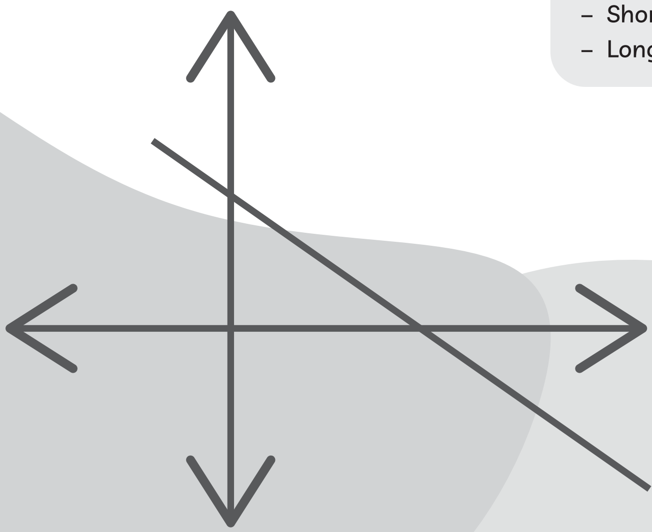
3

Pair of Linear Equations in Two Variables

Chapter Synopsis:



Objective Section **MAP**

PAIR OF LINEAR EQUATIONS IN TW0 VARIABLES

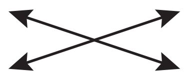
LINEAR EQUATION

- A equation in the form $ax + by + c = 0$, where a, b and c are real number, a and b are not both zero.
- Each solution (x, y) of a linear equation in two variables, $ax + by + c = 0$, corresponds to a point on the line representing the equation, and vice versa.

NATURE OF TWO STRAIGHT LINES IN A PLANE

If two lines are in plane than only one of the following three possibilities can happens:

(i) The two lines will intersect at one point.



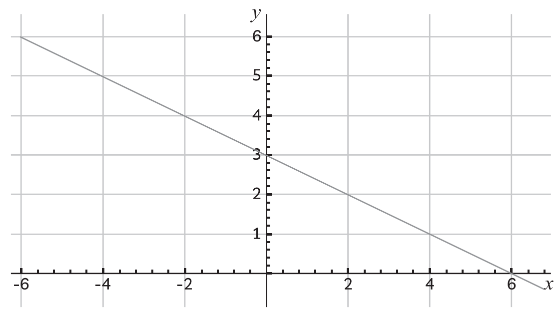
(ii) The two lines will not intersect, that means the lines are parallel.

(iii) The two lines will be coincident.

GRAPHICAL REPRESENTATION OF A LINEAR EQUATION

Graphical representation of a linear equation in two variables is a straight line.

Eg: $x + 2y - 4 = 0$



GENERAL FORM OF A LINEAR EQUATION IN TWO VARIABLES

The general form for a pair of linear equation in two variable x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where a_1, b_1, c_1, a_2, b_2 and c_2 are all real number and $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$

EQUATION REDUCIBLE TO A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Eg:
Solve the pair of equation by reducing them to a pair of linear equations.

$$\frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

Sol: *Put* $\frac{1}{x} = u$ *and* $\frac{1}{y} = v$

The pair of reduces to

$3u + 2v = 12$

$2u + 3v = 13$

The above pair of equations may be solved. After solving, back substitute to get the values of x and y.

A PAIR OF LINEAR EQUATION CAN BE REPRESENTED AND SOLVED BY THE FOLLOWING METHODS

Solution of a pair of linear equation in two variable

The solution of a linear equation in two variable 'x' and 'y' is a pair of values which makes the two sides of the equation equal.

Algebraic Method

Substitution Method

Eg:
Solve the following system by substitution

$4x + 7y = 5$
$x + y = 5$

Sol: *Let* $4x + 7y = 5$ .....(1)
And, $x + y = 5$ .....(2)
$y = 5 - x$ .....(3)
Substituting the value of y in eq. (1)
$4x + 7y = 5$
$4x + 7(5 - x) = 5$
$4x + 35 - 7x = 5$
$-3x = 5 - 35$
$-3x = -30$
$x = 10$
Now put the value of x in eq. (2)
$y = 5 - x$
$y = 5 - 10 = -5$
The solution is $(x, y) = (10, -5)$.

Elimination Method

Eg:
Solve the following system by elimination method

$2x + 3y = 8$ .....(1)
$3x + 2y = 7$ .....(2)

Sol: *On multiplying the eq. (1) by 3 and eq. (2) by 2*
$3\times$ (eq.1) –> $3\times(2x + 3y = 8)$ –> $6x + 9y = 24$
$2\times$ (eq.2) –> $2\times(3x + 2y = 7)$ –> $6x + 4y = 14$
Subtract the eq. (2) from the eq. (1)
$6x + 9y = 24$
$-(6x + 4y = 14)$ ($\therefore$ *eliminating x*)
$5y = 10$
$y = \frac{10}{5} = 2$

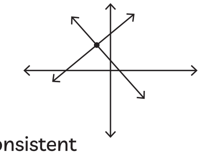
Put the value of y in eq. (1)
$2x + 3(2) = 8$
$2x + 6 = 8$
$2x = 2$
$x = 1$
Solution: $x = 1, y = 2$ *or* $(1, 2)$.

Graphical Method

If a pair of linear equation is given by
$a_1x + b_1y + c_1 = 0$ and
$a_2x + b_2y + c_2 = 0$

One Solution

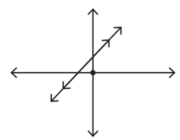
- $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ is equal to:
  Intersecting lines = Exactly one solution (Unique) = The pair of linear equation is consistent.



- Consistent
- Intersecting lines

Infinitely many Solution

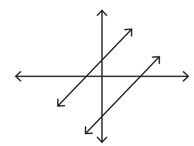
- $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is equal to:
  Coincident lines = Infinitely many solutions = The pair of linear equation is inconsistent.



- Inconsistent
- Coincident lines

No Solution

- $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ is equal to:
  Parallel lines = No solution = The pair of linear equation is dependent and consistent.



- Consistent
- Parallel lines

OBJECTIVE Type Questions

[**1** mark]

Multiple Choice Questions

1. **The pair of linear equations $\frac{3x}{2}+\frac{5y}{3}=7$ and $9x + 10y = 14$ is**

(a) consistent

(b) inconsistent

(c) consistent with one solution

(d) consistent with many solutions **[CBSE 2020]**

Ans. *(b) inconsistent*

Explanation:

For the given pair of equations, we have:

$$\frac{a_1}{a_2}=\frac{3/2}{9}=\frac{1}{6}$$

$$\frac{b_1}{b_2}=\frac{5/3}{10}=\frac{1}{6}$$

$$\frac{c_1}{c_2}=\frac{7}{14}=\frac{1}{2}$$

$$\therefore \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Hence, the pair of equations is inconsistent.

2. **Graphically, the pair of equations $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ represents two lines which are:**

(a) intersecting at exactly one point.

(b) intersecting at exactly two points.

(c) coincident

(d) parallel **[CBSE 2013]**

Ans. *(d) parallel*

Explanation: The given equations are:

$6x - 3y + 10 = 0$ ...(i)

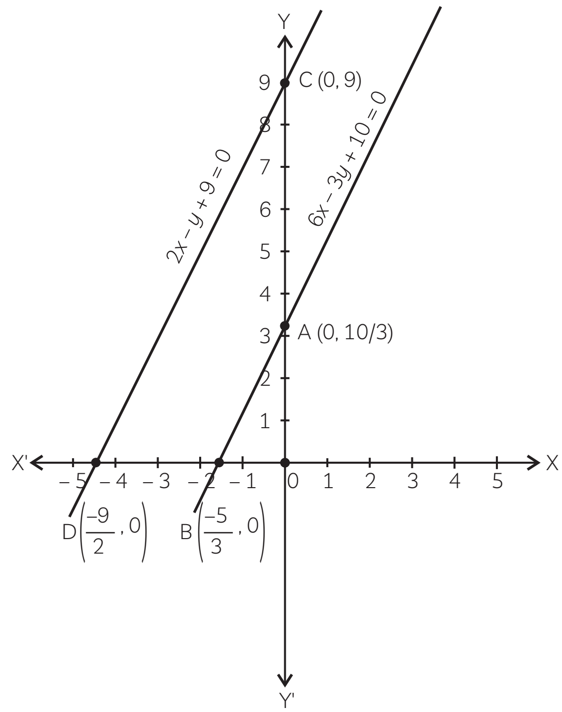
Also, $2x - y + 9 = 0$ ...(ii)

Table for $6x - 3y + 10 = 0$,

x	0	$-\frac{5}{3}$
y	$\frac{10}{3}$	0

Table for $2x - y + 9 = 0$,

x	0	$-\frac{9}{2}$
y	9	0



Hence, the pair of equations represents two parallel lines.

3. **If a pair of linear equations is consistent, then the lines will be:**

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting **[NCERT]**

Ans. *(c) intersecting or coincident*

Explanation: The conditions for a pair of linear equations to be consistent are:

- Intersecting lines having unique solution,

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

OR

- Coincident or dependent lines having infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

4. **The value of k for which the system of linear equation $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent is**

(a) $-\frac{14}{3}$ **(b)** $\frac{2}{5}$

(c) 5 **(d) 10**

Ans. *(d) 10*

Explanation:

The system of equations will be inconsistent if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Here, $a_1 = 1, b_1 = 2, c_1 = 3$

$a_2 = 5 \;\; b_2 = k, \;\; c_2 = -7$

$$\frac{1}{5} = \frac{2}{k} \neq \frac{3}{7}$$

i.e., when $\quad k = 10$

5. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is

(a) –2 (b) ≠2

(c) 3 (d) 2 **[CBSE 2020]**

Ans. *(d) 2*

Explanation: $x + y - 4 = 0$ and $2x + ky = 3$ has no solution, when:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$

$$\Rightarrow \quad k = 2$$

6. The pair of equations $y = 0$ and $y = -7$ has:

(a) one solution

(b) two solutions

(c) infinitely many solutions

(d) no solution **[NCERT]**

Ans. *(d) no solution*

Explanation: We know that equation of the form y = 'a' is a line parallel to the x-axis at a distance 'a' from it.

The given pair of equations are $y = 0$ and $y = -7$.

$y = 0$ is the equation of the x-axis and $y = -7$ is the equation of the line parallel to the x-axis. So, these two equations represent two parallel lines.

We know that parallel lines never intersect. So, there is no solution for these lines.

7. The pair of equations $x = a$ and $y = b$ graphically represents lines which are:

(a) parallel

(b) intersecting at (b, a)

(c) coincident

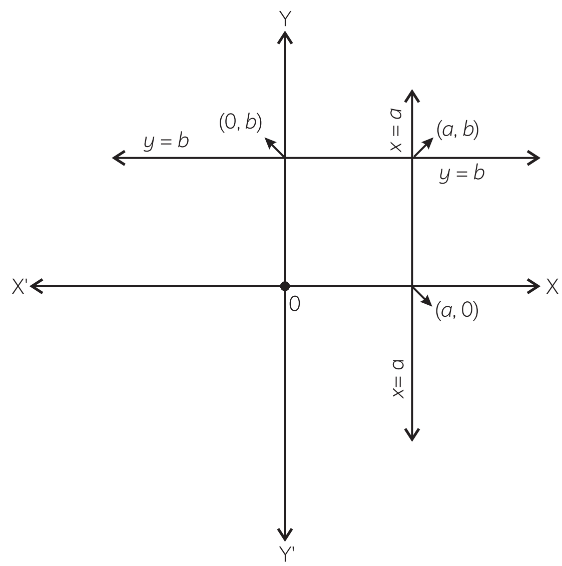
(d) intersecting at (a, b) **[NCERT]**

Ans. *(d) intersecting at (a, b)*

Explanation: We know that $x = a$ is the equation of a straight line parallel to the y-axis at a distance of 'a' from it.

Again, $y = b$ is the equation of a straight line parallel to the x-axis at a distance of 'b' from it.

So, the pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b) as shown below:



Hence, the two lines are intersecting at (a, b).

8. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel:

$3x - y - 5 = 0$

$6x - 2y - p = 0$

(a) all real values except 10

(b) 10

(c) $\frac{5}{2}$

(d) $\frac{1}{2}$

Ans. *(a) all real values except 10*

[CBSE Marking Scheme 2019]

9. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is:

(a) $-\frac{5}{4}$ (b) $\frac{2}{5}$

(c) $\frac{15}{4}$ (d) $\frac{3}{2}$

[CBSE 2015, 11, 10]

Ans. *(c)* $\frac{15}{4}$

Explanation: The given equation of lines are

$$3x + 2ky = 2$$

and $\quad 2x + 5y + 1 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3; b_1 = 2k; c_1 = -2$

$a_2 = 2; b_2 = 5; c_2 = 1$

$\frac{a_1}{a_2} = \frac{3}{2}; \frac{b_1}{b_2} = \frac{2k}{5}; \frac{c_1}{c_2} = \frac{-2}{1}$

We know that the condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{2k}{5}$$

$$\Rightarrow \quad 15 = 4k$$

$$\Rightarrow \quad k = \frac{15}{4}$$

For $k = \frac{15}{4}$,

$$\frac{2k}{5} = \frac{30}{20} = \frac{3}{2} \neq \frac{-2}{1}$$

Thus, $k = \frac{15}{4}$

10. **The pair of equations, $x = 0$ and $x = -4$ has**

(a) a unique solution

(b) no solution

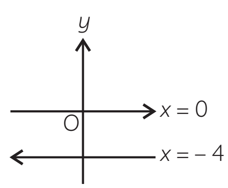
(c) infinitely many solution

(d) only solution (0, 0) **[CBSE 2020]**

Ans. *(b) no solution*

Explanation:

Since the lines represented by the given equations are parallel to each other, the pair of equations has no solution.



11. **One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be:**

(a) $10x + 14y + 4 = 0$

(b) $-10x - 14y + 4 = 0$

(c) $-10x + 14y + 4 = 0$

(d) $10x - 14y = -4$

Ans. *(d) $10x - 14y = -4$*

Explanation: In a pair of dependent linear equation, one equation is just a multiple of another equation. Thus, the second equation is

$k(-5x + 7y - 2) = 0$

Putting $k = 2$, we get

$\Rightarrow -10x + 14y - 4 = 0$

On moving it to the other side, we get

$\Rightarrow \quad 10x - 14y = -4$

$\therefore$ (D) option is correct.

12. **A pair of linear equations which has a unique solution $x = 2, y = -3$ is:**

(a) $x + y = -1$ and $2x - 3y = -5$

(b) $2x + 5y = -11$ and $4x + 10y = -22$

(c) $2x - y = 1$ and $3x + 2y = 0$

(d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

[NCERT]

Ans. *(d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$*

Explanation: If $x = 2$ and $y = -3$ is a unique solution of any pair of equation, then these values must satisfy that pair of equations.

Putting the values in the equations for every option and checking it -

For case (A):

The given equations are

$$x + y = -1$$

and $\quad 2x - 3y = -5$

Putting $x = 2, y = -3$ in the LHS of the equation

$$x + y = -1,$$

we get $\quad 2 - 3 = -1$ = RHS

Putting $x = 2, y = -3$ in the LHS of the equation

$$2x - 3y = -5,$$

we get $2 \times 2 - 3 \times (-3) = 4 + 9 = 13 \neq -5 \neq$ RHS

Since $x = 2, y = -3$ is satisfying only one of the two equations, option (A) is false.

Now, for case (B):

It is a pair of dependent linear equations and hence, has infinitely many solutions. (not a unique solution)

Now, for case (C):

The given equations are

$$2x - y = 1$$

and $\quad 3x + 2y = 0$

Putting $x = 2, y = -3$ in the LHS of the equation

$$2x - y = 1,$$

we get $2 \times 2 - (-3) = 4 + 3 = 7 \neq 1 \neq$ RHS

Putting $x = 2, y = -3$ in the LHS of the equation

$$3x + 2y = 0,$$

we get $3 \times 2 + 2 \times (-3) = 6 - 6 = 0$ = RHS

Since, $x = 2, y = -3$ is satisfying only one of the two equations, option (C) is false.

Now, for case (D):

The given equations are

$$x - 4y - 14 = 0$$

and $\quad 5x - y - 13 = 0$

Putting $x = 2, y = -3$ in the LHS of the equation

$$x - 4y - 14 = 0,$$

we get $2 - 4 \times (-3) - 14 = 2 + 12 - 14 = 0 =$ RHS

Putting $x = 2, y = -3$ in the LHS of the equation

$5x - y - 13 = 0,$

we get $5 \times 2 - (-3) - 13 = 10 + 3 - 13 = 0 =$ RHS

Since $x = 2, y = -3$ is satisfying both the equations, option (D) is true.

Hence, $x = 2, y = -3$ is the unique solution for these equations.

13. **If $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b respectively are:**

(a) 3 and 5 (b) 5 and 3

(c) 3 and 1 (d) −1 and −3

[CBSE 2010]

Ans. *(c) 3 and 1*

Explanation: Since $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, these values must satisfy the given pair of equations.

Putting the values in the equations, we have

$a - b = 2$ ...(i)

and $a + b = 4$ ...(ii)

Adding equations (i) and (ii), we get

$2a = 6$ or $a = 3$

Putting the value of a in equation (ii), we get

$3 + b = 4$ or $b = 1$

Hence, option (C) is correct.

14. **Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins respectively are:**

(a) 35 and 15 (b) 35 and 20

(c) 15 and 35 (d) 25 and 25

[NCERT]

Ans. *(d) 25 and 25*

Explanation: Let number of ₹ 1 coins = x

and number of ₹ 2 coins = y.

It is given that,

Total number of coins = $x + y = 50$ ...(i)

Also, amount of money with her

= (Number of ₹ 1 coins × 1)

+ (Number of ₹ 2 coins × 2)

Now, by the given condition:

$\Rightarrow x(1) + y(2) = 75$

$\Rightarrow x + 2y = 75$ ...(ii)

On subtracting eq. (i) from eq. (ii), we get

$\Rightarrow (x + 2y) - (x + y) = (75 - 50)$

$\Rightarrow y = 25$

Putting $y = 25$ in eq. (i), we get

$x + 25 = 50$

$\Rightarrow x = 25$

Hence, Aruna has 25 ₹ 1 coins and 25 ₹ 2 coins.

Fill in the Blanks/True False

Fill in the blanks/tables with suitable information or answer whether True/False

15. **The value of k for which pair of linear equations $3x + 2y = -5$ and $x - ky = 2$ has a unique solution is ....**

Ans. $k \neq -2/3$

Explanation: For unique solution

$\frac{3}{1} \neq \frac{2}{-k} \Rightarrow k \neq -2/3$

16. **The value of a so that the point (3, a), lies on the line represented by $2x - 3y = 5$ is ....**

Ans. $\frac{1}{3}$

Explanation: Given (3, a) lies on $2x - 3y = 5$

$\Rightarrow 2 \times 3 - 3a = 5 \Rightarrow 3a = 6 - 5 = 1$

$\Rightarrow a = \frac{1}{3}$

17. **The co-ordinate where the line $x - y = 8$ will intersect y-axis is ....**

Ans. *(0, – 8)*

Explanation: At y-axis, $x = 0$

$\therefore 0 - y = 8 \Rightarrow y = -8$

Point = (0, – 8)

18. **The value of k for which the pair of linear equations $kx + 3y = k - 2$ and $12x + ky = k$ has no solution is ....**

Ans. $k = \pm 6$

Explanation: Since, pair of linear equations has no solution

Then $\frac{k}{12} = \frac{3}{k} \Rightarrow \frac{k-2}{k}$ i.e., $k^2 = 36 \Rightarrow k = \pm 6$

19. **The graphical representation of the pair of equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ represents ....**

Ans. *Parallel lines*

Explanation: $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$

Here $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$

and $\frac{c_1}{c_2} = \frac{-4}{-12} = \frac{1}{3}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, lines are parallel or non-intersecting

20. **If $x + y = 2$ and $\frac{1}{x+y} = \frac{2}{5}$ then $x =$ ....**

Ans. $\frac{9}{4}$

Explanation: $x - y = 2$ and $\frac{1}{x+y} = \frac{2}{5}$

$$x - y = 2 \quad \text{...(i)}$$

$$x + y = \frac{5}{2} \quad \text{...(ii)}$$

Adding equations (i) and (ii), we get

$$(x - y) + (x + y) = 2 + \frac{5}{2}$$

$$2x = \frac{9}{2} \Rightarrow x = \frac{9}{4}$$

21. **The value of p for the following pair of linear equations $(p - 3)x + 3y = p$; $px + py = 12$ have infinitely many solutions is ....**

Ans. $p = 6$

Explanation: Given equations are

$(p - 3)\,x + 3y = p$ and $px + py = 12$

For infinitely many solution

$$\frac{p-3}{p} = \frac{3}{p} = \frac{p}{12}$$

$$\Rightarrow \quad \frac{p-3}{p} = \frac{3}{p} \text{ and } \frac{3}{p} = \frac{p}{12}$$

$$\Rightarrow \quad p - 3 = 3 \text{ and } p^2 = 36$$

$$\Rightarrow \quad p = 6 \text{ and } p = \pm 6$$

Common value is 6

So, $p = 6$

22. **If $x = a$, $y = b$ is the solution of the pair of equation $x - y = 2$ and $x + y = 4$ then the value of $3a + 4b$ is ....**

Ans. 13

Explanation: $x - y = 2$ ...(i)

$x + y = 4$ ...(ii)

On adding (i) and (ii), we get

$$2x = 6 \Rightarrow x = 3$$

and putting $x = 3$ in (i),

$$3 - y = 2 \Rightarrow y = 1$$

$$3a + 4b = 3x + 4y = 3 \times 3 + 4 \times 1 = 13$$

23. **For the pair of equations**

$\lambda x + 3y = -7$

$2x + 6y = 14$

to have infinitely many solutions, the value of λ should be 1. Is this statement true? Give reasons.

Ans.

No, for no value of λ will the given pair of linear equations has infinitely many solutions.

The given pair of linear equations is

$$\lambda x + 3y + 7 = 0$$

and $2x + 6y - 14 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = \lambda$, $b_1 = 3$, $c_1 = 7$;

$a_2 = 2$, $b_2 = 6$, $c_2 = -14$;

$$\frac{a_1}{a_2} = \frac{\lambda}{2};\ \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2};\ \frac{c_1}{c_2} = \frac{7}{-14} = -\frac{1}{2}$$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\lambda}{2} = \frac{1}{2} \text{ and } \frac{\lambda}{2} = -\frac{1}{2}$$

$$\Rightarrow \quad \lambda = 1 \text{ and } \lambda = -1$$

Since, λ does not have a unique value so for no value of λ will the given pair of linear equations have infinitely many solutions.

24. **For all real values of c, the pair of equations**

$x - 2y = 8$

$5x - 10y = c$

have a unique solution. Justify whether it is true or false. [NCERT]

Ans. *False*

The given pair of equations will not have a unique solution for any value of c.

The given pair of linear equations is

$$x - 2y - 8 = 0$$

and $5x - 10y - c = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1$, $b_1 = -2$, $c_1 = -8$;

$a_2 = 5$, $b_2 = -10$, $c_2 = -c$;

$$\frac{a_1}{a_2} = \frac{1}{5};\ \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5};\ \frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$$

But for $c = 40$ (any real value), the ratio will be

$$\frac{c_1}{c_2} = \frac{8}{40} = \frac{1}{5}$$

when $c = 40$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{5}$$

Thus, the given pair of linear equations will have infinitely many solutions for $c = 40$.

Also, when $c \neq 40$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the given pair of linear equations will have no solution for $c \neq 40$.

Hence, for any value of c, the system of linear equations does not have a unique solution.

Very short Questions

25. **Write the relationship between the coeffcients, if the following pair of equations are inconsistent.**

$ax + by + c = 0$; $a'x + b'y + c' = 0$.

Ans. The required relationship is:

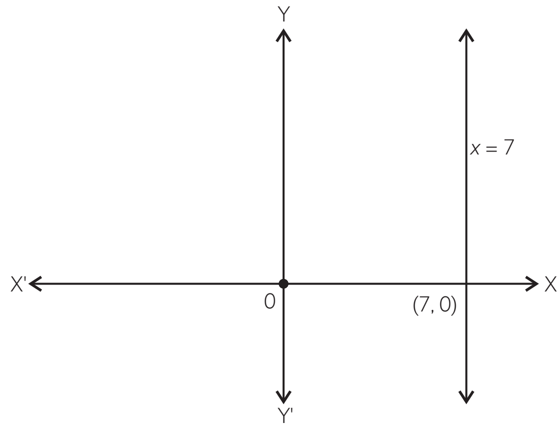
$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

26. **The line represented by $x = 7$ is parallel to the x-axis. Justify whether the statement is true or not.** **[NCERT]**

Ans. *False*

The line represented by $x = 7$ is not parallel to the x-axis.

Explanation: The line represented by $x = 7$ is of the form $x = a$. The graph of the equation is a line parallel to the y-axis and perpendicular to the x-axis.



Hence, the given statement is not true.

27. **When will the system $kx - y = 2$ and $6x - 2y = 3$ has a unique solution only?** **[Diksha]**

Ans. A pair of linear pair has unique solution only when,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Then, $\frac{k}{6} \neq \frac{-1}{-2}$

So, $k \neq 3$.

28. **Find the solution of $x + y = 3$ and $7x + 6y = 2$.**

Ans. $x + y = 3$ gives, $y = 3 - x$ ...(i)

So, $7x + 6y = 2$ gives $7x + 6(3 - x) = 2$

$\Rightarrow$ $7x + 18 - 6x = 2$

i.e. $x = -16$

From (i), $y = 3 + 16 = 19$

Thus, $x = -16$ and $y = 19$ is the required solution.

SHORT ANSWER (SA-I) Type Questions

[**2** marks]

29. **Find the value(s) of k for which the pair of equations $\begin{cases} kx + 2y = 3 \\ 3x + 6y = 10 \end{cases}$ has a unique solution.** **[CBSE 2019]**

Ans. Given: pair of equation is $kx + 2y = 3$

$3x + 6y = 10$

For a unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Here, $a_1 = k, b_1 = 2$

$a_2 = 3, b = 6$

$\therefore$ $\frac{k}{3} \neq \frac{2}{6}$

$\therefore$ $k \neq 1$

Hence, the pair of equation has a unique solution for all real values of k except 1.

30. **The larger of two supplementary angles exceeds the smaller by 18°. Find the angles.** **[CBSE 2019]**

Ans. Let, the smaller angle be 'x' and the larger angle be 'y'.

According to the given conditions:

$y = x + 18°$

or $-x + y = 18°$ ...(i)

and $x + y = 18°$ ...(ii)

(Sum of the supplementary angles is 180°)

Now, on adding equation (i) and (ii), we get:

$$\begin{array}{r} -x + y = 18° \\ x + y = 180° \\ \hline 2y = 198 \\ \hline \end{array}$$

$\Rightarrow \quad y = 99°$

Put the value of y in equation (i), we get

$$-x + 99 = 18°$$

$$\Rightarrow \quad x = 99 - 18°$$

$$= 81°$$

Hence, the two supplementary angles are 81° and 99°.

31. In a ΔABC, ∠A = x°, ∠B = $3x$° and ∠C = y°. If $3y° - 5x° = 30°$ prove that the triangle is right angled. [Diksha]

Ans. We know that,

$$\angle A + \angle B + \angle C = 180°$$

[Sum of interior angles of triangle ABC is 180°]

$$\Rightarrow \quad x + 3x + y = 180°$$

$$\Rightarrow \quad 4x + y = 180° \quad \text{...(i)}$$

and $\quad 3y - 5x = 30 \quad$ [Given] ...(ii)

Multiply equation (i) by 3

$$12x + 3y = 540° \quad \text{...(iii)}$$

Subtracting (ii) from (iii), we get

$$17x = 510$$

$$x = 30°$$

Putting value of x in equation (i), we get

$$4 \times 30° + y = 180°$$

$$y = 60°$$

$\therefore \quad \angle A = 30°,$

$\angle B = 3 \times 30° = 90°$

And $\angle C = 60°$

Hence, ΔABC is right angled triangle at B.

32. Find c if the system of equations $cx + 3y + (3 - c) = 0$; $12x + cy - c = 0$ has infinitely many solutions? [CBSE 2019]

Ans. Given equation is:

$$cx + 3y + (3 - c) = 0$$

$$12x + cy - c = 0$$

Condition for equations to have infinitely many solutions is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\quad a_1 = c, b_1 = 3, c_1 = 3 - c$

$a_2 = 12, b_2 = c, c_2 = -c$

$$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3 - c}{-c}$$

$$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6 \text{ or } c = -6 \quad \text{...(i)}$$

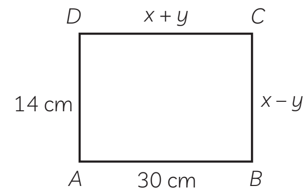
Also, $\quad -3c = 3c - c^2$

$$\Rightarrow \quad c = 6 \text{ or } c = 0 \quad \text{...(ii)}$$

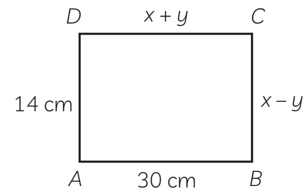
From (i) and (ii), we get, $c = 6$.

Hence, the value of $c = 6$.

33. In the figure, $ABCD$ is a rectangle. Find the values of x and y. [CBSE 2018]



Ans. Given: $ABCD$ is a rectangle



and $\quad AB = 30$ cm

$BC = x - y$

$CD = x + y$

$DA = 14$ cm

But the opposite sides of rectangle are equal.

$\therefore \quad AB = CD$

$$x + y = 30 \quad \text{...(i)}$$

and $\quad BC = AD$

$$x - y = 14 \quad \text{...(ii)}$$

On adding the equation (i) and (ii), we get:

$$x + y = 30$$

$$x - y = 14$$

$$2x = 44$$

$$\Rightarrow \quad x = 22$$

If we put the value of x in equation (i), we get:

$$22 + y = 30$$

$$\Rightarrow \quad y = 8$$

Hence, the value of 'x' and 'y' are 22 and 8, respectively.

34. For what value of k, does the system of linear equations

$2x + 3y = 7$

$(k - 1)x + (k + 2)y = 3k$

have an infinite number of solutions ? [CBSE 2019]

Ans. The given system of linear equation is :

$$2x + 3y = 7$$

$$(k - 1)x + (k + 2)y = 3k$$

For infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$a_1 = 2, b_1 = 3, c_1 = 7$

and $a_2 = (k - 1), b_2 = (k + 2), c_2 = 3k$

$$\Rightarrow \quad \frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$

$\Rightarrow \quad 2(k + 2) = 3(k - 1); 3 (3k) = 7(k + 2)$

$\Rightarrow \quad 2k - 3k = -3 - 4; 9k - 7k = 14$

$\Rightarrow \quad k = 7; k = 7$

Hence, the value of k is 7.

35. **If $2x + y = 23$ and $4x - y = 19$, find the values of $5y - 2x$ and $\frac{y}{x} - 2$.** **[NCERT]**

Ans. The given equations are

$$2x + y = 23 \quad ...(i)$$

$$4x - y = 19 \quad ...(ii)$$

On adding both equations, we get

$\Rightarrow \quad 6x = 42$

$\Rightarrow \quad x = 7$

Putting the value of x in eq. (i), we get

$\Rightarrow \quad 2(7) + y = 23$

$\Rightarrow \quad y = 23 - 14$

$\Rightarrow \quad y = 9$

We have $5y - 2x = 5(9) - 2(7)$

$= 45 - 14$

$= 31$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2$

$= -\frac{5}{7}$

Hence, the values of $(5y - 2x)$ and $\frac{y}{x} - 2$ are 31 and $-\frac{5}{7}$ respectively.

36. **Write an equation for a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?** **[NCERT]**

Ans. The given equations are

$$x + y = 2 \quad ...(i)$$

$$2x - y = 1 \quad ...(ii)$$

Adding eq. (i) and (ii), we have

$$3x = 3 \Rightarrow x = 1$$

Substituting $x = 1$ in eq. (i), we have

$$y = 1$$

So, the solution is $x = 1$ and $y = 1$ and the point that represents the solution is (1, 1).

We also know that an infinite number of lines can pass through a given point, say (1, 1).

Hence, infinite lines can pass through the intersection point of the linear equations $x + y = 2$ and $2x - y = 1$ i.e., PE(1, 1).

37. **A fraction becomes $\frac{1}{4}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.** **[CBSE 2020]**

Ans. Let the fraction be $\frac{a}{b}$

Then, according to the question,

$$\frac{a-1}{b} = \frac{1}{4} \text{ and } \frac{a}{b+8} = \frac{1}{4}$$

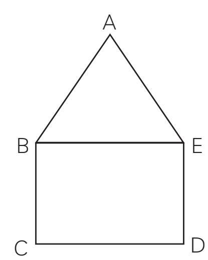
$\Rightarrow \quad 3a - b = 3$ and $4a - b = 8$

On solving these equations, we get:

$$a = 5, b = 12.$$

So, the fraction is $\frac{5}{12}$.

38. **In the figure, ABCDE is a pentagon with BE||CD and BC||DE. BC is perpendicular to CD. AB = 5 cm, AE = 5 cm, BE = 7 cm, BC = $x - y$ and CD = $x + y$. If the perimeter of ABCDE is 27 cm. Find the value of x and y, given $x, y \neq 0$.** **[CBSE 2019]**



Ans. $x + y = 7$ and $2(x - y) + x + y + 5 + 5 = 27$

□ $x + y = 7$ and $3x - y = 17$

Solving, we get, $x = 6$ and $y = 1$

[CBSE Marking Scheme 2019]

SHORT ANSWER (SA-II) Type Questions

[**3** marks]

39. **For which value(s) of λ do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have:**

(A) no solution?

(B) infinitely many solutions?

(C) a unique solution? **[NCERT]**

Ans.

The given pair of linear equations is

$$\lambda x + y - \lambda^2 = 0$$

and $x + \lambda y - 1 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$;

$a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$;

$$\frac{a_1}{a_2} = \frac{\lambda}{1};\ \frac{b_1}{b_2} = \frac{1}{\lambda};\ \frac{c_1}{c_2} = \frac{-\lambda^2}{-1} = \frac{\lambda^2}{1}$$

(A) For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{\lambda^2}{1}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \quad \text{and} \quad \frac{\lambda}{1} \neq \frac{\lambda^2}{1}$$

$$\lambda^2 - 1 = 0 \quad \text{and} \quad \lambda^2 \neq \lambda$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad (\lambda^2 - \lambda) \neq 0$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad \lambda(\lambda - 1) \neq 0$$

$$\lambda = 1, -1 \quad \text{and} \quad \lambda \neq 0, 1$$

Here, we take only $\lambda = -1$.

Hence for $\lambda = -1$, the pair of linear equations has no solution.

(B) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \quad \text{and} \quad \frac{\lambda}{1} = \frac{\lambda^2}{1}$$

$$\lambda^2 - 1 = 0 \quad \text{and} \quad \lambda^2 = \lambda$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad (\lambda^2 - \lambda) = 0$$

$$(\lambda - 1)(\lambda + 1) = 0 \quad \text{and} \quad \lambda(\lambda - 1) = 0$$

$$\lambda = 1, -1 \text{ and} \quad \lambda = 0, 1$$

$\lambda = 1$ satisfies both the equations.

Hence, for $\lambda = 1$, the pair of linear equations has infinitely many solutions.

(C) For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\lambda^2 - 1 \neq 0$$

$$(\lambda - 1)(\lambda + 1) \neq 0$$

$$\lambda \neq 1, -1$$

Hence, for all real values of λ except ± 1, the given pair of equations has a unique solution.

40. For which values of *a* and *b* will the following pair of linear equations have infinitely many solutions?

$x + 2y = 1$

$(a - b)x + (a + b)y = a + b - 2$

[CBSE 2013, 11]

Ans. The given pair of linear equations is

$$x + 2y = 1$$

and $$(a - b)x + (a + b)y = a + b - 2$$

$$x + 2y - 1 = 0$$

and $(a - b)x + (a + b)y - (a + b - 2) = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1$, $b_1 = 2$, $c_1 = -1$

$a_2 = (a - b)$, $b_2 = (a + b)$, $c_2 = -(a + b - 2)$

$$\frac{a_1}{a_2} = \frac{1}{a-b};\ \frac{b_1}{b_2} = \frac{2}{a+b};$$

$$\frac{c_1}{c_2} = \frac{-1}{-(a+b-2)} = \frac{1}{a+b-2}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{a-b} = \frac{2}{a+b} = \frac{1}{a+b-2}$$

Taking the first two parts

$$\frac{1}{a-b} = \frac{2}{a+b}$$

$$\Rightarrow \quad a + b = 2(a - b)$$

$$\Rightarrow \quad 2a - a = 2b + b$$

$$\Rightarrow \quad a = 3b \quad \text{...(i)}$$

Taking the last two parts,

$$\frac{2}{a+b} = \frac{1}{a+b-2}$$

$$\Rightarrow 2(a + b - 2) = (a + b)$$

$$\Rightarrow 2a + 2b - 4 = a + b$$

$$\Rightarrow \quad a + b = 4 \quad \text{...(ii)}$$

Putting the value of *a* from eq. (i) in eq. (ii), we get

$$\Rightarrow \quad 3b + b = 4$$

$$\Rightarrow \quad 4b = 4$$

$$\Rightarrow \quad b = 1$$

Putting the value of *b* in eq. (i), we get

$$a = 3(1) = 3$$

The values $(a, b) = (3, 1)$ satisfies all the parts.

Hence, the required values of a and b are 3 and 1 respectively for which the given pair of linear equations has infinitely many solutions.

41. Write a pair of linear equations which has the unique solution *x* = –1, *y* =3. How many such pairs can you write? [NCERT]

Ans. We know that the condition for the pair of system to have a unique solution is

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Let the equations be

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

It is given that $x = -1$ and $y = 3$ is the unique solution of these two equations, then it must satisfy the above equations.

$\Rightarrow \quad a_1(-1) + b_1(3) + c_1 = 0$

$\Rightarrow \quad -a_1 + 3b_1 + c_1 = 0$ ...(i)

and $\quad a_2(-1) + b_2(3) + c_2 = 0$

$\Rightarrow \quad -a_2 + 3b_2 + c_2 = 0$ ...(ii)

The restricted values of a_1, a_2 and b_1, b_2 are only

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \quad ...(iii)$$

So, all the real values of a_1, a_2, b_1, b_2 except those which satisfy eq. (iii) and satisfy eq. (i), and eq. (ii) will have the solution $x = -1$ and $y = 3$.

Hence, infinitely many pairs of linear equations are possible.

42. Solve the pair of equations:

$$\frac{2}{x}+\frac{3}{y}=11, \frac{5}{x}-\frac{4}{y}= 7$$

Hence, find the value of $5x - 3y$.

[CBSE 2020]

Ans. Given equations are

$$\frac{2}{x}+\frac{3}{y} = 11 \quad ...(i)$$

and $$\frac{5}{x}-\frac{4}{y} = -7 \quad ...(ii)$$

Eq (i) × 5 and eq (ii) × 2 give

$$\frac{10}{x}+\frac{15}{y} = 55 \quad ...(iii)$$

and $$\frac{10}{x}-\frac{8}{y} = -14 \quad ...(iv)$$

On subtracting eq (iv) from eq (iii), we have:

$$\frac{23}{y} = 69$$

$$\Rightarrow \quad y = \frac{1}{3}$$

On substituting this value $y = \frac{1}{3}$ in eq (i), we have:

$$\frac{2}{x}+9 = 11$$

i.e., $$\frac{2}{x} = 2$$

or $$x = 1$$

Thus, $x = 1$, $y = \frac{1}{3}$ is the required solution.

Hence, $5x - 3y = 5(1)-3\left(\frac{1}{3}\right) = 5 - 1 = 4$

43. Find the solution of the pair of equations $\frac{x}{10}+\frac{y}{5} - 1 = 0$ and $\frac{5}{4} = 15$. Hence, find λ, if $y = \lambda x + 5$. [NCERT]

Ans. The given pair of equations is

$$\frac{x}{10}+\frac{y}{5}-1 = 0$$

$\Rightarrow \quad x + 2y - 10 = 0$

$\Rightarrow \quad x + 2y = 10$ ...(i)

and $\quad \frac{x}{8}+\frac{y}{6} = 15$

$\Rightarrow \quad 3x + 4y - 360 = 0$

$\Rightarrow \quad 3x + 4y = 360$ ...(ii)

Multiplying eq. (i) by 2 and subtracting it from eq. (ii), we get

$(3x + 4y) - (2x + 4y) = 360 - 20$

$\Rightarrow \quad x = 340$

Putting the value of x in eq. (i), we get

$340 + 2y = 10$

$\Rightarrow \quad 2y = -330$

$\Rightarrow \quad y = -165$

It is given that $\quad y = \lambda x + 5$

Putting the values of x and y in the above equation, we get

$y = \lambda x + 5$

$\Rightarrow \quad -165 = \lambda(340) + 5$

$\Rightarrow \quad -\lambda(340) = 5 + 165$

$\Rightarrow \quad -\lambda(340) = 170$

$\Rightarrow \quad \lambda = \frac{-170}{340} = \frac{-1}{2}$

Hence, the solution of the pair of equations is $x = 340$, $y = -165$ and the required value of λ is $\frac{-1}{2}$.

44. By the graphical method, find whether the following pairs of equations are consistent or not. If consistent, solve them.

(A) $3x + y + 4 = 0$ and $6x - 2y + 4 = 0$

(B) $x - 2y = 6$ and $3x - 6y = 0$

(C) $x + y = 3$ and $3x + 3y = 9$ [CBSE 2014]

Ans. (A) $3x + y + 4 = 0$ and $6x - 2y + 4 = 0$

The given pair of equations is

$$3x + y + 4 = 0$$

and $$6x - 2y + 4 = 0$$

In $$3x + y + 4 = 0$$

$$y = -4 - 3x$$

When $x = 0$, then $y = -4$

When $x = -1$, then $y = -1$

When $x = -2$, then $y = 2$

x	0	–1	–2
y	–4	–1	2

And $$6x - 2y + 4 = 0$$

$$y = 3x + 2$$

When $x = 0$, then $y = 2$

When $x = -1$, then $y = -1$

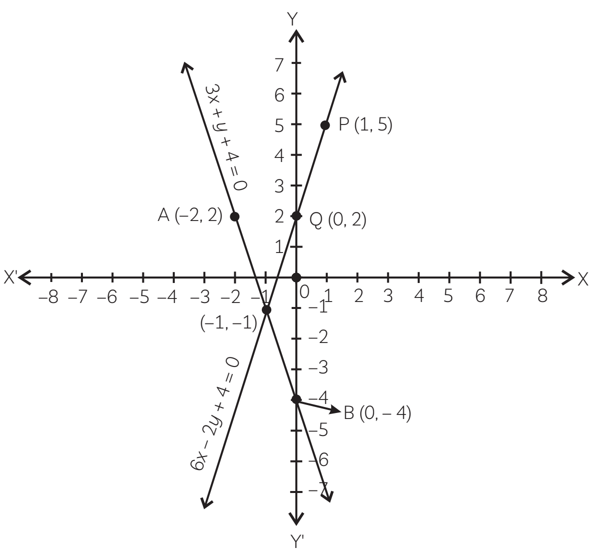
When $x = 1$, then $y = 5$

x	0	–1	1
y	2	–1	5

Plotting the points B(0, –4) and A(–2, 2), we get the straight line AB.

Plotting the points Q(0, 2) and P(1, 5) we get the straight line PQ.

The lines AB and PQ intersect at C(–1, –1).



Thus, the pair of equations is consistent and has solution $x = -1$, $y = -1$.

(B) $x - 2y = 6$ and $3x - 6y = 0$

The given pair of equations is

$$x - 2y = 6 \text{ and } 3x - 6y = 0$$

$\therefore$ $$x - 2y = 6$$

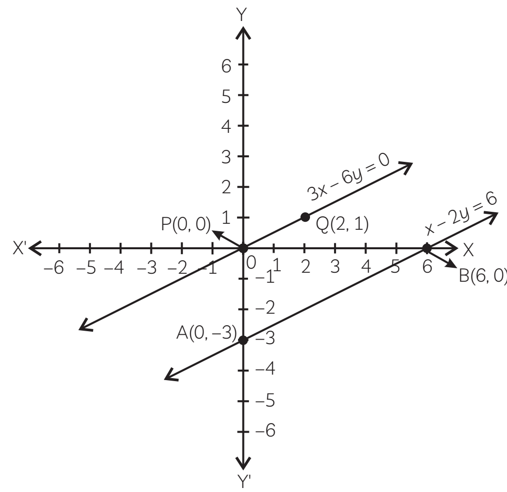
x	0	6
y	–3	0

and $$3x - 6y = 0$$

x	0	2
y	0	1

Plotting the points A(0, –3) & B(6, 0), we get line AB.

Again plotting the points P(0, 0) & Q(2, 1) we get line PQ.



Since, the lines are parallel, the pair of equations is inconsistent.

(C) $x + y = 3$ and $3x + 3y = 9$

The given pair of equations is

$$x + y = 3 \text{ and } 3x + 3y = 9$$

Now, $$x + y = 3$$

$\Rightarrow$ $$y = 3 - x$$

If $x = 0$, then $y = 3$

If $x = 3$, then $y = 0$

If $x = 2$, then $y = 1$

x	0	3	2
y	3	0	1

and $$3x + 3y = 9$$

$\Rightarrow$ $$y = \frac{(9-3x)}{3}$$

If $x = 0$ then $y = 3$

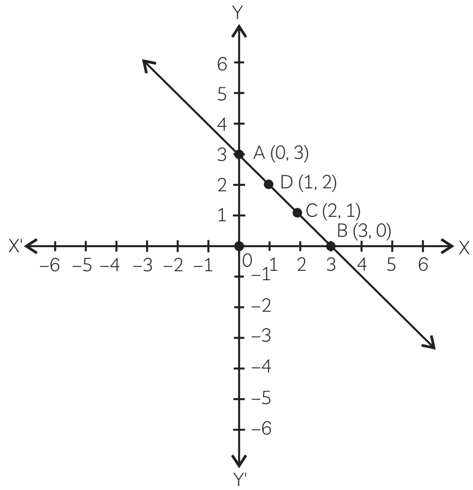
if $x = 1$, then $y = 2$

if $x = 3$, then $y = 0$.

x	0	1	3
y	3	2	0

Plotting the points A(0, 3) and B(3, 0), we get the line AB.

Again, plotting the points A(0, 3) and D(1, 2) and B(3, 0), we get the line ADB.



Since, the two lines are coincident, the pair of equations is consistent with infinitely many solutions.

45. The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages. [CBSE 2020]

Ans. Let 'x' (in years) be the present age of the father and 'y' (in years) be the present age of the son. Then, according to the given question:

$x = 3y + 3$ or $x - 3y = 3$ ...(i)

After 3 years,

Father's age = $x + 3$, son's age = $y + 3$

and $x + 3 = 2(y + 3) + 10$

or $x - 2y = 13$ ...(ii)

On solving the two equations, we get:

$$y = 10, \text{ and } x = 33.$$

Thus, the father's present age is 33 years and the son's present age is 10 years

46. Taxi charges in a city consist of fixed charges and the remaining charges depend upon the distance travelled. For a journey of 10 km, the charge paid is ₹ 75 and for a journey of 15 km, the charge paid is ₹ 110. Find the the fixed charge and charges per km. Also, find the charges of covering a distance of 35 km. [CBSE 2020]

Ans. Let the fixed charge be ₹ x and charges for per km be ₹ y.

As per the question,

$$x + 10y = 75 \text{ and } x + 15y = 110$$

On solving the two equations, we get

$$x = 5, y = 7$$

Thus, the fixed charge is ₹ 5 and the charge per km is ₹ 7.

Hence, charge for 35 km is ₹ [5 + 35(7)], i.e., ₹ 250.

47. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream. [CBSE 2020]

Ans. Let the speed of the stream be 'x' km/h; and the speed of rowing in still water be 'y' km/h.

Then, the speed of rowing in the downstream is '$x + y$' km/h.

And the speed of rowing in the upstream is '$y - x$' km/h

As per the question:

$$\frac{4}{y-x} = 2 \text{ and } \frac{4}{y-x} = 2$$

$\Rightarrow x + y = 10$ and $y - x = 2$

On solving the two equations, we get:

$x = 4$ and $y = 6$

Thus, the speed of rowing in still water is 6km/h, and the speed of the stream is 4 km/hr.

48. The angles of a triangle are x, y and 40°. The difference between the two angles x and y is 30°. Find x and y. [NCERT]

Ans. It is given that x, y and 40° are the angles of a triangle.

We know that the sum of all the angles of a triangle is 180°

$\Rightarrow \quad x + y + 40 = 180$

$\Rightarrow \quad x + y = 140$ ...(i)

Also, it is given that the difference of angles x and y is

$x - y = 30$ ...(ii)

Adding eq. (i) and (ii), we get

$(x + y) + (x - y) = 140 + 30$

$\Rightarrow \quad 2x = 170$

$\Rightarrow \quad x = 85$

Putting the value of x in eq. (i), we get

$\Rightarrow \quad 85 + y = 140$

$\Rightarrow \quad y = 140 - 85$

$y = 55$

Hence, the required values of x and y are 85° and 55° respectively.

49. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$. [CBSE 2020]

Ans.

$2y - x = 8$

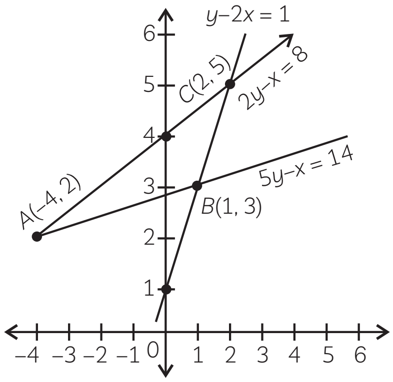
x	0	2
y	4	5

$5y - x = 14$

x	1	6
y	3	4

$y - 2x = 1$

x	0	1
y	1	3



Vertices of ΔABC are A (–4, 2), B (1, 3) and C (2, 5)

50. **A part of monthly hostel charges in a college hostel are fixed and the remaining depends on the number of days one has their meals in the mess. When a student A takes food for 25 days, he has to pay ₹ 4,500, whereas a student B who takes food for 30 days has to pay ₹ 5,200. Find the fixed charges per month and the cost of food per day. [CBSE 2019]**

Ans. Let, the fixed charge per student = ₹ x

Cost of food per day per student = ₹ y

According to the given condition,

$$x + 25y = 4500 \quad ...(i)$$

$$x + 30y = 5200 \quad ...(ii)$$

On subtracting equation (i) from equation (ii), we get

$$x + 30y = 5200$$
$$x + 25y = 4500$$
$$- \quad - \quad -$$
$$5y = 700$$

$\Rightarrow \quad y = \dfrac{700}{5} = 140$

Put the value of y in equation (i), we get:

$x + 25 \times 140 = 4500$

$\Rightarrow \quad x + 3500 = 4500$

$\Rightarrow \quad x = 1000$

Hence, the fixed charge per student is ₹ 1,000 and cost of food per day is ₹ 140.

51. **There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But, if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls. [NCERT]**

Ans. Let the number of students in hall A and B be x and y respectively.

By the given condition, to make the number of students equal in each hall, 10 students are sent from A to B

$\Rightarrow \quad x - 10 = y + 10$

$\Rightarrow \quad x - y = 20 \quad ...(i)$

Also, it is given that if 20 students are sent from B to A, the number of students in A becomes double the number of students in B

$\Rightarrow \quad (x + 20) = 2(y - 20)$

$x - 2y = -60 \quad ...(ii)$

Subtracting eq. (ii) from eq. (i), we get

$(x - y) - (x - 2y) = 20 - (-60)$

$\Rightarrow \quad y = 80$

Putting the value of y in eq. (i), we get

$x - 80 = 20$

$\Rightarrow \quad x = 100$

Hence, 100 students are in hall A and 80 students are in hall B.

52. **In a competitive examination, one mark is awarded for each correct answer, while $\frac{1}{2}$ mark is deducted for every wrong answer. Rahul answered 120 questions and got 90 marks. How many questions did he answer correctly? [CBSE 2011]**

Ans. Let x be the number of correct answers,

Then, marks awarded for correct answer = $x \times 1 = x$

Total no. of questions attempted = 120

Number of wrong answers = $(120 - x)$

Marks deducted for wrong answers

$$= (120 - x) \times \frac{1}{2} = \frac{120 - x}{2}$$

Total marks awarded to Rahul = 90

$\Rightarrow \quad x - \left(\dfrac{120 - x}{2}\right) = 90$

$\Rightarrow \quad x + \dfrac{x}{2} - 60 = 90$

$\Rightarrow \quad \dfrac{3x}{2} = 150$

$\Rightarrow \quad x = \dfrac{150 \times 2}{3}$

$\Rightarrow \quad x = 100$

Hence, Rahul answered 100 questions correctly.

53. **A father's age is three times the sum of the ages of his children. After 5 years, his age will be two times the sum of their ages. Find the present age of the father. [CBSE 2019]**

Ans. Let the sum of the ages of two children be x years and father's age be 'y' years.

According to the given condition:

$y = 3x$

or $y - 3x = 0$ ...(i)

After 5 years:

Father's age = $(y + 5)$ years

Sum of the ages of children = $(x + 5 + 5)$ years.

Then, $y + 5 = 2(x + 10)$

or $y - 2x - 15 = 0$ ...(ii)

On subtracting equation (i) from equation (ii), we get:

$$\begin{array}{r} y - 2x - 15 = 0 \\ y - 3x \quad\quad = 0 \\ - \quad + \quad\quad\quad \\ \hline x - 15 = 0 \\ \hline x = 15 \end{array}$$

If we put the value of x in equation (i), we get

$y - 3 \times 15 = 0$

$\Rightarrow \quad y = 45$

Hence, the present age of the father is 45 years.

54. Solve the following system of equations:

$$\frac{21}{x} + \frac{47}{y} = 110$$

$$\frac{47}{x} + \frac{21}{y} = 162, \quad x, y \neq 0$$

Ans. Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

$\Rightarrow 21a + 47b = 110$ and $47a + 21b = 162$

Adding and subtracting the two equations, we get

$a + b = 4$ and $a - b = 2$

Solving the above two equations, we get

$a = 3$ and $b = 1$

$\therefore x = \frac{1}{3}$ and $y = 1$

[CBSE Marking Scheme 2019]

55. The sum of reciprocals of a child's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Ans. Let the present age of the child (in years) be x. Then,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$\Rightarrow \quad 3(2x + 2) = (x - 3)(x + 5)$

$\Rightarrow \quad 6x + 6 = x^2 + 2x - 15$

$\Rightarrow \quad x^2 - 4x - 21 = 0$

$\Rightarrow \quad (x - 7)(x + 3) = 0$

$\Rightarrow \quad x - 7 = 0$ or $x + 3 = 0$

$\Rightarrow \quad x = 7$ or $x = -3$

($x = -3$ is a not possible)

Thus, her present age is 7 years.

56. A man wished to give ₹ 12 to each person and found that he fell short of ₹ 6 when he wanted to give to all the persons present. He, therefore, distributed ₹ 9 to each person and found that ₹ 9 were left over. How much money did he have and how many persons were there? [Diksha]

Ans. Let, number of persons = x

Money share per person = y

Therefore, total money = ₹ xy

According to the question,

$12 \times x = xy + 6$

$12x - 6 = xy$ ...(i)

and $9x = xy - 9$

$9x + 9 = xy$ ...(ii)

Equating (i) and (ii), we get

$12x - 6 = 9x + 9$

$3x = 15$

Put the value of x in equation (i). Then

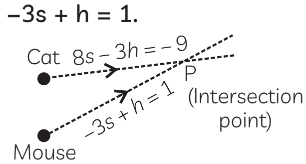
$12 \times 5 - 6 = x \times y$

$\Rightarrow \quad xy = 54$

So, he have ₹ 54 and there were 5 persons.

57. A computer animation below shows a cat moving in a straight line.

Its height, h metres, above the ground, is given by 8s – 3h = –9, where s is the time in seconds after it starts moving. In the same animation, a mouse starts to move at the same time as the cat and its movement is given by –3s + h = 1.



(A) Draw the graph of the two equations on the same sheet of graph paper;

(B) Will the mouse be able to catch the cat?

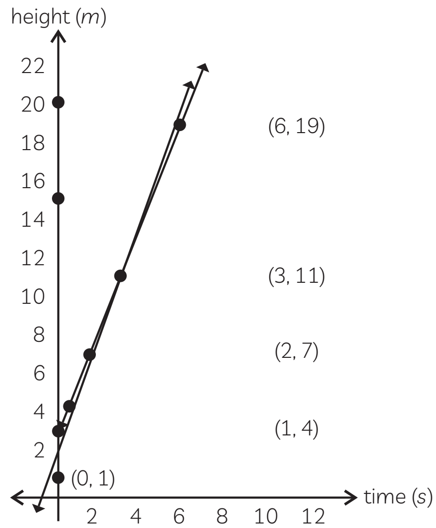
(C) If yes, after how much time and at what height?

Ans. (A) $-3s + h = 1$

$3s - 3h = -9$

s	0	1	2
h	1	4	7

s	0	3	6
h	3	11	19



(B) Yes, if find the several values of the variables *s* and *h* for cat as well as mouse, then the same values of s and h show their intersection point. It mean that the cat will definitely catch the mouse.

(C) As mentioned in above statement, the intersection point defines their time and height. Hence, after 6 seconds at a height of 19 m, the cat will catch the mouse.

After 6 seconds at a height of 19 m.

58. Find the solution of the pair of equations :

$$\frac{3}{x} + \frac{8}{y} = -1; \ \frac{1}{x} - \frac{2}{y} = 2, x, y \neq 0$$

[CBSE 2019]

Ans. Given, pair of equation is—

$$\frac{3}{x} + \frac{8}{y} = -1 \quad \text{...(i)}$$

$$\frac{1}{x} - \frac{2}{y} = 2 \quad \text{...(ii)}$$

If we multiply equation (ii) by 3 and subtract it from equation (i), we get

$$\frac{3}{x} + \frac{8}{y} = -1$$

$$\frac{3}{x} - \frac{6}{y} = 6$$

$$- \quad + \quad -$$

$$\frac{14}{y} = -7$$

$\Rightarrow$ $y = -2$

Put the value of $y = -2$ in equation (i), we get

$$\frac{3}{x} + \left(\frac{8}{-2}\right) = -1$$

$$\Rightarrow \quad \frac{3}{x} = -1 + 4$$

$$\Rightarrow \quad x = 1$$

Hence, the value of *x* and *y* are 1 and – 2 respectively.

59. Ratio between the girls and boys in a class of 40 students is 2 : 3. Five new students joined the class. How many of them must be boys so that the ratio between girls and boys becomes 4 : 5? [Diksha]

Ans. Let number of girls = $2x$

and number of boys = $3x$

Total students, $2x + 3x = 40$

$x = 8$

So, number of girls = $2 \times 8 = 16$

and number of boys = $3 \times 8 = 24$

Let out of 5 students, *y* denotes number of boys.

Then, number of girls = $5 - y$

According to question,

$$\frac{16+5-y}{24+y} = \frac{4}{5}$$

$$5(21 - y) = 4(24 + y)$$

$$105 - 5y = 96 + 4y$$

$$9y = 105 - 96$$
$$9y = 9$$
$$y = 1$$

So, there must be one boy among five new students.

60. **Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present ? [CBSE 2019]**

Ans. Let the present age of Sumit's son be 'x' years and the present age of Sumit be 'y' years.

According to the given conditions:

$$y = 3x$$

or $\quad -3x + y = 0 \quad$ ...(i)

Five years later:

Sumit's son's age = $(x + 5)$ years

Sumit's age = $(y + 5)$ years

$\therefore \quad (y + 5) = 2\frac{1}{2}(x + 5)$

$\Rightarrow \quad 2(y + 5) = 5(x + 5)$

$\Rightarrow \quad 2y + 10 = 5x + 25$

$\Rightarrow \quad -5x + 2y = 15 \quad$ ...(ii)

If we multiply equation (i) by 2 and subtract the equation (ii) from (i), we get

$$-6x + 2y = 0$$
$$-5x + 2y = 15$$
$$+ \quad - \quad -$$
$$-x = -15$$

or $\quad x = 15$

If we put the value of x in equation (i), we get

$$-3 \times 15 + y = 0$$

$\Rightarrow \quad y = 45$

Hence, Sumit's present age is 45 years and Sumit's son's present age is 15 years.

61. **A two digit number is 4 times the sum of the digits. It is also equal to 3 times the product of the digits. Find the number. [CBSE 2016]**

Ans. Let the digit at unit's place be x and at ten's place be 'y'.

Then, the number is $10y + x$

According to the question,

$$(10y + x) = 4(x + y)$$

$\Rightarrow \quad 10y + x = 4x + 4y$

$\Rightarrow \quad 6y - 3x = 0$

$\Rightarrow \quad -x + 2y = 0$

$\Rightarrow \quad x = 2y \quad$ ...(i)

and $\quad 10y + x = 3xy$

$\therefore \quad 10y + 2y = 3 \times 2y \times y \quad$ [from (i)]

$\Rightarrow \quad 12y = 6y^2$

$\Rightarrow \quad y = 2$

$\therefore \quad x = 4$

Hence, the number is 24.

62. **A and B each has a certain number of mangoes. A says to B, "If you give 30 of your mangoes, I will have twice as many as left with you." B replies "If you give me 10, I will have thrice as many as left with you". How many mangoes does each have? [Diksha]**

Ans.

Let number of mangoes with A be x.

Number of mangoes with B be y.

According to the question,

$$x + 30 = 2(y - 30)$$
$$x + 30 = 2y - 60$$

$x - 2y = -90 \quad$ ...(i)

and $\quad y + 10 = 3(x - 10)$

$$y + 10 = 3x - 30$$

$3x - y = 40 \quad$ ...(ii)

Multiplying equation (ii) by 2 and subtracting from equation (i), we get

$$-5x = -170$$
$$x = 34$$

Putting $x = 34$ in equation (i), we get

$$34 - 2y = -90$$
$$-2y = -90 - 34$$
$$-2y = -124$$
$$y = 62$$

So, number of mangoes with A are 34 and number of mangoes with B are 62.

LONG ANSWER Type Questions

[**4** marks]

63. **Determine, algebraically, the vertices of the triangle formed by the lines**

$3x - y = 3$,

$2x - 3y = 2$ and

$x + 2y = 8$ [NCERT]

Ans. The given equation of lines are:

$3x - y = 3 \quad$ ...(i)

$2x - 3y = 2 \quad$ ...(ii)

$x + 2y = 8 \quad$ ...(iii)

Let lines (i), (ii) and (iii) represent the side of a ΔABC i.e., AB, BC and CA respectively.

On solving lines (i) and (ii), we will get the intersection point B.

Multiplying eq. (i) by 3 and then subtracting eq. (ii), we get

$\Rightarrow (9x - 3y) - (2x - 3y) = 9 - 2$

$\Rightarrow \quad 7x = 7$

$\Rightarrow \quad x = 1$

Putting the value of x in eq. (i), we get

$\Rightarrow \quad 3 \times 1 - y = 3$

$\Rightarrow \quad y = 0$

Hence, the coordinate of point or vertex B is (1, 0).

On solving lines (ii) and (iii), we will get the intersection point C.

Multiplying eq. (iii) by 2 and then subtracting eq. (ii), we get

$(2x + 4y) - (2x - 3y) = 16 - 2$

$\Rightarrow \quad 7y = 14$

$\Rightarrow \quad y = 2$

Putting the value of y in eq. (iii), we get

$\Rightarrow \quad x + 2(2) = 8$

$\Rightarrow \quad x = 4$

Hence, the coordinate of point or vertex C is (4, 2).

On solving lines (iii) and (i), we will get the intersecting point A.

Multiplying eq. (i) by 2 and then adding eq. (iii), we get

$(6x - 2y) + (x + 2y) = 6 + 8$

$\Rightarrow \quad 7x = 14$

$\Rightarrow \quad x = 2$

Putting the value of x in eq. (i), we get

$\Rightarrow \quad 3 \times 2 - y = 3$

$\Rightarrow \quad y = 3$

Hence, the coordinate of point or vertex A is (2, 3).

Hence, the vertices of the ΔABC formed by the given lines are A (2, 3), B(1, 0) and C (4, 2).

64. **It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately ? [CBSE 2020]**

Ans. Let two pipes A and B of diameter d_1 and d_2 ($d_1 > d_2$) take x and y hours to fill the pool, respectively. Then,

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12} \quad ...(i)$$

and $$\frac{4}{x} + \frac{9}{y} = \frac{1}{12} \quad ...(ii)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore \quad u + v = \frac{1}{12} \quad ...(iii)$

and $\quad 4u + 9v = \frac{1}{2} \quad ...(iv)$

If we multiply equation (iii) by 4 and subtract it from (iv)

$$4u + 4v = \frac{1}{3}$$

$$4u + 9v = \frac{1}{2}$$

$$-5v = \frac{-1}{6}$$

$\therefore \quad v = \frac{1}{30}$

and $\quad u = \frac{1}{20}$

$\therefore \quad x = 20, y = 30$

Thus, the pipe with diameter d_1 takes 20 hours and the pipe with diamer d_2 takes 30 hours to fill the pool alone.

65. **Draw the graphs of the equations $x = 3$, $x = 5$ and $2x - y - 4 = 0$. Also, find the area of the quadrilateral formed by the lines and the x–axis. [NCERT]**

Ans. The given equation of the lines are

$x = 3 \quad x = 5 \quad$ and $\quad 2x - y - 4 = 0$

For line,

$2x - y - 4 = 0$

$\Rightarrow \quad y = 2x - 4$

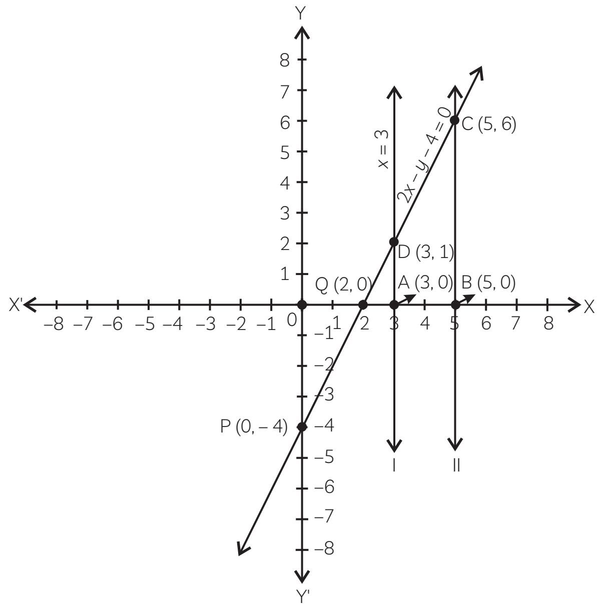
If $x = 0$, $\quad y = -4$

If $x = 2$, $\quad y = 0$

If $x = 4$, $\quad y = 4$

x	0	2	4
y	–4	0	4

Plotting $x = 3$ and $x = 5$ and $2x - y - 4 = 0$, we obtain three lines I, II and III respectively, forming a quadrilateral ABCD with the X-axis as shown below:



From the graph, we get,

$$AB = OB - OA = 5 - 3 = 2$$
$$AD = 2$$
$$BC = 6$$

We know that the quadrilateral ABCD is a trapezium.

Area of Quadrilateral ABCD

$= \frac{1}{2} \times$ (distance between parallel lines) $\times$ (sum of parallel sides)

$= \frac{1}{2} \times (AB) \times (AD + BC)$

$= \frac{1}{2} \times 2 \times (2 + 6)$

= 8 sq. units

Hence, the area of the required quadrilateral is 8 square units.

66. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus. [CBSE 2013, 11]

Ans. Let the speed of the rickshaw and the bus be x km/hr and y km/hr, respectively.

We know that

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

and
$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Case I:

Time taken by Ankita to travel 2 km by rickshaw,

$$t_1 = \frac{2}{x} \text{ hr}$$

Remaining distance = 14 – 2 = 12 km

Time taken by Ankita to travel remaining distance, i.e., 12 km by bus,

$$t_2 = \frac{12}{y} \text{ hr}$$

It is given that:

Total time taken by rickshaw and bus = $\frac{1}{2}$ hr

$$\Rightarrow \quad t_1 + t_2 = \frac{1}{2}$$

$$\Rightarrow \quad \frac{2}{x} + \frac{12}{y} = \frac{1}{2} \quad \text{...(i)}$$

Case II:

Time taken by Ankita to travel 4 km by rickshaw,

$$t_3 = \frac{4}{x} \text{ hr}$$

Remaining distance = 14 – 4 = 10 km

Time taken by Ankita to travel remaining distance i.e., 10 km by bus,

$$t_4 = \frac{10}{y} \text{ hr}$$

It is given that:

Total time taken by rickshaw and bus

$$= \left(\frac{1}{2} + \frac{9}{60}\right) \text{ hr}$$

$$= \left(\frac{1}{2} + \frac{3}{20}\right) \text{ hr}$$

$$t_3 + t_4 = \frac{2}{a+b} \quad \text{hr}$$

$$\frac{4}{x} + \frac{10}{y} = 13/20 \quad \text{...(ii)}$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Then eq. (i) and (ii) become

$$2u + 12v = \frac{1}{2} \quad \text{...(iii)}$$

$$4u + 10v = \frac{13}{20} \quad \text{...(iv)}$$

Multiplying eq. (iii) by 2 and then subtracting eq. (iv), we get

$$(4u + 24v) - (4u + 10v) = 1 - \frac{13}{20}$$

$$\Rightarrow \quad 14v = \frac{7}{20}$$

$$\Rightarrow \quad v = \frac{1}{40}$$

Putting the value of v in eq. (iii),

$$\Rightarrow \quad 2u + 12\left(\frac{1}{40}\right) = \frac{1}{2}$$

$$\Rightarrow \quad 2u = \frac{2}{10}$$

$$\Rightarrow \quad u = \frac{1}{10}$$

$$\Rightarrow \quad x = \frac{1}{u} = 10 \text{ km/hr}$$

$$\Rightarrow \quad y = \frac{1}{v} = 40 \text{ km/hr}$$

Hence, the speed of the rickshaw is 10 km/hr and the speed of bus is 40 km/hr.

67. A motorboat can travel 30 km upstream and 28 km downstream in 7 hrs. It can travel 21 km upstream and return in 5 hrs. Find the speed of the boat in still water and the speed of the stream. [CBSE 2019, 17, 12]

Ans. Let, the speed of the boat in still water = x km/hr

the speed of the stream = y km/hr

∴ The speed of the motorboat upstream = $(x - y)$ km/hr

And the speed of the motorboat downstream = $(x + y)$ km/hr

Case I:

We know that $\quad \text{time} = \frac{\text{distance}}{\text{speed}}$

Time taken by motorboat to travel 30 km upstream,

$$t_1 = \frac{30}{x-y} \text{ hrs}$$

Time taken by motorboat to travel 28 km downstream,

$$t_2 = \frac{28}{x+y} \text{ hrs}$$

According to the given condition,

$$t_1 + t_2 = 7 \text{ hrs}$$

$$\Rightarrow \quad \frac{30}{x-y} + \frac{28}{x+y} = 7 \quad \text{...(i)}$$

Case II:

Time taken by motorboat to travel 21 km upstream,

$$t_3 = \frac{21}{x-y} \text{ hrs}$$

Time taken by motorboat to travel 21 km downstream,

$$t_4 = \frac{21}{x+y} \text{ hrs}$$

According to the given condition,

$$t_3 + t_4 = 5 \text{ hrs}$$

$$\Rightarrow \quad \frac{21}{x-y} + \frac{21}{x+y} = 5 \quad \text{...(ii)}$$

Let $p = \frac{1}{x-y}$ and $q = \frac{1}{x+y}$

Putting these values in eq. (i) and eq. (ii) we get

$$30p + 28q = 7 \quad \text{...(iii)}$$

and $\quad 21p + 21q = 5$

$$\Rightarrow \quad p + q = \frac{5}{21} \quad \text{...(iv)}$$

Multiplying eq. (iv) by 28 and subtracting from eq. (iii), we get

$$(30p + 28q) - (28p + 28q) = 7 - \frac{140}{21}$$

$$\Rightarrow \quad 2p = 7 - \frac{20}{3}$$

$$\Rightarrow \quad 2p = \frac{1}{3}$$

$$\Rightarrow \quad p = \frac{1}{6}$$

Putting the value of p in eq. (iv), we get

$$\Rightarrow \quad \frac{1}{6} + q = \frac{5}{21}$$

$$\Rightarrow \quad q = \frac{5}{21} - \frac{1}{6}$$

$$= \frac{10-7}{42} = \frac{3}{42}$$

$$\Rightarrow \quad q = \frac{1}{14}$$

We know that

$p = \frac{1}{x-y}$ and $q = \frac{1}{x+y}$

$$\frac{1}{x-y} = \frac{1}{6}$$

$$(x - y) = 6 \quad \text{...(v)}$$

$$\frac{1}{x+y} = \frac{1}{14}$$

$$(x + y) = 14 \quad \text{...(vi)}$$

Adding eq. (v) and (vi), we get

$$2x = 20$$

$$x = 10$$

Putting the value of x in eq. (v), we get

$$10 - y = 6$$

$$y = 4$$

Hence, the speed of the motorboat in still water is 10 km/hr and the speed of the stream is 4 km/hr.

68. A shopkeeper sells a saree at a profit of 8% and a sweater at a discount of 10%, thereby getting a sum ₹ 1008. If she had sold the saree at a profit of 10% and the sweater at a discount of 8%, she would have got ₹ 1028. Find the cost of the saree and the list price (price before discount) of the sweater. [NCERT]

Ans. Let the cost price of a saree = ₹ x

and the list price of sweater = ₹ y

Case I:

(S. P. of saree at 8% profit) + (S.P. of a sweater at 10% discount) = ₹1008

$\Rightarrow (100 + 8)$ % of x + $(100 - 10)$ % of y = 1008

$$\Rightarrow \quad \frac{108x+90y}{100} = 1008$$

$$\Rightarrow \quad 108x + 90y = 100800$$

$$\Rightarrow \quad 6x + 5y = 5600$$

Multiplying above eq. by 46, we get

$$\Rightarrow \quad 276x + 230y = 257600 \quad \text{...(i)}$$

Case II:

(S.P. of saree at 10% profit) + (S.P. of a sweater at 8% discount) = ₹1028

$\Rightarrow (100 + 10)$% of x + $(100-8)$% of y = 1028

$\Rightarrow$110% of x + 92% of y = 1028

$\Rightarrow \quad 110x + 92y = 102800$

$\Rightarrow \quad 55x + 46y = 51400$

Multiplying above eq. by 5, we get

$\Rightarrow \quad 275x + 230y = 257000 \quad$ ...(ii)

Subtracting eq. (ii) from eq. (i), we get

$\Rightarrow (276x + 230y) - (275x + 230y)$

$= 257600 - 257000$

$x = 600$

Putting the value of x in the above equation, we get

$6x + 5y = 5600$

$\Rightarrow \quad 6(600) + 5y = 5600$

$\Rightarrow \quad 5y = 5600 - 3600$

$\Rightarrow \quad y = \frac{2000}{5}$

$\Rightarrow \quad y = 400$

Hence, the cost price of the saree and the list price (price before discount) of the sweater are ₹ 600 and ₹ 400, respectively.

69. Ruhi invested a certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

Ans. Let the money invested in scheme A = ₹ x

and the money invested in scheme B = ₹ y

Case I:

Ruhi invested ₹ x at 8% p.a. + Ruhi invested ₹ y at 9% p.a. and received ₹1860 as annual interest.

We know that simple interest,

$$SI = \frac{\text{Principle} \times \text{Rate} \times \text{Time}}{100}$$

Interest earned when ₹ x invested at 8% per annum on scheme A,

$$SI_1 = \frac{x \times 8 \times 1}{100} = \frac{8x}{100}$$

Interest earned when ₹ y invested at 9% per annum on scheme B,

$$SI_2 = \frac{y \times 9 \times 1}{100} = \frac{9y}{100}$$

Interest at 8% per annum on scheme A + Interest at 9% per annum on scheme B = 1860

$\Rightarrow \quad \frac{8x}{100} + \frac{9y}{100} = 1860$

$\Rightarrow \quad 8x + 9y = 186000 \quad$ ...(i)

Case II:

Ruhi invested ₹ y at 8% p.a. + Ruhi invested ₹ x at 9% p.a. and received ₹ (1860 + 20) as annual interest.

Interest earned when ₹ y invested at 8% per annum on scheme A,

$$SI_1 = \frac{y \times 8 \times 1}{100} = \frac{8y}{100}$$

Interest earned when ₹ x invested at 9% per annum on scheme B,

$$SI_2 = \frac{x \times 9 \times 1}{100} = \frac{9x}{100}$$

Interest at 8% per annum on scheme A + Interest at 9% per annum on scheme B = 1880

$\Rightarrow \quad \frac{8y}{100} + \frac{9x}{100} = 1880$

$\Rightarrow \quad 9x + 8y = 188000 \quad$ ...(ii)

Multiplying eq. (i) by 9 and eq. (ii) by 8 and then subtracting them, we get

$(72x + 81y) - (72x + 64y)$

$= 1674000 - 1504000$

$\Rightarrow \quad 81y - 64y = 170000$

$\Rightarrow \quad 17y = 170000$

$\Rightarrow \quad y = 10000$

Putting the value of y in eq. (i), we get

$\Rightarrow 8x + 9(10000) = 186000$

$\Rightarrow \quad 8x = 186000 - 90000$

$\Rightarrow \quad 8x = 96000$

$\Rightarrow \quad x = 12000$

Hence, Ruhi invested ₹ 12000 and ₹ 10000 in schemes A and B respectively.

70. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with a larger diameter takes 2 hours less than the tap with the smaller one to fill the tank separately. Find the time in which each tap can fill the tank. [CBSE 2019]

Ans. Let the smaller tap fills the tank in x hrs.

$\therefore$ The larger tap fills the tank in $(x - 2)$ hrs.

Time taken by both the taps together:

$$= \frac{15}{8} \text{ hrs.}$$

Now, work done by the smaller tap in an hour:

$$= \frac{1}{x}$$

Work done by the larger tap in an hour:

$$= \frac{1}{x - 2}$$

Now, according to the given condition:

$$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$$\Rightarrow \quad \frac{(x-2)+x}{x(x-2)} = \frac{8}{15}$$

$$\Rightarrow \quad \frac{2x-2}{x^2-2x} = \frac{8}{15}$$

$$\Rightarrow \quad 8x^2 - 16x = 30x - 30$$

$$\Rightarrow \quad 8x^2 - 46x + 30 = 0$$

$$\Rightarrow \quad 4x^2 - 23x + 15 = 0$$

$$\Rightarrow \quad (4x-3)(x-5) = 0$$

$x \neq \frac{3}{4}$ (not possible)

$\therefore x = 5$

Hence, the smaller tap will fill the tank in 5 hours.

The larger tap will fill the tank in (5 – 2) = 3 hours.

71. Rahul had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

Ans. Let the number of bananas in lot A = x

and the number of bananas in lot B = y

Case I:

Sold the first lot at the rate of ₹ 2 for 3 bananas + Sold the second lot at the rate of ₹ 1 per banana = Amount received

S.P. of 3 bananas of lot A = ₹ 2

$\Rightarrow$ S.P. of 1 banana of lot A = ₹ $\frac{2}{3}$

$\Rightarrow$ S.P. of x bananas of lot A = ₹ $\frac{2x}{3}$

S.P. of 1 banana of lot B = ₹ 1

$\Rightarrow$ S.P. of y bananas of lot B = ₹ y

As per given condition

$$\frac{2x}{3} + y = 400$$

$$2x + 3y = 1200 \quad \text{...(i)}$$

Case II:

Sold the first lot at the rate of ₹ 1 per banana + Sold the second lot at the rate of ₹ 4 for 5 bananas = Amount received

S.P. of 1 banana of lot A = ₹ 1

$\Rightarrow$ S.P. of x bananas of lot A = ₹ x

S.P. of 5 bananas of lot B = ₹ 4

$\Rightarrow$ S.P. of 1 banana of lot B = ₹ $\frac{4}{5}$

$\Rightarrow$ S.P. of y bananas of lot B = ₹ $\frac{4y}{5}$

As per the given condition

$$x + \frac{4y}{5} = 460$$

$$5x + 4y = 2300 \quad \text{...(ii)}$$

Multiplying eq. (i) by 4 and eq. (ii) by 3 and then subtracting them, we get

$$(8x + 12y) - (15x + 12y) = 4800 - 6900$$

$$\Rightarrow \quad -7x = -2100$$

$$\Rightarrow \quad x = 300$$

Putting the value of x in eq. (i), we get

$$2x + 3y = 1200$$

$$\Rightarrow \quad 2(300) + 3y = 1200$$

$$\Rightarrow \quad 3y = 1200 - 600$$

$$\Rightarrow \quad 3y = 600$$

$$\Rightarrow \quad y = 200$$

Total number of bananas

= Number of bananas in lot A

+ Number of bananas in lot B

= $(x + y)$

= (300 + 200)

= 500

Hence, the total number of bananas he had is 500.

72. The angles of a cyclic quadrilateral ABCD are $\angle A = (6x + 10)°$, $\angle B = (5x)°$, $\angle C = (x + y)°$ and $\angle D = (3y - 10)°$.

Find x and y and hence the values of the four angles. [NCERT]

Ans. It is given that,

$$\angle A = (6x + 10)°$$

$$\angle B = (5x)°$$

$$\angle C = (x + y)° \text{ and}$$

$$\angle D = (3y - 10)°.$$

We know that by property of cyclic quadrilateral:

Sum of opposite angles = 180°

$$\angle A + \angle C = 180°$$

$$\Rightarrow \quad (6x + 10) + (x + y) = 180°$$

$$\Rightarrow \quad 6x + 10 + x + y = 180°$$

$$\Rightarrow \quad 7x + y = 170° \quad \text{...(i)}$$

Also, $\quad \angle B + \angle D = 180°$

$$\Rightarrow \quad 5x + (3y - 10) = 180°$$

$$\Rightarrow \quad 5x + 3y = 180° + 10°$$

$\Rightarrow \quad 5x + 3y = 190°$ ...(ii)

Multiplying eq. (i) by 3 and then subtracting eq. (ii) from it, we get

$$3(7x + y) - (5x + 3y) = 3(170) - 190$$

$$\Rightarrow \quad 16x = 320$$

$$\Rightarrow \quad x = 20°$$

Putting the value of $x = 20°$ in eq. (i), we get

$$7(20) + y = 170$$

$$\Rightarrow \quad y = 30°$$

$$\angle A = (6x + 10)°$$
$$= (6 \times 20 + 10)°$$
$$= (120 + 10)° = 130°$$

$$\angle B = (5x)°$$
$$= (5 \times 20)°$$
$$= 100°$$

$$\angle C = (x + y)°$$
$$= (20 + 30)°$$
$$= 50°$$

$$\angle D = (3y - 10)°$$
$$= (3 \times 30 - 10)°$$
$$= (90 - 10)°$$
$$= 80°$$

Hence, the required values of x and y are 20° and 30° respectively, and the values of the four angles i.e., $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are 130°, 100°, 50°, and 80° respectively.