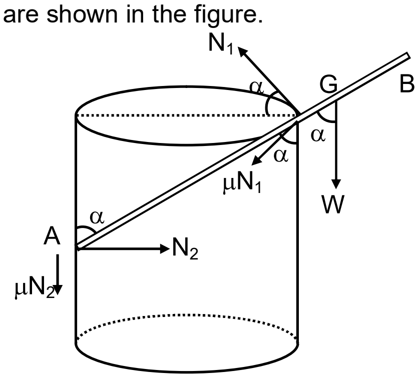
Total energy loss $\Delta E = \frac{1}{2}\left(\frac{m}{2}\right)u^2\left(1-\frac{1}{4}\right) + \frac{1}{2}\frac{m}{2}\left(\frac{3u}{4}\right)^2\left(1-\frac{1}{9}\right)$

dqy ÅtkZ esa gkfu $\Delta E = \frac{1}{2}\left(\frac{m}{2}\right)u^2\left(1-\frac{1}{4}\right) + \frac{1}{2}\frac{m}{2}\left(\frac{3u}{4}\right)^2\left(1-\frac{1}{9}\right)$

$$= \frac{1}{4}mu^2\left(\frac{3}{4}\right) + \frac{1}{4}mu^2\left(\frac{9}{16}\right)\left(\frac{8}{9}\right)$$

$$= \frac{3mu^2}{16} + \frac{1}{8}mu^2 = .\ \frac{5}{16}mu^2$$

Sol.8 Suppose the radius of the cylinder is R and length of rod is 2ℓ. Consider the case when the end A slides up. Forces acting on the rod are shown in the figure.



Resolving forces horizontally and vertically, we have

$N_2 = N_1 \cos\alpha + \mu N_1 \sin\alpha$ .....(i)

and $N_1 \sin\alpha = \mu N_2 + \mu N_1 \cos\alpha + W$ .....(ii)

Taking moment about A,

$N_1(2R \operatorname{cosec}\alpha) = W(\ell \sin\alpha)$ .....(iii)

From equations (i), (ii) and (iii), we get

$2R = \ell[(1 - \mu^2)\sin\alpha - 2\mu\cos\alpha]\sin^2\alpha$ .....(iv)

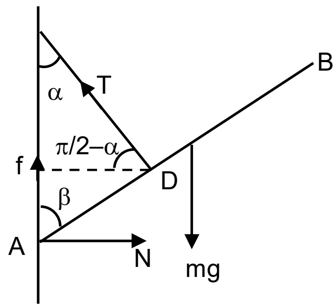
Similarly, when the rod makes least angle β, we get

$2R = \ell[(1 - \mu^2)\sin\beta + 2\mu\cos\beta]\sin^2\beta$ .....(v)

From equations (iv) and (v), we get

$$\mu = \tan\left[\frac{1}{2}\tan^{-1}\left(\frac{\sin^3\alpha - \sin^3\beta}{\sin^2\alpha\cos\alpha + \sin^2\beta\cos\beta}\right)\right]$$

Sol.9



Translational equilibrium

$mg = f + T\cos\alpha$ ....(i)

$N = T\sin\alpha$ .... (ii)

Rotational equilibrium w.r.t D

$$mg\,\frac{\ell}{4}\sin\beta + f\,\frac{\ell}{4}\sin\beta = N\,\frac{\ell}{4}\cos\beta$$

$mg + f = N\cot\beta$ .... (iii)

from (i) (ii) and (iii)

$f = N(\cot\beta - \cot\alpha) \le \mu N$

$\mu \ge \cot\beta - \cot\alpha$

If $\alpha = \beta$ $\quad f = 0$

$\alpha > \beta$ $\quad f = +$ vertically upward.

Sol. **(10, 11 & 12)**

The inertia moments of the three cylinders are:

$$I_1 = \frac{1}{2}\rho_1\pi\left(R^4 - r^4\right)h,\ I_2 = \frac{1}{2}\rho_2\pi R^4 h = \frac{1}{2}mR^2,\ I_3 = \frac{1}{2}\rho_2\pi(R^4 - r^4)h$$

Because the three cylinders have the same mass :

$m = \rho_1(R^2 - r^2)\,h = \rho_2\pi R^2 h$

it results:

$$r^2 = R^2\left(1 - \frac{\rho_2}{\rho_1}\right) = R^2\left(1 - \frac{1}{n}\right), n = \frac{\rho_1}{\rho_2}$$

The inertia moments can be written:

$$I_1 = I_2\left(2 - \frac{1}{n}\right) > I_2,\ I_3 = I_2\left(2 - \frac{1}{n}\right).\frac{1}{n} = \frac{I_1}{n}$$

In the expression of the inertia momentum I_3 the sum of the two factors is constant:

$$\left(2 - \frac{1}{n}\right) + \frac{1}{n} = 2$$

independent of n, so that their products are maximum when these factors are equal:

$2 - \frac{1}{n} = \frac{1}{n}$; it results n = 1, and the products $\left(2 - \frac{1}{n}\right).\frac{1}{n} = 1$ In fact n > 1, so that the products is les than 1. It results :

$$I_1 > I_2 > I_3$$

For a cylinder rolling over freely on the inclined plane (fig. 1.1) we can write the equations:

$mg\sin\alpha - F_f = ma$

$N - mg\cos\alpha = 0$

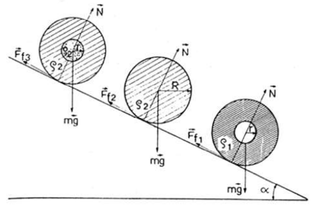
$F_f R = I\varepsilon$

where ε is the angular acceleration. If the cylinder doesn't slide we have the condition:

$a = \varepsilon R$

Solving the equation system (6-8) we find:

$$a = \frac{g\sin\alpha}{1 + \frac{I}{mR^2}},\ F_f = \frac{mg\sin\alpha}{1 + \frac{mR^2}{I}}$$



In the case of the cylinders from this problem, the condition necessary so that none of them slides is obtained for maximum I:

$$\tan\alpha < \mu\left(1+\frac{mR^2}{I_1}\right) = \mu\frac{4n-1}{2n-1}$$

The accelerations of the cylinders are:

$$a_1 = \frac{2g\sin\alpha}{3+\left(1-\frac{1}{n}\right)},\ a_2 = \frac{2g\sin\alpha}{3}\quad a_3 = \frac{2g\sin\alpha}{3-\left(1-\frac{1}{n}\right)^2}$$

The relation between accelerations:

$a_1 < a_2 < a_3$

In the case than all the three cylinders slide:

$F_f = \mu N = \mu mg\cos\alpha$

and from (7) results :

$$\varepsilon = \frac{R}{I}\mu mg\cos\alpha$$

for the cylinders of the problem:

$$\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = \frac{1}{I_1} : \frac{1}{I_2} : \frac{1}{I_3} = 1 : \left(1-\frac{1}{n}\right) : n$$

$\varepsilon_1 < \varepsilon_2 < \varepsilon_3$

In the case that one of the cylinders is sliding:

mg sin $\alpha - F_f = ma$, $F_f = \mu mg\cos\alpha$,

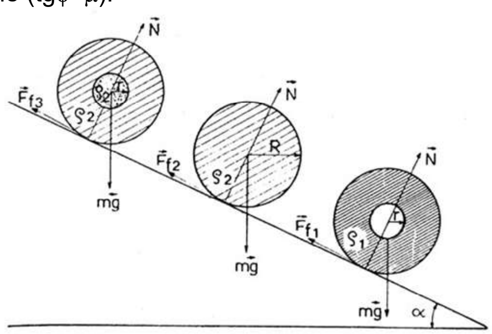
$\alpha = g(\sin\alpha - \mu\cos\alpha)$

Let $\vec{F}$ be the total force acting on the liquid mass ml inside the cylinder (fig.1.2), we can write:

$F_x + m_l g\sin\alpha = m_l\alpha = m_l g(\sin\alpha - m\cos\alpha)$, $F_y - m_l g\cos\alpha = 0$

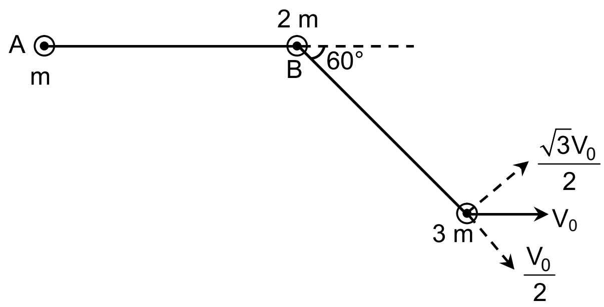
$$F = \sqrt{F_x^2 + F_y^2} = m_l g\cos\alpha\ .\ \sqrt{1+\mu^2} = m_l g\frac{\cos\alpha}{\cos\phi}$$

where ϕ is the friction angle ($tg\phi=\mu$).

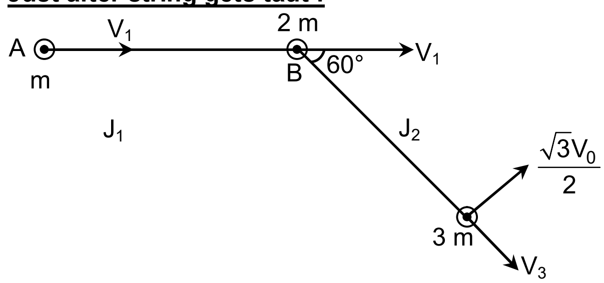


Sol. (13, 14 & 15)

<u>Just before string gets taut :</u>



Just after string gets taut :



Impulse momentum theorem :

$3mV_0/2 - J_2 = 3mV_3$ .... (1)

$V_3 = V_1 \cos 60 + V_2 \cos 30 \Rightarrow V_3 = \dfrac{V_1 + \sqrt{3}V_2}{2}$ .... (2)

$J_2 \cos 60 = 3mV_1$ .... (3)

$J_2 \sin 60 = 2mV_2$ .... (4)

$J_1 = mV_1$ ....(5)

$$\tan 60 = \frac{2V_2}{3V_1} = \sqrt{3}$$

$$2V_2 = 3\sqrt{3}V_1$$

$$V_3 = \frac{V_1 + \sqrt{3}V_2}{2} = \frac{V_1 + \sqrt{3}\left(\dfrac{3\sqrt{3}V_1}{2}\right)}{2}$$

$$= \frac{V_1 + \dfrac{9V_1}{2}}{2} = \frac{11V_1}{4}$$

$$V_1 = \frac{2V_0}{19}$$

$$V_2 = \frac{3\sqrt{3}}{2}\left(\frac{2V_0}{19}\right) = \frac{3\sqrt{3}}{19}V_0$$

$$V_3 = \frac{11}{4}\left(\frac{2V_0}{19}\right) = \frac{11V_0}{38}$$

$$J_2 = \frac{12mv_0}{19}$$

$$K_I = \frac{3}{2}mv_0^2$$

$$K_f = \frac{627}{2888} mv_0^2$$

$$\text{loss} = K_I - K_f = \frac{3705}{2888} mv_0^2$$

Solution : (16, 17 & 18)
***Situation* I:** slipping throughout the impact.
a) *Calculation of the velocity at the instant before impact*
Equating the potential gravitational energy to the kinetic energy at the instant before impact we can arrive at the pre-impact velocity v_0:

$$mgh = \frac{mv_0^2}{2} \qquad .....(1)$$

from which we may solve for v_0 as follows:

$$v_0 = \sqrt{2gh} \qquad .....(2)$$

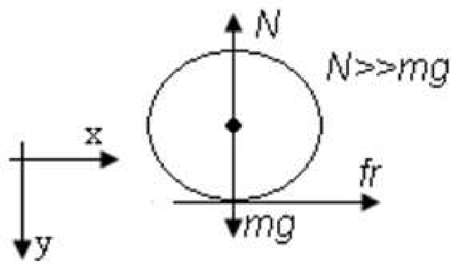
b) *Calculation of the vertical component of the velocity at the instant after impact*
Let v_{2x} and v_{2y} be the horizontal and vertical components, respectively, of the velocity of the mass center an instant after impact. The height attained in the vertical direction will be αh and then:

$$v_{2y}^2 = 2g\alpha h \qquad .....(3)$$

from which, in terms of α (or the restitution coefficient $c = \sqrt{\alpha}$

$$v_{2y} = \sqrt{2g\alpha h} = cv_0 \qquad .....(4)$$

General equations for the variations of linear and angular momenta in the time interval of the Impact



Considering that the linear impulse of the forces is equal to the variation of the linear momentum and that the angular impulse of the torques is equal to the variation of the angular momentum, we have:

$$I_y = \int_{t_1}^{t_2} N(t)dt = mv_0 + mv_{2y} = m(1+c)\sqrt{2gh} \qquad .....(5)$$

$$I_x = \int_{t_1}^{t_2} f_r(t)dt = mv_{2x} \qquad .....(6)$$

$$I_\theta = \int_{t_1}^{t_2} Rf_r(t)dt = R\int_{t_1}^{t_2} f_r(t)dt = I(\omega_0 - \omega_2) \qquad .....(7)$$

Where I_x, I_y and I_θ are the linear and angular impulses of the acting forces and ω_2 is the angular velocity after impact. The times t_1 and t_2 correspond to the beginning and end of impact.

Variants
At the beginning of the impact the ball will always be sliding because it has a certain angular velocity ω_0. There are, then, two possibilities:

I. The entire impact takes place without the friction being able to spin the ball enough for it to stop at the contact point and go into pure rolling motion.

II. For a certain time t $\in$ (t_1, t_2), the point that comes into contact with the floor has a velocity equal to zero and from that moment the friction is zero. Let us look at each case independently.

Case I
In this variant, during the entire moment of impact, the ball is sliding and the friction relates to the normal force as:

$$fr = \mu_k N(t) \qquad .....(8)$$

Substituting (8) in relations (6) and (7), and using (5), we find that:

$$I_x = \mu_k \int_{t_1}^{t_2} N(t)dt = \mu_k I_y = \mu_k (1+c)\sqrt{2gh} = mv_{2x} \qquad .....(9)$$

and

$$I_\theta = R\mu_k \int_{t_1}^{t_2} N(t)dt = R\mu_k m(1+c)\sqrt{2gh} = I(\omega_0 - \omega_2) \qquad .....(10)$$

which can give us the horizontal component of the velocity v_{2x} and the final angular velocity in the form:

$$V_{2x} = \mu_k (1+c)\sqrt{2gh} \qquad .....(11)$$

$$\omega_2 = \omega_0 - \frac{\mu_k mR(1+c)}{I}\sqrt{2gh} \qquad .....(12)$$

With this we have all the basic magnitudes in terms of data. The range of validity of the solution under consideration may be obtained from (11) and (12). This solution will be valid whenever at the end of the impact the contact point has a velocity in the direction of the negative *x*. That is, if:

$$\omega_2 R > v_{2x}$$

$$\omega_0 - \frac{\mu_k mR(1+c)}{I}\sqrt{2gh} > \frac{\mu_k (1+c)}{R}\sqrt{2gh}$$

$$\omega_0 > \frac{\mu_k \sqrt{2gh}}{R}(1+c)\left(\frac{mR^2}{I}+1\right) \qquad .....(13)$$

so, for angular velocities below this value, the solution is not valid.

Case II
In this case, rolling is attained for a time *t* between the initial time t_1 and the final time t_2 of the impact. Then the following relationship should exist between the horizontal component of the velocity v_{2x} and the final angular velocity:

$$\omega_2 R = v_{2x} \qquad .....(14)$$

Substituting (14) and (6) in (7), we get that:

$$mRv_{2x} = I\left(\omega_0 - \frac{v_{2x}}{R}\right) \qquad .....(15)$$

which can be solved for the final values:

$$V_{2x} = \frac{I\omega_0}{mR + \frac{I}{R}} = \frac{I\omega_0 R}{mR^2 + I} = \frac{2}{7}\omega_0 R \qquad .....(16)$$

and:

$$\omega_2 = \frac{I\omega_0}{mR^2 + I} = \frac{2}{7}\omega_0 \qquad .....(17)$$

Calculation of the tangents of the angles
Case I
For tan θ we have, from (4) and (11), that:

$$\tan\theta = \frac{v_{2x}}{v_{2y}} = \frac{\mu_k (1+c)\sqrt{2gh}}{c\sqrt{2gh}} = \mu_k \frac{(1+c)}{c}$$

$$\tan\theta = \mu_k \frac{(1+c)}{c} \qquad .....(18)$$

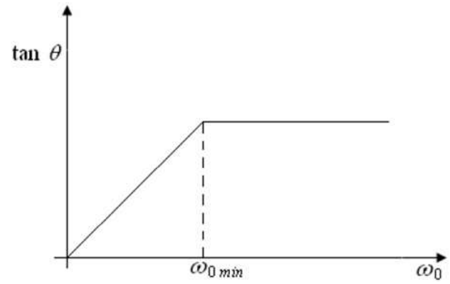
i.e., the angle is independent of ω_0

Case II
Here (4) and (16) determine for tan θ that :

$$\tan\theta = \frac{v_{2x}}{v_{2y}} = \frac{I\omega_0 R}{I + mR^2}\frac{1}{c\sqrt{2gh}} = \frac{I\omega_0 R}{(I + mR^2)c\sqrt{2gh}}$$

$$\tan\theta = \frac{2\omega_0 R}{7c\sqrt{2gh}} \qquad .....(19)$$

then (18) and (19) give the solution.



We see that θ does not depend on ω_0 if $\omega_0 > \omega_0$ min ; where ω_{0min} is given as

$$\omega_{0\,min} = \frac{\mu_k (1+c)\sqrt{2gh}\left(1+\frac{,mR^2}{I}\right)}{R}$$

$$\omega_{0\,min} = \frac{7\mu_k (1+c)\sqrt{2gh}}{2R}$$

Calculation of the distance to the second point of impact

Case I

The rising and falling time of the ball is:

$$t_v = 2\frac{v_{2y}}{g} = \frac{2c\sqrt{2gh}}{g} = 2c\sqrt{\frac{2h}{g}}$$

The distance to be found, then, is;

$$d_I = v_{2x}t_v = \mu_k (1+c)\sqrt{2gh}\,2c\sqrt{\frac{2h}{g}}$$

$$d_I = 4\mu_k (1+c)ch$$

which is independent of ω_0.

Case II

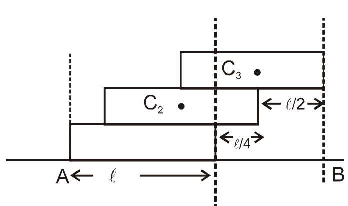
In this case, the rising and falling time of the ball will be the one given in (21). Thus the distance we are trying to find may be calculated by multiplying t_v by the velocity v_{2x} so that:

$$d_{II} = v_{2x}t_v = \frac{I\omega_0}{mR^2 + I}2c\sqrt{\frac{2h}{g}} = \frac{2\omega_0 Rc}{1+\frac{5}{2}}\sqrt{\frac{2h}{g}}$$

$$d_{II} = \frac{4}{7}c\sqrt{\frac{2hR}{g}}R\omega_0$$

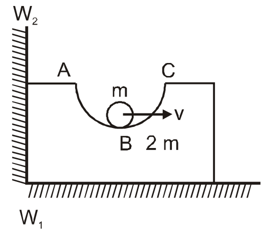
Thus, the distance to the second point of impact of the ball increases linearly with ω_0.

Sol.19



$$(AB)_{max} = \ell + \frac{\ell}{4} + \frac{\ell}{2} = \frac{7\ell}{4}$$

Sol.20 $mg(R - r) = \frac{7}{10}mV^2 = \frac{7P^2}{10m}$



$P = m\sqrt{\frac{10g(R-r)}{7}}$ = Impulse on the system by wall