CALORIMETRY AND THERMAL EXPANSION

Contents

JEE (Advance) Syllabus

Thermal expansion of solids, liquids and gases; Calorimetry, latent heat; Heat conduction in one dimension

JEE (Main) Syllabus

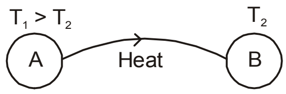
Thermal expansion of solids, liquids and gases; Calorimetry, latent heat; Heat conduction in one dimension

Note: ✍ Marked Questions can be used for Revision.

CALORIMETRY AND THERMAL EXPANSION

1. HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of the receiving body. It should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of a body" are meaningless.



When we say that a body is heated it means that its molecules begin to move with greater kinetic energy.
S.I. unit of heat energy is joule (J). Another common unit of heat energy is calorie (cal).

1 calorie = 4.18 joules.

1 calorie : The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 ºC at one atmospheric pressure is 1 calorie.

Mechanical Equivalent of Heat

In early days heat was not recognized as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write,

$$\mathbf{W = JH}$$

where J is called mechanical equivalent of heat. J is expressed in joule/calorie. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

SOLVED EXAMPLE

Example 1. What is the change in potential energy (in calories) of a 10 kg mass after 10 m fall ?

Solution : Change in potential energy

$$\Delta U = mgh = 10 \times 10 \times 10$$
$$= 1000 \text{ J}$$
$$= \frac{1000}{4.186} \text{ cal} \quad \textbf{Ans.}$$

2. SPECIFIC HEAT

Specific heat of substance is equal to heat gain or released by that substance to raise or fall its temperature by 1ºC for a unit mass of substance.
When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to:

(a) the mass of the body $\Delta Q \propto m$

(b) rise or fall of temperature of the body $\Delta Q \propto \Delta T$

$$\Delta Q \propto m \, \Delta T \quad \text{or} \quad \mathbf{\Delta Q = m\, s\, \Delta T}$$

or $dQ = m\, s\, dT$ or $Q = m \int s\, dT$.

where s is a constant and is known as the specific heat of the body $s = \frac{Q}{m\Delta T}$. S.I. unit of s is joule/kg-kelvin and C.G.S. unit is cal./gm °C.

Specific heat of water : S = 4200 J/kg°C = 1000 cal/kg°C = 1 Kcal/kg°C = 1 cal/gm°C

Specific heat of steam = half of specific heat of water = specific heat of ice

SOLVED EXAMPLE

Example 2. Heat required to increases the temperate of 1 kg water by 20°C

Solution : heat required = $\Delta Q = ms\Delta\theta$

$\because$ S = 1 cal/gm°C = 1 Kcal/kg°C

= 1 × 20 = 20 Kcal.

Heat capacity or Thermal capacity :

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°. If 'm' is the mass and 's' the specific heat of the body, then **Heat capacity = m s**.

Units of heat capacity in: CGS system is, **cal °C^{-1}**; SI unit is, **JK^{-1}**

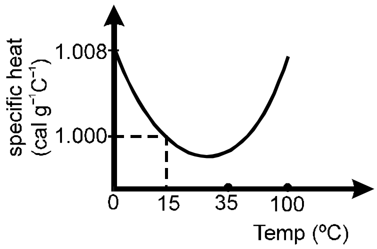
Important Points:

(a) We know, $s = \frac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$), then $s = Q/0 = \infty$. Thus the specific heat of a substance when it melts or boils at constant temperature is infinite.

(b) If the temperature of the substance changes without the transfer of heat (Q = 0) then $s = \frac{Q}{m\Delta T} = 0$. Thus when liquid in the thermos flask is shaken, its temperature increases without the transfer of heat and hence the specific heat of liquid in the thermos flask is zero.

(c) To raise the temperature of saturated water vapours, heat (Q) is withdrawn. Hence, specific heat of saturated water vapours is negative. (This is for your information only and not in the course)

(d) The slight variation of specific heat of water with temperature is shown in the graph at 1 atmosphere pressure. Its variation is less than 1% over the interval form 0 to 100°C.



Relation between Specific heat and Water equivalent:

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$ms\,\Delta T = m_W S_W \Delta T \quad \Rightarrow \quad m_W = \frac{ms}{s_W}$

In calorie $s_W = 1$

$\therefore \quad m_W = ms$

m_w is also represent by W

so $\quad W = ms$.

Phase change:

Heat required for the change of phase or state,

Q = mL , L = latent heat.

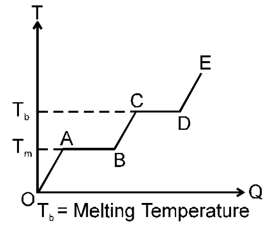
Latent heat (L): The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

Latent heat of Fusion (L_f): The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion. Latent heat of fusion of ice is 80 kcal/kg

Latent heat of vaporization (L_v): The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization. Latent heat of vaporization of water is 540 kcal kg^{-1}.

Latent heat of ice : L = 80 cal/gm = 80 Kcal/kg = 4200 × 80 J/kg

Latent heat of steam : L = 540 cal/gm = 540 Kcal/kg = 4200 × 540 J/kg



T_b = Melting Temperature
T_m = Boiling Temperature

The given figure, represents the change of state by different lines

OA – solid state , AB – solid + liquid state (Phase change)

BC – liquid state , CD – liquid + vapour state (Phase change)

DE – vapour state

$\Delta Q = ms\Delta T$

slope $\frac{\Delta T}{\Delta Q} = \frac{1}{ms}$ $\Rightarrow$ $\frac{\Delta T}{\Delta Q} \propto \frac{1}{S}$

where mass (m) of substance constant slope of T – Q graph is inversely proportional to specific heat, if in given diagram

(slope) OA > (slope) DE

then $(s)_{OA} < (s)_{DE}$

when $\Delta Q = mL$

If (length of AB) > (length of CD)

then (latent heat of AB) > (latent heat of CD)

SOLVED EXAMPLE

Example 3. Find the amount of heat released if 1 kg steam at 200ºC is converted into –20ºC ice.

Solution : Heat released ΔQ = heat release to convert steam at 200 ºC into 100ºC steam + heat release to convert 100º C steam into 100ºC water + heat release to convert 100º water into 0ºC water + heat release to convert 0 ºC water into – 20ºC ice.

$$\Delta Q = 1 \times \frac{1}{2} \times 100 + 540 \times 1 + 1 \times 1 \times 100 + 1 \times 80 + 1 \times \frac{1}{2} \times 20$$

= 780 Kcal.

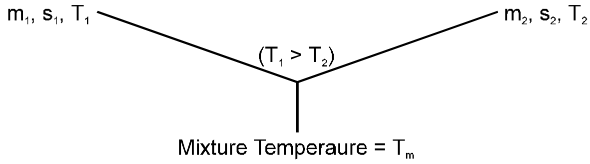
3. CALORIMETRY

The branch of thermodynamics which deals with the measurement of heat is called calorimetry. A simple calorimeter is a vessel generally made of copper with a stirrer of the same material. The vessel is kept in a wooden box to isolate it thermally from the surrounding. A thermometer is used to measure the temperature of the contents of the calorimeter. Object at different temperatures are made to come in contact with each other in the calorimeter. As a result, heat is exchanged between the object as well as with the calorimeter. Neglecting any heat exchange with the surrounding.

Law of Mixture:

When two substances at different temperatures are mixed together, then exchange of heat continues to take place till their temperatures become equal. This temperature is then called final temperature of mixture. Here, **Heat taken by one substance = Heat given by another substance**

$\Rightarrow m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$



SOLVED EXAMPLE

Example 4. An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it loses 25% energy to surroundings. Then find the temperature rise of the block. (Take sp. heat of iron 470 J/kg ºC)

Solution : $mS\Delta\theta = \frac{1}{4}\,mgh \quad \Rightarrow \quad \Delta\theta = \frac{10 \times 10}{4 \times 470}$

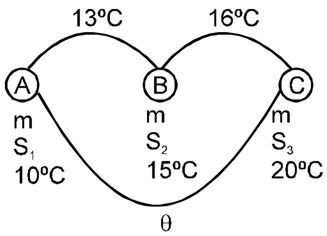
Zeroth law of thermodynamics :

If objects A and B are separately in thermal equilibrium with a third object C , then objects A and B are in thermal equilibrium with each other.

SOLVED EXAMPLE

Example 5. The temperature of equal masses of three different liquids A, B, and C are 10ºC 15ºC and 20ºC respectively. The temperature when A and B are mixed is 13ºC and when B and C are mixed, it is 16ºC. What will be the temperature when A and C are mixed?

Solution :



when A and B are mixed

$mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13)$

$3S_1 = 2S_2$ .....(1)

when B and C are mixed

$S_2 \times 1 = S_3 \times 4$ ......(2)

when C and A are mixed

$S_1(\theta - 10) = S_3 \times (20 - \theta)$ ....(3)

by using equation (1), (2) and (3)

we get $\theta = \frac{140}{11}$ ºC

Example 6. If three different liquid of different masses specific heats and temperature are mixed with each other and then what is the temperature mixture at thermal equilibrium..

$m_1, s_1, T_1 \rightarrow$ specification for liquid

$m_2, s_2, T_2 \rightarrow$ specification for liquid

$m_3, s_3, T_3 \rightarrow$ specification for liquid.

Solution : Total heat lost or gain by all substance is equal to zero

$$\Delta Q = 0$$

$$m_1s_1(T - T_1) + m_2s_2(T - T_2) + m_3s_3(T - T_3) = 0$$

so $$T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$$

Example 7. In following equation calculate value of H 1 kg ice at –20ºC = H + 1 Kg water at 100ºC, here H means heat required to change the state of substance.

Solution : Heat required to convert 1 kg ice at – 20ºC into 1 kg water at 100ºC

= 1 kg ice at – 20ºC to 1 kg ice at 0ºC ice at 0ºC + 1 kg water at 0ºC + 1 kg water at 0ºC to 1 kg water at 100ºC

$= 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 100 = 190$ Kcal. So H = – 190 Kcal

Negative sign indicate that 190 Kcal heat is with drawn from 1 kg water at 100ºC to convert it into 1 kg ice at – 20ºC

Example 8. 1 kg ice at –20ºC is mixed with 1 kg steam at 200ºC. Then find equilibrium temperature and mixture content.

Solution : Let equilibrium temperature is 100 ºC heat required to convert 1 kg ice at –20ºC to 1 kg water at 100ºC is equal to

$H_1 = 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 1 \times 100 = 190$ Kcal

heat release by steam to convert 1 kg steam at 200ºC to 1 kg water at 100ºC is equal to

$H_2 = 1 \times \frac{1}{2} \times 100 + 1 \times 540 = 590$ Kcal

1 kg ice at – 20ºC = H_1 + 1kg water at 100ºC ......(1)

1 kg steam at 200ºC = H_2 + 1kg water at 100ºC ....(2)

by adding equation (1) and (2)

1 kg ice at –20ºC + 1 kg steam at 200ºC = $H_1 + H_2$ + 2 kg water at 100ºC.

Here heat required to ice is less than heat supplied by steam so mixture equilibrium temperature is 100ºC then steam is not completely converted into water.

So mixture has water and steam which is possible only at 100ºC

mass of steam which converted into water is equal to

$$m = \frac{190 - 1 \times \frac{1}{2} \times 100}{540} = \frac{7}{27} \text{ kg}$$

so mixture content

mass of steam = $1 - \frac{7}{27} = \frac{20}{27}$ kg

mass of water = $1 + \frac{7}{27} = \frac{34}{27}$ kg

4. THERMAL EXPANSION

Most materials expand when their temperature is increased. Rails roads tracks, bridges all have some means of compensating for thermal expansion. When a homogeneous object expands, the distance between any two points on the object increases. Figure shows a block of metal with a hole in it. **The expanded object is like a photographic enlargement.** That in the hole expands in the same proportion as the metal, it does not get smaller

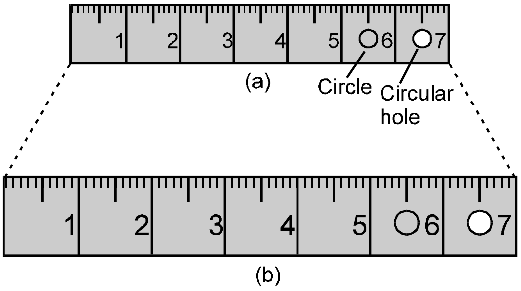
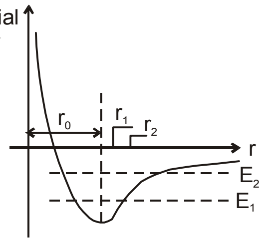


Fig. The same steel ruler two different temperatures. When it expands, the scale, the numbers, the thickness, and the diameters of the circle and circular hole are all incrased by the same factor. (The expansion has been exaggerated for clarity.)



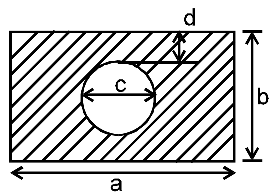
Thermal expansion arises because the well is not symmetrical about the equilibrium position r_0. As the temperature rise the energy of the atom increases. The average position when the energy is E_2 is not the same as that when the energy is E_1.

At the atomic level, thermal expansion may be understood by considering how the potential energy of the atoms varies with distance. The equilibrium position of an atom will be at the minimum of the potential energy well if the well is symmetric. At a given temperature each atom vibrates about its equilibrium position and its average remains at the minimum point. If the shape of the well is not symmetrical the average position of an atom will not be at the minimum point. When the temperature is raised the amplitude of the vibrations increases and the average position is located at a greater inter atomic separation. This increased separation is manifested as expansion of the material.

Almost all solids and liquids expand as their temperature increases. Gases also expand if allowed. Solids can change in length, area or volume, while liquids change in their volumes.

SOLVED EXAMPLE

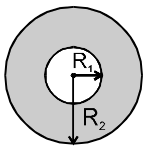
Example 9. A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.



Solution : Distance between any two point on an object increases with increase in temperature.

So, all dimension a, b, c and d will increase

Example 10. In the given figure, when temperature is increased then which of the following increases

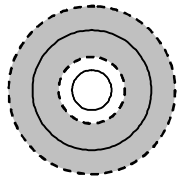


(A) R_1 (B) R_2 (C) $R_2 - R_1$

Solution : All of the above

- - - - - represents expanded Boundary

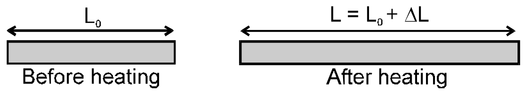
--------- represents original Boundary



As the intermolecular distance between atoms increases on heating hence the inner and outer perimeter increases. Also if the atomic arrangement in radial direction is observed then we can say that it also increases hence all A,B,C are true.

5. LINEAR EXPANSION

When the rod is heated, its increase in length ΔL is proportional to its original length L_0 and change in temperature ΔT where ΔT is in ºC or K.



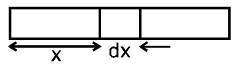
$dL = \alpha L_0\, dT \quad \Rightarrow \quad \Delta L = \alpha\, L_0\, \Delta T \quad$ If $\alpha\, \Delta T << 1$

$\alpha = \dfrac{\Delta L}{L_0\, \Delta T}$ where α is called the coefficient of linear expansion whose unit is $ºC^{-1}$ or K^{-1}.

$L = L_0\, (1 + \alpha\, \Delta\, T)$. Where L is the length after heating the rod.

Variation of α with temperature and distance

(a) If α varies with distance, $\alpha = ax + b$.



Then total expansion = $\int (ax + b)\, \Delta T\, dx$.

(b) If α varies with temperature, $\alpha = f\,(T)$. Then $\Delta L = \int \alpha\, L_0\, dT$

Note : Actually thermal expansion is always 3-D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one dimension and it is known as linear expansion.

SOLVED EXAMPLE

Example 11. What is the percentage change in length of 1m iron rod if its temperature changes by 100ºC. α for iron is 2×10^{-5}/ºC.

Solution : percentage change in length due to temperature change

$$\%\ell = \frac{\Delta \ell}{\ell} \times 100 = \alpha \Delta \theta \times 100$$

$$= 2 \times 10^{-5} \times 100 \times 100$$

$$= 0.2\%$$ **Ans.**

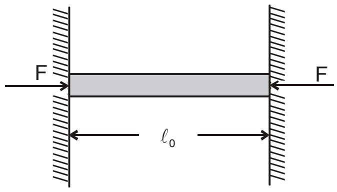
Thermal stress of a material:

If the rod is free to expand then there will be no stress and strain. Stress and strain is produced only when an object is restricted to expand or contract according to change in temperature. When the temperature of the rod is decreased or increased under constrained condition , compressive or tensile stresses are developed in the rod. These stresses are known as thermal stresses.

$$\textbf{Strain} = \frac{\Delta L}{L_0} = \frac{\text{final length} - \text{original length}}{\text{original length}} = \alpha \, \Delta T \,,$$

Note : Original and final length should be at same temperature.

Consider a rod of length ℓ_0 which is fixed between to rigid end separated at distance ℓ_0 now if the temperature of the rod is increased by $\Delta\theta$ then the strain produced in the rod will be :



$$\text{strain} = \frac{\text{length of the rod at new temperature - natural length of the rod at new temperature}}{\text{natural length of the rod at new temperature}}$$

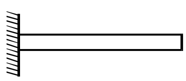
$$= \frac{\ell_0 - \ell_0(1+\alpha\Delta\theta)}{\ell_0(1+\alpha\Delta\theta)} = \frac{-\ell_0\alpha\Delta\theta}{\ell_0(1+\alpha\Delta\theta)}$$

$\because$ α is very small so

strain = – $\alpha \, \Delta \, \theta$ (negative sign in the answer represents that the length of the rod is less than the natural length that means is compressed by the ends.)

SOLVED EXAMPLE

Example 12. In the given figure a rod is free at one end and other end is fixed. When we change the temperature of rod by $\Delta\theta$, then strain produced in the rod will be



(A) $\alpha\Delta\theta$ (B) $\frac{1}{2}\alpha\Delta\theta$ (C) zero (D) information incomplete

Solution : Here rod is free to expand from one side by so by changing temperature no strain will be produced in the rod.

Hence ans. is (C)

Example 13. An iron ring measuring 15.00 cm in diameter is to be shrunk on a pulley which is 15.05 cm in diameter. All measurements refer to the room temperature 20°C. To what minimum temperature should the ring be heated to make the job possible? Calculate the strain developed in the ring when it comes to the room temperature. Coefficient of linear expansion of iron = 12×10^{-6}/°C.

Solution : The ring should be heated to increase its diameter from 15.00 cm to 15.05 cm.

Using $\ell_2 = \ell_1 (1 + \alpha \Delta\theta)$,

$$= \frac{0.05 \text{ cm}}{15.00 \text{ cm} \times 12 \times 10^{-6} /°\text{C}} = 278°\text{C}$$

The temperature = 20°C + 278°C = 298°C.

The strain developed = $\frac{\ell_2 - \ell_1}{\ell_1} = 3.33 \times 10^{-3}$.

Example 14. A steel rod of length 1m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

Solution : in absence of external force no strain or stress will be created hence rod is free to move.

Example 15. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is in natural length at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = 1.2×10^{-5}/°C.

Solution : as we known that strain

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta\ell}{\ell_0}$$

$\therefore$ Strain = $\alpha \Delta\theta$

$= 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$

here strain is compressive strain because final length is smaller than initial length.

Example 16. A steel wire of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is 1.2×10^{-5}/°C and its Young's modulus is 2.0×10^{11} N/m^2.

Solution : here final length is more than original length so that strain is tensile and tensile force is given by

$F = AY \alpha \Delta t = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20$ **= 24 N**

Variation of time period of pendulum clocks:

The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum every time it reaches to its extreme position the second hand of the clock advances by one second that means second hand moves by two seconds when one oscillation in complete

Let $T = 2\pi\sqrt{\frac{L_0}{g}}$ at temperature θ_0 and $T' = 2\pi\sqrt{\frac{L}{g}}$ at temperature θ.

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L[1+\alpha\Delta\theta]}{L}} = 1 + \frac{1}{2}\alpha\Delta\theta$$

Therefore change (loss or gain) in time per unit time lapsed is

$$\frac{T'-T}{T} = \frac{1}{2}\alpha\Delta\theta$$

gain or loss in time in duration of 't' in

$\mathbf{\Delta t = \frac{1}{2}\alpha\Delta\theta\, t}$, if T is the correct time then

(a) $\theta < \theta_0$, $T' < T$ clock becomes fast and gain time
(b) $\theta > \theta_0$, $T' > T$ clock becomes slow and loose time

SOLVED EXAMPLE

Example 17. A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at 20°C, how fast or slow will it go in 24 hours at 40°C? Coefficient of linear expansion of iron = 1.2×10^{-6}/°C.

Solution : The time difference occurred in 24 hours (86400 seconds) is given by

$$\Delta t = \frac{1}{2}\alpha\Delta\theta\, t$$

$$= \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 86400 = 1.04 \text{ sec.}$$ **Ans.**

This is loss of time as θ is greater than θ_0. As the temperature increases, the time period also increases. Thus, the clock goes slow.

Measurement of length by metallic scale:

Case (i)

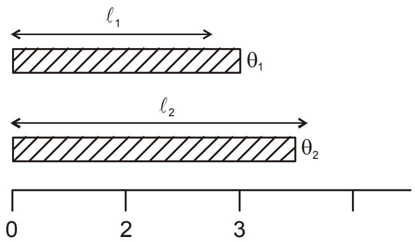
When object is expanded only

$\ell_2 = \ell_1 \{1 + \alpha_0(\theta_2 - \theta_1)$

ℓ_1 = actual length of object at θ_1ºC = measure length of object at θ_1ºC.

ℓ_2 = actual length of object at θ_2ºC = measure length of object at θ_2ºC.

α_0 = linear expansion coefficient of object.



Case (ii)

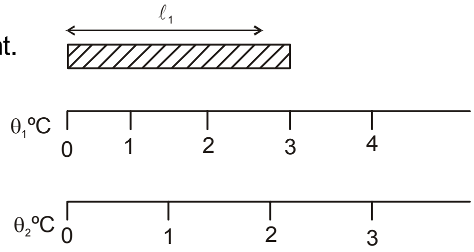
When only measurement instrument is expanded actual length of object will not change but measured value (MV) decreases.

MV = ℓ_1 { $1 - \alpha_S (\theta_2 - \theta_1)$}

α_S = linear expansion coefficient of measuring instrument.

at θ_1 C MV = 3

at θ_2 C MV = 2.2



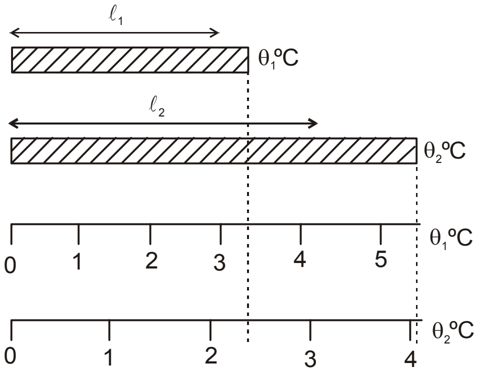
Case (iii)

If both expanded simultaneously

MV = {$1 + (\alpha_0 - \alpha_s) (\theta_2 - \theta_1)$)

(i) If $\alpha_0 > \alpha_s$, then measured value is more then the actual value at θ_1°C

(ii) If $\alpha_0 < \alpha_s$, then measured value is less then the actual value at θ_1°C



at θ_1°C MV = 3.4

θ_2°C MV = 4.1

Measured value = calibrated value × {1 + $\alpha\ \Delta\ \theta$}

where $\alpha = \alpha_0 - \alpha_s$

α_o = coefficient of linear expansion of object material, α_s = coefficient of linear expansion of scale material

$\Delta\theta = \theta - \theta_C$

θ = temperature at the time of measurement θ_C = temperature at the time of calibration.

For scale, **true measurement = scale reading [1 + α ($\theta - \theta_0$)]**

If **$\theta > \theta_0$ true measurement > scale reading**

$\theta < \theta_0$ true measurement < scale reading

SOLVED EXAMPLE

Example 18. A bar measured with a Vernier caliper is found to be 180mm long. The temperature during the measurement is 10ºC. The measurement error will be if the scale of the Vernier caliper has been graduated at a temperature of 20ºC : ($\alpha = 1.1 \times 10^{-5}$ °C^{-1}. Assume that the length of the bar does not change.)

(A) 1.98×10^{-1} mm (B*) 1.98×10^{-2} mm (C) 1.98×10^{-3} mm (D) 1.98×10^{-4} mm

Solution : True measurement = scale reading [1 + α ($\theta - \theta_0$)]

= 180 [1– 10 × 1.1 × 10^{-5}]

error = 180 – 180 [1 – 1.1 × 10^{-4}] = 1.98 × 10^{-2} mm

6. SUPERFICIAL OR AREAL EXPANSION

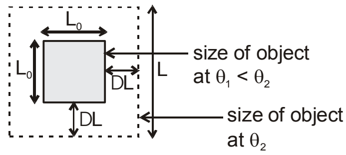
When a solid is heated and its area increases, then the thermal expansion is called superficial or areal expansion. Consider a solid plate of area A_0. When it is heated, the change in area of the plate is directly proportional to the original area A_0 and the change in temperature ΔT.

$dA = \beta A_0\, dT$ or $\mathbf{\Delta A = \beta A_0 \Delta T}$

$\beta = \dfrac{\Delta A}{A_0 \Delta T}$ Unit of β is $°C^{-1}$ or K^{-1}.

$A = A_0 (1 + \beta \Delta T)$

where A is area of the plate after heating,



SOLVED EXAMPLE

Example 19. A plane lamina has area $2m^2$ at 10ºC then what is its area at 110ºC It's superficial expansion is 2×10^{-5}/C

Solution : $A = A_0 (1 + \beta \Delta \theta) = 2\{1 + 2 \times 10^5 \times (110 - 10)\}$
$= 2 \times \{1 + 2 \times 10^{-3}\}$ **Ans.**

7. VOLUME OR CUBICAL EXPANSION

When a solid is heated and its volume increases, then the expansion is called volume expansion or cubical expansion. Let us consider a solid or liquid whose original volume is V_0. When it is heated to a new volume, then the change ΔV

$dV = \gamma V_0\, dT$ or $\mathbf{\Delta V = \gamma V_0 \Delta T}$

$\gamma = \dfrac{\Delta V}{V_0 \Delta T}$ Unit of γ is $°C^{-1}$ or K^{-1}.

$\mathbf{V = V_0 (1 + \gamma \Delta T)}$ where V is the volume of the body after heating

SOLVED EXAMPLE

Example 20. The volume of glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficient of cubical expansion of mercury and glass are 1.8×10^{-4}/°C and 9.0×10^{-6}/°C respectively.

Solution : Let volume of glass vessel at 20ºC is V_g and volume of mercury at 20ºC is V_m
so volume of remaining space is $= V_g - V_m$
It is given constant so that

$V_g - V_m = V_g' - V_m'$

where V_o' and V_m' are final volumes.

$V_g - V_m = V_g\{1 + \gamma_g \Delta\theta\} - V_m\{1 + \gamma_{Hg} \Delta\theta\} \Rightarrow \quad V_g \gamma_g = V_m \gamma_{Hg}$

$\Rightarrow \quad V_m = \dfrac{100 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}} \quad \Rightarrow \quad V_m = 50$ cc.

8. RELATION BETWEEN α , β AND γ

(i) For isotropic solids: $\boldsymbol{\alpha : \beta : \gamma = 1 : 2 : 3}$ **or** $\boldsymbol{\dfrac{\alpha}{1} = \dfrac{\beta}{2} = \dfrac{\gamma}{3}}$

(ii) For non-isotropic solid $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$. Here α_1, α_2 and α_3 are coefficient of linear expansion in X, Y and Z direction.

SOLVED EXAMPLE

Example 21. If percentage change in length is 1% with change in temperature of a cuboid object ($\ell \times 2\ell \times 3\ell$) then what is percentage change in its area and volume.

Solution : percentage change in length with change in temperature = % ℓ

$$\frac{\Delta\ell}{\ell}\times 100 = \alpha\Delta\theta \times 100 = 1$$

change in area

$$\Rightarrow \% A = \frac{\Delta A}{A} \times 100 = \beta\Delta\theta \times 100 \qquad \Rightarrow 2\,(\alpha\,\Delta\,\theta \times 100)$$

% A = 2 % **Ans.**

change in volume

$$\% V = \frac{\Delta V}{V} \times 100 = V\,\Delta\,\theta \times 100 \quad = 3\,(\alpha\,\Delta\,\theta \times 100)$$

% V = 3 % **Ans.**

9. VARIATION OF DENSITY WITH TEMPERATURE

As we known that mass = volume × density .

Mass of substance does not change with change in temperature so with increase of temperature, volume increases so density decreases and vice-versa.

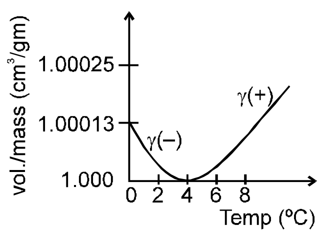
$$\mathbf{d = \frac{d_0}{(1+\gamma\Delta T)}}.$$

For solids values of γ are generally small so we can write $d = d_0\,(1 - \gamma\,\Delta T)$ (using binomial expansion).

Note : (i) γ for liquids are in order of 10^{-3}.

(ii) **Anomalous expansion of water :**

For water density increases from 0 ºC to 4 ºC so γ is negative and for 4 ºC to higher temperature γ is positive. At 4 ºC density is maximum. This anamolous behaviour of water is due to presence of three types of molecules i.e. H_2O, $(H_2O)_2$ and $(H_2O)_3$ having different volume/mass at different temperatures.



This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches , the water temperature decreases initially at the surface. The water there sinks because of its increase density. Consequently , the surface reaches 0^0C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4^0C.

SOLVED EXAMPLE

Example 22. The densities of wood and benzene at 0°C are 880 kg/m^3 and 900 kg/m^3 respectively. The coefficients of volume expansion are 1.2 × 10^{-3}/°C for wood and 1.5 × 10^{-3}/°C for benzene. At what temperature will a piece of wood just sink in benzene?

Solution : At just sink gravitation force = upthrust force

$\Rightarrow \quad mg = F_B \quad \Rightarrow \quad V\rho_1 g = V\rho_2 g \quad \Rightarrow \quad \rho_1 = \rho_2$

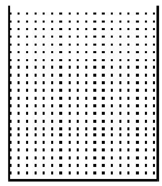
$$\Rightarrow \quad \frac{880}{1+1.2\times10^{-3}\theta} = \frac{900}{1+1.5\times10^{-3}\theta} \quad \Rightarrow \quad \theta = 83^\circ\text{ C}$$

10. APPARENT EXPANSION OF A LIQUID IN A CONTAINER

Initially container was full . When temperature change by ΔT,

volume of liquid $V_L = V_0\,(1 + \gamma_L\,\Delta T)$

volume of container $V_C = V_0\,(1 + \gamma_C\,\Delta T)$



So overflow volume of liquid relative to container

$\Delta V = V_L - V_C \qquad \mathbf{\Delta V = V_0\,(\gamma_L - \gamma_C)\,\Delta T}$

So, coefficient of apparent expansion of liquid w.r.t. container

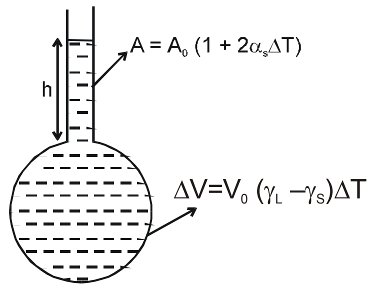
$\gamma_{apparent} = \gamma_L - \gamma_C$.

In case of expansion of liquid + container system:

if $\gamma_L > \gamma_C \longrightarrow$ level of liquid rise

if $\gamma_L < \gamma_C \longrightarrow$ level of liquid fall

Increase in height of liquid level in tube when bulb was initially not completely filled



$$h = \frac{\text{volume of liquid}}{\text{area of tube}} = \frac{V_0(1+\gamma_L\Delta T)}{A_0(1+2\alpha_S\Delta T)} = h_0\,\{1 + (\gamma_L - 2\alpha_S)\,\Delta T\}$$

$$\mathbf{h = h_0\,\{1 + (\gamma_L - 2\alpha_S)\,\Delta T\}}$$

where h_0 = original height of liquid in container

α_S = linear coefficient of expansion of container.

SOLVED EXAMPLE

Example 23. A glass vessel of volume 100 cm^3 is filled with mercury and is heated from 25°C to 75°C. What volume of mercury will overflow? Coefficient of linear expansion of glass = 1.8 × 10^{-6}/°C and coefficient of volume expansion of mercury is 1.8 × 10^{-4}/°C.

Solution : $\Delta V = V_0(\gamma_L - \gamma_C)\,\Delta T = 100 \times \{1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}\} \times 50$

$\Delta V = 0.87\text{ cm}^3$ Ans.

11. VARIATION OF FORCE OF BUOYANCY WITH TEMPERATURE

If body is submerged completely inside the liquid

For solid, Buoyancy force $F_B = V_0\, d_L\, g$

V_0 = Volume of the solid inside liquid,

d_L = density of liquid

Volume of body after increase its temperature $V = V_0\,[1 + \gamma_S\, \Delta\theta]$,

Density of body after increase its temperature $d'_L = \dfrac{d_L}{[1+\gamma_L\ \Delta\theta]}$.

Buoyancy force of body after increase its temperature, $F'_B = V\, d'_L\, g$, $\dfrac{F'_B}{F_B} = \dfrac{[1+\gamma_S\,\Delta\theta]}{[1+\gamma_L\ \Delta\theta]}$,

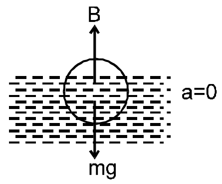
if $\gamma_S < \gamma_L$ then $F'_B < F_B$

(Buoyant force decreases) or apparent weight of body in liquid gets increased

$[W - F'_B > W - F_B]$.

SOLVED EXAMPLE

Example 24. A body is float inside liquid if we increases temperature then what changes occur in Buoyancy force. (Assume body is always in floating condition)



Solution : Body is in equilibrium

so mg = B

and gravitational force does not change with change in temperature. So Buoyancy force remains constant.

By increasing temperature density of liquid decreases so volume of body inside the liquid increases to kept the Buoyance force constant for equal to gravitational force)

Example 25. In previous question discuss the case when body move downward, upwards and remains at same position when we increases temperature.

Solution : Let f = fraction of volume of body submerged in liquid.

$$f = \frac{\text{volume of body submerged in liquid}}{\text{total volume of body}}$$

$f_1 = \dfrac{v_1}{v_0}$ at θ_1°C

$f_2 = \dfrac{v_2}{v_0(1+3\alpha_S\Delta\theta)}$ at θ_2°C

for equilibrium $mg = B = v_1 d_1 g = v_2 d_2 g$.

so $v_2 = \dfrac{v_1 d_1}{d_2}$ $\because$ $d_2 = \dfrac{d_1}{1+\gamma_L\Delta\theta}$ $= v_1(1 + \gamma_L\,\Delta\theta)$ $\therefore$ $f_2 = \dfrac{v_1(1+\gamma_L\Delta\theta)}{v_0(1+3\alpha_s\Delta\theta)}$

where $\Delta\theta = \theta_2 - \theta_1$

Case I : Body move downward if $f_2 > f_1$

means $\gamma_L > 3\alpha_S$

Case II : Body move upwards if $f_2 < f_1$

means $\gamma_L < 3\alpha_S$

Case III : Body remains at same position

if $f_2 = f_1$

means $\gamma_L = 3\alpha_S$

12. BIMETALLIC STRIP

It two strip of different metals are welded together to form a bimetallic strip, when heated uniformly it bends in form of an arc, the metal with greater coefficient of linear expansion lies on convex side. The radius of arc thus formed by bimetal is :

$$\ell_0 (1 + \alpha_1\Delta\theta) = \left(R - \frac{d}{2}\right)\theta$$

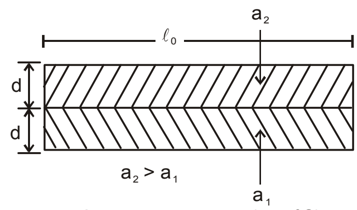
$$\ell_0 (1 + \alpha_2\Delta\theta) = \left(R + \frac{d}{2}\right)\theta$$

$$\Rightarrow \frac{1+\alpha_2\Delta\theta}{1+\alpha_1\Delta\theta} = \frac{R+\frac{d}{2}}{R-\frac{d}{2}}$$

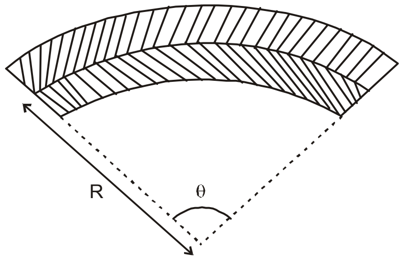
$$\Rightarrow R = \frac{d}{(\alpha_2 - \alpha_1)\Delta\theta}$$

$\Delta\theta$ = change in temperature

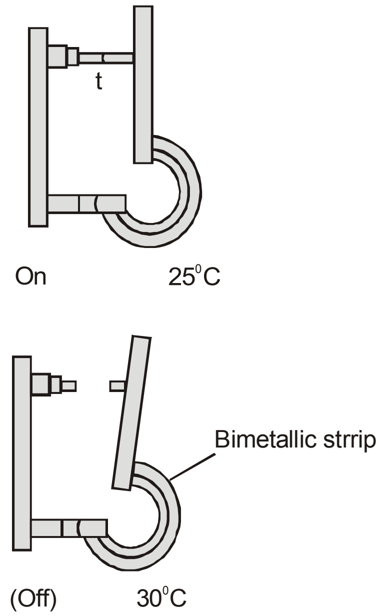
$= \theta_2 - \theta_1$



Lower temperature (at θ_1°C)



Higher temperature (at θ_2°C)



A bimetallic strip, consisting of a strip of brass and a strip of steel welded together, at temperature T_0 in figure (a) and figure (b). The strip bends as shown at temperatures above the reference temperature. Below the reference temperature the strip bends the other way. Many thermostats operate on this principle, making and breaking an electrical circuit as the temperature rises and falls.

13. APPLICATIONS OF THERMAL EXPANSION

(a) A small gap is left between two iron rails of the railway.

(b) Iron rings are slipped on the wooden wheels by heating the iron rings

(c) Stopper of a glass bottle jammed in its neck can be taken out by heating the neck.

(d) The pendulum of a clock is made of invar [an alloy of zinc and copper].

14. TEMPERATURE

Temperature may be defined as the **degree of hotness or coldness** of a body. Heat energy flows from a body at higher temperature to that at lower temperature until their temperatures become equal. At this stage, the bodies are said to be in thermal equilibrium.

Measurement of Temperature

The branch of thermodynamics which deals with the measurement of temperature is called thermometry. A thermometer is a device used to measure the temperature of a body. The substances like liquids and gases which are used in the thermometer are called thermometric substances.

Different Scales of Temperature

A thermometer can be graduated into following scales.

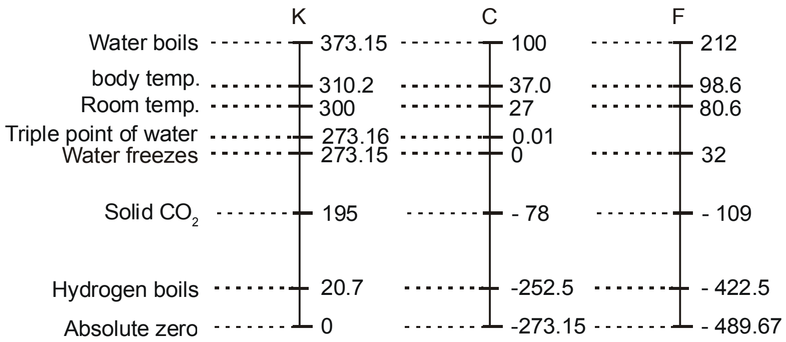
(a) The Centigrade or Celsius scale (ºC)

(b) The Fahrenheit scale (ºF)

(c) The Reaumer scale (ºR)

(d) Kelvin scale of temperature (K)

Comparison between Different Temperature Scales



The formula for the conversion between different temperature scales is:

$$\frac{K-273}{100}=\frac{C}{100}=\frac{F-32}{180}=\frac{R}{80}$$

General formula for the conversion of temperature from one scale to another:

$$\frac{\text{Temp on one scale}(S_1)-\text{Lower fixed point}(S_1)}{\text{Upper fixed point}(S_2)-\text{Lower fixed point}(S_1)}$$

$$=\frac{\text{Temp. on other scale}(S_2)-\text{Lower fixed point}(S_2)}{\text{Upper fixed point}(S_2)-\text{Lower fixed point}(S_2)}$$

Thermometers

Thermometers are device that are used to measure temperatures. All thermometers are based on the principle that some physical property of a system changes as the system temperature changes.

Required properties of good thermometric substance.

(1) Non-sticky (absence of adhesive force)

(2) Low melting point (in comparison with room temperature)

(3) High boiling temperature

(4) Coefficient of volumetric expansion should be high (to increase accuracy in measurement).

(5) Heat capacity should be low.

(6) Conductivity should be high

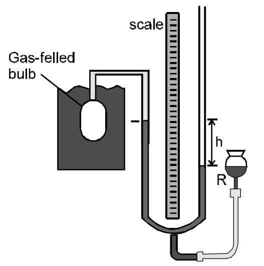
Mercury (Hg) suitably exhibits above properties.

Types of Thermometers

Type of thermometer and its range	Thermometric property	Advantages	Disadvantages	Particular Uses
Mercury-in-glass – 39ºC to 450ºC	Length of column of mercury in capillary tube	(i) Quick and easy to (direct reading) (ii) Easily portable	(i) Fragile (ii) Small size limits (iii) Limited range	(i) Every laboratory use where high accuracy is not required. (ii) Can be calibrated against constant-volume gas thermometer for more accurate work
Constant-volume gas thermometer – 270º to 1500ºC	Pressure of a fixed mass of gas at constant volume	(i) Very accurate (ii) Very sensitive (iii) Wide range (iv) Easily reproducible	(i) Very large volume of bulb (ii) Slow to use and inconvenient	(i) Standard against which others calibrated (ii) He, H_2 or N_2 used depending on range (iii) can be corrected to the ideal gas scale (iv) Used as standard below-183ºC
Platinum resistance –180º to 1150ºC	Electrical resistance of a platinum coil	(i) Accurate (ii) Wide range	Not suitable for varying temperature (i.e., is slow to respond to changes)	(i) Best thermometer for small steady temperature differences (ii) Used as standard between 183ºC and 630ºC.
Thermocouple –250ºC to 1150ºC	Emf produced between junctions of dissimilar metals at different temperatures for measurement of emfs	(i) Fast response because of low heat capacity. (ii) wide range (iii) can be employed for remote readings using long leads.	Accuracy is lost if emf is measured using a moving-coil voltmeter (as may be necessary for rapid changes when potentiometer is unsuitable)	(i) Best thermometer for small steady temperature differences (ii) Can be made direct reading by calibrating galvanometer (iii) Used as standard between 630ºC and 1063ºC
Radiation pyrometer above 1000ºC	Colour of radiation emitted by a hot body	Does not come into contact when temperature is measured	(i) Cumbersome (ii) Not direct reading (needs a trained observer)	(i) Only thermometer possible for very high temperatures (ii) Used as standard above 1063ºC.

The constant-volume gas thermometer

The standard thermometer, against which all other thermometers are calibrated, is based on the pressure of a gas in a fixed volume. Figure shows such a constant volume gas thermometer; it consists of a gas-filled bulb connected by a tube to a mercury monometer.



A constant volume gas thermometer, its bulb immersed in a liquid whose temperature T is to be measured.

$$T = (273.16\text{ K})\left(\lim_{gas\to 0}\frac{P}{P_3}\right)$$

P = Pressure at the temperature being measured, P_3 = pressure when bulb in a triple point cell.

SOLVED EXAMPLE

Example 26.

The readings of a thermometer at 0ºC and 100ºC are 50 cm and 75 cm of mercury column respectively. Find the temperature at which its reading is 80 cm of mercury column?

Solution : By using formula

$$\frac{80-50}{75-50} = \frac{T-0}{100-0}$$

$\Rightarrow$ T = 120ºC

MISCELLANEOUS SOLVED EXAMPLE

Problem 1. A bullet of mass 10 gm in moving with speed 400m/s. Find its kinetic energy in calories ?

Solution : $\Delta k = \frac{1}{2} \times \frac{10}{1000} \times 400 \times 400 = 800 \quad \Rightarrow \quad \frac{800}{4.2} = 191.11$ Cal.

Problem 2. Calculate amount of heat required to convert 1 kg steam from 100ºC to 200ºC steam

Solution : Heat required = $1 \times \frac{1}{2} \times 100 = 50$ kcal

Problem 3. Calculate heat required to raise the temperature of 1 g of water through 1ºC ?

Solution : heat required $= 1 \times 10^{-3} \times 1 \times 1 = 1 \times 10^{-3}$ kcal = 1 cal

Problem 4. 420 J of energy supplied to 10 g of water will raise its temperature by

Solution : $\frac{420 \times 10^{-3}}{4.20} = 10 \times 10^{-3} \times 1 \times \Delta t = 10^\circ$ C

Problem 5. The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3 . Find the ratio of their thermal capacities for unit volume ?

Solution : $\frac{\rho_1}{\rho_2} = \frac{3}{4}, \frac{s_1}{s_2} = \frac{4}{3}$

ratio $= \frac{m \times s}{m/\rho} \quad \Rightarrow \quad \frac{\theta_1}{\theta_2} = \frac{s_1}{s_2} \times \frac{\rho_1}{\rho_2} = 1 : 1.$

Problem 6. Heat releases by 1 kg steam at 150ºC if it convert into 1 kg water at 50ºC.

Solution : $H = 1 \times \frac{1}{2} \times 50 + 1 \times 540 + 1 \times 1 \times 50$

$= 540 + 75 = 615$ Kcal

Heat release = 615 Kcal.

Problem 7. 200 gm water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature is increase by 20ºC. Find the heat supplied to the water.

Solution : $H = 200 \times 10^{-3} \times 1 \times 20 = 4$ Kcal.

Heat supplied = 4000 cal

Problem 8. A bullet of mass 5 gm is moving with speed 400 m/s. strike a target and energy. Then calculate rise of temperature of bullet. Assuming all the lose in kinetic energy is converted into heat energy of bullet if its specific heat is. 500J/kgºC.

Solution : Kinetic energy $= \frac{1}{2} \times 5 \times 10^{-3} \times 400 \times 400$

$ms\,\Delta T = 5 \times 10^{-3} \times 500 \times \Delta T$

$\Delta T = 160^\circ$ C

Rise in temperature is 160 ºC

Problem 9. 1 kg ice at –10ºC is mixed with 1 kg water at 100ºC. Then find equilibrium temperature and mixture content.

Solution : Heat taken by 1 kg Ice = Heat given by 1 kg water

$1 \times \frac{1}{2} \times 10 + 1 \times 80 + 1 \times T = 1 \times (100 - T)$

$85 = 100 - 2T \quad \Rightarrow \quad 2T = 15$

$\theta = \frac{15}{2} = 7.5$ ºC , water

Problem 10. 1 kg ice at –10º is mixed with 1kg water at 50ºC. Then find equilibrium temperature and mixture content.

Solution : Heat taken by ice = 5 Kcal + 80 Kcal = 85 Kcal

Heat given by water = 1 × 1 × 50 = 50 Kcal

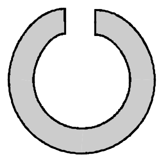
Heat taken > Heat given so, ice will not complete melt let m g ice melt then

$$1 \times \frac{1}{2} \times 10 + 80\,m = 50$$

$$80\,m = 45 \qquad \Rightarrow \qquad m = \frac{45}{80}$$

$$\text{Content of mixture} \begin{Bmatrix} \text{water} & \left(1+\frac{45}{80}\right)\text{kg} \\ \text{ice} & \left(1-\frac{45}{80}\right)\text{kg} \end{Bmatrix} \text{ and temperature is } 0^{\circ}\text{C}$$

Problem 11. A small ring having small gap is shown in figure on heating what will happen to size of gap.



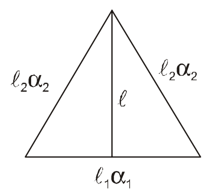
Solution :

Gap will also increase. The reason is same as in above example.

Problem 12. An isosceles triangle is formed with a thin rod of length ℓ_1 and coefficient of linear expansion α_1, as the base and two thin rods each of length ℓ_2 and coefficient of linear expansion α_2 as the two sides. If the distance between the apex and the midpoint of the base remain unchanged as the temperature is varied show that $\frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$.

Solution :

$$\ell = \sqrt{\left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2}$$



$$\ell^2 = \left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2$$

$$0 = \frac{2\ell_1}{2}\frac{1}{2}\frac{d\ell_1}{dT} + 2\ell_2 \times \frac{d\ell_2}{dT}$$

$$\ell_1 \times \frac{\ell_1\alpha_1}{2} \times \Delta T = 2\ell_2 \times \ell_2\,\alpha_2\Delta T \qquad \Rightarrow \qquad \frac{\ell_1^2}{\ell_2^2} = 4\,\frac{\alpha_2}{\alpha_1} \Rightarrow \frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}.$$

Problem 13. A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is 1.0×10^{-5} /°C.

Solution : $\ell_t = 10(1 + 1 \times 10^{-5} \times 35)$

$= 10.0035$ m

Problem 14. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = 1.2×10^{-5}/°C.

Solution : $\frac{\Delta \ell}{\ell} = \frac{\ell_0 \alpha \Delta\theta}{\ell_0}$ = 3.6×10^{-4}

Problem 15. If rod is initially compressed by $\Delta\ell$ length then what is the strain on the rod when the temperature
(a) is increased by $\Delta\theta$ (b) is decreased by $\Delta\theta$.

Solution: (a) Strain = $\frac{\Delta\ell}{\ell} + \alpha\Delta\theta$ (b) Strain = $\left|\frac{\Delta\ell}{\ell} - \alpha\Delta\theta\right|$

Problem 16. A pendulum clock having copper rod keeps correct time at 20°C. It gains 15 seconds per day if cooled to 0°C. Calculate the coefficient of linear expansion of copper.

Solution : $\frac{15}{24 \times 60 \times 60} = \frac{1}{2}\alpha \times 20$

$\Rightarrow$ $\alpha = \frac{1}{16 \times 3600}$ = 1.7×10^{-5}/°C

Problem 17. A meter scale made of steel is calibrated at 20°C to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is 1.1×10^{-5}/°C.

Solution : $\ell_t = 1\,(1 - 1.1 \times 10^{-5} \times 10) = 0.99989$ cm

Problem 18. A uniform solid brass sphere is rotating with angular speed ω_0 about a diameter. If its temperature is now increased by 100ºC. What will be its new angular speed. (Given $\alpha_B = 2.0 \times 10^{-5}$ perºC)

(A) $\frac{\omega_0}{1-0.002}$ (B) $\frac{\omega_0}{1+0.002}$ (C*) $\frac{\omega_0}{1+0.004}$ (D) $\frac{\omega_0}{1-0.004}$

Solution : $I_0\,\omega_0 = I_t\,\omega t$

$Mr_0^2\,\omega_0 = Mr_0^2\,(1 + 2\alpha\Delta T)\omega_t$

$\omega_t = \frac{\omega_0}{1+0.004}$.

Problem 19. The volume occupied by a thin - wall brass vessel and the volume of a solid brass sphere are the same and equal to 1,000 cm^3 at 0ºC. How much will the volume of the vessel and that of the sphere change upon heating to 20ºC ? The coefficient of linear expansion of brass is $\alpha = 1.9 \times 10^{-5}$.

Solution : $\Delta V = V_0\,3\alpha\,\Delta T = 1.14$ cm^3

1.14 cm^3 for both

Problem 20. A thin copper wire of length L increases in length by 1% , when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from T_1 to T_2 ?

(A) 1% (B) 3% (C) 4% (D*) 2%

Solution : $L_f = L\,(1 + \alpha\Delta t) \Rightarrow \frac{L_f}{L} \times 100 = (1 + \alpha\Delta t) \times 100 = 1\%$

$A_f = 2L \times L\,(1 + 2\,\alpha\Delta t) \Rightarrow \frac{A_f}{2L \times L} \times 100 = (1 + 2\alpha\,\Delta t) \times 100 = 2\%$

Problem 21. The density of water at 0°C is 0.998 g/cm^3 and at 4°C is 1.000 g/cm^3. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

Solution : $d_t = \frac{d_0}{1+\gamma\Delta t}$

$\Rightarrow \quad 1 = \frac{0.998}{1+\gamma \times 4} = -5 \times 10^{-4} / {}^0C$

Problem 22. A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. it is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1.6 cm^3 of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass = 6.5 × 10^{-6}/°C

Solution : $\Delta V = V_{Hg} - V_V$

$1.6 = 10^3 \gamma_\ell \times 10 - 10^3 \times 3 \times 6.5 \times 10^{-6} \times 10$

$\gamma_L = (1.6 + 0.195) \times 10^{-4} = 1.795 \times 10^{-4} / {}^0C$

Problem 23. A metal ball immersed in alcohol weighs W_1 at 0^0C and W_2 at 50^0C. The coefficient of cubical expansion of the metal is less than alcohol. Assuming that density of the metal is large compared to that of the alcohol, find which of W_1 and W_2 is greater?

Solution : $\gamma_M < \gamma_\ell$ so, $\frac{F'_B}{F_B} = \frac{[1+\gamma_S\Delta\theta]}{[1+\gamma_\ell\Delta\theta]} \Rightarrow F'_B < F_B$

so Apparent weight increases

so, $W_2 > W_1$

Problem 24. In figure which strip brass or steel have higher coefficient of linear expansion.

Solution : Brass Strip

Problem 25. The upper and lower fixed points of a faulty thermometer are 5ºC and 105º C .If the thermometer reads 25º C , what is the actual temperature ?

Solution : $\frac{25-5}{100} = \frac{C-0}{100}$

C = 20º C

Problem 26 At what temperature is the Fahrenheit scale reading equal to twice of Celsius ?

Solution : $\frac{F-32}{180} = \frac{C-0}{100}$

$\frac{2x-32}{180} = \frac{x-0}{100}$

$10x - 160 = 9x$

x = 160º C

Problem 27. Temperature of a patient is 40º C . Find the temperature on Fahrenheit scale ?

Solution : $\frac{F-32}{180} = \frac{40-0}{100} \quad \Rightarrow \quad F = 104^\circ F$

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

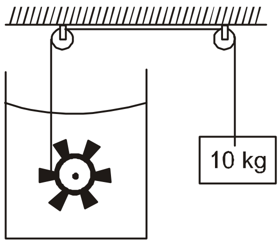
Section (A) : Calorimetry

A-1. An aluminium container of mass 100 gm contains 200 gm of ice at – 20°C. Heat is added to the system at the rate of 100 cal/s. Find the temperature of the system after 4 minutes (specific heat of ice = 0.5 and L = 80 cal/gm, specific heat of Al = 0.2 cal/gm/°C)

A-2. From what height should a piece of ice (0°C) fall so that it melts completely? Only one-quarter of the energy produced is absorbed by the ice as heat. (Latent heat of ice = 3.4×10^5 J kg^{-1}, g = 10 m/s^2)

A-3. A copper cube of mass 200g slides down on a rough inclined plane of inclination 37° at a constant speed. Assume that any loss in mechanical energy goes into the copper block as thermal energy. Find the increase in the temperature of the block as it slides down 60 cm. Specific heat capacity of copper = 420 J/kg-K.

A 4. A paddle wheel is connected with a block of mass 10 kg as shown in figure. The wheel is completely immersed in liquid of heat capacity 4000 J/K. The container is adiabatic. For the time interval in which block goes down 1 m slowly calculate



(a) Work done on the liquid

(b) Heat supplied to the liquid

(c) Rise in the temperature of the liquid

Neglect the heat capacity of the container and the paddle. (g =10 m/s^2)

A 5 300 g of water at 25°C is added to 100 g of ice at 0°C. Find the final temperature of the mixture. **[1989; 2M]**

Section (B) : Thermal Expansion

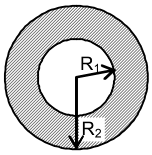
B-1. What should be the sum of lengths of an aluminium and steel rod at 0°C is, so that at all temperatures their difference in length is 0.25m. (Take coefficient of linear expansion for aluminium and steel at 0°C as 22×10^{-6} / °C and 11×10^{-6} / °C respectively.)

B 2. A steel tape is correctly calibrated at 20 °C and is used to measure the length of a table at 30 °C. Find the percentage error in the measurement of length.[α_{steel} = 11×10^{-6}/ °C]

B 3. The temperature of a metal ball is raised. Arrange the percentage change in volume, surface area and radius in ascending order.

B 4. A brass disc fits in a hole in a steel plate. Would you heat or cool the system to loosen the disc from the hole? Assume that $\alpha_s < \alpha_b$.

B 5. Temperature of plate is increased by $\Delta\theta$ then find new



(a) inner radius

(b) outer radius

(c) the difference in outer and inner radius and show that it is positive

(d) area of plate material

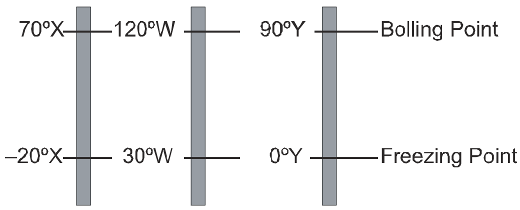
(assume coefficient of expansion is α)

B 6. We have a hollow sphere and a solid sphere of equal radii and of the same material. They are heated to raise their temperature by equal amounts. How will the change in their volumes, due to volume expansions, be related? Consider two cases (i) hollow sphere is filled with air, (ii) there is vacuum inside the hollow sphere.

Section (C) : Temperature

C 1 . What is the temperature at which we get the same reading on both the centigrade and Fahrenheit scales?

C 2. The figure shows three temperature scales with the freezing and boiling points of water indicated.



(a) Rank the size of a degree on these scales, greatest first.

(b) Rank the following temperatures, highest first : 50ºX, 50ºW and 50ºY.

PART - II : OBJECTIVE QUESTIONS

Section (A) : Calorimetry

A 1. A block of mass 2.5 kg is heated to temperature of 500°C and placed on a large ice block. What is the maximum amount of ice that can melt (approx.). Specific heat for the body = 0.1 Cal/gm°C.

(A) 1 kg (B) 1.5 kg (C) 2 kg (D) 2.5 kg

A 2. A small quantity, mass m, of water at a temperature θ (inºC) is poured on to a large mass M of ice which is at its melting point. If c is the specific heat capacity of water and L the latent heat of fusion of ice, then the mass of ice melted is given by :

(A) $\frac{ML}{mc\theta}$ (B) $\frac{mc\theta}{ML}$ (C) $\frac{Mc\theta}{L}$ (D) $\frac{mc\theta}{L}$

A 3. A thermally isolated vessel contains 100 g of water at 0ºC. When air above the water is pumped out, some of the water freezes and some evaporates at 0ºC itself. then the mass of the ice formed if no water is left in the vessel. Latent heat of vaporization of water at 0ºC = 2.10×10^6 J/kg and latent heat of fusion of ice = 3.36×10^5 J/kg.

(A) 86.2 g (B) 13.8 g (C) 76.2 g (D) 65.6 g

A 4. 20 gm ice at –10 ºC is mixed with m gm steam at 100 ºC. Minimum value of m so that finally all ice and steam converts into water. (Use s_{ice}=0.5 cal/gmºC,s_{water}=1 cal/gmºC,L (melting)=80 cal/gm and L (vaporization) = 540 cal/gm)

(A) $\frac{85}{32}$ gm (B) $\frac{85}{64}$ gm (C) $\frac{32}{85}$ gm (D) $\frac{64}{85}$ gm

A 5. 2 kg ice at – 20 ºC is mixed with 5 kg water at 20 ºC . Then final amount of water in the mixture will be : [Specific heat of ice = 0.5 cal/gm ºC, Specific heat of water = 1 cal/gm ºC, Latent heat of fusion of ice = 80 cal/gm] **[JEE-2003 (Scr.), 3/84,–1]**

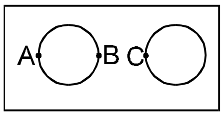
(A) 6 kg (B) 7 kg (C) 3.5 kg (D) 5 kg

Section (B) : Thermal Expansion

B 1. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, the real length of the given piece of wood must be:

(A) 25 cm (B) <25 cm (C) >25 cm (D) can not say

B 2. Two large holes are cut in a metal sheet. If this is heated, distances AB and BC, (as shown)



(A) both will increase (B) both will decrease

(C) AB increases, BC decreases (D) AB decreases, BC increases

B 3. A steel scale is to be prepared such that the millimeter intervals are to be accurate within 6×10^{-5} mm. The maximum temperature variation from the temperature of calibration during the reading of the millimeter marks is ($\alpha = 12 \times 10^{-6}$ /ºC)

(A) 4.0°C (B) 4.5°C (C) 5.0°C (D) 5.5°C

B 4. Expansion during heating –

(A) occurs only in a solid (B) increases the density of the material

(C) decreases the density of the material (D) occurs at the same rate for all liquids and solids.

B 5. If a bimetallic strip is heated, it will

(A) bend towards the metal with lower thermal expansion coefficient.

(B) bend towards the metal with higher thermal expansion coefficient.

(C) twist itself into helix.

(D) have no bending.

B 6. Two rods, one of aluminium and the other made of steel, having initial length ℓ_1 and ℓ_2 are connected together to form a single rod of length $\ell_1 + \ell_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperature are raised by t°C, then find the ratio $\frac{\ell_1}{(\ell_1+\ell_2)}$. **[JEE-2003 (Scr.), 3/84,–1]**

(A) $\frac{\alpha_s}{\alpha_a}$ (B) $\frac{\alpha_a}{\alpha_s}$ (C) $\frac{\alpha_s}{(\alpha_a+\alpha_s)}$ (D) $\frac{\alpha_a}{(\alpha_a+\alpha_s)}$

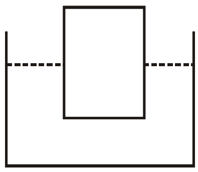
Section (C) : Temperature

C 1. A difference of temperature of 25° C is equivalent to a difference of :

(A) 45° F (B) 72° F (C) 32° F (D) 25° F

PART - III : MATCH THE COLUMN

1. A cylindrical isotropic solid of coefficient of thermal expansion α and density ρ floats in a liquid of coefficient of volume expansion γ and density d as shown in the diagram **[in sheet 2008-09]**



Column I	Column II
(A) volume of cylinder inside the liquid remains constant	(p) $\gamma = 0$
(B) volume of cylinder outside the liquid remains constant	(q) $\gamma = 2\alpha$
(C) Height of cylinder outside the liquid remains constant	(r) $\gamma = 3\alpha \frac{d}{\rho}$
(D) Height of cylinder inside the liquid remain constant	(s) $\gamma = (2\alpha + \alpha \frac{d}{\rho})$

2. In the following question column - I represents some physical quantities & column-II represents their units, match them

	Column I		Column II
(A)	Coefficient of linear expansion	(p)	Cal/°C
(B)	Water equivalent	(q)	gm
(C)	heat capacity	(r)	$(°C)^{-1}$
(D)	Specific heat	(s)	Cal/g°C

Exercise # 2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A long solid cylinder is radiating power. It is remoulded into a number of smaller cylinders, each of which has the same length as original cylinder. Each small cylinder has the same temperature as the original cylinder. The total radiant power emitted by the pieces is twice that emitted by the original cylinder. How many smaller cylinders are there ? Neglect the energy emitted by the flat faces of cylinder.

(A) 3 (B) 4 (C) 5 (D) 6

2. Steam at 100° C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15° C till the temperature of the calorimeter and its contents rises to 80° C . The mass of the steam condensed in kilogram is : **[JEE '1986, 2]**

(A) 0.130 (B) 0.065 (C) 0.260 (D) 0.135

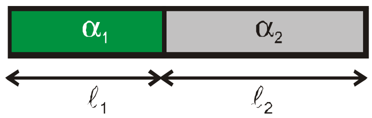
3. A metal ball of specific gravity 4.5 and specific heat 0.1 cal/gm-°C is placed on a large slab of ice at 0°C. Half of the ball sinks in the ice. The initial temperature of the ball is :-

(Latent heat capacity of ice = 80 cal/g, specific gravity of ice = 0.9)

4. If I is the moment of inertia of a solid body having α -coefficient of linear expansion then the change in I corresponding to a small change in temperature ΔT is

(A) $\alpha I \Delta T$ (B) $\frac{1}{2}\alpha I \Delta T$ (C) $2\alpha I \Delta T$ (D) $3\alpha I \Delta T$

5. Two rods having length ℓ_1 and ℓ_2, made of materials with the linear coefficient of expansion α_1 and α_2, were welded together. The equivalent coefficients of linear expansion for the obtained rod :-



(A) $\frac{\ell_1\alpha_2+\ell_2\alpha_1}{\ell_1+\ell_2}$ (B) $\frac{\ell_1\alpha_1+\ell_2\alpha_2}{\alpha_1+\alpha_2}$ (C) $\frac{\ell_1\alpha_1+\ell_2\alpha_2}{\ell_1+\ell_2}$ (D) $\frac{\ell_2\alpha_1+\ell_1\alpha_2}{\alpha_1+\alpha_2}$

6. The volume thermal expansion coefficient of an ideal gas at constant pressure is

(A) T (B) T^2 (C) $\frac{1}{T}$ (D) $\frac{1}{T^2}$

7. A metal ball immersed in water weighs w_1 at 5ºC and w_2 at 50ºC. The coefficient of cubical expansion of metal is less than that of water. Then

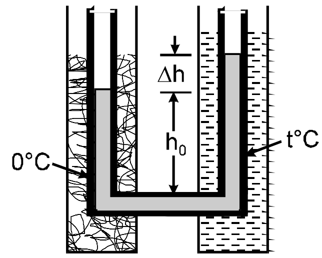
(A) $w_1 > w_2$ (B) $w_1 < w_2$ (C) $w_1 = w_2$ (D) data is insufficient

8. A piece of metal floats on mercury . The coefficient of volume expansion of the metal and mercury are γ_1 & γ_2 respectively . If the temperatures of both mercury and the metal are increased by an amount ΔT , the fraction of the volume of the metal submerged in mercury changes by the factor.

(Ratio of final fraction to the initial fraction) **[JEE '1991, 2]**

(A) $\frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}$ (B) $\frac{1+\gamma_1\Delta T}{1+\gamma_2\Delta T}$ (C) $1+(\gamma_1+\gamma_2)\Delta T$ (D) None of these

9. Two vertical glass tubes filled with a liquid are connected at their lower ends by a horizontal capillary tube. One tube is surrounded by a bath containing ice and water at 0°C and the other by hot water at t°C. The difference in the height of the liquid in the two columns is Δh, and the height of the column at 0°C is h_0. Coefficient of volume expansion of the liquid is.



(A) $\frac{\Delta h}{h_0 t}$ (B) $\frac{2\Delta h}{h_0 t}$ (C) $\frac{2h_0}{\Delta h t}$ (D) $\frac{h_0}{\Delta h t}$

10. A small pond of depth 0.5 m deep is exposed to a cold winter with outside temperature of 263 K. Thermal conductivity of ice is K = 2.2 W m^{-1} K^{-1}, latent heat L = 3.4 × 10^5 Jkg^{-1} and density ρ = 0.9 × 10^3 kgm^{-3}. Take the temperature of the pond to be 273 K. The time taken for the whole pond to freeze is about. **[Olympiad (Stage-1) 2017]**

(A) 20 days (B) 25 days (C) 30 days (D) 35 days

11. Two identical thin metal strips, one of aluminum and the other of iron are riveted together to form a bimetallic strip. The temperature is raised by 50 ºC. If the central planes of the two strips are separated by 2mm and the coefficients of thermal expansion of aluminum and iron are respectively 30 × 10^{-6}/ºC and 10 × 10^{-6} / ºC the average radius of curvature of the bimetallic strip is about.

[Olympiad 2014 (stage-1)]

(A) 50 cm (B) 100 cm (C) 150 cm (D) 200 cm

12. Two thin rods of length l_1 and l_2 at a certain temperature are joined to each other end to end. The composite rod is then heated through a temperature θ.The coefficients of linear expansion of the two rods are α_1 and α_2 respectively. Then, the effective coefficient of linear expansion of the composite rod is: **[Olympiad 2015 (stage-1)]**

(A) $\frac{\alpha_1 + \alpha_2}{2}$ (B) $\sqrt{\alpha_1\alpha_2}$ (C) $\frac{l_1\alpha_2 + l_2\alpha_1}{l_1 + l_2}$ (D) $\frac{l_1\alpha_1 + l_2\alpha_2}{l_1 + l_2}$

PART - II : NUMERICAL TYPE QUESTIONS

1. A pitcher contains 20 kg of water. 0.5 gm of water comes out on the surface of the pitcher every second through the pores and gets evaporated taking energy from the remaining water. Calculate the approximate time (in s) in which temperature of the water decreases by 5ºC .Neglect backward heat transfer from the atmosphere to the water.

Specific heat capacity of water = 4200J/KgºC

Latent heat of vaporization of water = 2.27 × 10^6 J/Kg

2. The temperature of 100 gm of water is to be raised from 24° C to 90° C by adding steam to it. Find the mass (in g) of the steam required for this purpose . **[JEE '1996, 2]**

3. An electrical heating coil was placed in a calorimeter containing 360 gm of water at 10° C . The coil consumes energy at the rate of 90 watt. The water equivalent of the calorimeter and the coil is 40 gm. Calculate what will be the temperature (in °C) of water after 10 minutes. **[REE '1985, 7]**
J = 4.2 Joules/cal.

4. A one liter flask contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume (in L) of mercury in the flask? Coefficient of linear expansion of glass = 9×10^{-6} /° C. Coefficient of volume expansion of mercury = 1.8×10^{-4} /° C. **[JEE '1991, 3]**

5. A steel rod 25 cm long has a cross-sectional area of 0.8 cm^2. What force (in N) would be required to stretch this rod by the same amount as the expansion produced by heating it through 10°C? (Coefficient of linear expansion of steel is 10^{-5}/°C and Young's modulus of steel is - 2×10^{10} N/m^2.) **[JEE 1989, 3]**

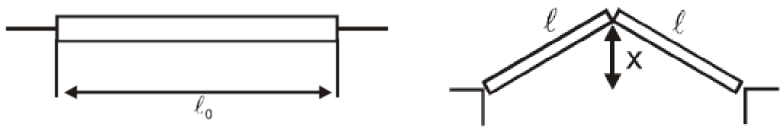
6. A metal piece weighing 15g is heated to 100ºC and then immersed in a mixture of ice and water at the thermal equilibrium. The volume of the mixture is found to be reduced by 0.15 cm^3 with the temperature of mixture remaining constant. Find the specific heat (in cal/gmºC) of the metal. Given specific gravity of ice = 0.92, latent heat of fusion of ice = 80 cal/gm.

7. A simple seconds pendulum is constructed out of a very thin string of thermal coefficient of linear expansion $\alpha = 20 \times 10^{-4}$ / ºC and a heavy particle attached to one end. The free end of the string is suspended from the ceiling of an elevator at rest. The pendulum keeps correct time at 0ºC. When the temperature rises to 50ºC, the elevator operator of mass 60kg being a student of Physics accelerates the elevator vertically, to have the pendulum correct time. Find the apparent weight (in N) of the operator when the pendulum keeps correct time at 50 ºC. (Take g = 10 m/s^2)

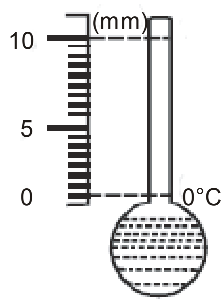
8. Earth receives 1400 W/m^2 of solar power . If all the solar energy falling on a lens of area 0.2 m^2 is focused on to a block of ice of mass 280 grams, find the time taken (in s) to melt the ice. (Latent heat of fusion of ice = 3.3×10^5 J/kg) **[JEE 1997, 2]**

9. A 50 gm lead bullet, specific heat 0.02 cal/g is initially at 30° C . It is fired vertically upwards with a speed of 840 m/sec & on returning to the starting level strikes a cake of ice at 0° C. How much (in g) ice is melted . Assume that all energy is spent in melting only.[Latent heat of ice = 80 cal/g] **[REE 1988, 5]**

10. As a result of temp rise of 32ºC, a bar with a crack at its centre buckles upward. If the fixed distance ℓ_0 is 4 m, and coefficient of linear expansion of bar in 25×10^{-6} ºc^{-1}. Find the rise x (in cm) of the centre.



11. Level of a certain liquid at 0ºC and 100 ºC are 0 and 10 mm on a given fixed scale (as shown in fig.) coefficient of volume expansion this liquid varies with temperature as $\gamma = \gamma_0\left(1+\frac{T}{100}\right)$ (where T in ºC). Find the level (in mm) of liquid at 48ºC.



12. A simple seconds pendulum is constructed out of a very thin string of thermal coefficient of linear expansion $\alpha = 20 \times 10^{-4}$ /ºC and a heavy particle attached to one end. The free end of the string is suspended from the ceiling of an elevator at rest. The pendulum keeps correct time at 0 ºC. When the temperature rises to 50 ºC, the elevator operator of mass 60kg being a student of Physics accelerates the elevator vertically, to have the pendulum correct time. Find the apparent weight (kgwt) of the operator when the pendulum keeps correct time at 50 ºC. (Take g = 10 m/s^2)

13. The specifc heat of a substance varies with temperature according to $c = 0.2 + 0.16\,T + 0.024\,T^2$ with T in ºC and c is cal/gK. Find the energy (in cal) requied to raise the temp of 2g substance from 0º to 5ºC

14. 50g of Ice at 0ºC is mixed with 200g of water at 0ºC. 6 kcal heat is given to system [Ice + water].Find the temperature (in ºC) of the system .

15. Two identical calorimeter A and B contain equal quantity of water at 20°C. A 5 gm piece of metal X of specific heat 0.2 cal g^{-1} $(C°)^{-1}$ is dropped into A and a 5 gm piece of metal Y into B. The equilibrium temperature in A is 22°C and in B 23°C. The initial temperature of both the metals is 40°C. Find the specific heat of metal Y in cal g^{-1} $(C°)^{-1}$.

PART - III : ONE OR MORE THAN ONE CORRECT OPTIONS

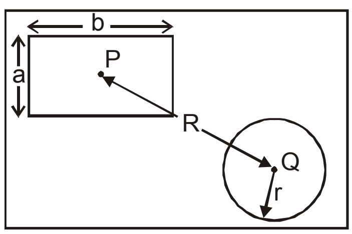
1. A uniform cylinder of steel of mass *M*, radius *R* is placed on frictionless bearings and set to rotate about its axis with angular velocity ω_0. After the cylinder has reached the specified state of rotation, it is heated from temperature T_0 to $(T_0 + \Delta T)$ without any mechanical contact . If $\frac{\Delta I}{I}$ is the fractional change in moment of inertia of the cylinder and $\frac{\Delta\omega}{\omega_0}$ be the fractional change in the angular velocity of the cylinder and α be the coefficient of linear expansion, then

(A) $\frac{\Delta I}{I} = \frac{2\Delta R}{R}$ (B) $\frac{\Delta I}{I} = \frac{2\Delta\omega}{\omega_0}$ (C) $\frac{\Delta\omega}{\omega_0} = -2\alpha\Delta T$ (D) $\frac{\Delta I}{I} = -\frac{2\Delta R}{R}$

2. When two non reactive samples at different temperatures are mixed in an isolated container of negligible heat capacity the final temperature of the mixture can be :

(A) lesser than lower or greater than higher temperature
(B) equal to lower or higher temperature
(C) greater than lower but lesser than higher temperature
(D) average of lower and higher temperatures

3. There is a rectangular metal plate in which two cavities in the shape of rectangle and circle are made, as shown with dimensions. P and Q are the centres of these cavities. On heating the plate, which of the following quantities increase ?



(A) πr^2 (B) ab (C) R (D) b

4*. When m gm of water at 10°C is mixed with m gm of ice at 0°C, which of the following statements are false?
(A) The temperature of the system will be given by the equation
$m \times 80 + m \times 1 \times (T - 0) = m \times 1 \times (10 - T)$
(B) Whole of ice will melt and temperature will be more than 0°C but lesser than 10°C
(C) Whole of ice will melt and temperature will be 0°C
(D) Whole of ice will not melt and temperature will be 0°C

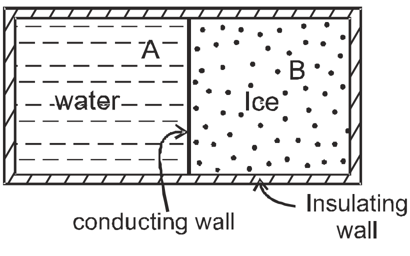
5*. Two identical beakers with negligible thermal expansion are filled with water to the same level at 4°C. If one says A is heated while the other says B is cooled, then :
(A) water level in A must rise (B) water level in B must rise
(C) water level in A must fall (D) water level in B must fall

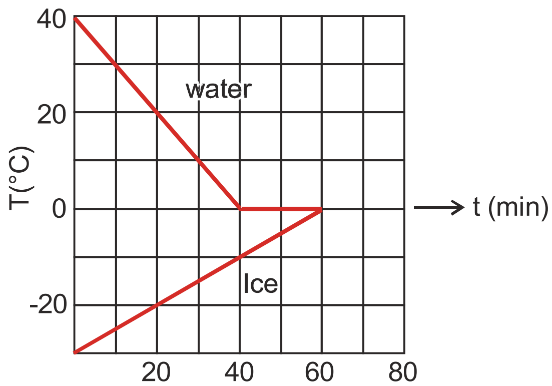
6*. A bimetallic strip is formed out of two identical strips, one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B .On heating, the temperature of the strips goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is:
[JEE 1999, 2/200]
(A) Proportional to ΔT (B) inversely proportional to ΔT
(C) proportional to $|\alpha_B - \alpha_C|$ (D) inversely proportional to $|\alpha_B - \alpha_C|$

PART - IV : COMPREHENSION

Comprehension # 1

A 0.60 kg sample of water and a sample of ice are placed in two compartments A and B that are separated by a conducting wall, in a thermally insulated container. The rate of heat transfer from the water to the ice though the conducting wall is constant P, until thermal equilibrium is reached. The temperature T of the liquid water and the ice are given in graph as functions of time t. Temperature of the compartments remain homogeneous during whole heat transfer process.
Given specific heat of ice = 2100 J/kg-K
Given specific heat of water = 4200 J/kg-K
Latent heat of fusion of ice = 3.3×10^5 J/kg





1. The value of rate P is
(A) 42.0 W (B) 36.0 W (C) 21.0 W (D) 18.0 W

2. The initial mass of the ice in the container is equal to
(A) 0.36 kg (B) 1.2 kg (C) 2.4 kg (D) 3.6 kg

3. The mass of the ice formed due to conversion from the water till thermal equilibrium is reached, is equal to
(A) 0.12 kg (B) 0.15 kg (C) 0.25 kg (D) 0.40 kg

Comprehension # 2

In a container of negligible heat capacity, 200 gm ice at 0°C and 100 gm steam at 100°C are added to 200 gm of water that has temperature 55°C. Assume no heat is lost to the surroundings and the pressure in the container is constant 1.0 atm. (Latent heat of fusion of ice = 80 cal/gm, Latent heat of vaporization of water = 540 cal/gm, Specific heat capacity of ice = 0.5 cal/gm-K, Specific heat capacity of water = 1 cal/gm-K)

4. What is the final temperature of the system ?

(A) 48°C (B) 72°C (C) 94°C (D) 100°C

5. At the final temperature, mass of the total water present in the system, is

(A) 472.6 gm (B) 483.3 gm (C) 493.6 gm (D) 500 gm

6. Amount of the steam left in the system, is equal to

(A) 16.7 gm (B) 12.0 gm (C) 8.4 gm

(D) 0 gm, as there is no steam left.

Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

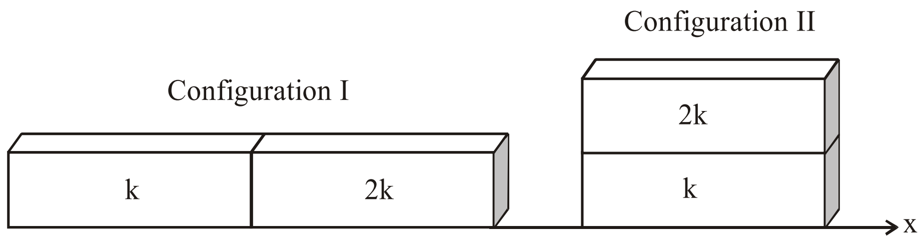
1. A piece of ice (heat capacity = 2100 J kg^{-1} $^{o}C^{-1}$ and latent heat = 3.36 × 10^5 J kg^{-1}) of mass m grams is at –5 ºC at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is : **[JEE 2010, 3/163]**

2. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5} /°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of 'm' in kg is nearly. **[JEE 2011, 4/160]**

3. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is **[JEE 2012]**

(A) $\left(\frac{65}{2}\right)^{1/4} T$ (B) $\left(\frac{97}{4}\right)^{1/4} T$ (C) $\left(\frac{97}{2}\right)^{1/4} T$ (D) $(97)^{1/4} T$

4. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity k and the other 2k. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is :- **[JEE-Advance-2013]**



(A) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0 s

5. One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is :- **[JEE-Advance-2013]**

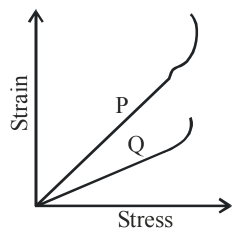
(A) 0.25 (B) 0.50 (C) 2.00 (D) 4.00

6. Parallel rays of light of intensity I = 912 Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to :- **[JEE-Advance-2014]**

(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

7. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is. **[JEE-Advance-2015]**

8. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are):- **[JEE-Advance-2015]**



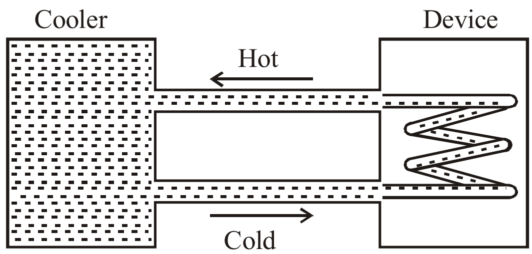
(A) P has more tensile strength than Q

(B) P is more ductile than Q

(C) P is more brittle than Q

(D) The Young's modulus of P is more than that of Q

9. A water cooler of storage capacity 120 litres can cool water at constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is : **[JEE-Advance-2016]**



(Specific heat of water is 4.2 kJ $kg^{-1}\ K^{-1}$ and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

10. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C ? **[JEE-Advance-2016]**

11. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\,K^{-1}$, the change in length of the wire PQ is **[JEE-Advance-2016]**

(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

12. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of σT_0^4 = 460 Wm^{-2} (where σ is the Stefan-Boltzmann constant). Which of the following options is/are **correct** ? **[JEE-Advance-2017]**

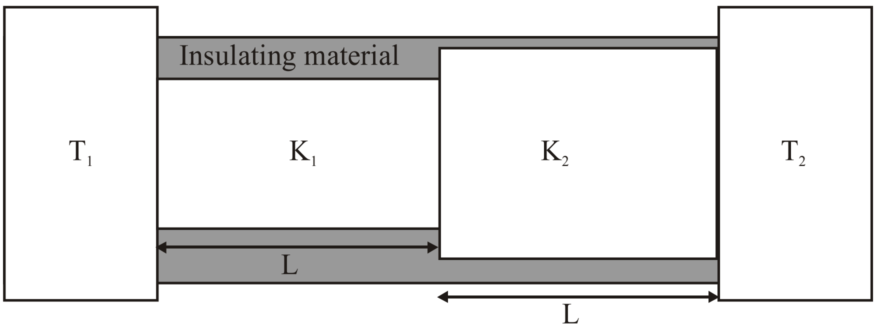
(A) The amount of energy radiated by the body in 1 second is close to 60 Joules

(B) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(D) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

13. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = _________. **[JEE-Advance-2018]**



14. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ______ °C. **[JEE-Advance-2019]**

[Neglect the heat exchange with surrounding]

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The specific heat capacity of a metal at low temperature (T) is given as :**[AIEEE 2011, 11 May; 4/120, –1]**

$$C_p\ (kjK^{-1}\ kg^{-1}) = 32\left(\frac{T}{400}\right)^3$$

A 100 gram vessel of this metal is to be cooled from 20ºK to 4ºK by a special refrigerator operating at room temperature (27ºC). The amount of work required to cool the vessel is :

(1) greater than 0.148 kJ (2) between 0.148 kJ and 0.028 kJ

(3) less than 0.028 kJ (4) equal to 0.002 kJ

2. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by tºC, the linear stress developed in its is :

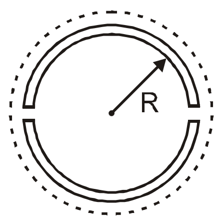
[AIEEE 2011, 11 May; 4/120, –1]

(1) $\frac{Y}{\alpha t}$ (2) $Y\alpha t$ (3) $\frac{1}{(Y\alpha t)}$ (4) $\frac{\alpha t}{Y}$

3. An aluminium sphere of 20 cm diameter is heated from 0ºC to 100ºC. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{Al} = 23 \times 10^{-6}$/ºC) **[AIEEE 2011, 11 May; 4/120, –1]**

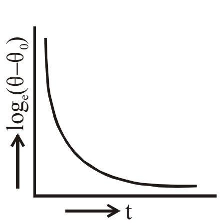
(1) 2.89 cc (2) 9.28 cc (3) 49.8 cc (4) 28.9 cc

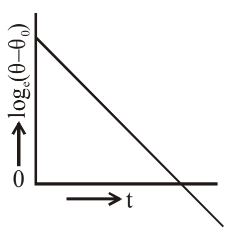
4. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surroundifng temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is:
[AIEEE 2012 ; 4/120, –1]

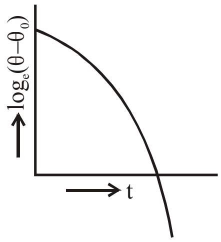


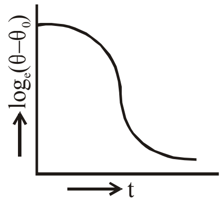
(1) $2\pi SY\alpha\Delta T$ (2) $SY\alpha\Delta T$ (3) $\pi SY\alpha\Delta T$ (4) $2SY\alpha\Delta T$

5. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :- **[AIEEE 2012]**

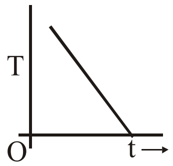
(1) 

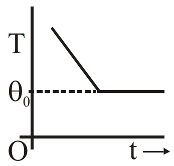
(2) 

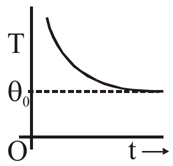
(3) 

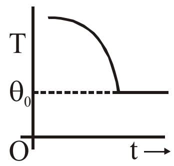
(4) 

6. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 the graph between the temperature T of the metal and time t will be closed to: **[JEE-Main- 2013]**

(1) 

(2) 

(3) 

(4) 

7. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is :

(For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

[JEE-Main-2014]

(1) 2.2×10^7 Pa (2) 2.2×10^6 Pa (3) 2.2×10^8 Pa (4) 2.2×10^9 Pa

8. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is : **[JEE-Main-2014]**

(1) 4.8 cal/s (2) 6.0 cal/s (3) 1.2 cal/s (4) 2.4 cal/s

9. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :- (g = gravitational acceleration) **[JEE-Main-2015]**

(1) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (2) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (3) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (4) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

10. A pendulume clock loses 12s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coeffecient of linear expansion (α) of the metal of the pendulum shaft are respectively :- **[JEE-Main-2016]**

(1) 55°C ; $\alpha = 1.85 \times 10^{-2}$ / °C (2) 25°C ; $\alpha = 1.85 \times 10^{-5}$ / °C

(3) 60°C ; $\alpha = 1.85 \times 10^{-4}$ / °C (4) 30°C ; $\alpha = 1.85 \times 10^{-3}$ / °C

11. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by : (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C)

[JEE-Main-2017]

(1) 1250°C (2) 825°C (3) 800°C (4) 885° C

12. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of : **[JEE-Main-2017]**

(1) 81 (2) $\frac{1}{81}$ (3) 9 (4) $\frac{1}{9}$

13. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by : **[JEE-Main-2017]**

(1) $\frac{3\alpha}{PK}$ (2) $3PK\alpha$ (3) $\frac{P}{3\alpha K}$ (4) $\frac{P}{\alpha K}$

14. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is : **[JEE-Main-2018]**

(1) $\frac{Ka}{3mg}$ (2) $\frac{mg}{3Ka}$ (3) $\frac{mg}{Ka}$ (4) $\frac{Ka}{mg}$

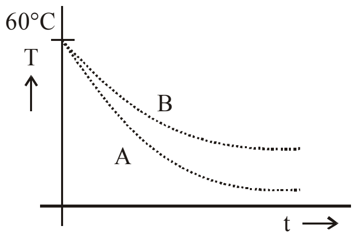
15. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is : **[JEE (Main) 2019, Jan.; 4/120, –1]**

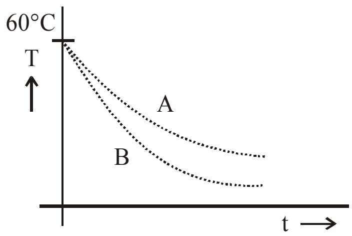
(1) $\frac{F}{2A\alpha\Delta T}$ (2) $\frac{F}{A\alpha(\Delta T-273)}$ (3) $\frac{F}{A\alpha\Delta T}$ (4) $\frac{2F}{A\alpha\Delta T}$

16. Ice at –20° C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to
(Specific heat of water = 4.2 J/g/°C) **[JEE (Main) 2019, Jan.; 4/120, –1]**
Specific heat of Ice = 2.1 J/g/°C
Heat of fusion of water at 0°C = 334 J/g)

(1) 50 g (2) 40 g (3) 60 g (4) 100 g

17. Two identical breakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same) **[JEE(Main) 2019, April; 4/120, –1]**

(1)


(2)


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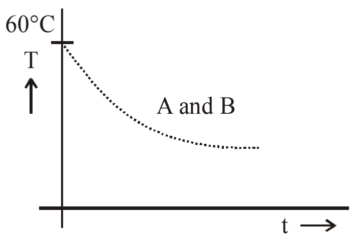

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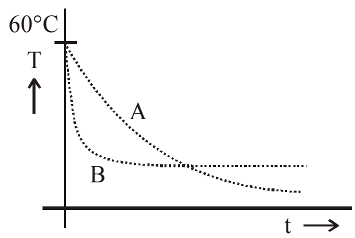


18. A thermally insulated vessel contains 150g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to : (Latent heat of vaporization of water = 2.10×10^6 J kg^{-1} and Latent heat of Fusion of water = 3.36×10^5 J kg^{-1})

[JEE(Main) 2019, April; 4/120, –1]

(1) 130 g (2) 35 g (3) 20 g (4) 150 g

19. A leak proof cylinder of length 1m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to : **[JEE(Main) 2020, Jan.; 4/100, –1]**

(1) 1.01 (2) 1.04 (3) 1.03 (4) 1.26

20. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in litres) are taken from each containers and mixed (assume no loss of heat during the process) **[JEE(Main) 2020, Jan.; 4/100, –1]**

C_1	C_2	C_3	T
$1l$	$2l$	–	60°C
–	$1l$	$2l$	30°C
$2l$	–	$1l$	60°C
$1l$	$1l$	$1l$	θ

The value of θ (in °C to the nearest integer) is ....

21. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to___________. **[JEE(Main) 2020, Sep.; 4/100, –1]**

22. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixure is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} °C^{-1}) **[JEE(Main) 2020, Sep.; 4/100, –1]**

(1) 2.6 (2) 2 (3) 4 (4) 3.2

Answers

Exercise # 1

PART - I

Section (A) :

A-1. 25.5°C
A-2. 136 km
A-3. $\frac{3}{350} = 8.6 \times 10^{-3}$ °C
A 4. (a) 100 J (b) 0 (c) 1/40 °C
A 5 0°C

Section (B) :

B-1. 0.75m
B 2. **Ans :** 1.1×10^{-2}
B 3. %R < %A < %V
B 4. We will cool the system.
B 5. (a) $R_1' = R_1 (1 + \alpha\Delta\theta)$

(b) $R_2' = R_2 (1 + \alpha\Delta\theta)$

(c) $R_2' - R_1' = (R_2 - R_1)(1 + \alpha\Delta\theta)$

(d) $A' = (\pi R_2^2 - \pi R_1^2)(1 + 2\alpha\Delta\theta) = A(1 + 2\alpha\Delta\theta)$

B 6. (i) hollow sphere > solid sphere
(ii) hollow sphere = solid sphere

Section (C) :

C 1 . – 40°C or – 40ºF.
C 2. (a) All tie (b) 50ºX, 50ºY, 50ºW.

PART - II

Section (A):

A 1. (B) **A 2.** (D) **A 3.** (A) **A 4.** (A)
A 5. (A)

Section (B) :

B 1. (B) **B 2.** (A) **B 3.** (C) **B 4.** (C)
B 5. (A) **B 6.** (C)

Section (C) :

C 1. (A)

PART - III

1. (A) – (p) ; (B) – (r) ; (C) – (s) ; (D) – (q)
2. (A) – (r) ; (B) – (q) ; (C) – (p) ; (D) – (s)

Exercise # 2

PART - I

1. (B) **2.** (A) **3.** (C) **4.** (C)
5. (C) **6.** (C) **7.** (B) **8.** (A)
9. (A) **10.** (A) **11.** (D) **12.** (D)

PART - II

1. 370 **2.** 12 **3.** 42.14 **4.** 0.15
5. 160 **6.** 0.092 **7.** 660 **8.** 330
9. 52.875 **10.** 8 **11.** 4 **12.** 66
13. 8 **14.** 8 **15.** 0.32

PART - III

1. (A, C) **2.** (B, C, D)
3. (A, B, C, D) **4.** (A, B, C)
5. (A, B) **6.** (B, D)

PART - IV

1. (A) **2.** (C) **3.** (B) **4.** (D)
5. (B) **6.** (A)

Exercise # 3

PART - I

1. 8 gm **2.** 3 **3.** (C) **4.** (A)
5. (C) **6.** (A) **7.** 2 **8.** (A,B)
9. (B) **10.** 9 **11.** (A) **12.** (C)
13. 4 [3.99, 4.01] **14.** 270.00

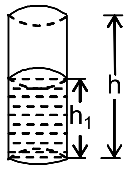
PART - II

1. (4) **2.** (2) **3.** (4) **4.** (4)
5. (2) **6.** (3) **7.** (3) **8.** (1)
9. (3) **10.** (2) **11.** (4) **12.** (3)
13. (3) **14.** (2) **15.** (3) **16.** (2)
17. (1) **18.** (3) **19.** (1) **20.** 50
21. 20 **22.** (2)

RANKER PROBLEMS

SUBJECTIVE QUESTIONS

1. Consider a cylindrical container of cross section area 'A', length 'h' having coefficient of linear expansion α_C. The container is filled by liquid of volume expansion coefficient γ_L up to height h_1. When temperature of the system is increased by $\Delta\theta$ then



(a) Find out new height, area and volume of cylindrical container and new volume of liquid.

(b) Find the height of liquid level when expansion of container is neglected.

(c) Find the relation between γ_L and α_c for which volume of container above the liquid level.

(i) increases (ii) decreases (iii) remains constant.

(d) If $\gamma_\ell > 3\alpha_c$ and $h = h_1$ then calculate, the volume of liquid overflow.

(e) If the surface of a cylindrical container is marked with numbers for the measurement of liquid level of liquid filled inside it. Assuming correct marking at initial temperature if we increase the temperature of the system by $\Delta\theta$ then

(i) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of liquid

(ii) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of container.

(iii) Find relation between γ_L and α_C so that height of liquid level with respect to ground

(1) increases (2) decreases (3) remains constant.

2. The time represented by the clock hands of a pendulum clock depends on the number of oscillations performed by pendulum. Every time it reaches to its extreme position the second hand of the clock advances by one second that means second hand moves by two second when one oscillation is completed.

(a) How many number of oscillations completed by pendulum of clock in 15 minutes at calibrated temperature 20ºC

(b) How many number of oscillations are completed by a pendulum of clock in 15 minutes at temperature of 40ºC if $\alpha = 2 \times 10^{-5}$ /ºC

(c) What time 7is represented by the pendulum clock at 40ºC after 15 minutes if the initial time shown by the clock is 12 : 00 pm ?

(d) If the clock gains two seconds in 15 minutes in correct clock then find - (i) Number of extra oscillations (ii) New time period (iii) change in temperature.

3. A thermally insulated, closed copper vessel contains water at 15ºC. When the vessel is shaken vigorously for 15 minutes, the temperature rises to 17ºC. The mass of the vessel is 100g and that of the water is 200g. The specific heat capacities of copper and water are 420 J/kg-K and 4200 J/kg-K respectively. Neglect any thermal expansion. (a) How much heat is transferred to the liquid-vessel system? (b) How much work has been done on this system? (c) How much is the increase in internal energy of the system?

4. One gram of water (volume = 1 cm^3) becomes 1671 cm^3 of steam when boiled at a pressure of one atmosphere . Latent heat of vaporization at this pressure is 539 cal/gm . Compute the work done.

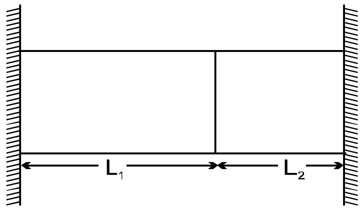
[1 atm = 1.013×10^5 Nm^{-2}] **[REE '86, 3]**

5. The brass scale of a barometer gives correct reading at 0° C. Coefficient of thermal expansion of brass is 0.00002/° C. The barometer reads 75 cm at 27° C. What is the correct atmospheric pressure at 27°C?

[JEE '89, 2]

6. A clock with an iron pendulum keeps correct time at 20° C. How much will it lose or gain in a day if the temperature changes to 40° C? (Coefficient of cubical expansion of iron = 0.000036/° C) **[JEE '90, 3]**

7. Two rods of different metals having same area of cross section A are placed end to end between two massive platforms, as shown in the figure. The first rod has a length L_1, coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for the second rod are L_2, α_2, and Y_2. The temperature of both the rods is now increased by T° C. Find the force with which the rods act on each other (at the higher temperature) in terms of given quantities. Also find the lengths of the rods at the higher temperature. Assume that there is no change in the cross sectional area of the rods and that the rods do not bend. There is no deformation of the walls. **[JEE '90, 5]**

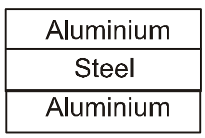


8. A composite rod is made by joining a copper rod end to end with a second rod of different material but of the same cross section. At 25° C the composite rod is 1 m in length of which the length of the copper rod is 30 cm. At 125° C the length of the composite rod increases by 1.91 mm. When the composite rod is not allowed to expand by holding it between two rigid walls it is found that the length of the two constituents do not change with the rise of temperature. Find the Young's modulus and the linear expansion of the second rod given that Young's modulus of for copper = 1.3×10^{11} N/m^2 and the coefficient of linear expansion of copper = 1.7×10^{-5} /° C. **[JEE '90, 4]**

9. A piece of metal weighs 46 g in air. When it is immersed in a liquid of specific gravity 1.24 at 27° C it weighs 30 g. When the temperature of liquid is raised to 42° C the metal piece weighs 30.5 g. Specific gravity of liquid at 42° C is 1.20. Calculate the coefficient of linear expansion of the metal.

[JEE '91, 3]

10. Two Aluminium rods and a steel rod of equal cross-sectional area and equal length ℓ_0 are joined rigidly side by side as shown in figure. Initially the rods are at 0°C. Find the length of the rod at the temperature θ if young's modulus of elasticity of the aluminium and steel are Y_a and Y_s respectively and coefficient of linear expansion of aluminium and steel are α_a and α_s respectively.



11. The alternate discs of iron and carbon , having same area of cross–section , are cemented together to make a cylindrical conductor whose temperature coefficient of resistivity is zero. If the change in temperature in two alternate discs is the same , determine the ratio of their thickness and the ratio of heat produced in them. The resistivity of iron and carbon at 20°C are 1×10^{-7} and 3×10^{-5} Ω-m and their temperature coefficient of resistance are 5×10^{-3} and -7.5×10^{-4} per °C, respectively. Neglect thermal expansion. **[REE 1988]**

12. Consider a metal scale of length 30 cm and an object. The scale is calibrated for temp 20ºC.

(a) What is the actual length of division which is shown as 1 cm by scale at 40ºC.

Given $\alpha_s = 2\times10^{-5}$ /ºC.

(b) What will be the reading of scale at 40ºC if the actual length of object is 10 cm.

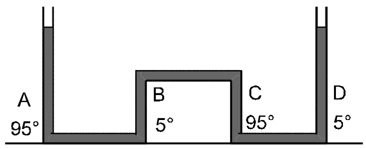
(c) What will be the actual length of object at 40 ºC if its measured length is 10 cm.

(d) What is % error in measurement for part (b) and (c) .

(e) If the linear expansion coefficient of object is $\alpha_0 = 4 \times 10^{-5}$ and neglecting the expansion of scale, then answers of (b) and (c) parts.

(f) If $\alpha_0 = 4 \times 10^{-5}$ and $\alpha_s = 2 \times 10^{-5}$ then find answers of (b) and (c) part.

13. The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B & C are 49 cm each . The two outer columns A & D are open to the atmosphere . A & C are maintained at a temperature of 95º C while the columns B & D are maintained at 5º C . The height of the liquid in A & D measured from the base line are 52.8 cm & 51 cm respectively. Determine the coefficient of thermal expansion of the liquid . **[JEE '97, 5]**



Answers

1. (a) $h_f = h\{1+\alpha_c\,\Delta\theta\}$
$A_f = A\{1 + 2\alpha_c\,\Delta\theta\}$
$v_f = Ah\{1 + 3\alpha_c\,\Delta\theta\}$
volume of liquid $V_W = Ah_1(1+\gamma_L\,\Delta\theta)$

(b) $h_f = h_1\{1 + \gamma_L\,\Delta\theta\}$

(c) (i) $3h\,\alpha_c > h_1\gamma_\ell$ (ii) $3h\,\alpha_c < h_1\gamma_\ell$ (iii) $3h\,\alpha_c = h_1\gamma_\ell$

(d) $\Delta V = Ah(\gamma_L - 3\alpha_C)\Delta\theta$

(e) (i) $h_f = h_1(1 - 3\alpha_C\Delta\theta)$ (ii) $h_f = h_1(1 + \gamma_L\Delta\theta)$
(iii) (1) $\gamma_L > 2\alpha_c$ (2) $\gamma_L < 2\alpha_c$ (3) $\gamma_L = 2\alpha_c$

2. (a) 450 (b) 449 (c) 12:14:59 pm (d) (i) 1 (ii) $\frac{900}{451}$ s (iii) $\frac{10^5}{450}$ ºC

3. (a) zero (b) 1764 J (c) 1764 J

4. 169.171 J

5. 75.0405 cm

6. 10.368 s

7. $F = \frac{AT(L_1\alpha_1 + L_2\alpha_2)Y_1Y_2}{L_1Y_2 + L_2Y_1}$, Length of the first rod = $L_1 + \frac{L_1L_2T(Y_1\alpha_1 - Y_2\alpha_2)}{L_1Y_2 + L_2Y_1}$, Length of the second rod = $L_2 + \frac{L_1L_2T(Y_2\alpha_2 - Y_1\alpha_1)}{L_1Y_2 + L_2Y_1}$]

8. $Y_2 = 1.105 \times 10^{11}$ N/m^2, $\alpha_2 = 2 \times 10^{-5}$/º C]

9. $\alpha = \frac{1}{43200}$ /º C = 2.31×10^{-5} /º C

10. $\ell_0\left[1 + \frac{2Y_a\alpha_a + Y_s\alpha_s}{2Y_a + Y_s}\,\theta\right]$

11. $\frac{t_F}{t_C} = 45$, $\frac{H_F}{H_C} = \frac{3}{20}$

12. (a) $\ell = 1\{1 + 2 \times 10^{-5} \times 20\}$

(b) $\ell = 10\{1 - 4 \times 10^{-4}\}$

(c) $\ell = 10\{1 + 4\times10^{-4}\}$

(d) $\%\,\ell_1 = -4 \times 10^{-2}\,\%$

$\%\,\ell_2 = \frac{-4\times10^{-2}}{1+4\times10^{-4}}\,\% \cong -4\times 10^{-2}\,\%$

(e) $\ell_1 = 10\{1 + 20 \times 4 \times 10^{-5}\}$
$\ell_2 = 10\{1 - 20 \times 4 \times 10^{-5}\}$

(f) $\ell_1 = 10\{1 + 40 \times 10^{-5}\}$
$\ell_2 = 10\{1 - 40 \times 10^{-5}\}$

13. $\gamma = 2 \times 10^{-4}$ / °C

SELF ASSESSMENT PAPER

JEE (ADVANCED) TEST PAPER - 1

SECTION-1 : ONE OPTION CORRECT TYPE (Maximum Marks - 12)

1. A liquid with coefficient of volume expansion γ is filled in a container of a material having the coefficient of linear expansion α .If the liquid overflows on heating, then –

(A) $\gamma > 3\alpha$ (B) $\gamma < 3\alpha$ (C) $\gamma = 3\alpha$ (D) none of these

2. A steel rod 25 cm long has a cross-sectional area of 0.8 cm^2. Force required to stretch this rod by the same amount as the expansion produced by heating it through 10ºC is:

(Coefficient of linear expansion of steel is 10^{-5}/ºC and Young's modulus of steel is 2×10^{10} N/m^2.)

(A) 160 N (B) 360 N (C) 106 N (D) 260 N

3. Time taken by a 836 W heater to heat one liter of water from 10ºC to 40ºC is :

(A) 50 s (B) 100 s (C) 150 s (D) 200 s

4. 2 liters water at 27°C is heated by a 1 kW heater in an open container. On an average heat is lost to surroundings at the rate 160 J/s. The time required for the temperature to reach 77°C is

(A) 8 min 20 sec (B) 10 min (C) 7 min (D) 14 min

SECTION-2 : ONE OR MORE THAN ONE CORRECT TYPE (Maximum Marks - 32)

5. Which one of the following statements is **NOT** the explanation for the fact that metal pipes that carry water often burst during cold winter months?

(A) Water contracts upon freezing while the metal expands at lower temperatures.

(B) The metal contracts to a greater extent than the water.

(C) The interior of the pipe contracts less than the outside of the pipe.

(D) Both the metal and the water expand, but the water expands to a greater extent.

6. A rod 'PQ' of length 'ℓ' is pivoted at an end P and freely rotated in a horizontal plane at an angular speed 'ω' about a vertical axis passing through P. If coefficient of linear expansion of material of rod is α and temperature of rod is increased by ΔT. Select the ***CORRECT*** alternatives.

(A) Angular momentum of rod about axis of rotation will increase.

(B) Angular momentum of rod about axis of rotation will remains constant.

(C) Final angular velocity of rod after heating is $\omega[1 + 2\alpha\Delta T]$.

(D) Final angular velocity of rod after heating is $\omega[1 – 2\alpha\Delta T]$.

7. A metal rod is shaped into ring with a small gap in between ends. If this rod is heated :

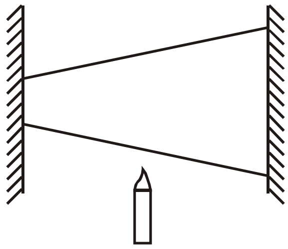
(A) The length of the rod will increase.

(B) The gap will decrease

(C) The gap will increase

(D) The diameter of the ring will increase in the same ratio as the length of the rod.

8. A rod is made of uniform material & a nonuniform cross-section area as shown in the figure, when the rod is heated. Identify the **INCORRECT** statements.



(A) Force of compression in the rod is maximum at mid-section and minimum at ends.

(B) Compressive thermal stress in rod is maximum at left end

(C) Since rod is fixed at both ends so no elongation and hence no strain is induced

(D) Maximum stress in the rod is at right end.

9. Supercooling is a state where liquids do not solidiby even below their normal freezing point. Supercooled water can remain in the liquid state as low as –40°C. If a small piece of ice is dropped then water suddenly freezes. If super cooled water at – 10°C suddenly starts freezing then in a very short interval a mixture of water and ice both at 0°C is formed. If 1 kg of water freezes then 334 kJ of energy is released. Specific heat of ice = 2.1 kJ $kg^{-1}k^{-1}$, specific heat of water (in supercooled state also) = 4.2 kJ $kg^{-1}K^{-1}$, Latent heat of ice at 0°C = 334 kJ kg^{-1}. Consider 1 kg of water and mark the correct statements :-

(A) Latent heat of fusion at –10°C is nearly 313 kJ kg^{-1}.

(B) Latent heat of fusion at –10°C is 334 $kJkg^{-1}$.

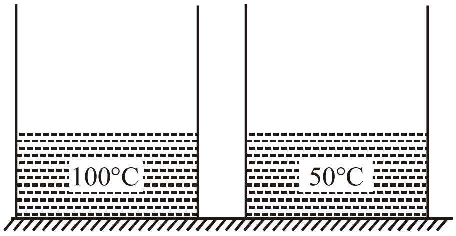
(C) If supercooled water suddenly freezes at –10°C then approximately 0.67 kg of ice is formed.

(D) If supercooled water suddenly freezes at –10°C then approximately 0.13 kg of ice is formed.

10. In a heat insulated vessel there is tap water at 15 °C. We place in it 'n' ice cubes of –15 °C taken out of the deep-freezer. $S_{ice} = \frac{S_{water}}{2}$. Each ice cube has same mass as mass of water in vessel.

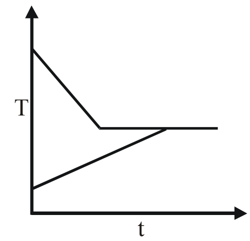
(A) If n = 1, some ice will melt.
(B) If n = 1, some water will freeze.
(C) If n = 2, some water will freeze.
(D) If n = 2, final temperature is 0°C

11. 2 kg water at 100°C and 2.5 kg water at 50°C is kept in two identical containers A and B respectively of water equivalent 0.5 kg. If water of container A is poured into container B the final temperature of mixture is T_1 and if water of container B is poured into container A the final temperature is T_2. (heat loss is negligible)



(A) $15T_1 = 14T_2$
(B) $T_1 = T_2$
(C) $T_1 = 70°C$
(D) $T_2 = 70°C$

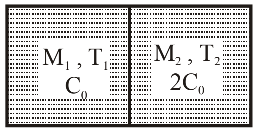
12. A sample A of liquid water and a sample B of ice of identical mass are kept in two neighbouring chambers in an otherwise insulated container. The chambers can exchange heat with each other. The graph of temperatures of the two chambers is plotted with time. $S_{ice} = \frac{S_{water}}{2}$:-



(A) Finally the contents in sample A is water.
(B) Equilibrium temperature is freezing point of water
(C) Ice melts partly
(D) Finally the contents in sample B is ice only.

SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. A thermally insulated vessels contains two liquids with initial temperature T_1 and T_2 and specific heats C_0 and $2C_0$, separated by a non conducting partition. The partition is removed, and the difference between the initial temperature of one of the liquids and the temperature T established in the vessel turns out to be equal to half the difference between the initial temperatures of the liquids. Determine the ratio of $\frac{M_1}{M_2}$ of the masses of the liquids.



14. When an automobile is braking, the friction between the brake drums made up of iron and the brake shoes converts translational kinetic energy into heat. A 2100 kg automobile brakes from 216 km/h to 0 km/h. If each of the four brake drums has a mass of 10 kg and specific heat of iron is 0.1 kcal/kg·°C, how much does the temperature of the brake drums rise (in S.I. units)? Assume that all the heat accumulates in the brake drums (there is not enough time for the heat to leak away into the air) and that the heat in all brake drums is the same. (Take 1 kcal = 4200 J)

15. A gas thermometer is used as a standard thermometer for measurement of temperature . When the gas container of the thermometer is immersed in water at its triple point 273.16 K, the pressure in the gas thermometer reads 3×10^4 N/m^2. When the gas container of the same thermometer is immersed in another system, the gas pressure reads 3.5×10^4 N/m^2. Find the temperature (in ºC) of this system.

16. 300 g of water at 25°C is added to 100 g of ice at 0°C; find the final temperature (in ºC) of the mixture.

17. In the following equation calculate the value of H (in Kcal).
1 kg steam at 200ºC = H + 1 kg water at 100ºC (S_{steam} = Constant = 0.5 Cal/gm°C)

18. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Find the final temperature of the mixture (in Kelvin).
Given, L_{fusion} = 80 cal/gm = 336 J/gm, $L_{vaporization}$ = 540 cal/gm = 2268 J/gm,
S_{ice} = 2100 J/kg K = 0.5 cal/gm K and S_{water} = 4200 J/kg K = 1 cal/gmK

Answers

1.	(A)	**2.**	(A)	**3.**	(C)	**4.**	(A)	**5.**	(A,B,C,D)	**6.**	(B,D)
7.	(A,C,D)	**8.**	(A,C,D)	**9.**	(B,D)	**10.**	(A,D)	**11.**	(A,C)	**12.**	(B, D)
13.	2	**14.**	225	**15.**	45.536	**16.**	0	**17.**	590	**18.**	273