Chapter 5

Arches, Cables, Matrix Methods

CHAPTER HIGHLIGHTS

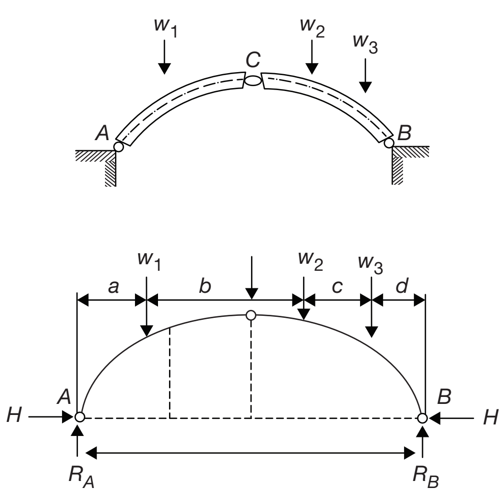
- *Introduction*
- *Arches*
- *Cables*
- *Matrix method of structural analysis*

Introduction

In this chapter the concept on arches, cables and matrix method of structural analysis are discussed.

Arches

Three-hinged Arch



Three-hinged arch

The figure shows a three-hinged arch with two hinges A and B at supports and the third hinge usually at the highest point of arch known as crown.

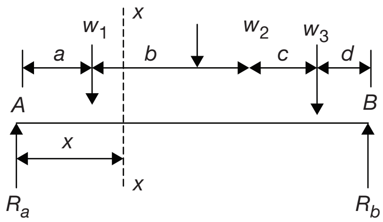
- The hinges, A and B may or may not be at the same level.
- The height of the crown (highest point of arch) above the level of two hinges A and B, when they are at same level is called the rise of arch.
- The horizontal distance between lower hinges (hinges at A and B) is called the span of the arch.

Calculation of Reactions

- When the lower hinges are at same level, the vertical reactions R_A and R_B are calculated similar to that of a simply supported beam of the same span carrying the same load.
- The horizontal component of the reaction at either lower end is called the horizontal thrust at the support.
- When the loading on arch is entirely vertical, the horizontal thrust at each of support must be same.
- The horizontal thrust 'H' can be computed by equating the bending moment at the crown hinge 'C' to zero.
- The bending moment at the section X of the arch is given by (for the given figure)

$$\boxed{M_x = R_a x - w_1(x-a) - Hy}$$

For a simply supported beam of same loading, the bending moment at the same section X would be,



$$M = R_a x - w_1(x-a)$$

Therefore,

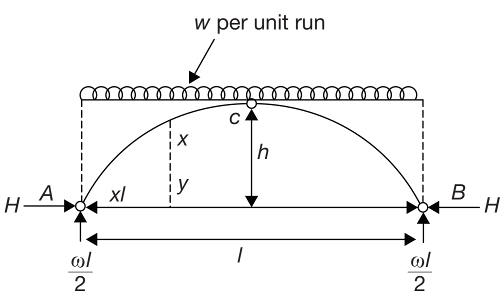
$$\text{Arch moment} = \text{Beam moment} - Hy$$

NOTE

The sectional requirement for an arch is less than that of a beam due to the less bending moment.

Special Cases

1. A three-hinged parabolic arch of span 'l' and rise 'h' carries a uniformly distributed load of 'w' per unit run over the span.



- Equation to the arch with the end A as origin is,

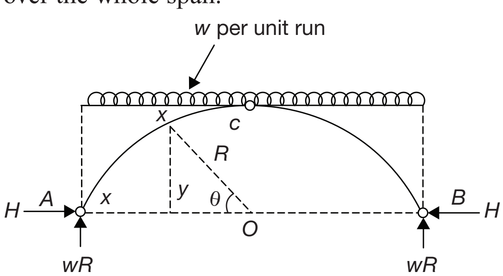
$$y = \frac{4h}{l^2}(l-x)$$

- Horizontal thrust at each support,

$$H = \frac{wl^2}{8h}(l-x)$$

- In this case, arch is not subject to any bending moment at any section.

2. A three hinged semi-circular arch of the radius 'R' carries a uniformly distributed load of w per unit run over the whole span.



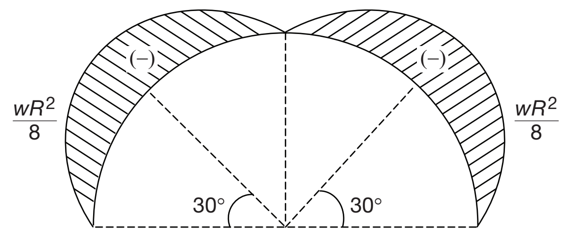
- Horizontal thrust at each support,

$$H = \frac{wR}{2}$$

- The bending moment at any section x,

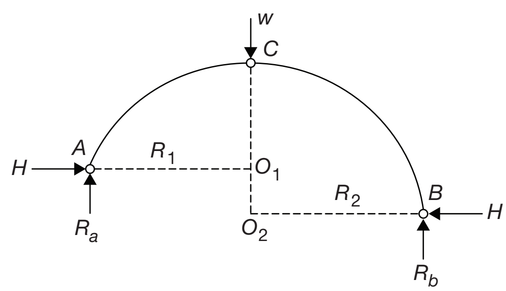
$$M_x = w\,Rx - \frac{wx^2}{2} - Hy$$

- The maximum bending moment occurs at $\theta = 30°$ and its value is $\frac{wR^2}{8}$.
- Distance of point of maximum bending moment from the crown $= R\cos 30° = \frac{R\sqrt{3}}{2}$.



Bending moment diagram

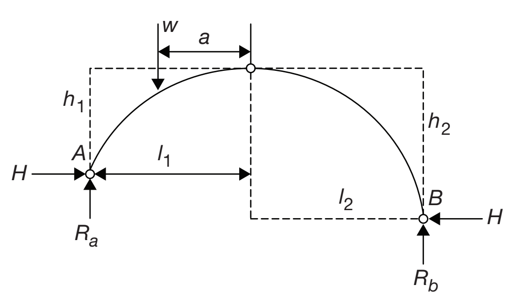
3. A three-hinged arch consisting of two quandrantal parts AC and CB of radii R_1 and R_2 carrying a concentrated load 'w' on the crown as shown in the figure below.



In this case,

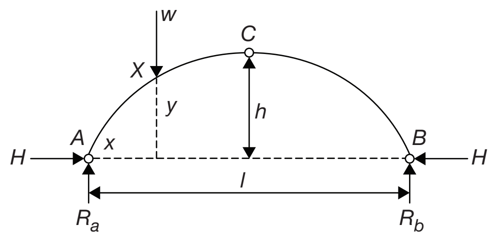
- Horizontal thrust at each support, $H = \frac{w}{2}$
- Reactions, $R_a = R_b = \frac{w}{2}$

4. The horizontal thrust 'H' at each support for the three hinged arch as shown in the figure below is:



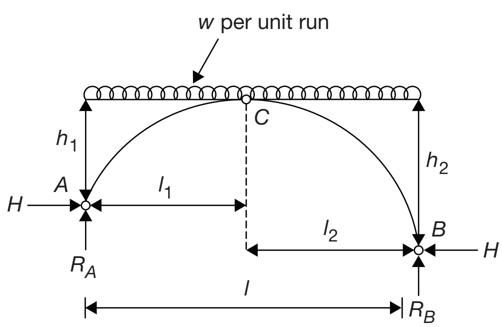
$$\boxed{H = \frac{wl_2(l_1 - a)}{h_1 l_2 + h_2 l_1}}$$

5. A symmetrical three-hinged parabolic arch of span 'l' and rise 'h' carries a point load 'w' which may be placed anywhere on the span.



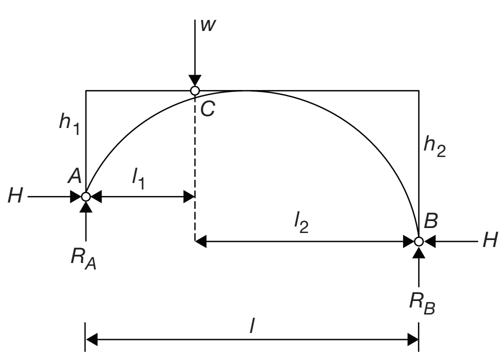
- Horizontal thrust, $\boxed{H = \frac{wx}{2h}}$
- Absolute maximum bending moment occurs at a distance $\frac{l}{2\sqrt{3}}$ on either side of the crown.

6. A three-hinged parabolic arch of span 'l' has its abutments at depth h_1 and h_2 below the crown and carries a uniformly distributed load of 'w' per unit over the whole span as shown in the figure below.



Horizontal thrust, $\boxed{H = \frac{wl^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}}$

7. A three-hinged parabolic arch of span 'l' has its abutments A and B at depths h_1 and h_2 below the crown 'C' and also carries a concentrated load 'w' at the crown as shown in the figure below.

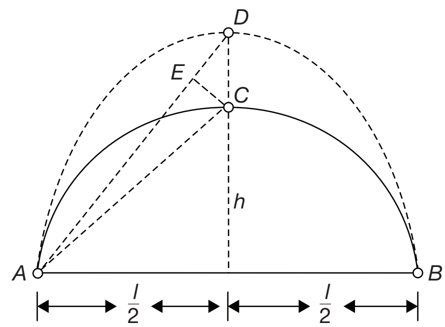


Horizontal thrust at each support,

$$H = \frac{wl}{(\sqrt{h_1} + \sqrt{h_2})^2}.$$

Temperature Effect on Three-hinged Arches

- Increase in temperature causes an increase in the length of the arch since the hinge at crown is not connected to any permanent object.



Increase in the rise of the arch, (CD)

$$\boxed{\delta = \frac{l^2 + 4h^2}{4h}\alpha T}$$

Effect of Temperature Rise on the Horizontal Thrust

- Stresses are not produced in the arch due to the temperature change alone.
- As there is a rise in arch due to the temperature change, the horizontal thrust for the arch already carrying a load will also alter.
- The decrease in the horizontal thrust due to the rise in temperature is given by:

$$\boxed{dH = -\frac{dh}{h}(H)}$$

Where

dh = Increase in rise of arch due to rise in temperature.

$$= \frac{l^2 + 4h^2}{4h}\alpha T$$

h = Rise of arch before the temperature increase.

H = Horizontal thrust due to loading before rise in temperature.

Two-hinged Arches

Two-hinged arch is statically indeterminate to first degree.

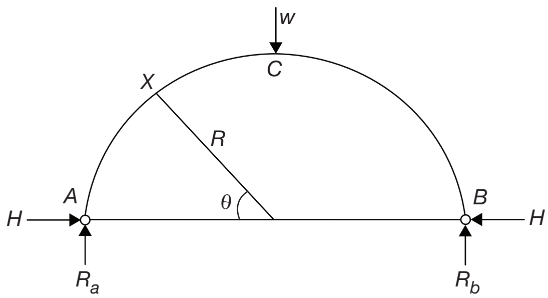
Support reactions:

- The vertical reactions can be determined by taking moments about either hinge.
- The horizontal thrust at each support may be determined from the condition that the horizontal displacement of either hinge with respect to the other is zero and is given by:

$$H = \frac{\int \frac{M \cdot y \cdot ds}{EI}}{\int \frac{y^2}{EI} ds}$$

Special Cases

1. A two-hinged semi-circular arch with the load 'w' at the crown as shown below.



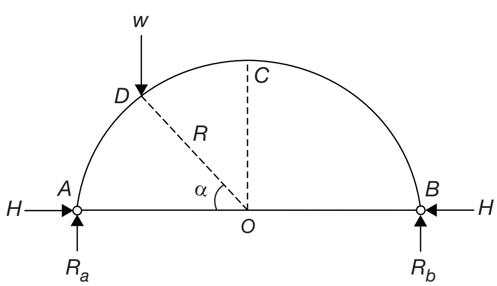
Horizontal thrust, $H = \frac{w}{\pi}$

Horizontal thrust is independent of the magnitude of the radius of arch.

Vertical deflection of the crown,

$$\delta = \frac{wR^3}{8\pi EI}(3\pi^2 - 8\pi - 4)$$

2. A two-hinged semi-circular arch of radius 'R' carries a load 'w' at a section, the radius vector corresponding to which makes an angle 'α' with the horizontal as shown below.

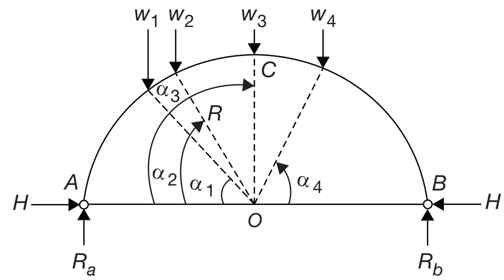


Horizontal thrust at each support,

$$H = \frac{w}{\pi}\sin^2\alpha$$

When $\alpha = \frac{\pi}{2}; H = \frac{w}{\pi}$

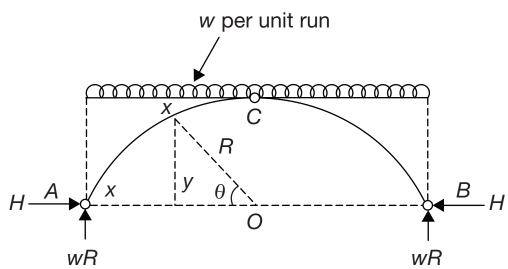
3. When a semi-circular two-hinged arch is subjected to loads w_1, w_2, w_3, w_4, … at section corresponding which the radius vector making an angle α_1, α_2, α_3, α_4 with the horizontal as shown in the following figure.



Horizontal thrust at each support,

$$H = \Sigma \frac{w}{H}\sin^2\alpha$$

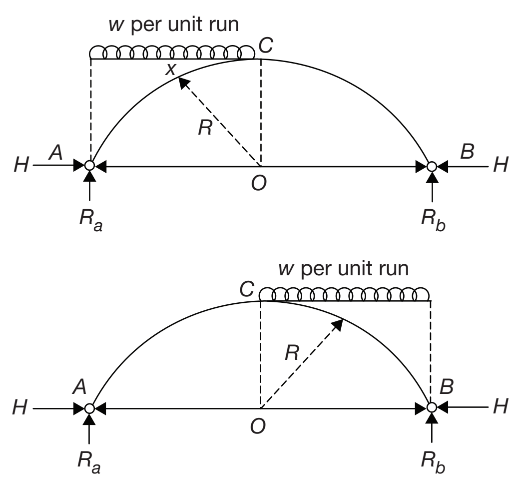
4. A two-hinged semi-circular arch of radius 'R' carries a uniformly distributed load of 'w' per unit run over the whole span as shown below.



Horizontal thrust at each support is given by

$$H = \frac{4}{3} \cdot \frac{wR}{\pi}$$

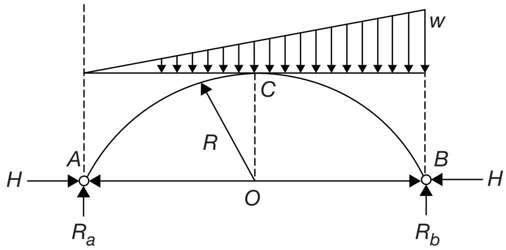
5. A two-hinged semi-circular arch carries a uniformly distributed load of 'w' per unit run over the left half or over the right half of its span as shown below.

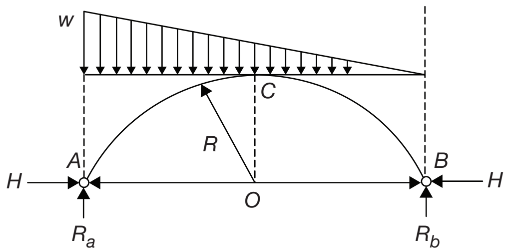


Horizontal thrust at each support when one half of the span is loaded,

$$H = \frac{2}{3}\frac{wR}{\pi}$$

6. A two-hinged semi-circular arch of radius 'R' carries a uniformly varying load from zero at the one end to 'w' per unit run at other end as shown below.

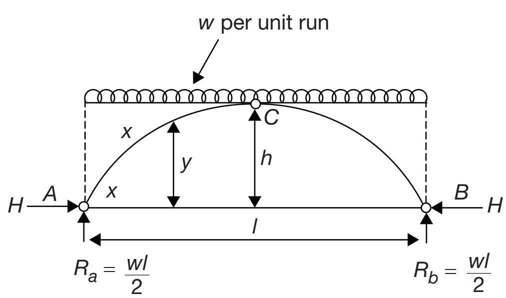




Horizontal thrust at each support due to anyone of the triangular load systems

$$H = \frac{2}{3}\frac{wR}{\pi}$$

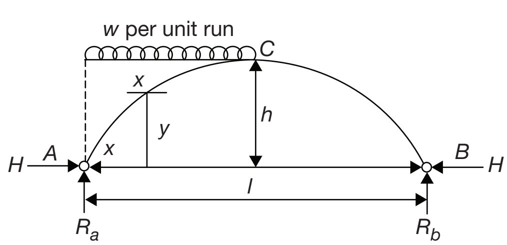
7. A two-hinged parabolic arch of span 'l' and rise 'h' carries a uniformly distributed load of 'w' per unit run over the whole span as shown below.

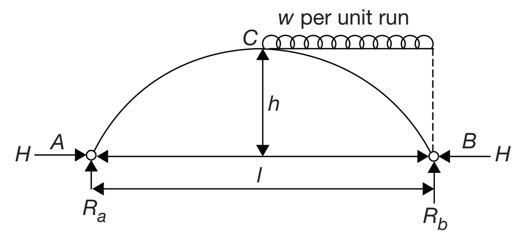


Horizontal thrust at each support,

$$H = \frac{wl^2}{8h}$$

8. A two-hinged parabolic arch carries a uniformly distributed load of 'w' per unit run over the left half or over the right half of its span shown in the following figure.

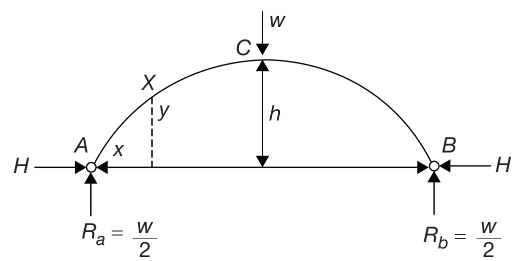




Horizontal thrust at each span when one half of the span is loaded

$$H = \frac{wl^2}{16h}$$

9. A two-hinged parabolic arch of span 'l' and rise 'h' carries a concentrated load 'w' at the crown as shown below.



The horizontal thrust at each support,

$$H = \frac{25}{128}\frac{wl}{h}$$

Temperature Effect on Two-hinged Arches

- Due to the increase in temperature (T), an horizontal thrust 'H' will be developed at each support as the horizontal displacement are not allowed due to presence of hinges at each end.
- Horizontal thrust (H) for the two hinged arch subjected to the rise in temperature is,

$$H = \frac{\alpha T l}{\int \frac{y^2 ds}{EI}}$$

- If the arch section is of uniform rigidity,

$$H = \frac{EI\alpha T l}{\int y^2 ds}$$

Special Cases

1. For a semi-circular two hinged arch:

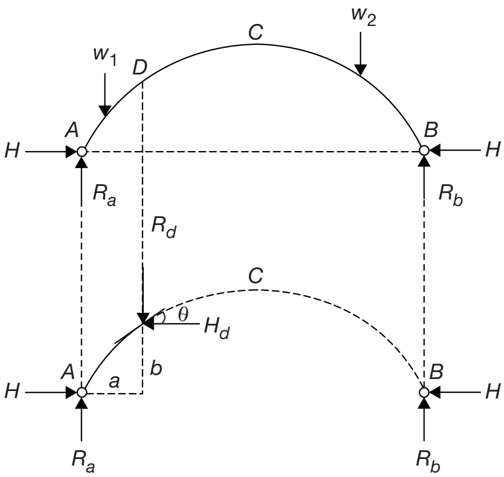
$$H = \frac{4EI\alpha T}{\pi R^2}$$

2. For the parabolic two-hinged arch,

$$H = \frac{15}{8}\frac{EI\alpha T}{h^2}$$

Normal Thrust and Radial Shear

Consider the equilibrium of the part '*AD*' of the arch as shown below.



This part is in equilibrium under the action of the following:

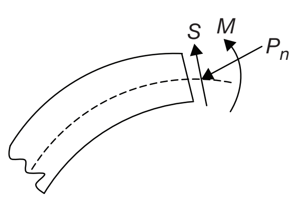
1. Reactions R_a and H at A.
2. External loads between A and D.
3. Reacting forces R_d and H_d providing by the part DB on the part DA at D.
4. Reacting moment (bending moment at D).
   - Let the tangent to the center line of the arch at '*D*' be inclined at θ to the horizontal.
   - The component of reacting forces at '*D*' along the tangent '*D*' is called the normal thrust at *D*.

$$\text{Normal thurst } D = P_n = H_d \cos\theta + R_d \cos\theta$$

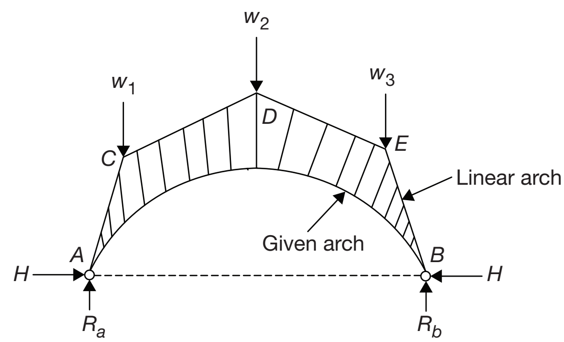
   - The component of reacting forces at '*D*' perpendicular to the tangent at '*D*' is called the radial shear or simply shear at '*D*'.

$$\text{Radial shear at } D = S = H_d \sin\theta - R_d \cos\theta$$

Figure shows the arch sections subjected to normal thrust P_n, radial shear S and bending moment 'M'.



Linear Arch or Theoretical Arch

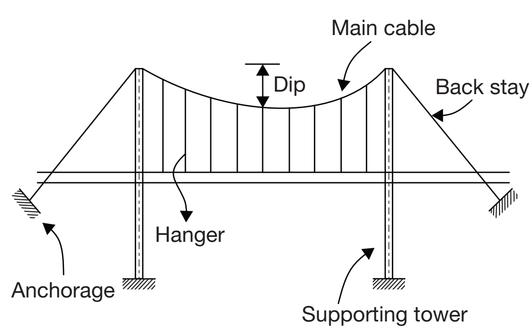


- The structure *ACDEB* is called the linear arch or the theoretical arch.
- The shape of the linear arch follows the shape of the free bending moment diagram for a beam of the same span and subject to same loading.
- The joints of the linear arch are in equilibrium and the different members of the linear arch are subjected to axial compressive forces.
- The bending moment at any section of an arch is proportional to the ordinate or the intercept between the given arch and the linear arch. This principle is called Eddy's theorem.

Cables

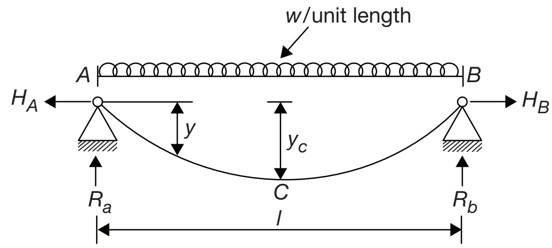
- Cables form the main load carrying element in many structures like suspension bridges, suspension roofs and trolley wheels.
- In case of suspension bridges, the deck loads are transmitted to the cables through closely spaced hangers.
- If the number of hangers is large, the load transmitted to the cables can be approximately to a uniformly distributed load taking the shape of parabola.
- A cable under the given loading takes the shape of a funicular polygon which to some extent represents the bending moment diagram of a simple beam.

Following figure shows the schematic diagram of a suspension bridge.



Cable Subjected to Uniformly Distributed Load

- A cable of span 'l' suspended from supports A and B at some level as shown below:



Cable subjected to UDL

- Let 'C' be the lowest point of the cable and the sag of cable at 'C' be y_c.

Reaction components: The vertical reaction components at supports A and B are,

$$R_A = R_B = \frac{\omega l}{2}$$

Horizontal reaction components $H_A = H_B = H$ can be obtained by taking moments about C and setting $M_C = 0$.

$$\boxed{H = \frac{Wl^2}{8y_c}}$$

Equation of the cable:

$$\boxed{y = \frac{4y_c}{l^2}(l-x)}$$

This is a second order parabola.

NOTE

The deflected shape of the cable under its own weight is not exactly a parabola but a catenary or a cosh function.

Tension in the Cable

The maximum cable tension occurs at supports since H is constant all along and vertical reaction R_A or R_B is maximum at supports.

$$T_A = \sqrt{R_A^2 + H^2} \text{ or } T_B = \sqrt{R_B^2 + H^2}$$

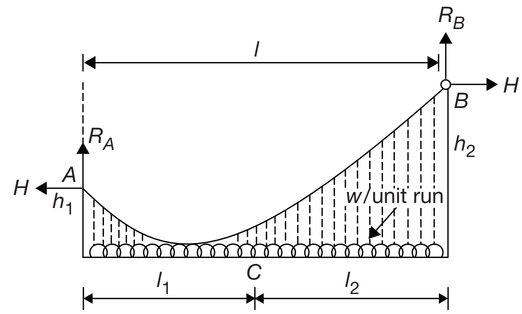
$$T_{\max} = \sqrt{\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8y_c}\right)^2}$$

(occurring near each support)

Minimum tension $= T_{\min} = T_C = H$ (occurring at middle point of cable)

Tension in Cable Supported at Different Levels

- Consider a cable ACB stretched between two supports A and B at different levels and subjected to a uniformly distributed load as shown in the figure below.



Since ACB as a parabolic,

$\frac{x^2}{y}$ = constant with 'C' as a origin.

$$\frac{l_1^2}{h_1} = \frac{l_2^2}{h_2} \Rightarrow \frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\boxed{l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \text{ and } l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}}$$

Reaction components: Let R_A and R_B be the vertical reaction components and H be the horizontal reaction at each support.

Taking moments about 'C',

On LHS, $$\boxed{R_A = H\frac{h_1}{l_1} + \frac{\omega l_1}{2}}$$

On RHS, $$\boxed{R_A = H\frac{h_2}{l_2} + \frac{\omega l_2}{2}}$$

Horizontal reaction can be obtained by adding R_A and R_B,

$$\boxed{H = \frac{\omega l^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}}$$

Tension in the cable at A,

$$\boxed{T_A = \sqrt{R_A^2 + H^2}}$$

Tension in the cable at B,

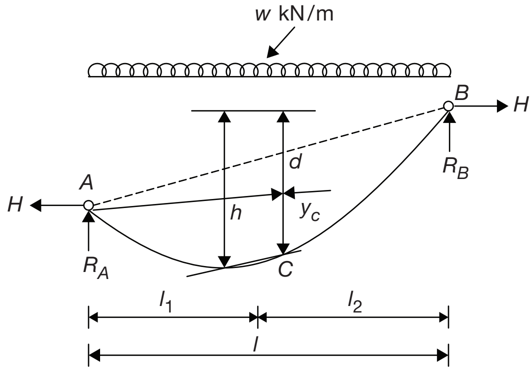
$$\boxed{T_B = \sqrt{R_B^2 + H^2}}$$

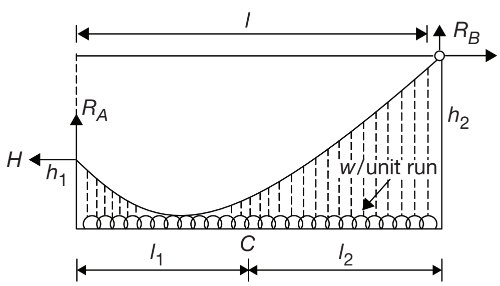
Length of the Cable

1. **Cable supports at the same level:** Total length of the cable when supports are at the same level is given by:

$$L = 1 + \frac{8}{3}\frac{y_c^2}{l}$$

2. Cable supports at different levels:





Cable supported at ends at different levels

Both the figures represent the same with supports at different level.

On observation,

$$h = y_c + d = h_2, y_c = h_1$$

Length of cable, $$L = l + \frac{4}{3}\left(\frac{h^2}{l_1} + \frac{y_c^2}{l_2}\right)$$

Temperature stresses in the cable: Let the span and dip of a cable be 'l' and 'h' respectively.

Due to the rise in temperature t°C,

$$\text{Increase in length of cable, } dL = \alpha t l$$

$$\text{Increase in dip of cable, } dh = \frac{3}{16}\alpha t\frac{l^2}{h}$$

Let df be the change in stress in the cable,

$$\frac{df}{f} = \frac{dH}{H} = -\frac{dh}{h} = -\frac{3}{16} \propto t\frac{l^2}{h^2}$$

Where

f = Stress in the cable $= \frac{T_{\max}}{A}$

dH = Change in the horizontal reaction due to rise of temperature.

dh = Change in dip of cable.

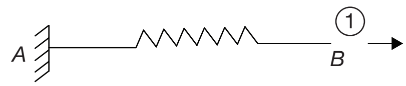
Matrix Method of Structural Analysis

Flexibility and Stiffness

- Flexibility of a structure is defined as the displacement caused by a unit force.
- Stiffness is defined as the force required for a unit displacement.

Flexibility Matrix

Consider a structural element with a single degree of freedom as shown below.



The flexibility of the spring is defined as the displacement δ_{11} at coordinate 1 due to a unit force at coordinate 1.

If a force P, produces a displacement Δ_1 at coordinate 1,

$$\text{Flexibility} = \frac{\Delta_1}{p_1} = f_{11}$$

Stiffness Matrix

For the above case, the stiffness of the spring is defined as the force P_1 required for a unit displacement at coordinate 1.

$$\text{Stiffness} = \frac{P_1}{\Delta_1} = k_{11}$$

NOTES

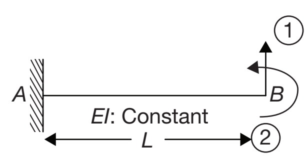
1. The inverse of stiffness matrix is flexibility matrix.
2. The product of stiffness and flexibility is equal to one.

Properties of Flexibility and Stiffness Matrix

1. The stiffness and flexibility matrix are square matrixes of order n.
2. The flexibility and stiffness matrices are symmetrical matrices. This is in accordance with the Maxwells reciprocal theorem.
3. The elements lying on the leading diagonal are always positive.
4. The order of flexibility and stiffness matrix will depend on the number of coordinate assigned, which will depend upon degree of indeterminacy.

Procedure to Develop Flexibility Matrix

Consider a beam with the coordinates marked as shown in the figure.



Step 1: The order of the flexibility matrix will depend on the number of coordinates assigned. In this case number of coordinates is equal to two. Therefore, flexibility matrix is of second order as shown below.

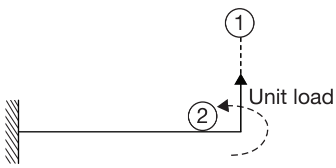
$$F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}_{2\times2}$$

Where

f_{11} = Displacement at (1) due to unit load at (1)

f_{21} = Displacement at (2) due to unit load at (1)

Step 2: In order to develop the first column of flexibility matrix apply unit load in direction of (1) and measure the displacements in direction (1) and (2).



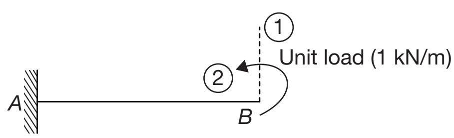
f_{11} = Displacement in direction (1) due to unit load in direction (1)

$$\boxed{f_{11} = \frac{L^3}{3EI}}$$

f_{21} = Displacement in direction (2) due to unit load in direction (1)

$$\boxed{F_{21} = \frac{L^2}{2EI}}$$

Step 3: To generate second column of flexibility matrix apply unit load in direction (2) only and measure displacement in direction (1) and (2).



For a cantilever beam with unit moment at free end,

$$\theta = \frac{ML}{EI} \text{ and } \delta = \frac{ML^2}{2EI}$$

f_{12} = Displacement in direction of (1) due to unit load in direction (2)

$$f_{12} = \frac{ML^2}{2EI} = \frac{L^2}{2EI}[\because M = 1]$$

f_{22} = Displacement in direction of (2) due to unit load in direction of (2).

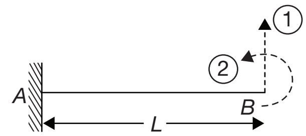
$$f_{22} = \frac{ML}{EI} = \frac{L}{EI}[\because M = 1]$$

Step 4: Flexibility matrix

$$f = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \begin{bmatrix} \frac{L^3}{3EI} & \frac{L^2}{2EI} \\ \frac{L^2}{2EI} & L/EI \end{bmatrix}$$

Procedure to Develop Stiffness Matrix

Consider a beam with the coordinates marked as shown below.



Step 1: The order of stiffness matrix depends on the number of coordinates. In this case, number of coordinates is equal to two therefore, stiffness matrix is of order '2'.

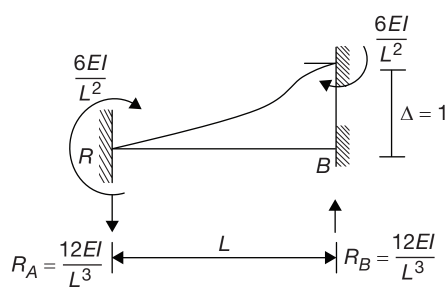
$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}_{2\times2}$$

Step 2: In order to develop the first column of stiffness matrix, given unit displacement in direction (1) Only without any displacement in direction of other coordinates and measure the force developed in (1) and (2).

k_{11} = Force or reaction developed in direction (1) due to unit displacement in direction (1) only.

k_{21} = Force or reaction developed in direction (2) due to unit displacement in direction (1) only.

For k_{11} and k_{21}, restrain the structure at B and given a unit upward displacement as shown below.



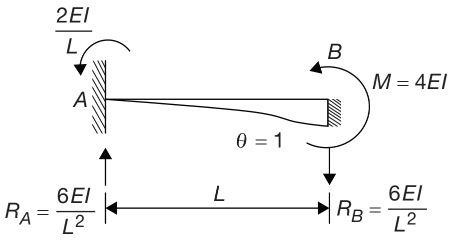
$$\boxed{k_{11} = R_B = \frac{12EI}{L^3}; k_{21} = \frac{-6EI}{L^2}}$$

Step 3: To develop second column of stiffness matrix given unit displacement in direction (2) only without any displacement in other coordinate directions and measure the forces or reactions in coordinates (1) and (2).

k_{12} = Force in direction (1) due to unit displacement in direction (2).

k_{22} = Force in direction (2) due to unit displacement in direction (2).

For k_{12} and k_{22}, provide hinge at B as shown in the following figure.



$$k_{22} = \frac{4EI}{L}$$

$$k_{12} = R_B = \frac{-6EI}{L^2}$$

Step 4: Stiffness matrix

$$[K] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{2EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

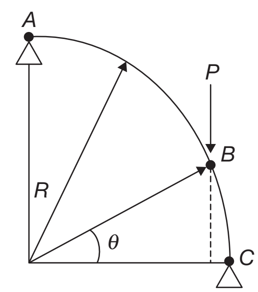
Flexibility and stiffness values of a prismatic member with respect to the four types of displacements

Type of Displacement, Δ	Flexibility	Stiffness
Axial	$\frac{L}{AE}$	$\frac{AE}{L}$
Transverse		
(a) Far end fixed	$\frac{L^3}{12EI}$	$\frac{12EI}{L^3}$
(b) Far end hinged	$\frac{L^3}{3EI}$	$\frac{3EI}{L^3}$
Bending or flexural		
(a) Far end fixed	$\frac{L}{4EI}$	$\frac{4EI}{L}$
(b) Far end hinged	$\frac{L}{3EI}$	$\frac{3EI}{L}$
Torsional	$\frac{L}{GK}$	$\frac{GK}{L}$

Exercises

1. Consider the following statements relating to structural analysis:
I. Flexibility matrix and its transpose are equal.
II. Elements of main diagonal of stiffness matrix are always positive.
III. For unstable structures, coefficients in leading diagonal matrix can be negative.
Which of these statements is/are correct?
(A) I, II and III
(B) I and II only
(C) II and III only
(D) III only

2. A three-hinged arch shown in the figure is quarter of a circle. If the vertical and horizontal components of reaction at A are equal, the value of θ is



(A) 60°
(B) 45°
(C) 30°
(D) None in (0°, 90°)

3. In a linear elastic structural element
(A) stiffness is directly proportional to flexibility.
(B) stiffness is inversely proportional to flexibility.
(C) stiffness is equal to flexibility.
(D) stiffness and flexibility are not related.

4. For linear elastic frame, if stiffness matrix is doubled with respect to the existing stiffness matrix, the deflection of the resulting frame will be
(A) twice the existing value.
(B) half the existing value.
(C) the same as existing value.
(D) indeterminate value.

5. The order for the flexibility matrix for a structure is,
(A) equal to the number of redundant forces.
(B) more than the number of redundant forces.
(C) less than the number of redundant forces.
(D) equal to the number of redundant forces plus three.

6. The stiffness matrix of a beam element is given as $\frac{2EI}{L}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix}$. Then flexibility matrix is

(A) $\frac{L}{2EI}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (B) $\frac{L}{6EI}\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

(C) $\frac{L}{3EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ (D) $\frac{L}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

7. A three-hinged parabolic arch of span 'l' and rise 'h' is subjected to a UDL of intensity 'W', then the horizontal thrust at the supports is

(A) $\frac{Wl^2}{8h}$ (B) $\frac{Wl}{h}$

(C) $\frac{Wl}{8h^2}$ (D) $\frac{Whl}{8}$

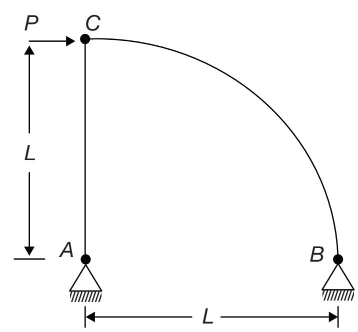
8. The stiffness matrix of a beam element is $\left(\frac{2EI}{L}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Which one of the following is its flexibility matrix?

(A) $\left(\frac{L}{2EI}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (B) $\left(\frac{L}{6EI}\right)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\left(\frac{L}{5EI}\right)\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$ (D) $\left(\frac{L}{6EI}\right)\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

9. Reaction at support 'B' of the structure shown is



(A) P (B) $P\sqrt{2}$

(C) $\frac{P}{\sqrt{2}}$ (D) $\frac{P}{2}$

10. A two-hinged semicircular arch of radius R carries a concentrated load W at the crown. The horizontal thrust is

(A) $\frac{W}{2\pi}$ (B) $\frac{W}{\pi}$

(C) $\frac{2W}{3\pi}$ (D) $\frac{4W}{3\pi}$

11. **Assertion (A):** Any arch cannot practically be built to the shape of the theoretical arch.

Reason (R): The shape of the theoretical arch is affected by loads moving on it.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not a correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

12. Match List I with List II and select the correct answer using the codes given below the lists:

	List I		List II
a.	The shape of a cable suspended between two supports is defined by	**1.**	a catenary
b.	The cable uniformly loaded along the horizontal span takes the shape of	**2.**	a little load
c.	The cable uniformly loaded along its length assumes the shape of	**3.**	a parabola
d.	The grider in a suspension bridge transmits to its supports	**4.**	applied loads

Codes:

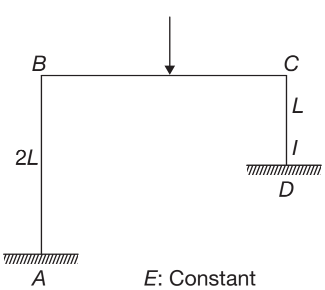
	a	b	c	d		a	b	c	d
(A)	1	2	4	3	(B)	1	3	4	2
(C)	4	3	1	2	(D)	4	2	1	3

13. A two-hinged semicircular arch of radius R varies a concentrated load W at crown. The horizontal thrust is _______.

(A) $\frac{W}{2\pi}$ (B) $\frac{W}{\pi}$

(C) $\frac{2W}{3\pi}$ (D) $\frac{4W}{3\pi}$

14. A rigid-jointed plane frame shown in the figure _______.



(A) will not sway
(B) will sway to left
(C) will sway to right
(D) None of these

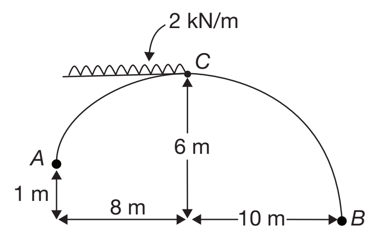
15. A three-hinged parabolic arch is carrying UDL of 10 kN/m over its entire span. At any section the arch is subjected to _______.

I. normal thrust
II. SF and normal thrust
III. BM
IV. SF and BM

Which of these statements is/are correct?

(A) Only I
(B) II and III
(C) Only II
(D) Only IV

16. The horizontal thrust at support A in a three-hinged arch shown in the figure is _______.



(A) 4.5 kN (B) 5.5 kN
(C) 6 kN (D) 6.5 kN

17. If the flexibility matrix for a beam is written as

$$[A] = \frac{L^3}{6EI}\begin{bmatrix} 2 & 5 \\ 5 & 16 \end{bmatrix}$$

What is the corresponding stiffness matrix?

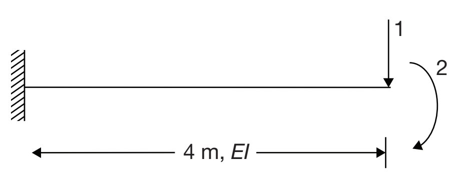
(A) $\frac{6EI}{L^3}\begin{bmatrix} 16 & -5 \\ 5 & 2 \end{bmatrix}$ (B) $\frac{6EI}{7L^3}\begin{bmatrix} 16 & 5 \\ -5 & 2 \end{bmatrix}$

(C) $\frac{6EI}{L^3}\begin{bmatrix} 16 & -5 \\ -5 & 2 \end{bmatrix}$ (D) $\frac{6EI}{7L^3}\begin{bmatrix} 16 & -5 \\ -5 & 2 \end{bmatrix}$

18. A cable carrying a load of 10 kN/m run of horizontal span, is stretched at supports 150 m apart. The supports are at same level and the central dip of is 10 m. Find the greatest tension developed in cable.

(A) 750 kN
(B) 2810 kN
(C) 2910 kN
(D) 3510 kN

19. The flexibility matrix of the beam shown in the figure is _______.



(A) $\begin{bmatrix} \frac{64}{3EI} & \frac{-8}{EI} \\ \frac{-8}{EI} & \frac{64}{EI} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{64}{3EI} & \frac{8}{EI} \\ \frac{8}{EI} & \frac{16}{EI} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{64}{3EI} & \frac{-8}{EI} \\ \frac{-8}{EI} & \frac{4}{EI} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{64}{3EI} & \frac{8}{EI} \\ \frac{8}{EI} & \frac{4}{EI} \end{bmatrix}$

20. In a two-hinged arch an increase in temperature induces
(A) maximum bending at the crown.
(B) uniform bending moment in the arch rib.
(C) no bending moment in the arch rib.
(D) maximum bending moment at hinges.

21. A symmetrical three-hinged parabolic arch of span L and rise h is hinged at springing and crown. It is subjected to a UDL W throughout the span. What is the bending moment at a section $L/4$ from the left support?
(A) $WL^2/16$ (B) $WL^3/8h$
(C) zero (D) $WL^2/8$

22. The flexibility matrix of a beam element is given as $\left(\frac{L}{6EI}\right)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Then the stiffness matrix is ______.

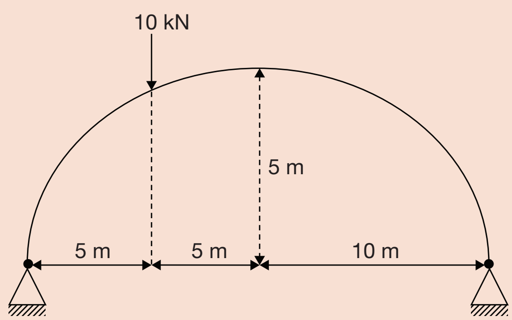
(A) $\left(\frac{2EI}{L}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (B) $\left(\frac{L}{2EI}\right)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\frac{2EI}{L}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ (D) $\frac{4EI}{3L}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

Previous Years' Questions

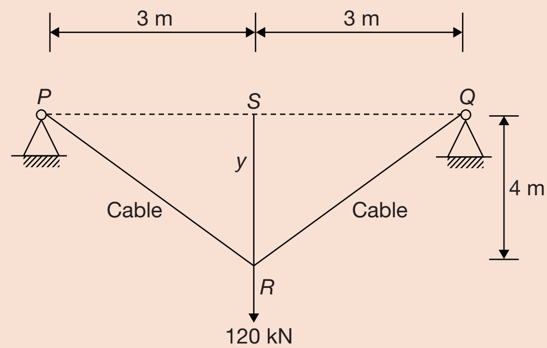
1. The stiffness coefficient k_{ij} indicates **[GATE, 2007]**
(A) force at i due to a unit deformation at j.
(B) deformation at j due to a unit force at i.
(C) deformation at i due to a unit force at j.
(D) force at j due to a unit deformation at i.

2. A three-hinged parabolic arch having a span of 20 m and a rise of 5 m carries a point load of 10 kN at quarter span from the left end as shown in the figure. The resultant reaction at the left support and its inclination with the horizontal are respectively **[GATE, 2010]**



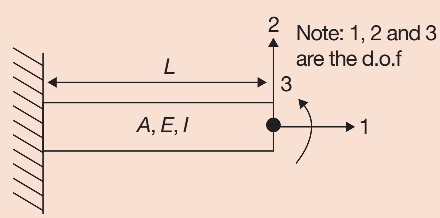
(A) 9.01 kN and 56.31°
(B) 9.01 kN and 33.69°
(C) 7.50 kN and 56.31°
(D) 2.50 kN and 33.69°

3. The tension (in kN) in a 10 m long cable shown in the figure, neglecting its self-weight, is **[GATE, 2014]**



(A) 120 (B) 75
(C) 60 (D) 45

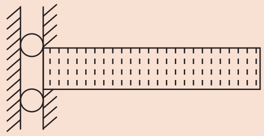
4. For the beam shown below, the stiffness coefficient K_{22} can be written as **[GATE, 2015]**



(A) $\frac{6EI}{L^2}$ (B) $\frac{12EI}{L^3}$

(C) $\frac{3EI}{L}$ (D) $\frac{EI}{6L^2}$

5. A guided support as shown in the figure below is represented by three springs (horizontal, vertical and rotations) with stiffness, k_x, k_y and k_θ respectively. The limiting values of k_x, k_y and k_θ are **[GATE, 2015]**



(A) ∞, 0, ∞
(B) ∞, ∞, ∞
(C) 0, ∞, ∞
(D) ∞, ∞, 0

Answer Keys

Exercises

1. B **2.** D **3.** B **4.** B **5.** A **6.** D **7.** A **8.** B **9.** B **10.** B
11. A **12.** C **13.** B **14.** A **15.** A **16.** D **17.** D **18.** C **19.** D **20.** A
21. C **22.** A

Previous Years' Questions

1. A **2.** A **3.** B **4.** B **5.** A