Chapter 14

TRIGONOMETRIC EQUATIONS & INVERSE TRIGONOMETRIC FUNCTIONS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. The minimum value of $27^{\cos 2x}.\ 81^{\sin 2x}$ is

(a) 1 (b) $\frac{1}{9}$

(c) $\frac{1}{81}$ (d) $\frac{1}{243}$

2. The set of values of x for which the identity

$\cos^{-1} x + \cos^{-1}\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right) = \frac{\pi}{3}$ holds good, is

(a) $[0, 1]$ (b) $\left[0, \frac{1}{2}\right]$

(c) $\left[\frac{1}{2}, 1\right]$ (d) $\{-1, 0, 1\}$

3. $3\cos^{-1} x - \pi x - \frac{\pi}{2} = 0$ has

(a) one solution
(b) one and only one solution
(c) no solution
(d) more than one solution

4. The value of 'a' for which $ax^2 + \sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(x^2 - 2x + 2) = 0$ has a real solution, is

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{2}$

(c) $\frac{2}{\pi}$ (d) none of these

5. If $A = 2\tan^{-1}(2\sqrt{2}-1)$ and

$B = 3\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right)$, then

(a) $A = B$
(b) $A < B$
(c) $A > B$
(d) cannot be determined without tables

6. The greatest and the least values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are respectively

(a) $-\frac{\pi}{2}, \frac{\pi}{2}$ (b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$

(c) $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ (d) $\frac{\pi^3}{32}, \frac{\pi^3}{8}$

7. If $ax + b(\sec(\tan^{-1} x)) = c$ and $ay + b(\sec(\tan^{-1} y)) = c$,

then $\frac{x+y}{1-xy} =$

(a) $\frac{ac}{a^2+c^2}$ (b) $\frac{2ac}{a-c}$

(c) $\frac{2ac}{a^2-c^2}$ (d) $\frac{a+c}{1-ac}$

8. $\sin^{-1}\left(a - \frac{a^2}{3} + \frac{a^3}{9} + ...\right) + \cos^{-1}(1 + b + b^2 + ...) = \frac{\pi}{2}$ when

(a) $a = -3$ & $b = 1$ (b) $a = 1$ & $b = -\frac{1}{3}$

(c) $a = \frac{1}{6}$ & $b = \frac{1}{2}$ (d) none of these

9. The number of real solutions of the equation

$\tan^{-1}\sqrt{x^2-3x+2} + \cos^{-1}\sqrt{4x-x^2-3} = \pi$ is

(a) one (b) two
(c) zero (d) infinite

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

10. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\dfrac{3\pi}{2}$ and $f(1)=2, f(p+q)$ $=f(p).f(q)\ \forall\ p, q \in R$, then $f\left(\dfrac{x+y+z}{xyz}\right)$ equals to

(a) 1 (b) 2 (c) 3 (d) 8

11. If $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\dfrac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta$ $=-1, \theta\in[0, 2\pi]$, then

(a) $\theta\in\left(0, \dfrac{\pi}{2}\right)-\left\{\dfrac{\pi}{4}\right\}$ (b) $\theta\in\left(\dfrac{\pi}{2}, \pi\right)-\left\{\dfrac{3\pi}{4}\right\}$

(c) $\theta\in\left(\pi, \dfrac{3\pi}{2}\right)-\left\{\dfrac{5\pi}{4}\right\}$ (d) $\theta\in(0, \pi)-\left\{\dfrac{\pi}{4}, \dfrac{\pi}{2}\right\}$

12. If $u=\cot^{-1}\sqrt{\tan\alpha}-\tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\dfrac{\pi}{4}-\dfrac{u}{2}\right)$ may be equal to

(a) $\sqrt{\tan\alpha}$ (b) $\sqrt{\cot\alpha}$

(c) $\tan\alpha$ (d) $\cot\alpha$

13. If $5^x+(2\sqrt{3})^{2x}\geq 13^x$, then the solution set for x is

(a) $[2,\infty)$ (b) $\{2\}$ (c) $(-\infty, 2]$ (d) $[0, 2]$

14. The equation

$\sin x+\cos x=\min\left\{\sqrt{2a^2-4a+5}\right\}, a\in R$ has

(a) no solution for all $a\in R$

(b) no solution of $a\leq 0$

(c) no solution for $a\geq 2$

(d) none of these

15. The number of solutions of $\log_{\sin x}2^{\tan x}>0$ in the interval $\left(0, \dfrac{\pi}{2}\right)$ is

(a) 0 (b) 1

(c) 2 (d) inifinite

16. The number of all possible 5-tuples $(a_1, a_2, a_3, a_4, a_5)$ such that $a_1+a_2\sin x+a_3\cos x+a_4\sin 2x+a_5\cos 2x=0$ holds for all x is

(a) zero (b) 1 (c) 2 (d) infinite

17. Number of solutions of the equation $|\cos x|=2[x]$ is

(a) zero (b) 1

(c) 2 (d) infinitely may

18. The most general solution of $2^{\sin x}+2^{\cos x}=2^{1-\frac{1}{\sqrt{2}}}$ is

(a) $n\pi+\dfrac{\pi}{4}$ (b) $n\pi-\dfrac{\pi}{4}$

(c) $(2n+1)\pi+\dfrac{\pi}{4}$ (d) $2n\pi+\dfrac{\pi}{4}$

19. The number of solutions of the equation

$\sin^5 x-\cos^5 x=\dfrac{1}{\cos x}-\dfrac{1}{\sin x}; (\sin x\neq\cos x),\ 0<x<2\pi$

is

(a) 0 (b) 1

(c) 3 (d) infinite

20. $|\tan x+\sec x|=|\tan x|+|\sec x|$ where $x\in[0, 2\pi]$ if and only if x belongs to the interval

(a) $[0, \pi]$ (b) $\left[0, \dfrac{\pi}{2}\right)\cup\left(\dfrac{\pi}{2}, \pi\right]$

(c) $\left[\pi, \dfrac{3\pi}{2}\right)\cup\left(\dfrac{3\pi}{2}, 2\pi\right]$ (d) $(\pi, 2\pi]$

21. The values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x}=1$; are in A.P. with common difference

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{8}$

(c) $\dfrac{3\pi}{8}$ (d) $\dfrac{5\pi}{8}$

22. The set of all x in $(-\pi, \pi)$ satisfying $|4\sin x-1|<\sqrt{5}$ is given by

(a) $\left(-\dfrac{\pi}{10}, \dfrac{\pi}{10}\right)$ (b) $\left(-\dfrac{\pi}{5}, \dfrac{\pi}{10}\right)$

(c) $\left(-\dfrac{\pi}{10}, \dfrac{3\pi}{10}\right)$ (d) None of these

MARK YOUR RESPONSE					
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ		

23. If m and n ($> m$) are positive integers, the number of solutions of the equation $n\,|\sin x| = m\,|\cos x|$ in $[0, 2\pi]$ is :

(a) m (b) n

(c) $m\,n$ (d) none of these

24. $\sin^{-1}(\sin 5) > x^2 - 4x$ holds if

(a) $x < 2 - \sqrt{9 - 2\pi}$

(b) $-1 < x < 5$

(c) $x \in (-\infty, -1) \cup (5, \infty)$

(d) $x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$

25. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ where x is a variable, has real roots if p lies in the interval

(a) $(0, 2\pi)$ (b) $(-\pi, 0)$

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

26. If $[\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1}x] = 1$, where [.] denotes the greatest integer function, then x belongs to the interval

(a) [tan sin cos 1, tan sin cos sin 1]

(b) [tan sin cos 1, tan sin cos sin 2]

(c) $[-1, 1]$

(d) [sin cos tan 1, sin cos sin tan 1]

27. The expression $\cos 3\theta + \sin 3\theta + (2\sin 2\theta - 3)(\sin\theta - \cos\theta)$ is positive for all θ in

(a) $\left(2n\pi - \frac{3\pi}{4}, 2n\pi + \frac{\pi}{4}\right) \cup \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{6}\right)$

(b) $\left(2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3}\right)$

(c) $\left(2n\pi - \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right)$

(d) none of these

28. If $[\sin x] + [\sqrt{2}\cos x] = -3$, $x \in [0, 2\pi]$ ([.] denotes the greatest integer function), then x belongs to

(a) $\left[\frac{5\pi}{4}, 2\pi\right]$ (b) $\left(\frac{5\pi}{4}, 2\pi\right)$

(c) $\left(\pi, \frac{5\pi}{4}\right)$ (d) $\left[\pi, \frac{5\pi}{4}\right]$

29. $|\cos x| = \cos x - 2\sin x$ if

(a) $x = n\pi$

(b) $x = 2n\pi$ or $(2n+1)\pi + \frac{\pi}{4}$

(c) $x = n\pi + \frac{\pi}{4}$

(d) $x = n\pi$ or $n\pi + \frac{\pi}{4}$

30. The point of intersection of the curves $y = \sin\frac{\pi x}{2}$ and $y = \left[\sin\frac{\pi}{10} + \cos\frac{\pi}{10}\right]$, where [.] denotes greatest integer function, is given by ($n \in I$)

(a) $(4n+1, 1)$ (b) $(4n-1, -1)$

(c) $(4n-1, 1)$ (d) $(2n, 0)$

31. Solution set of $[\sin^{-1}x] > [\cos^{-1}x]$, where [.] denotes the greatest integer function is

(a) $[\sin 1, 1]$ (b) $\left[\frac{1}{\sqrt{2}}, 1\right]$

(c) $(\cos 1, \sin 1)$ (d) None of these

32. If α is a positive real root of the equation $2x^4 + 4x^3 \sin A \sin B - x^2(\cos 2A + \cos 2B) + 4x\cos A\cos B - 2 = 0$, then $\alpha =$

(a) $\cos A\cos B + 1$ (b) $\sin A\sin B + 1$

(c) $\cos(A - B) + 2$ (d) none of these

33. If $\sin x \neq \pm 1$ and $\cos y \neq \pm 1$, then the equation $\sin^4 x + \cos^4 y + 2 = 4\sin x\cos y$ has

(a) no solution (b) exactly one solution

(c) exactly two solutions (d) infinite solutions

34. $x = n\pi - \tan^{-1}3$ is a solution of the equation

$$12\tan 2x + \frac{\sqrt{10}}{\cos x} + 1 = 0 \text{ for}$$

(a) no value of n (b) all integral values of n

(c) even values of n (d) odd values of n

35. The equation $\int_0^x (t^2 - 8t + 13)\,dt = x\sin\frac{a}{x}$ has a solution if $\sin\frac{a}{6} =$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

MARK YOUR RESPONSE					
	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ
	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ
	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ		

36. The value of 'b' such that the equation

$$\frac{b\cos x}{2\cos 2x-1}=\frac{b+\sin x}{(\cos^2 x-3\sin^2 x)\tan x}$$ possess solutions, belong to the set

(a) $\left(-\infty,\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},\infty\right)$

(c) $(-\infty,\infty)$ (d) $\left(-\infty,\frac{1}{2}\right)\cup(1,\infty)$

37. If the equation

$$x^2+\left\{\frac{2(\sin\theta-a)-1}{2}\right\}x+\left\{\frac{1-16(\sin\theta+a)}{16}\right\}=0$$ has real and distinct roots then

(a) $\sin\theta>-\frac{3}{4}$ (b) $\sin\theta>\frac{25}{32}$

(c) $\sin\theta\le-\frac{1}{2}$ (d) $-1\le\sin\theta\le 1$

38. The value of tan $[\sin^{-1}\{\cos(\sin^{-1}x)\}],[\tan[\cos^{-1}\{\sin(\cos^{-1}x)\}]$ is equal to

(a) x (b) x^2

(c) $x\left(\frac{\pi}{2}-x\right)$ (d) 1

39. If S_n denotes the sum to n terms of the series

$\cot^{-1}\frac{7}{4}+\cot^{-1}\frac{19}{4}+\cot^{-1}\frac{39}{4}+.....$ then

(a) $S_n=\tan^{-1}\frac{n}{2n+5}$ (b) $S_n=\cot^{-1}\frac{n+5}{2n}$

(c) $S_n=\cot^{-1}\frac{4n}{2n+5}$ (d) $S_\infty=\cot^{-1}\frac{1}{2}$

40. The value of $\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)$

$+\tan^{-1}(\cot^3 A)$, for $0<a<\frac{\pi}{4}$, is

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(\cot A)$

(c) $4\tan^{-1}(1)$ (d) $2\tan^{-1}(2)$

41. If $(\alpha+\pi)$ and $(\beta-\pi)$ are two distinct solutions of the equations $\tan^2\left(\frac{\theta}{2}\right)-\frac{2b}{c-a}\tan\left(\frac{\theta}{2}\right)+\frac{c+a}{c-a}=0$, then α, β are the solutions of the equation

(a) $a\cos\theta-b\sin\theta-c=0$

(b) $a\cos\theta+b\sin\theta-c=0$

(c) $a\cos\theta+b\sin\theta+c=0$

(d) $a\cos\theta-b\sin\theta+c=0$

42. The number of solutions of the equation $\sin^3x\cos x+\sin^2x\cos^2x+\sin x\cos^3x=1$, in the interval $[0, 2\pi]$, is

(a) 4 (b) 2

(c) 1 (d) 0

43. If $A=\tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ and $B=\tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$, then $A-B$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

44. A general solution of the equation

$$\frac{3\sin\theta-\sin 3\theta}{1+\cos\theta}+\frac{3\cos\theta+\cos 3\theta}{1-\sin\theta}=(\cos\theta-\sin\theta)$$ is

(a) $\theta=n\pi$ (b) $\theta=n\pi+\frac{\pi}{2}$

(c) $\theta=n\pi\pm\frac{\pi}{2}$ (d) $\theta=2n\pi$

45. Exhaustive set of values of parameter 'a' so that $\sin^{-1}x-\tan^{-1}x=a$ has a solution is

(a) $\left[-\frac{\pi}{6},\frac{\pi}{6}\right]$ (b) $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$

(c) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (d) $\left(0,\frac{\pi}{2}\right)$

MARK YOUR RESPONSE					
	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ
	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ

46. General solution of the equation $1 + \sin^4 2x = \cos^2 6x$ is

(a) $\frac{n\pi}{3}$ (b) $\frac{n\pi}{2}$

(c) $n\pi$ (d) $n\pi + \frac{\pi}{3}, (n \in \pi)$

47. If $\cos 3x + \sin\left(2x - \frac{7\pi}{6}\right) = -2$, then $x =$

(a) $\frac{\pi}{3}(6m-1)$ (b) $\frac{\pi}{3}(6m+1)$

(c) $\frac{\pi}{3}(2m+1)$ (d) none of these

(where $m \in I$)

48. The value of x which satisfies equation

$2\tan^{-1} 2x = \sin^{-1}\frac{4x}{1+4x^2}$ is

(a) $\left[\frac{1}{2}, \infty\right)$ (b) $\left(-\infty, -\frac{1}{2}\right]$

(c) $[-1, 1]$ (d) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

49. The number of solutions of the equation

$\cos^{-1}\left(\frac{1+x^2}{2x}\right) - \cos^{-1} x = \frac{\pi}{2} + \sin^{-1} x$ is given by

(a) 0 (b) 1
(c) 2 (d) 3

MARK YOUR RESPONSE	46. ⓐⓑⓒⓓ	47. ⓐⓑⓒⓓ	48. ⓐⓑⓒⓓ	49. ⓐⓑⓒⓓ	

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

Boundedness of different trigonometric functions are used in solving many trigonometric equations

Consider an equation $\sin x + \sin y = 2$ ...(1)

We know that $\sin x \le 1$ and $\sin y \le 1$ for all x, y

So, $\sin x + \sin y \le 2$ for all x and y

Therefore, $\sin x + \sin y = 2$ if and only if $\sin x = 1$ and $\sin y = 1$

$\Rightarrow \quad x = 2n\pi + \frac{\pi}{2}$ and $y = 2m\pi + \frac{\pi}{2}$

Which is the required solution of given equation. To solve the equation (1), we have used the boundedness of $\sin x$ rather than using conventional methods of solving equations

In general we employ one or more of the following extreme value conditions.

1. $-1 \le \sin x \le 1 \Rightarrow |\sin x| \le 1$ and $\sin^2 x \le 1$
2. $-1 \le \cos x \le 1 \Rightarrow |\cos x| \le 1$ and $\cos^2 x \le 1$
3. $-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$

$\Rightarrow |a\sin x + b\cos x| \le \sqrt{a^2+b^2}$

1. Number of roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $[0, 2\pi]$ is

(a) 0 (b) 1
(c) 2 (d) 4

2. The smallest positive number p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has a solution in $[0, 2\pi]$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4\sqrt{2}}$ (d) $\frac{\pi}{2\sqrt{2}}$

3. The value of 'a' for which the equation $a^2 - 2a + \sec^2 \pi(a + x) = 0$ has solution is

(a) 1 (b) 2
(c) 0 or 1 (d) 1 or 2

4. The values of the k for which the equation $\sin x + \cos(k + x) + \cos(k - x) = 2$ has real solutions, satisfy

(a) $\sin^2 k \ge \frac{1}{2}$ (b) $\cos^2 k \le \frac{1}{4}$

(c) $\sin^2 k \le \frac{1}{4}$ (d) $\forall k \in R$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	

PASSAGE-2

The function $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\cot^{-1}x$, $\text{cosec}^{-1}x$ and $\sec^{-1}x$ are called inverse circular functions. Each of the inverse circular function is multivalued. To make each inverse circular function single valued let us define the principal values as follow.

$\sin^{-1}x \in \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right], \cos^{-1}x \in [2\pi, 3\pi]$

(in both cases $x \in [-1, -1]$)and $\tan^{-1}x \in \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$, $x\ (-\infty, \infty)$.

5. Number of possible solutions for the equation

$\sin^{-1}x + \cos^{-1}y = \frac{11\pi}{2}$ is /are

(a) 0 (b) 1
(c) 2 (d) infinite

6. $\sin^{-1}\left(\cos\frac{5\pi}{4}\right)$ is equal to

(a) $-\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$
(c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

7. Range of values of x for which $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ holds, is

(a) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$
(c) $[-1, 1]$ (d) $\left[-\infty, -\frac{1}{\sqrt{2}}\right]$

MARK YOUR RESPONSE	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ		

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options:

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. Statement-1 : $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}\ \forall x \in R$

Statement-2 : $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\ \forall x \in R$

2. Statement-1 : The value of $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is $\frac{\pi}{4}$.

Statement-2 : If $x > 0, y > 0$ then

$\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right) = \frac{\pi}{4}$

3. Statement-1 : If $e^{-\pi/2} < \theta < \frac{\pi}{2}$ then $\cos(\log\theta) > \log(\cos\theta)$

Statement-2 : Whenever defined $\cos(\log\theta) \geq 0$

4. Statement-1 : The smallest positive root of the equation $\tan x - x = 0$ lies in $\left(\pi, \frac{3\pi}{2}\right)$

Statement-2 : $y = x$ is tangent to $y = \tan x$ at (0, 0).

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. If $\tan^{-1}y = 4\tan^{-1}x$, then y is not finite if

(a) $x^2 = 3 + 2\sqrt{2}$ (b) $x^2 = 3 - 2\sqrt{2}$

(c) $x^4 = 6x^2 - 1$ (d) $x^4 = 6x^2 + 1$

2. A solution (x, y) of the system of equations $x - y = \frac{1}{3}$ and $\cos^2(\pi x) - \sin^2(\pi y) = \frac{1}{2}$ is given by

(a) $\left(\frac{7}{6}, \frac{5}{6}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{3}\right)$

(c) $\left(-\frac{5}{6}, -\frac{7}{6}\right)$ (d) $\left(\frac{13}{6}, \frac{11}{6}\right)$

3. $\sqrt{\cos 2x} + \sqrt{1 + \sin 2x} = 2\sqrt{\sin x + \cos x}$ if

(a) $\sin x + \cos x = 0$

(b) $x = 2n\pi$

(c) $x = n\pi - \frac{\pi}{4}$

(d) $x = 2n\pi \pm \cos\left(\frac{\pi}{5}\right)$

4. If $\cos^4 x + \sin^4 x - \sin 2x + \frac{3}{4}\sin^2 2x = y$, then

(a) $y = 1$ if $x = \frac{15\pi}{2}$ (b) $y \neq 0$ for any value of x

(c) $y = 0$ if $x = 15\pi$ (d) $y = 1$ if $\sin 2x = 0$

5. $\cos(x - y) - 2\sin x + 2\sin y = 3$ if $(n, k \in I)$

(a) $\sin x = \sin y$

(b) $x + y = 2n\pi, x - y = (4k + 1)\pi$

(c) $x = 2k\pi - \frac{\pi}{2}, y = 2n\pi + \frac{\pi}{2}$

(d) $\cos(x - y) = -1$

6. If the equation $\sin^{-1}(x^2 + x + 1) + \cos^{-1}(ax + 1) = \frac{\pi}{2}$ has exactly two solutions then a can not have the integral value

(a) -1 (b) 0 (c) 1 (d) 2

7. The value (s) of x satisfying the equation $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ is/are given by (n is any integer)

(a) $n\pi$ (b) $n\pi + 1$

(c) $n\pi - 1$ (d) $2n\pi + 1$

8. If $\tan^{-1}(\sin^2\theta + 2\sin\theta + 2) + \cot^{-1}(4^{\sec^2\phi} + 1) = \frac{\pi}{2}$ has solution for some θ and ϕ then

(a) $\sin\theta = -1$ (b) $\sin\theta = 1$

(c) $\cos\phi = 1$ (d) $\cos\phi = -1$

9. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then

(a) $\sin\alpha - \cos\alpha = \pm\sqrt{2}\sin\theta$

(b) $\sin\alpha + \cos\alpha = \pm\sqrt{2}\cos\theta$

(c) $\cos 2\theta = \sin 2\alpha$

(d) $\sin 2\theta + \cos 2\alpha = 0$

10. The equation $2\sin\frac{x}{2}\cos^2 x - 2\sin\frac{x}{2}\sin^2 x = \cos^2 x - \sin^2 x$ has a root for which

(a) $\sin 2x = 1$ (b) $\sin 2x = -1$

(c) $\cos x = 1/2$ (d) $\cos 2x = -1/2$

11. If $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$, then $\sin 2\theta$ is equal to

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

12. The value of $x \in [0, 2\pi]$, for which $2^{1+|\sin x|+|\sin 2x|+|\sin 3x|+\ldots} = 2$ is

(a) 0 (b) π

(c) 2π (d) $\frac{3\pi}{2}$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ			

13. The value of $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation

$(\sqrt{3})^{\sec^2\theta} = 2\tan^2\theta + \tan^4\theta$ is

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{3}$

14. If $\left(\cos^2 x + \frac{1}{\cos^2 x}\right)(1+\tan^2 2y)(3+\sin 3z) = 4$, then

(a) x may be a multiple of π

(b) x can not be an even multiple of π

(c) z can be a multiple of π

(d) y can be a multiple of $\pi/2$.

15. $\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$. Then θ is equal to

(a) $\pi/12$ (b) $5\pi/12$ (c) $-3\pi/12$ (d) $-7\pi/12$

16. A set of value of x satisfying the equation

$\cos^2\left(\frac{1}{2}px\right) + \cos^2\left(\frac{1}{2}qx\right) = 1$ form an arithmetic progression with common difference

(a) $\frac{2\pi}{p+q}$ (b) $\frac{2\pi}{p-q}$ (c) $\frac{\pi}{p+q}$ (d) 2π

MARK YOUR RESPONSE	13.ⓐⓑⓒⓓ	14.ⓐⓑⓒⓓ	15.ⓐⓑⓒⓓ	16.ⓐⓑⓒⓓ	

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

1. **Match the following columns :**

Column-I		Column-II
(A) The equation $\cos 2x + a\sin x = 2a - 7$ possess a solution if a belongs to the interval	p.	$[1, \infty)$
(B) The equation $7\cos x + 5\sin x = 2k + 1$ has a solution if k belongs to the interval	q.	$[2, 6]$
(C) If $\cos^4 x + a\cos^2 x + 1 = 0$ has at least one solution then a belongs to the interval	r.	$(-\infty, -2]$
(D) If the equation $\lvert\sin 2x\rvert - \lvert x\rvert - a = 0$ does not have any real solution then a belongs to the interval	s.	$[-4, 3]$

2. **Match the following columns :**

Column-I		Column-II
(A) If the equation $x^2 + 4 + 3\sin(ax + b) - 2x = 0$ has at least one real solution, where $a, b \in [0, 2\pi]$ then $\sin(a + b)$ can be equal to	p.	-1
(B) If $\sin^{-1} x \le \cos^{-1} x$ then x can be equal to	q.	0
(C) The number of the ordered pairs (x, y) satisfying $\lvert y\rvert = \cos x$ and $y = \sin^{-1}(\sin x)$, where $-2\pi \le x \le 3\pi$ is equal to	r.	1
(D) If $n \in N$ and the set of equations $\cos^{-1} x + (\sin^{-1} y)^2 = \frac{n\pi^2}{4}$ and $(\sin^{-1} y)^2 - \cos^{-1} x = \frac{\pi^2}{16}$ is consistent, then n can be equal to	s.	5

MARK YOUR RESPONSE

1.

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

2.

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

3. Match the following columns :

Column-I		Column-II
(A) The minimum value of $9^3 . 27^{\cos 2x} . 81^{\sin 2x}$	p.	1
(B) Number of solutions of the equation $\cos^7 x + \sin^4 x = 1$ $x \in [0, 2\pi]$	q.	2
(C) Value of a for which the equations $a^2 - 2a + \sec^2 \pi (a + x) = 0$ has a solution	r.	3
(D) If $\cos(p \sin x) = \sin(p \cos x)$, then the minimum positive value of $\frac{4\sqrt{2}}{\pi} p$ is	s.	4

MARK YOUR RESPONSE

3.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

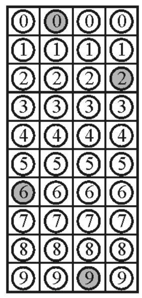
F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.



1. If the equation $\sec\theta + \operatorname{cosec}\theta = c$ has two real roots between 0 and 2π, then the least integer which c^2 cannot exceed is equal to

2. The number of pairs (x, y) satisfying the equation $\sin x + \sin y = \sin(x + y)$ and $|x| + |y| = 1$ is equal to

3. If α be the smallest positive root of the equation $\sqrt{\sin(1-x)} = \sqrt{\cos x}$, then the approximate integral value of α is equal to

4. The number of ordered pairs (x, y) satisfying the equations : $\sin x \cos y = 1$ and $x^2 + y^2 \le 9\pi^2$ is equal to

5. If θ_1, θ_2, θ_3 are three values lying in $[0, 3\pi)$ for which $\tan\theta = \lambda$,

then $\left|\tan\frac{\theta_1}{3}\tan\frac{\theta_2}{3} + \tan\frac{\theta_2}{3}\tan\frac{\theta_3}{3} + \tan\frac{\theta_1}{3}\tan\frac{\theta_3}{3}\right|$ is equal to _____

MARK YOUR RESPONSE

1.	2.	3.	4.	5.
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(d)	**10**	(d)	**19**	(a)	**28**	(c)	**37**	(b)	**46**	(b)
2	(c)	**11**	(d)	**20**	(b)	**29**	(b)	**38**	(d)	**47**	(b)
3	(b)	**12**	(a)	**21**	(a)	**30**	(a)	**39**	(d)	**48**	(d)
4	(b)	**13**	(c)	**22**	(c)	**31**	(a)	**40**	(c)	**49**	(b)
5	(c)	**14**	(d)	**23**	(d)	**32**	(d)	**41**	(a)		
6	(c)	**15**	(a)	**24**	(d)	**33**	(a)	**42**	(d)		
7	(c)	**16**	(b)	**25**	(d)	**34**	(d)	**43**	(c)		
8	(b)	**17**	(a)	**26**	(a)	**35**	(a)	**44**	(d)		
9	(c)	**18**	(c)	**27**	(a)	**36**	(a)	**45**	(b)		

B COMPREHENSION TYPE

1	(d)	**3**	(a)	**5**	(b)	**7**	(b)
2	(d)	**4**	(c)	**6**	(d)		

C REASONING TYPE

1	(d)	**2**	(a)	**3**	(c)	**4**	(b)

D MULTIPLE CORRECT CHOICE TYPE

1	(a,b,c)	**4**	(a,b,d)	**7**	(a,b,c)	**10**	(a, b, c, d)	**13**	(a, b)	**16**	(a, b)
2	(a,c,d)	**5**	(b, c)	**8**	(a,c,d)	**11**	(a, b)	**14**	(a, d)		
3	(a,b,c)	**6**	(a,c,d)	**9**	(a,b,c,d)	**12**	(a, b, c)	**15**	(a,b,c,d)		

E MATRIX-MATCH TYPE

1. A - q; B - s; C - r; D - p, q

2. A - p; B - p, q; C - s; D - r

3. A - r; B - s; C - p; D - q

F NUMERIC/INTEGER ANSWER TYPE

1	8	**2**	6	**3**	6	**4**	13	**5**	3

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(d)** Let $y = 27^{\cos 2x}\ 81^{\sin 2x} = 3^{3\cos 2x + 4\sin 2x}$

Now $-\sqrt{3^2+4^2} \le 3\cos 2x + 4\sin 2x \le \sqrt{3^2+4^2}$

or $-5 \le 3\cos 2x + 4\sin 2x \le 5$

$\therefore\ 3^{-5} \le 3^{3\cos 2x + 4\sin 2x} \le 3^5$

2. **(c)** **Case 1 :** If $0 \le x \le \frac{1}{2}$, then

$$\cos^{-1}\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right) = \cos^{-1}\left\{x.\frac{1}{2}+\sqrt{1-x^2}.\frac{\sqrt{3}}{2}\right\}$$

$$= \cos^{-1}x - \cos^{-1}\frac{1}{2}$$

$\therefore$ Equation is

$$\cos^{-1}x + \cos^{-1}x - \cos^{-1}\frac{1}{2} = \frac{\pi}{3} \Rightarrow x = \frac{1}{2}$$

Case 2 : If $\frac{1}{2} \le x \le 1$, then

$$\cos^{-1}\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right) = \cos^{-1}\frac{1}{2} - \cos^{-1}x$$

$\therefore$ Equation is $\cos^{-1}x + \cos^{-1}\frac{1}{2} - \cos^{-1}x = \frac{\pi}{3}$,

which is identity

Hence the identity holds good for $x \in \left[\frac{1}{2}, 1\right]$.

3. **(b)** $x = \frac{1}{2}$ is clearly a solution of the given equation which can be obtained by trial and error method. The given equation can be written as $3\cos^{-1}x = \pi x + \frac{\pi}{2}$.

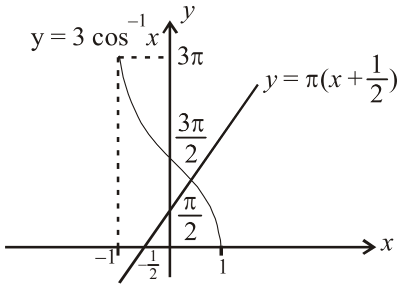
Since the L.H.S. of (1) is a decreasing function and R.H.S. of (1) is an increasing function of x. The equation (1) has either no solution or only one solution. Here, $x = \frac{1}{2}$ is one and only one solution of the given equation.

ALTERNATIVELY

The given equation can be rewritten as

$$3\cos^{-1}x = \pi x + \frac{\pi}{2}$$

From the following graph we find that the curves $y = 3\cos^{-1}x$ and $y = \pi x + \frac{\pi}{2}$ intersect at exactly one point



4. **(b)** The given is

$ax^2 + \sin^{-1}\{(x-1)^2+1\} + \cos^{-1}\{(x-1)^2+1\} = 0$

$\because\ -1 \le (x-1)^2 + 1 \le 1 \Rightarrow x = 1$

So, we have $a + \frac{\pi}{2} = 0 \Rightarrow a = -\frac{\pi}{2}$

5. **(c)** We have $A = 2\tan^{-1}(2\sqrt{2}-1) = 2\tan^{-1}(1.828)$

$$\Rightarrow A > 2\tan^{-1}\sqrt{3} \Rightarrow A > \frac{2\pi}{3}$$

Next

$$\sin^{-1}\left(\frac{1}{3}\right) < \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{6} \Rightarrow \sin^{-1}\frac{1}{3} < \frac{\pi}{2}$$

Also $3\sin^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left[3.\frac{1}{3} - 4\left(\frac{1}{3}\right)^3\right]$

$$= \sin^{-1}\left(\frac{23}{27}\right) = \sin^{-1}(0.852)$$

$$\Rightarrow 3\sin^{-1}\left(\frac{1}{3}\right) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow 3\sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{3}$$

Further

$$\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}(0.6) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{3}$$

Hence, $B = 3\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$.

Hence $A > B$

6. **(c)** $(\sin^{-1} x)^3 + (\cos^{-1} x)^3 = (\sin^{-1} x + \cos^{-1} x)^3 - 3\sin^{-1} x \cos^{-1} x(\sin^{-1} x + \cos^{-1} x)$

$$= \left(\frac{\pi}{2}\right)^3 - 3\sin^{-1} x \cos^{-1} x\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x\right]$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32}$$

$$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1} x - \frac{\pi}{4}\right)^2$$

$\Rightarrow$ the least value is $\frac{\pi^3}{32}$ and since

$$\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \le \left(\frac{3\pi}{4}\right)^2$$

$\Rightarrow$ the greatest value is $\frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$

7. **(c)** Let $\tan^{-1} x = \alpha$ and $\tan^{-1} y = \beta \Rightarrow \tan\alpha = x, \tan\beta = y$

The given system of equations is : $a \tan\alpha + b \sec\alpha = c$ and $a \tan\beta + b \sec\beta = c$

$\therefore$ α and β are roots of $a \tan\theta + b \sec\theta = c$

$$\Rightarrow (b\sec\theta)^2 = (c - a\tan\theta)^2$$

$$\Rightarrow (a^2 - b^2)\tan^2\theta - 2ac\tan\theta + c^2 - b^2 = 0$$

$\therefore \tan\alpha + \tan\beta = \frac{2ac}{a^2 - b^2}$ and

$$\tan\alpha \tan\beta = \frac{c^2 - b^2}{a^2 - b^2}$$

$\therefore x + y = \frac{2ac}{a^2 - b^2}$ and $xy = \frac{c^2 - b^2}{a^2 - b^2}$

$\Rightarrow 1 - xy = \frac{a^2 - c^2}{a^2 - b^2}$ $\qquad \therefore \frac{x+y}{1-xy} = \frac{2ac}{a^2 - c^2}$

8. **(b)** The given relation is possible when

$$a - \frac{a^2}{3} + \frac{a^3}{9} + ... = 1 + b + b^2 + ...$$

Also

$$-1 \le a - \frac{a^2}{3} + \frac{a^3}{9} + ... \le 1 \ \& -1 \le 1 + b + b^2 + ... \le 1$$

$\Rightarrow |b| < 1 \Rightarrow |a| < 3$ and $\frac{a}{1 + \frac{a}{3}} = \frac{1}{1-b}$

$\Rightarrow \frac{3a}{a+3} = \frac{1}{1-b}$, there are infinitely many solution. But in given options it is satisfied only when $a = 1$ and $b = -\frac{1}{3}$.

9. **(c)** Since

$$\sqrt{x^2 - 3x + 2} \ \ge \ 0 \Rightarrow 0 \le \tan^{-1}\sqrt{x^2 - 3x + 2} < \frac{\pi}{2}$$

Since

$$\sqrt{4x - x^2 - 3} \ge 0 \Rightarrow 0 < \cos^{-1}\sqrt{4x - x^2 - 3} \le \frac{\pi}{2}$$

$\Rightarrow 0 <$ L.H.S. $< \pi \Rightarrow$ The given equation has no solution.

10. **(d)** $-\frac{\pi}{2} < \sin^{-1} x \le \frac{\pi}{2}$

$$\therefore \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

$$\Leftrightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2} \Leftrightarrow x = y = z = 1$$

Also $f(p+q) = f(p)\,.\,f(q)\ \forall\ p, q \in \boldsymbol{R}$ ...(1)

Given, $f(1) = 2$

From (1), $f(1+1) = f(1).f(1) \Rightarrow f(2) = 2^2 = 4$ ...(2)

From (2), $f(2+1) = f(2).f(1) = 2^2.2 = 2^3 = 8$

Now given expression $= f(3) = 8$

11. **(d)** $\frac{\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)}(1 + \sin\theta\cos\theta) - \frac{|\sin\theta|\cos\theta}{1} - 2 = -1$

$$\because \sin\theta \ne \cos\theta, \theta \ne \frac{\pi}{4}, \frac{5\pi}{4}, \sin\theta \ne 0, \cos\theta \ne 0$$

$1 + \sin\theta\cos\theta - \cos\theta\,|\sin\theta| = 1$

$\cos\theta\,(\sin\theta - |\sin\theta|) = 0$ $\cos\theta = 0$ or $\sin\theta = |\sin\theta|$

$\Rightarrow \cos\theta = 0$ or $\sin\theta \ge 0$

$$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or } \theta \in (0, \pi) - \left\{\frac{\pi}{4}\right\}$$

but $\theta \ne \frac{n\pi}{2}$

$$\therefore \theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) - \left\{\frac{\pi}{4}\right\}$$

12. **(a)** Let $\sqrt{\tan\alpha} = \tan x$, then

$$u = \cot^{-1}(\tan x) - \tan^{-1}(\tan x) = \frac{\pi}{2} - x - x = \frac{\pi}{2} - 2x$$

$$\Rightarrow 2x = \frac{\pi}{2} - u \Rightarrow x = \frac{\pi}{4} - \frac{u}{2}$$

$$\Rightarrow \tan x = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) \Rightarrow \sqrt{\tan\alpha} = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$$

13. (c) The given inequality is

$$5^x + 12^x \geq 13^x \Rightarrow \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1$$

$$\Rightarrow\ (\cos\alpha)^x + (\sin\alpha)^x \geq 1,\ \text{ where } \cos\alpha = \frac{5}{13}$$

Clearly equality holds if $x = 2$

Further, cos α and sin α are proper fractions, therefore, if $x < 2$ then $(\cos\alpha)^x$ and $(\sin\alpha)^x$ both increase

So, $(\cos\alpha)^x + (\sin\alpha)^x \geq 1$ *if* $x \leq 2 \Rightarrow x \in (-\infty, 2]$.

14. (d) $a^2 - 4a + 5 = (a-2)^2 + 1 \geq 1$

$$\Rightarrow\ \min\left\{\sqrt{2}, a^2 - 4a + 5\right\} \in [1, \sqrt{2}]$$

further $-\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}$

Hence equation has infinitely many solutions for any a.

15. (a) $\log_{\sin x} 2^{\tan x} > 0 \Rightarrow \sin x > 0$ and $\sin x \neq 1$

$$\Rightarrow\ 0 < x < \frac{\pi}{2}$$

Now, $0 < \sin x < 1$ in $\left(0, \frac{\pi}{2}\right)$, then inequality reduces to

$2^{\tan x} < 1$ (not possible)

$\therefore$ No solutions. $\qquad (\because \tan x > 0)$

16. (b) Since the equation

$a_1 + a_2 \sin x + a_3 \cos x + a_4 \sin 2x + a_5 \cos 2x = 0$

holds for all values of x,

$a_1 + a_3 + a_5 = 0$ (on putting $x = 0$)

$a_1 - a_3 + a_5 = 0$ (on putting $x = \pi$)

$\Rightarrow\ a_3 = 0$ and $a_1 + a_5 = 0$ ...(1)

Putting $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$, we get

$a_1 + a_2 - a_5 = 0$ and $a_1 - a_2 - a_5 = 0$ ...(2)

$\Rightarrow a_2 = 0$ and $a_1 - a_5 = 0$ (1) and (2) give

$a_1 = a_2 = a_3 = a_5 = 0$

The given equation reduces to $a_4 \sin 2x = 0$. This is true for all values of x, therefore $a_4 = 0$

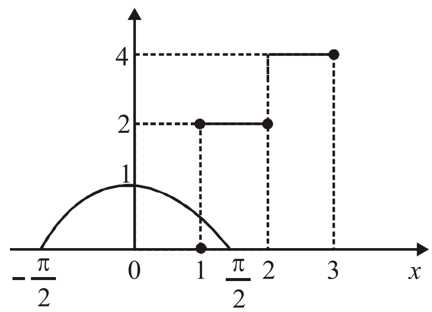
Hence, $a_1 = a_2 = a_3 = a_4 = a_5 = 0$

Thus the number of 5-tuples is one.

17. (a) We have $\frac{|\cos x|}{2} = [x]$ also $\frac{|\cos x|}{2} \in \left[0, \frac{1}{2}\right]$

$\therefore\ [x] = 0 \Rightarrow x \in [0, 1)$

But $\cos x = 0 \Rightarrow x = \frac{\pi}{2} > 1$



$\therefore\ x \in \phi$

ALTERNATIVELY *:* We have $|\cos x| = 2[x]$

If we consider the graphs of $y = |\cos x|$ and $y = 2[x]$, these graphs don't cut each other for any real value of x.

Hence number of solution is zero.

18. (c) $\because$ A.M. $\geq$ G.M. $\therefore\ \frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x}.2^{\cos x}}$

$\therefore\ 2^{\sin x} + 2^{\cos x} \geq 2.\sqrt{2^{\sin x + \cos x}}$. Equality holds if $\sin x = \cos x$ but minimum value of $\cos x + \sin x$ is $-\sqrt{2}$.

$\therefore\ 2^{\sin x} + 2^{\cos x} \geq 2\sqrt{2^{-\sqrt{2}}} = 2^{1-\frac{1}{\sqrt{2}}}$, equality holds if $\sin x = \cos x = -\frac{1}{\sqrt{2}}$

But the given equation is

$2^{\sin x} + 2^{\cos x} = 2^{1-\frac{1}{\sqrt{2}}}$, which can hold only if $2^{\sin x} = 2^{\cos x}$

$$\Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = (2n+1)\pi + \frac{\pi}{4}$$

19. (a) The given equation can be written as

$$\sin^5 x - \cos^5 x = \frac{\sin x - \cos x}{\sin x \cos x}$$

$$\Rightarrow\ \sin x \cos x\left[\frac{\sin^5 x - \cos^5 x}{\sin x - \cos x}\right] = 1$$

$$\Rightarrow \frac{1}{2}\sin 2x\,[\sin^4 x + \sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos^3 x + \cos^4 x] = 1$$

$\Rightarrow\ \sin 2x\,[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x + \sin x \cos x(\sin^2 x + \cos^2 x) + \sin^2 x \cos^2 x] = 2$

$\Rightarrow\ \sin 2x\,[1 - \sin^2 x \cos^2 x + \sin x \cos x] = 2$

$$\Rightarrow\ \sin 2x\left[1 - \frac{1}{4}\sin^2 2x + \frac{1}{2}\sin 2x\right] = 2$$

$\Rightarrow\ \sin^3 2x - 2\sin^2 2x - 4\sin 2x + 8 = 0$

$\Rightarrow\ (\sin 2x - 2)^2(\sin 2x + 2) = 0$

$\Rightarrow\ \sin 2x = \pm 2$, which is not possible for any x.

20. **(b)** $|\tan x + \sec x| = |\tan x| + |\sec x|$ iff sec and tan x both have same sign.

$\Rightarrow$ sec x. tan $x \geq 0 \Rightarrow \dfrac{\sin x}{\cos^2 x} \geq 0$

$\Rightarrow \sin x \geq 0$, but $\cos x \neq 0$ $\Rightarrow x \in \left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$

21. **(a)** We have $\sin x\sqrt{8\cos^2 x} = 1 \Rightarrow \sin x\,|\cos x| = \dfrac{1}{2\sqrt{2}}$

Case I : When $\cos x > 0$

In this case, $\sin x \cos x = \dfrac{1}{2\sqrt{2}} \Rightarrow \sin 2x = \dfrac{1}{\sqrt{2}}$

$\Rightarrow 2x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$

$\Rightarrow x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$. As x lies between 0 and 2π

and $\cos x > 0$, so, $x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}$

Case II : When $\cos x < 0$
In this case,

$\sin x\,|\cos x| = \dfrac{1}{2\sqrt{2}} \Rightarrow \sin x \cos x = -\dfrac{1}{2\sqrt{2}}$

or $\sin 2x = -\dfrac{1}{\sqrt{2}}$

$\Rightarrow x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8} \Rightarrow x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}$
as $\cos x < 0$
Thus the values of x satisfying the given equation which lie between 0 and 2π are $\dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}$.

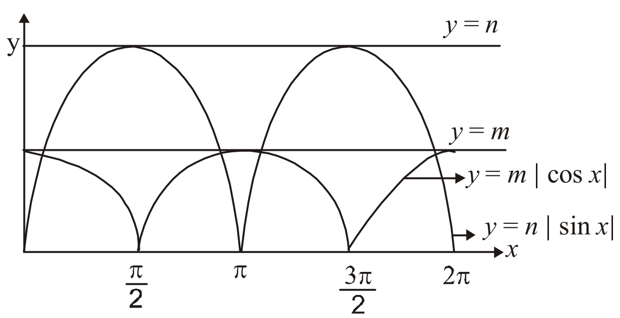
These are in A.P. with common difference $\dfrac{\pi}{4}$.

22. **(c)** $|4\sin x - 1| < \sqrt{5} \Rightarrow -\sqrt{5} < 4\sin x - 1 < \sqrt{5}$

$1 - \sqrt{5} < 4\sin x < 1 + \sqrt{5}$

$\dfrac{1-\sqrt{5}}{4} < \sin x < \dfrac{1+\sqrt{5}}{4} \Rightarrow -\dfrac{\pi}{10} < x < \dfrac{3\pi}{10}$

23. **(d)** See the graph of $y = n\,|\sin x|$ and $y = n\,|\cos x|$



The curves $y = n\,|\sin x|$ and $y = m\,|\cos x|$ intersect at 4 points in $[0, 2\pi]$

24. **(d)** $\therefore\ \dfrac{3\pi}{2} < 5 < \dfrac{5\pi}{2}$ $\qquad \therefore \sin^{-1}(\sin 5) = 5 - 2\pi$

Given $\sin^{-1}(\sin 5) > x^2 - 4x \Rightarrow x^2 - 4x < 5 - 2\pi$

$\Rightarrow x^2 - 4x + (2\pi - 5) < 0$

Roots of $x^2 - 4x + 2\pi - 5 = 0$ are $2 \pm \sqrt{9 - 2\pi}$
$\therefore x^2 - 4x + 2\pi - 5 < 0$

$\Rightarrow 2 - \sqrt{9-2\pi} < x < 2 + \sqrt{9-2\pi}$

25. **(d)** The equation has real roots if :

$\cos^2 p - 4(\cos p - 1)\sin p \geq 0$

or $\cos^2 p - 4\cos p \sin p + 4\sin p \geq 0$

or $(\cos p - 2\sin p)^2 - 4\sin^2 p + 4\sin p \geq 0$

or $(\cos p - 2\sin p)^2 + 4\sin p\,(1 - \sin p) \geq 0$ ...(1)

since $(\cos p - 2\sin p)^2 \geq 0$, $1 - \sin p \geq 0$ for all values of p and for $p \in [0, \pi]$, $\sin p \geq 0$ so that the discriminant is non-negative.

26. **(a)** We have $[\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x] = 1$

$\Rightarrow 1 \leq \sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x \leq \dfrac{\pi}{2}$

$\Rightarrow \sin 1 \leq \cos^{-1}\sin^{-1}\tan^{-1} x \leq 1$

$\Rightarrow \cos\sin 1 \geq \sin^{-1}\tan^{-1} x \geq \cos 1$

$\Rightarrow \sin\cos\sin 1 \geq \tan^{-1} x \geq \sin\cos 1$

$\Rightarrow \tan\sin\cos\sin 1 \geq x \geq \tan\sin\cos 1$.

Hence $x \in [\tan\sin\cos 1, \tan\sin\cos\sin 1]$

27. **(a)** $\cos 3\theta + \sin 3\theta + (2\sin 2\theta - 3)(\sin\theta - \cos\theta)$
$= 4\cos^3\theta - 3\cos\theta + 3\sin\theta - 4\sin^3\theta$
$\quad + 2\sin 2\theta(\sin\theta - \cos\theta) - 3\sin\theta + 3\cos\theta$
$= 4(\cos^3\theta - \sin^3\theta) + 2\sin 2\theta(\sin\theta - \cos\theta)$
$= 4(\cos\theta - \sin\theta)(\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta - \sin\theta\cos\theta)$

$= 4\sqrt{2}\sin\left(\dfrac{\pi}{4} - \theta\right) > 0 \Rightarrow \sin\left(\theta - \dfrac{\pi}{4}\right) < 0$

$\sin\left(\theta - \dfrac{\pi}{4}\right)$ is negative only for

$(2n-1)\pi < \theta - \dfrac{\pi}{4} < 2n\pi$

i.e., $2n\pi - \dfrac{3\pi}{4} < \theta < 2n\pi + \dfrac{\pi}{4}$

$\Rightarrow \theta \in \left(2n\pi - \dfrac{3\pi}{4}, 2n\pi + \dfrac{\pi}{4}\right), n \in \boldsymbol{I}$

Note that

$\left(2n\pi-\frac{\pi}{2}, 2n\pi+\frac{\pi}{6}\right) \subset \left(2n\pi-\frac{3\pi}{4}, 2n\pi+\frac{\pi}{4}\right)$ for all $n \in \mathbf{I}$

Thus, $\left(2n\pi-\frac{3\pi}{4}, 2n\pi+\frac{\pi}{4}\right) \cup \left(2n\pi-\frac{\pi}{2}, 2n\pi+\frac{\pi}{6}\right)$

$$= \left(2n\pi-\frac{3\pi}{4}, 2n\pi+\frac{\pi}{4}\right)$$

28. **(c)** There is only one possibility for the validity of the given equation, i.e.,

$[\sin x] = -1$ *and* $[\sqrt{2}\cos x] = -2 \Rightarrow -1 \le \sin x < 0$

$\Rightarrow x \in (\pi, 2\pi)$

Also $-2 \le \sqrt{2}\cos x < -1$

$\Rightarrow \cos x < -\frac{1}{\sqrt{2}} \Rightarrow x \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ *so* $x \in \left(\pi, \frac{5\pi}{4}\right)$

29. **(b)** $|\cos x| = \cos x - 2\sin x$

$\Rightarrow \cos x = \cos x - 2\sin x$ if $\cos x \ge 0$

$\Rightarrow \sin x = 0 \Rightarrow x = 2n\pi$ (as $\cos x \ge 0$), $n \in \boldsymbol{I}$

Next $|\cos x| = \cos x - 2\sin x$

$\Rightarrow -\cos x = \cos x - 2\sin x$ if $\cos x < 0$

$\Rightarrow \cos x - \sin x = 0 \Rightarrow \tan x = 1$

Now, $\cos x < 0$ and $\tan x = 1 \Rightarrow \tan x = \tan\frac{5\pi}{4}$

$\Rightarrow x = 2n\pi + \left(\frac{5\pi}{4}\right) = (2n+1)\pi + \frac{\pi}{4}$

30. **(a)** $\sin 18^\circ + \cos 18^\circ = \sqrt{2}\sin(45^\circ + 18^\circ) = \sqrt{2}\sin 63^\circ$

Now, $\sin 63^\circ > \sin 45^\circ$

$\Rightarrow \sqrt{2}\sin 63^\circ > 1$. Also, $\sqrt{2}\sin 63^\circ < \sqrt{2}$

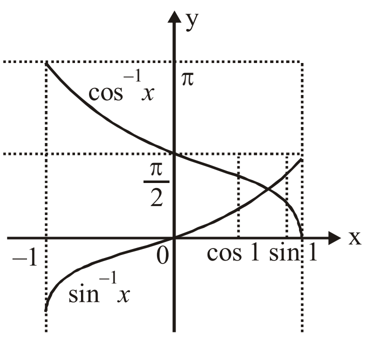
So, $[\sqrt{2}\sin 63^\circ] = 1$. For point of intersection

$\sin\frac{\pi x}{2} = 1 \Rightarrow \frac{\pi x}{2} = (4n+1)\frac{\pi}{2}$

$\Rightarrow x = 4n + 1$, where $n \in \boldsymbol{I}$.

31. **(a)** $[\sin^{-1} x] > [\cos^{-1} x] \Rightarrow x > 0$

Clearly, $[\cos^{-1} x] = \begin{cases} 1, & x \in [0, \cos 1] \\ 0, & x \in [\cos 1, 1] \end{cases}$



$[\sin^{-1} x] = \begin{cases} 0, & x \in [0, \sin 1] \\ 1, & x \in [\sin 1, 1] \end{cases}$

Also, $\sin 1 > \cos 1$

$\therefore$ In $[\sin 1, 1]$, $[\sin^{-1} x] = 1$, $[\cos^{-1} x] = 0$

Hence $[\sin^{-1} x] > [\cos^{-1} x] \Rightarrow x \in [\sin 1, 1]$

32. **(d)** The given equation can be written as

$2x^4 + 2x^3[\cos(A-B) - \cos(A+B)] - 2x^2\cos(A+B)\cos(A-B) + 2x[\cos(A+B) + \cos(A-B)] - 2 = 0$

$\Rightarrow x\cos(A-B)[x^2 - x\cos(A+B) + 1] + x^2[x^2 - x\cos(A+B) + 1] - x^2 + x\cos(A+B) - 1 = 0$

$\Rightarrow [x^2 - x\cos(A+B) + 1][x^2 + x\cos(A-B) - 1] = 0$

$\Rightarrow$ Either $x^2 - x\cos(A+B) + 1 = 0$ ...(1)

or $x^2 + x\cos(A-B) - 1 = 0$ ...(2)

Since the discriminant $\cos^2(A+B) - 4$ of (1) is negative, (1) is not satisfied by any real value of x.

From (2), we get

$$x = \frac{-\cos(A-B) \pm \sqrt{\cos^2(A-B) + 4}}{2}$$

Since α is a positive real root of the given equation

$$\text{so } \alpha = \frac{-\cos(A-B) + \sqrt{\cos^2(A-B) + 4}}{2}$$

33. **(a)** $\sin^4 x + \cos^4 y + 2 = 4\sin x\cos y$

$\Rightarrow (\sin^4 x - 2\sin^2 x + 1) + (\cos^4 y - 2\cos^2 y + 1) + 2\sin^2 x + 2\cos^2 y - 4\sin x\cos y = 0$

$\Rightarrow (\sin^2 x - 1)^2 + (\cos^2 y - 1)^2 + (\sin x - \cos y)^2 = 0$

which holds if $\sin^2 x = 1$ or $\sin x = \pm 1$,

$\cos^2 y = 1$ or $\cos y = \pm 1$

and $\sin x = \cos y$, i.e., $\sin x = \cos y = 1$

or $\sin x = \cos y = -1$

which is not true. Hence the given equation has no solution.

34. **(d)** $x = n\pi - \tan^{-1} 3 \Rightarrow \tan^{-1} 3 = n\pi - x$

$\Rightarrow \tan(n\pi - x) = 3 \Rightarrow -\tan x = 3$

$\Rightarrow \tan 2x = \frac{2\tan x}{1 - \tan^2 x} = \frac{3}{4}$ and

$\cos x = \pm\frac{1}{\sqrt{1 + \tan^2 x}} = \pm\frac{1}{\sqrt{10}}$

on substituting these value in the given equation we find only $\cos x = -\frac{1}{\sqrt{10}}$ satisfies the equation, so that the given equation holds for values of x for which $\tan x = -3$ and $\cos x = -\frac{1}{\sqrt{10}}$ which is possible if x lies in the second quadrant only and so n must be an odd integer.

35. **(a)** $\int_0^x (t^2 - 8t + 13)\, dt = x \sin \frac{a}{x}$

$\Rightarrow \left[\frac{t^3}{3} - \frac{8t^2}{2} + 13t\right]_0^x = x \sin \frac{a}{x}$

$\Rightarrow \frac{x^3}{3} - 4x^2 + 13x = x \sin \frac{a}{x}$

$\Rightarrow \frac{1}{3}[x^2 - 12x + 39] = \sin \frac{a}{x}$

$\left[\because x \neq 0 \text{ as } \sin \frac{a}{x} \text{ is not defined for } x = 0\right]$

$\Rightarrow \frac{1}{3}(x-6)^2 + 1 = \sin \frac{a}{x}$ ...(1)

Since L.H.S. of (1) is ≥ 1 and R.H.S. of (1) ≤ 1

The equation (1) holds if $\frac{1}{3}(x-6)^2 + 1 = 1 = \sin \frac{a}{x}$

$\therefore$ $x = 6$ is solution of (1) if $\sin \frac{a}{6} = 1$.

NOTE : *Commonly students try to differentiate both sides, which must be avoided.*

36. **(a)** For the domain of definition of the above equation, we have

(i) $2 \cos 2x - 1 \neq 0 \Rightarrow x \neq n\pi \pm \frac{\pi}{6}$

(ii) $\tan x \neq 0 \Rightarrow x \neq \pm \frac{n\pi}{2}$ [for odd multiples of $\frac{\pi}{2}$, $\tan x$ is not defined]

(iii) $\cos^2 x - 3 \sin^2 x \neq 0 \Rightarrow x \neq n\pi \pm \frac{\pi}{6}$

Also, $2 \cos 2x - 1 = 2(\cos^2 x - \sin^2 x) - (\cos^2 x + \sin^2 x) = \cos^2 x - 3 \sin^2 x$

Now, the given equation reduces to $b \sin x = b + \sin x \Rightarrow \sin x = \frac{b}{b-1}$ ...(1)

$\therefore$ $-1 \leq \sin x \leq 1 \Rightarrow -1 \leq \frac{b}{b-1} \leq 1$

$\Rightarrow \frac{b}{b-1} + 1 \geq 0$ and $\frac{b}{b-1} - 1 \leq 0$

$\Rightarrow \frac{2b-1}{b-1} \geq 0$ and $\frac{1}{b-1} \leq 0 \Rightarrow b \leq \frac{1}{2}$

or $b > 1$ and $b < 1$ $\Rightarrow b \leq \frac{1}{2}$

when $b = \frac{1}{2}$, $\sin x = 1$, which is not possible

$\therefore$ $b < \frac{1}{2}$.

37. **(b)** $x^2 + \frac{(2(\sin\theta - a) - 1)x}{2} + \left(\frac{1 - 16(\sin\theta + a)}{16}\right) = 0$

$\because$ roots are distinct $\Rightarrow D > 0$

$\Rightarrow \left(\frac{2(\sin\theta - a) - 1}{2}\right)^2 - \left(\frac{1 - 16(\sin\theta + a)}{4}\right) > 0$

or $\frac{4(\sin^2\theta + a^2 - 2a\sin\theta) + 1 - 4(\sin\theta - a)}{4} - \frac{1}{4} + 4(\sin\theta + a) > 0$

or $a^2 + \sin^2\theta - 2a\sin\theta + a - \sin\theta + 4(\sin\theta + a) > 0$.

If $a^2 + (5 - 2\sin\theta)\, a + \sin^2\theta + 3\sin\theta > 0$

$\Rightarrow D < 0$

$\therefore$ $(5 - 2\sin\theta)^2 - 4(\sin^2\theta + 3\sin\theta) < 0$ $\quad \sin\theta > \frac{25}{32}$

$\Rightarrow \quad \theta \in \left(2n\pi + \sin^{-1}\frac{25}{32}.(2n+1)\pi - \sin^{-1}\frac{25}{32}\right).n \in 1$

38. **(d)** The given expression

$= \tan(\sin^{-1}(\cos(\sin^{-1} x))).\tan(\cos^{-1}(\sin(\cos^{-1} x)))$

$= \tan(\sin^{-1}(\cos(\cos^{-1}\sqrt{1-x^2}))).$

$\tan(\cos^{-1}(\sin(\sqrt{1-x^2})))$

$= \tan(\sin^{-1}\sqrt{1-x^2}).\tan(\cos^{-1}\sqrt{1-x^2})$

$= \tan\left(\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right).\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)$

$= \frac{x}{\sqrt{1-x^2}}.\frac{\sqrt{1-x^2}}{x} = 1$

39. **(d)** $T_r = \cot^{-1}\left(\frac{4r^2+3}{4}\right) = \tan^{-1}\frac{1}{1 + r^2 - \frac{1}{4}}$

$= \tan^{-1}\frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + \left(r + \frac{1}{2}\right)\left(r - \frac{1}{2}\right)}$

$= \tan^{-1}\left(r + \frac{1}{2}\right) - \tan^{-1}\left(r - \frac{1}{2}\right)$

$S_n = \sum T_r = \tan^{-1}\left(n + \frac{1}{2}\right) - \tan^{-1}\frac{1}{2} = \tan^{-1}\left(\frac{4n}{2n+5}\right)$

$S_\infty = \lim_{n\to\infty} S_n = \tan^{-1} 2 = \cot^{-1}\frac{1}{2}$.

40. **(c)** We have

$$\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A)$$

$$=\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}\left(\frac{\cot A+\cot^3 A}{1-\cot^4 A}\right)+\pi$$

$(\because \cot A > 1)$

$$=\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)+\pi+\tan^{-1}\left(\frac{\cot A}{1-\cot^2 A}\right)$$

$$=\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)+\pi-\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)$$

$$=\pi=4.\frac{\pi}{4}=4\tan^{-1}(1)$$

41. **(a)** Since $(\alpha+\pi)$ and $(\beta-\pi)$ are two solutions of the given equation, so $\tan\left(\frac{\alpha+\pi}{2}\right)$ and $\tan\left(\frac{\beta-\pi}{2}\right)$ are the roots of the equation. It means that $-\cot\frac{\alpha}{2}$ and $-\cot\frac{\beta}{2}$ are the roots of the equation

$$1-\frac{2b}{c-a}\cot\frac{\theta}{2}+\frac{c+a}{c-a}\cot^2\frac{\theta}{2}=0$$

$\Rightarrow \cot\frac{\alpha}{2},\cot\frac{\beta}{2}$ are the roots of the equation

$$(c-a)+2b\cot\frac{\theta}{2}+(c+a)\cot^2\frac{\theta}{2}=0$$

$\Rightarrow \alpha, \beta$ are the solutions of the equation

$$(c-a)\sin^2\frac{\theta}{2}+2b\sin\frac{\theta}{2}\cos\frac{\theta}{2}+(c+a)\cos^2\frac{\theta}{2}=0$$

$\Rightarrow \alpha, \beta$ are the solutions of the equation

$a\cos\theta-b\sin\theta-c=0$

42. **(d)** The given equation can be written as

$\sin x\cos x\,[\sin^2 x+\sin x\cos x+\cos^2 x]=1$

or $\sin x\cos x\,[1+\sin x\cos x]=1$

or $\sin 2x\,[2+\sin 2x]=4$

$$\Rightarrow \sin 2x=\frac{-2\pm\sqrt{4+16}}{2}=-1\pm\sqrt{5}$$

which is not possible.

43. **(c)** Here $A-B=\tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)-\tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$

$$=\tan^{-1}\left[\frac{3xk-4xk+2x^2+2k^2-xk}{\sqrt{3}\left(2k^2-xk+2x^2-xk\right)}\right]$$

$$=\tan^{-1}\left[\frac{2k^2+2x^2-2xk}{\sqrt{3}(2k^2+2x^2-2xk)}\right]=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}.$$

44. **(d)** The given equation can be rewritten as

$$\frac{4\sin^3\theta}{1+\cos\theta}+\frac{4\cos^3\theta}{1-\sin\theta}=4(\cos\theta-\sin\theta)$$

or $4\sin\theta\,(1-\cos\theta)+4\cos\theta\,(1+\sin\theta)$

$=4\cos\theta-4\sin\theta$

or $\sin\theta=0\Rightarrow\theta=n\pi$.

For $\theta=(2m+1)\pi,\ 1+\cos\theta=0$.

which is not possible. Hence $\theta=2n\pi$.

45. **(b)** Let $f(x)=\sin^{-1}x-\tan^{-1}x,\ |x|\le 1$

$$\Rightarrow f'(x)=\frac{1}{\sqrt{1-x^2}}-\frac{1}{1+x^2}>0\ \forall x\in(-1,1)$$

$$f(-1)=\sin^{-1}(-1)-\tan^{-1}(-1)=-\frac{\pi}{2}+\frac{\pi}{4}=-\frac{\pi}{4}$$

$$f(1)=\sin^{-1}(1)-\tan^{-1}(1)=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}.$$

46. **(b)** RHS ≤ 1 and LHS ≥ 1. Solution exists if and only if

$\sin^4 2x=0$ and $\cos^2 6x=1\Rightarrow\ x=\frac{n\pi}{2},n\in 1$

47. **(b)** $1+\cos 3x+1-\left[\cos\left(\frac{\pi}{2}+\left(2x-\frac{7\pi}{6}\right)\right)\right]=0$

$$2\cos^2\frac{3x}{2}+1-\cos\left(2x-\frac{2\pi}{3}\right)=0$$

$$2\cos^2\frac{3x}{2}+2\sin^2\left(x-\frac{\pi}{3}\right)=0$$

$$\cos\frac{3x}{2}=0,\ \sin\left(x-\frac{\pi}{3}\right)=0$$

$$x=\frac{\pi}{3},\pi \text{ and } x=\frac{\pi}{3},\frac{2\pi}{3},\frac{7\pi}{3},......$$

$\therefore\ x=\frac{\pi}{3}$ is the common value which satisfies both

$$\therefore\ x=2n\pi+\frac{\pi}{3}=(6n+1)\frac{\pi}{3}.$$

48. **(d)** $-\frac{\pi}{2}\le 2\tan^{-1}2x\le\frac{\pi}{2}\Rightarrow-\frac{\pi}{4}\le\tan^{-1}2x\le\frac{\pi}{4}$

$\Rightarrow\ -1\le 2x\le 1\qquad\Rightarrow -\frac{1}{2}\le x\le\frac{1}{2}.$

49. **(b)** $\cos^{-1}\left(\frac{1+x^2}{2x}\right)=\frac{\pi}{2}+(\sin^{-1}x+\cos^{-1}x)$

$$\Rightarrow\ \cos^{-1}\left(\frac{1+x^2}{2x}\right)=\pi.$$

Also, $1+x^2\ge 2x$ if $x>0$ and $1+x^2\ge -2x$ if $x<0$

$\Rightarrow\ \left|\frac{1+x^2}{2x}\right|\ge 1\Rightarrow\frac{1+x^2}{2x}$ can take values -1 or 1

out of these only $x=-1$ satisfies the equation.

B COMPREHENSION TYPE

1. **(d)** $\cos^7 x \le \cos^2 x$ and $\sin^4 x \le \sin^2 x$

$\Rightarrow \cos^7 x + \sin^4 x \le \cos^2 x + \sin^2 x = 1$

So, the given equation is satisfied if and only if

$\cos^7 x = 0$ and $\sin^2 x = 1$ or $\cos^7 x = 1$ and $\sin^2 x = 0$

$\Leftrightarrow x = (2n+1)\frac{\pi}{2}$ or $x = 2m\pi$

$\because 0 \le x \le 2\pi$, so $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

2. **(d)** $\cos(p\sin x) = \sin(p\cos x) = \cos\left(\frac{\pi}{2} - p\cos x\right)$

$\Rightarrow p\sin x = 2n\pi \pm \left(\frac{\pi}{2} - p\cos x\right), n \in I$

Consider the positive sign, we have

$p(\sin x + \cos x) = (4n+1)\frac{\pi}{2}$

$\Rightarrow \sin x + \cos x = \frac{(4n+1)\pi}{2p}$

$\because |\cos x + \sin x| \le \sqrt{2} \Rightarrow \left|\frac{(4n+1)\pi}{2p}\right| \le \sqrt{2}$

or $p \ge \frac{|(4n+1)|\pi}{2\sqrt{2}}$ (for positive values of p)

p is smallest positive if $n = 0$, giving $p = \frac{\pi}{2\sqrt{2}}$

Now consider the negative sign, we have

$p(\sin x - \cos x) = \frac{(4n-1)\pi}{2}$,. So, $\left|\frac{(4n-1)\pi}{2p}\right| \le \sqrt{2}$

or $p \ge \frac{|4n-1|}{2\sqrt{2}}$

p is smallest positive if $n = 0$, giving again $p = \frac{\pi}{2\sqrt{2}}$

3. **(a)** The equation is $a^2 - 2a + \sec^2 \pi(a + x) = 0$

$\Rightarrow a^2 - 2a + 1 - 1 + \sec^2 \pi(a+x) = 0$

$\Rightarrow (a-1)^2 + \tan^2 \pi(a+x) = 0$

Above equation holds if and only if

$a - 1 = 0$ and $\tan \pi(a+x) = 0 \Rightarrow a = 1$ and $\pi(a+x) = n\pi$

i.e., $a = 1$ and $x = (n-1)\pi = m\pi$, $m \in I$

4. **(c)** The equation is $\sin x + 2\cos x \cos k = 2$, since,

$-\sqrt{1+4\cos^2 k} \le \text{LHS} \le \sqrt{1+4\cos^2 k}$

So, for real solutions

$\sqrt{1+4\cos^2 k} \ge 2 \Rightarrow \cos^2 k \ge \frac{3}{4} \Rightarrow \sin^2 k \le \frac{1}{4}$

5. **(b)** Extreme value of $\sin^{-1} x = \frac{5\pi}{2}$ and $\cos^{-1} y = 3\pi$

$\Rightarrow \sin^{-1}x + \cos^{-1}y \le \frac{11\pi}{2}$

$\therefore$ Only 1 possible ordered pairs $(1, -1)$.

6. **(d)** Clearly we need $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

The value of θ from the interval $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ for which

$\sin\theta = -\frac{1}{\sqrt{2}}$ is $\frac{7\pi}{4}$.

7. **(b)** We have $2\theta = \sin^{-1}(\sin 2\theta)$ [let $\sin^{-1} x = \theta$].

$\Rightarrow \frac{3\pi}{2} \le 2\theta \le \frac{5\pi}{2} \Rightarrow \frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}$

$\Rightarrow -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$.

C REASONING TYPE

1. **(d)** If $x < 0$ $\tan^{-1}\left(\frac{1}{x}\right) = -\pi + \cot^{-1} x$

$\tan^{-1} x + \tan^{-1}\frac{1}{x} = \tan^{-1} x - \pi + \cot^{-1} x \;= -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$

A is false R is true.

2. **(a)** $\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right)$

$= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{1-\frac{x}{y}}{1+\frac{x}{y}}\right)$

$= \tan^{-1}\left(\frac{x}{y}\right) + \frac{\pi}{4} - \tan^{-1}\left(\frac{x}{y}\right) = \left(\frac{\pi}{4}\right)$

Taking $\frac{x}{y} = \frac{3}{4}$, we get $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$

3. **(c)** $e^{-\pi/2} < \theta < \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} < \log\theta < \log\frac{\pi}{2} < \frac{\pi}{2}$

$\therefore \cos(\log\theta) > 0$

However, $0 < \cos\theta < 1 \Rightarrow \log(\cos\theta) < 0$

$\therefore \cos(\log\theta) > \log\cos\theta$ if $e^{-\pi/2} < \theta < \frac{\pi}{2}$

Clearly $\cos(\log\theta)$ can be negative if

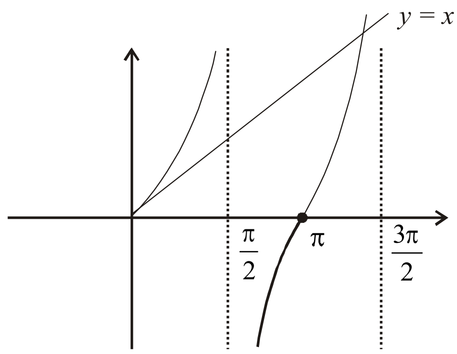
$\log\theta < \frac{\pi}{2} \Rightarrow \theta > e^{\pi/2}$

4. **(b)** $y = \tan x \Rightarrow \left(\frac{dy}{dx}\right)_{(0,0)} = \sec^2 0 = 1$

$\therefore$ $y = x$ is a tangent to $y = \tan x$ at (0, 0)

From the graph it is clear than $y = x$ intersects

$y = \tan x$ in $\left(\pi, \frac{3\pi}{2}\right)$



D MULTIPLE CORRECT CHOICE TYPE

1. **(a,b,c)** If we put $x = \tan\theta$, the given equality becomes $\tan^{-1} y = 4\theta$

$\Rightarrow y = \tan 4\theta = \frac{2\tan 2\theta}{1-\tan^2 2\theta} = \frac{2\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$

$= \frac{2\times 2x(1-x^2)}{(1-x^2)^2 - 4x^2} = \frac{4x(1-x^2)}{1-6x^2+x^4}$

so that y is finite if $x^4 - 6x^2 + 1 \neq 0$

$\Rightarrow x^2 \neq \frac{6 \pm \sqrt{36-4}}{2} \neq 3 \pm 2\sqrt{2}$

2. **(a,c,d)** $\cos^2(\pi x) - \sin^2(\pi y) = \frac{1}{2}$

$\Rightarrow \cos\pi(x+y)\cos\pi(x-y) = \frac{1}{2}$

$\Rightarrow \cos\pi(x+y)\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ $\quad\left[\because x - y = \frac{1}{3}\right]$

$\Rightarrow \cos\pi(x+y) = 1$

$\Rightarrow \pi(x+y) = 2n\pi \Rightarrow x + y = 2n$

Now, $x + y = 2n$ and $x - y = \frac{1}{3}$

$\Rightarrow x = n + \frac{1}{6},\; y = n - \frac{1}{6}\; (n \in \boldsymbol{I})$

$\therefore (x, y) = \left(n + \frac{1}{6}, n - \frac{1}{6}\right)$

(a), (c), (d) correspond $n = 1, -1, 2$ respectively and hence are correct solutions.

But (b) corresponds to $n = \frac{1}{2} \notin \boldsymbol{I}$.

So (b) is not the correct solution.

3. **(a,b,c)** The given equation can be written as

$$\sqrt{\cos^2 x - \sin^2 x} + \sqrt{(\cos x + \sin x)^2} = 2\sqrt{\cos x + \sin x}$$

$$\Rightarrow \sqrt{\cos x + \sin x}[\sqrt{\cos x - \sin x} + \sqrt{\cos x + \sin x}] = 2\sqrt{\cos x + \sin x}$$

$\Rightarrow$ Either $\cos x + \sin x = 0 \Rightarrow \tan x = -1$

$\Rightarrow x = n\pi - \dfrac{\pi}{4}$ $(n \in \boldsymbol{I})$ $\Rightarrow$ (a) and (c) are correct

or $\sqrt{\cos x - \sin x} + \sqrt{\cos x + \sin x} = 2$

$\Rightarrow 2\cos x + 2\sqrt{\cos^2 x - \sin^2 x} = 4$

$\Rightarrow \cos^2 x - \sin^2 x = (2 - \cos x)^2$

$\Rightarrow \cos^2 x + 4\cos x - 5 = 0$

$\Rightarrow \cos x = \dfrac{-4 \pm \sqrt{16+20}}{2} = -5$ or 1

But $\cos x \neq -5$ so $\cos x = 1$

$\Rightarrow x = 2n\pi$ and thus (b) is correct.

4. **(a,b,d)** $y = \sin^4 + \cos^4 x - \sin 2x + \dfrac{3}{4}\sin^2 2x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x - \sin 2x + \dfrac{3}{4}\sin^2 2x$

$= 1 - \dfrac{1}{2}\sin^2 2x - \sin 2x + \dfrac{3}{4}\sin^2 2x$

$= 1 + \dfrac{1}{4}\sin^2 2x - \sin 2x$

$= \dfrac{1}{4}(\sin^2 2x - 4\sin 2x + 4) = \dfrac{1}{4}(\sin 2x - 2)^2$

since $\sin 2x \neq 2$, $y \neq 0$ for any value of x and $y = 1$ if $\sin 2x = 0$

$\Rightarrow y = 1$ if $x = \dfrac{n\pi}{2}$ for any integer n.

So, (a), (b) and (d) are correct solutions.

5. **(b, c)** $\cos(x - y) - 2\sin x + 2\sin y = 3$

$\Rightarrow 1 - 2\sin^2\dfrac{x-y}{2} - 4\sin\dfrac{x-y}{2}\cos\dfrac{x+y}{2} - 3 = 0$

$\Rightarrow \sin^2\dfrac{x-y}{2} + 2\sin\dfrac{x-y}{2}\cos\dfrac{x+y}{2} + 1 = 0$

$$\Rightarrow \sin\frac{x-y}{2} = \frac{-2 \pm \sqrt{4\cos^2\frac{x+y}{2} - 4}}{2}$$

For real values of $\sin\dfrac{x-y}{2}$, we have

$\cos^2\dfrac{x+y}{2} = 1$

$\Rightarrow \sin^2\dfrac{x+y}{2} = 0$ or $\sin\dfrac{x+y}{2} = 0 \Rightarrow x + y = 2n\pi$

and then $\sin\dfrac{x-y}{2} = -1$, $\Rightarrow x - y = (4k+1)\pi$.

If $\sin x = \sin y$ then the given equation becomes $\cos(x - y) = 3$ which is not correct; (a) is not correct.

Next if $x = 2k\pi - \dfrac{\pi}{2}$ and $y = 2n\pi + \dfrac{\pi}{2}$.

Then, $\cos(x - y) - 2\sin x + 2\sin y = -1 + 2 + 2 = 3$ so (c) is correct finally if $\cos(x - y) = -1$, the given equation becomes $\sin x - \sin y = -2$ which is not true for all values of x and y so (d) is not correct.

6. **(a,c,d)** The given equation holds if $x^2 + x + 1 = ax + 1$ and

$-1 \le x^2 + x + 1 \le 1$

$\Rightarrow x(x + 1 + a) = 0$ and $-1 \le x \le 0$

$\Rightarrow x = 0$ or $a - 1$ and $-1 \le x \le 0$

$\therefore x = 0$ is one solution and for another different solution $-1 \le a - 1 < 0$

$\Rightarrow 0 \le a < 1$. So only integral value a can have is 0.

7. **(a,b,c)** The solution of $y = \sqrt{y}$ is $y = 0$ or $y = 1$

If $\sin^{-1}|\sin x| = 1 \Rightarrow x = 1$ or $\pi - 1$ (in the interval $(0, \pi)$)

But $y = \sin^{-1}|\sin x|$ is periodic with period π, so $x = n\pi + 1$ or $n\pi - 1$

Again if $\sin^{-1}|\sin x| = 0 \Rightarrow x = n\pi$

8. **(a,c,d)** The equation holds if

$\sin^2\theta + 2\sin\theta + 2 = 4^{\sec^2\phi} + 1$

Now LHS $= (\sin\theta + 1)^2 + 1 \le 5$ and RHS ≥ 5

$(\because \sec^2\phi \ge 1)$

So, LHS = RHS $\Rightarrow \sin\theta = -1$ and $\sec^2\phi = 1$

9. **(a, b, c, d)**

$\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$ ...(1)

$\Rightarrow \dfrac{1 + \tan\theta}{1 - \tan\theta} = \tan\alpha$

$\therefore \sin 2\alpha = \dfrac{2\tan\alpha}{1 + \tan^2\alpha}$

$$= \frac{2\left(\frac{1+\tan\theta}{1-\tan\theta}\right)}{1 + \left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2} = \frac{2(1-\tan^2\theta)}{2(1+\tan^2\theta)} = \cos 2\theta$$

$\cos 2\alpha = \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$

$$= \frac{1-\left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2}{1+\left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2} = \frac{-4\tan\theta}{2(1+\tan^2\theta)}$$

$\therefore\ \cos 2\alpha = -\cos 2\theta \Rightarrow \cos 2\alpha + \cos 2\theta = 0$

Again, squaring (1), we get

$\tan^2\theta(\sin\alpha+\cos\alpha)^2$

$= (\sin\alpha - \cos\alpha)^2$

$= (\sin\alpha+\cos\alpha)^2 - 4\sin\alpha\cos\alpha$

$\Rightarrow\ (\tan^2\theta - 1)(\sin\alpha+\cos\alpha)^2$

$= -2\sin 2\alpha = -2\cos 2\theta$

$\Rightarrow\ (\tan^2\theta-1)(\sin\alpha+\cos\alpha)^2 = -2\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow\ (\sin\alpha+\cos\alpha)^2 = 2\cos^2\theta$

$\Rightarrow\ \sin\alpha+\cos\alpha = \pm\sqrt{2}\cos\theta$

Similarly $\sin\alpha - \cos\alpha = \pm\sqrt{2}\sin\theta$

10. **(a, b, c, d)** $2\sin\frac{x}{2}\cos^2 x - 2\sin\frac{x}{2}\sin^2 x$

$= \cos^2 x - \sin^2 x$

$\Rightarrow \left(2\sin\frac{x}{2}-1\right)\left(\cos^2 x - \sin^2 x\right) = 0$

Either

$\sin\frac{x}{2} = \frac{1}{2} \Rightarrow \cos x = 1 - 2\sin^2\frac{x}{2} = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore\ \cos 2x = 2\cos^2 x - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

or $\cos^2 x - \sin^2 x = 0 \Rightarrow \cos 2x = 0$

$\therefore\ \sin 2x = \pm 1$

11. **(a, b)** $\cos(\pi\sin\theta) = \sin(\pi\cos\theta) = \cos\left(\frac{\pi}{2} - \pi\cos\theta\right)$

$\Rightarrow\ \pi\sin\theta = 2n\pi \pm \left(\frac{\pi}{2} - \pi\cos\theta\right)$

$\Rightarrow\ \cos\theta + \sin\theta = 2n + \frac{1}{2}$

or $\sin\theta - \cos\theta = 2n - \frac{1}{2}$

$\because \sqrt{2} \le \sin\theta \pm \cos\theta \le \sqrt{2}$

$\therefore\ \cos\theta + \sin\theta = \frac{1}{2}$ or $\sin\theta - \cos\theta = -\frac{1}{2}$

Squaring above we get $\sin 2\theta = -\frac{3}{4}$ or $\frac{3}{4}$

12. **(a, b, c)** We have $2^{1+|\sin x|+|\sin 2x|+|\sin 3x|+\ldots} = 2$.

Hence $1 + |\sin x| + |\sin 2x| + |\sin 3x| + \ldots = 1$

$\Rightarrow |\sin x| = 0 = |\sin 2x| = |\sin 3x| = \ldots$

$\Rightarrow x = 0$ or $x = \pi$ or $x = 2\pi$.

13. **(a, b)** The given equation can be written as

$(\sqrt{3})^{\sec^2\theta} = 2\tan^2\theta + \tan^4\theta = \tan^2\theta\,(2 + \tan^2\theta)$

$= (\sec^2\theta - 1)(\sec^2\theta + 1) = \sec^4\theta - 1 \ge 0$

$\Rightarrow \sec^2\theta = 2$

or $\cos^2\theta = \frac{1}{2} \Rightarrow\ \theta = \pm\frac{\pi}{4}$

14. **(a, d)** $\left(\cos^2 x + \frac{1}{\cos^2 x}\right)\left(1+\tan^2 2y\right)\left(3 + \sin 3z\right) = 4$

Since, $\cos^2 x + \frac{1}{\cos^2 x} \ge 2,\ 1 + \tan^2 2y \ge 1,$

$2 \le 3 + \sin 3z \le 4$

So, the only possibility is

$\cos^2 x + \frac{1}{\cos^2 x} = 2,\ 1 + \tan^2 2y = 1,$

$3 + \sin 3z = 2$

$\Rightarrow \cos x = \pm 1 \Rightarrow x = n\pi.$

$\tan 2y = 0 \Rightarrow\ y = \frac{m\pi}{2}$

$\sin 3z = -1 \Rightarrow z = (4k-1)\,\frac{\pi}{6}$; $m, n, k \in 1$

15. **(a,b,c,d)** Rewriting the given equation we get

$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = 1$ or $\tan 3\theta = \tan\frac{\pi}{4}$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{4} \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}, n \in 1$

16. **(a, b)** The given equation can be written as

$1 + \cos px + 1 + \cos qx = 2$ leading to

$\cos\left(\frac{p+q}{2}\right)x\cos\left(\frac{p-q}{2}\right)x = 0$

$\cos\left(\frac{p+q}{2}\right)x = 0$ gives the set

$x = \frac{(2n+1)\pi}{p+q}, n = 0, \pm 1, \pm 2\ldots.$ which forms an A.P.

with common difference $= \frac{2\pi}{p+q}$.

$\cos\left(\frac{p-q}{2}\right)x = 0$ gives the set

$x = \frac{(2n+1)\pi}{p-q}, n = 0, \pm 1, \pm 2\ldots.$ which forms an A.P.

with common difference $= \frac{2\pi}{p-q}$.

E — MATRIX-MATCH TYPE

1. A - q; B - s; C - r; D - p, q

(A) The given equation can be written as

$1 - 2\sin^2 x + a\sin x = 2a - 7$

$\Rightarrow\ 2\sin^2 x - a\sin x + 2a - 8 = 0$

$$\Rightarrow\ \sin x = \frac{a \pm \sqrt{a^2 - 8(2a-8)}}{4} = \frac{a \pm (a-8)}{4}$$

$$\Rightarrow\ \sin x = \frac{a-4}{2} \text{ which is possible if}$$

$$-1 \le \frac{a-4}{2} \le 1 \ \ or\ 2 \le a \le 6.$$

(B) $7\cos x + 5\sin x = 2k + 1 \ \Rightarrow \sin(x+\alpha) = \dfrac{2k+1}{\sqrt{74}}$,

where $\sin\alpha = \dfrac{7}{\sqrt{74}}$ and $\cos\alpha = \dfrac{5}{\sqrt{74}}$.

So, $-1 \le \dfrac{2k+1}{\sqrt{74}} \le 1 \ \Rightarrow\ \dfrac{-\sqrt{74}-1}{2} \le k \le \dfrac{\sqrt{74}-1}{2}$

(C) Let $t = \cos^2 x\ \Rightarrow\ t \in [0, 1]$ and equation is

$t^2 + at + 1 = 0$.

This quadratic has atleast one root in [0, 1] if

$a^2 - 4 \ge 0$ and $1 + a + 1 \le 0\ \Rightarrow a \le -2$

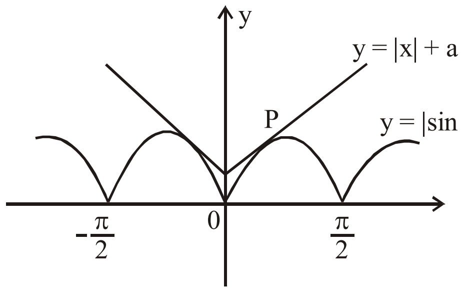
(D) The equation is $|\sin 2x| = |x| + a$. In the limiting situation the lines $y = |x| + a$ must be tangents to $y = |\sin 2x|$.

At the point of contact P,

$$\frac{d}{dx}(\sin 2x) = 1 \Rightarrow 2\cos 2x = 1\ \Rightarrow x = \frac{\pi}{6}.$$

So, P is $\left(\dfrac{\pi}{6}, \dfrac{\sqrt{3}}{2}\right)$.

The point also lies on $y = x + a\ \Rightarrow a = \dfrac{3\sqrt{3}-\pi}{6}$



$\therefore$ The lines neither cut nor touch the curve if $a > \dfrac{3\sqrt{3}-\pi}{6}$.

2. A - p; B - p, q; C - s; D - r

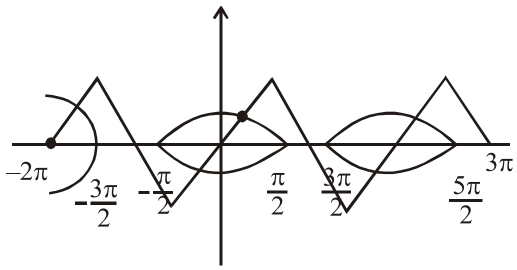
(A) $x^2 + 4 + 3\sin(ax+b) - 2x = 0 \Rightarrow (x-1)^2 + 3\{1 + \sin(ax+b)\} = 0$

The above equation holds if and only if $x = 1$ and $\sin(ax+b) = -1 \Rightarrow \sin(a+b) = -1$

(B) $\cos^{-1} x \ge \sin^{-1} x\ \Rightarrow\ \dfrac{\pi}{2} - \sin^{-1} x \ge \sin^{-1} x$

or $\sin^{-1} x \le \dfrac{\pi}{4} \Rightarrow -1 \le x \le \dfrac{1}{\sqrt{2}}$

(C) The graphs of $|y| = \cos x$ and $y = \sin^{-1}(\sin x)$ intersect at five points in $[-2\pi, 3\pi]$.



(D) Adding we get, $\left(\sin^{-1} y\right)^2 = \dfrac{4n+1}{32}\pi^2$

$$\Rightarrow\ 0 \le \frac{4n+1}{32}\pi^2 \le \frac{\pi^2}{4}\ \Rightarrow -\frac{1}{4} \le n \le \frac{7}{4}$$

Also, $\cos^{-1} x = \dfrac{4n-1}{32}\pi^2$

$$\Rightarrow\ 0 \le \frac{4n-1}{32}\pi^2 \le \pi\ \Rightarrow \frac{1}{4} \le n \le \frac{8}{\pi} + 1$$

Hence, $n = 1$

3. A - r; B - s; C - p; D - q

(A) $9^3 27^{\cot 2x}.81^{\sin 2x} = 3^6.3^{3\cos 2x}.3^{4\sin 2x}$

$= 3^{6+3\cos 2x+4\sin 2x}$ is minimum if

$6 + 3\cos 2x + 4\sin 2x$ is minimum.

Also $6 - 5 \le 6 + 3\cos 2x + 4\sin 2x \le 6 + 5$

$\Rightarrow 6 + 3\cos 2x + 4\sin 2x \ge 1$

$\Rightarrow (A) \Rightarrow$ (r)

(B) The given equation is $\cos^7 x = 1 - \sin^4 x$
$= (1 - \sin^2 x)(1 + \sin^2 x) = \cos^2 x(1 + \sin^2 x)$

so that either $\cos^2 x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

or $\cos^5 x = 1 + \sin^2 x$
$\Rightarrow \sin^2 x = 0, \cos^5 x = 1$ simultaneously
$\Rightarrow x = 0, 2\pi$.
Hence $(B) \Rightarrow$ (s).

(C) Here $a^2 - 2a + \sec^2\pi(a + x) = 0$
or $\sec^2\pi(a + x) = 2a - a^2 \geq 1$
or $0 \geq (a - 1)^2 \Rightarrow a = 1$.
Hence (C) $\Rightarrow$ (p)

(D) Here $\cos(p \sin x) = \sin(p \cos x) = \cos\left(\frac{\pi}{2} - p\cos x\right)$

$\Rightarrow p \sin x = \frac{\pi}{2} - p\cos x$

or $p(\sin x + \cos x) = \frac{\pi}{2}$

or $(\sin x + \cos x) = \frac{\pi}{2p} \leq \sqrt{2} \Rightarrow 2 \leq \frac{4\sqrt{2}p}{\pi}$

Hence (D) $\Rightarrow$ (q).

F NUMERIC/INTEGER ANSWER TYPE

1. **Ans : 8**

Given $\sec\theta + \operatorname{cosec}\theta = c \Rightarrow \sqrt{1+\tan^2\theta} + \sqrt{1+\cot^2\theta} = c$

$\Rightarrow \sqrt{1+t^2} + \sqrt{1+\frac{1}{t^2}} = c$, where $\tan\theta = t$

$\Rightarrow \sqrt{1+t^2}\left(\frac{1+t}{t}\right) = c \Rightarrow (t^2+t+1)^2 = (c^2+1)t^2$

$\Rightarrow t^2 + t + 1 \mp t\sqrt{c^2+1} = 0$

$\Rightarrow t^2 + t + 1 + t\sqrt{c^2+1} = 0$ or $t^2 + t + 1 - t\sqrt{c^2+1} = 0$

Discriminant of first equation, $D_1 = (1+\sqrt{c^2+1})^2 - 4$

Discriminant of second equation, $D_2 = (1-\sqrt{c^2+1})^2 - 4$

Now, $D_1 = 1 + 2\sqrt{c^2+1} + 1 - 4 = 2(\sqrt{c^2+1} - 2) > 0$

$\therefore$ The equation always has two real roots. It will have only two roots provided

$D_2 < 0 \Rightarrow 1 + c^2 + 1 - 2\sqrt{c^2+1} - 4 < 0$

$\Rightarrow c^2 - 2 < 2\sqrt{c^2+1}$. It is always true if $c^2 \leq 2$. If $c^2 > 2$, then on squaring we get

$c^4 - 4c^2 + 4 < 4c^2 + 4 \Rightarrow c^2(c^2 - 8) < 0 \Rightarrow c^2 < 8$

So, the equation has two and only two roots if $c^2 < 8$.
Alternate solution

2. **Ans : 6**
From the given equation we have,

$2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = 2\sin\frac{x+y}{2}\cos\frac{x+y}{2}$

$\Rightarrow \sin\frac{x+y}{2}\left[\cos\frac{x-y}{2} - \cos\frac{x+y}{2}\right] = 0$

$\Rightarrow \sin\frac{x+y}{2} \times 2\sin\frac{x}{2}\sin\frac{y}{2} = 0$

which holds if either $\sin\frac{x+y}{2} = 0$ or $\sin\frac{x}{2} = 0$

or $\sin\frac{y}{2} = 0$

Also since $|x| + |y| = 1 \Rightarrow |x| \leq 1, |y| \leq 1$. So, the required solution is
Either $x + y = 0$ or $x = 0$ or $y = 0$
if $x = 0, y = \pm 1$ and if $y = 0, x = \pm 1$

and if $x + y = 0 \Rightarrow y = -x \Rightarrow |y| = |-x| = |x| = \frac{1}{2}$

so that the required pairs (x, y) are

$(0, \pm 1), (\pm 1, 0), \left(\frac{1}{2}, -\frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right)$ which are 6 in number.

3. **Ans : 6**
The given equation is possible if $\sin(1 - x) \geq 0$ and $\cos x \geq 0$. From the given equation, on squaring both the sides we get

$\sin(1-x) = \cos x = \sin\left(\frac{\pi}{2} - x\right)$

$\Rightarrow 1 - x = n\pi + (-1)^n\left(\frac{\pi}{2} - x\right)$ where $(n \in I)$

But for $n = 2m$, $(m \in I)$ we get no value of x.

$\therefore 1 - x = (2m+1)\pi - \frac{\pi}{2} + x \Rightarrow x = \frac{1}{2} - \frac{4m+1}{4}\pi$

Now, $m \geq 0 \Rightarrow x < 0$;

If $m = -1 \Rightarrow x = \frac{1}{2} + \frac{3\pi}{4}$

$$\Rightarrow \sin(1-x)=\sin\left(\frac{1}{2}-\frac{3\pi}{4}\right)=-\sin\left(\frac{1}{2}+\frac{\pi}{4}\right)<0$$

Hence rejected

If $m=-2 \Rightarrow x=\frac{1}{2}+\frac{7\pi}{4}$

$$\Rightarrow \sin(1-x)=\sin\left(\frac{1}{2}+\frac{\pi}{4}-2\pi\right)>0 \text{ and}$$

$$\cos x=\cos\left\{\frac{1}{2}-\left(\frac{\pi}{4}-\frac{1}{2}\right)\right\}$$

Hence, $x=\frac{1}{2}+\frac{7\pi}{4}$ is the smallest +ve root of the given equation.

So, $\alpha=\frac{1}{2}+\frac{7\pi}{4}=\frac{1}{2}+\frac{7}{4}\times\frac{22}{7}=6$

4. Ans : 13

$\sin x\cos y=1 \Leftrightarrow \sin x=1$ and $\cos y=1$ or $\sin x=-1$ and $\cos y=-1$

Case I : $\sin x=\cos y=1$

$$\Rightarrow x=2n\pi+\frac{\pi}{2} \text{ and } y=2m\pi,\ m,n\in I$$

Now $x^2+y^2\le 9\pi^2 \Rightarrow (4n+1)^2+(4m)^2\le 36$

When $m=0$, $n=-1, 0, 1$ and when $m=\pm 1$, $n=-1, 0$.

So, 7 solutions

Case II :

$$\sin x=\cos y=-1 \Rightarrow x=2p\pi-\frac{\pi}{2} \text{ and } y=2q\pi+\pi$$

$$\therefore (4p-1)^2+(4q+2)^2\le 36$$

When $q=-1$, $p=-1, 0, 1$ and when $q=0$, $p=-1, 0, 1$.

So, 6 solutions

Thus total 7 + 6 = 13 pairs are possible.

5. Ans : 3

$$\tan\theta=\frac{3\tan\frac{\theta}{3}-\tan^3\frac{\theta}{3}}{1-3\tan^2\frac{\theta}{3}}=\lambda$$

$$\tan^3\frac{\theta}{3}-3\lambda\tan^2\frac{\theta}{3}-3\tan\frac{\theta}{3}+\lambda=0$$

Thus $\tan\frac{\theta_1}{3}$, $\tan\frac{\theta_2}{3}$ and $\tan\frac{\theta_3}{3}$ are roots of the above equation

$$\therefore \Sigma\tan\frac{\theta_1}{3}\tan\frac{\theta_2}{3}=-3$$

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