11th Sept,
THURSDAY

10. STRESS DISTRIBUTION

→ Boussinesq's Theory

* Assumptions :
  - Soil is homogenous.
  - Isotropic soil.
  - Semi infinite.
  - Elastic medium.
  - Point load.

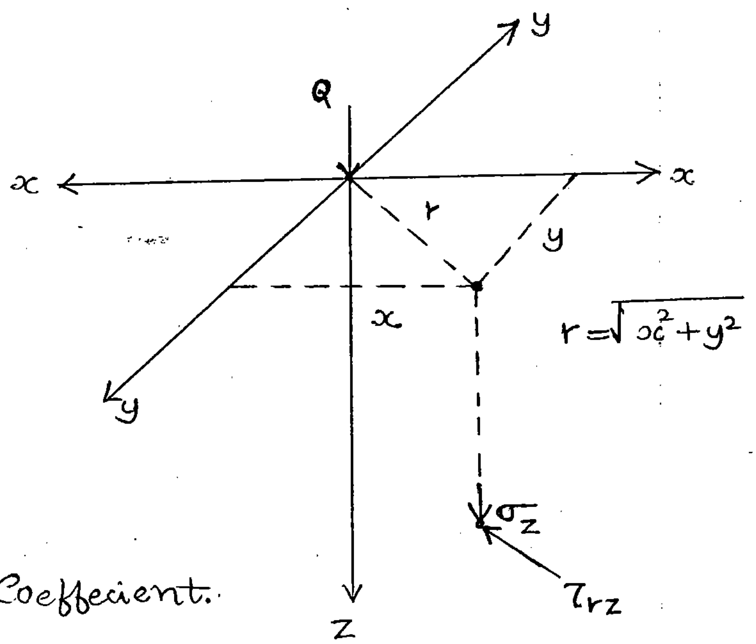
Homogenous means at different locations, soil has same elastic properties in same direction. (same E, μ)

Isotropic means at a single point, soil has same elastic properties in different directions.

Semi-infinite means material bounded by a horizontal plan and extending to infinite length in all directions to one side of horizontal plane.

Vertical stress;

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{3}{2\pi} \left[\frac{1}{1+\left(\frac{r}{z}\right)^2} \right]^{5/2}$$



$$\sigma_z = \frac{Q}{z^2} \cdot k_B.$$

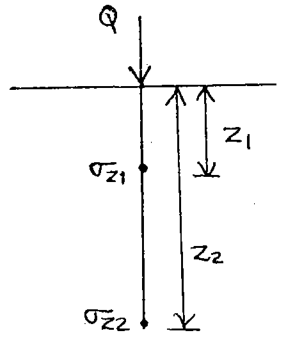
k_B → Boussinesq Influence Coeffecient.

- if $r = 0$ (vertically below the load).

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{3}{2\pi}.$$

$$\Rightarrow \sigma_z \propto \frac{1}{z^2}$$

$$\frac{\sigma_{z1}}{\sigma_{z2}} = \left(\frac{z_2}{z_1}\right)^2$$



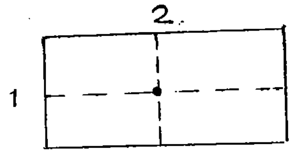
- Radial Shear Stress,

$$\tau_{rz} = \sigma_z \cdot \frac{r}{z}$$

Vertically below the load, $\tau_{rz} = 0$.

Q. A rectangular footing 1 m x 2m size has a load intensity of 10 t/m² on the ground surface.. Determine the vertical stress at 3 m below ground level a) below CG of the footing b) below the corner of footing, using Boussinesq's Theory.

a) below CG of footing,



$$Q = 10 \times 1 \times 2 = 20 \text{ t. (acting at CG)}$$

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{3}{2\pi}$$

$$= \frac{20}{3^2} \cdot \frac{3}{2\pi} = 1.06 \text{ t/m}^2$$

b) below corner of footing,

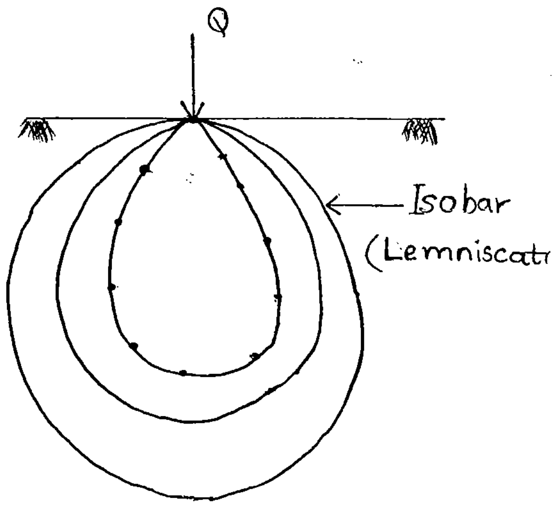
$$r = \sqrt{0.5^2 + 1^2} = 1.11 \text{ m.}$$

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{3}{2\pi}\left(\frac{1}{1+\left(\frac{r}{z}\right)^2}\right)^{5/2} = \frac{20}{9^2} \cdot \frac{3}{2\pi}\left(\frac{1}{1+\left(\frac{1.118}{3}\right)^2}\right)^{5/2}$$

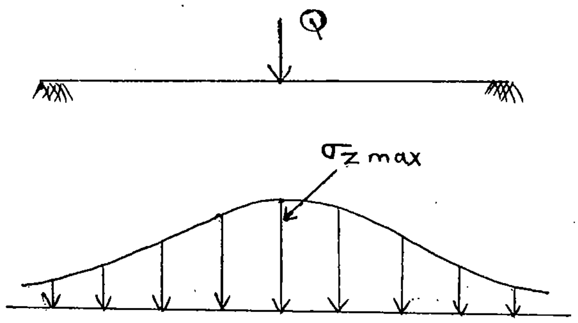
$$= \underline{\underline{0.7665 \text{ t/m}^2}}$$

→ Isobar

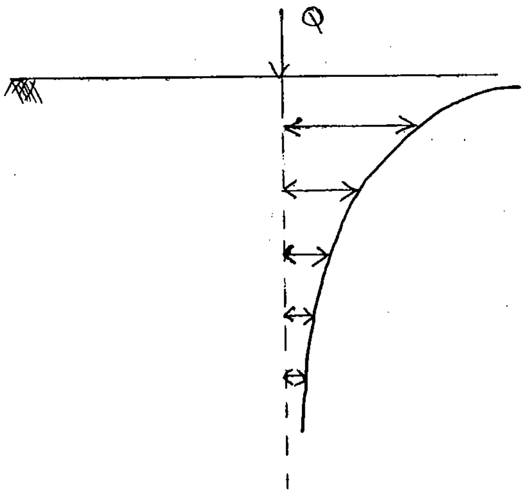
It is a curve or contour connecting all points below the ground surface of equal vertical stress.

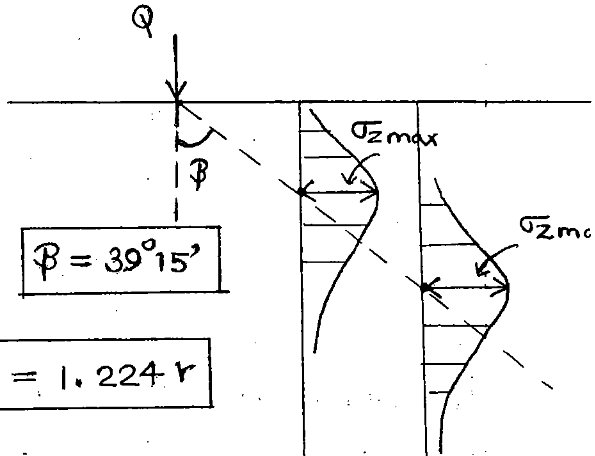


* σ_z variation on a Horizontal Plane.

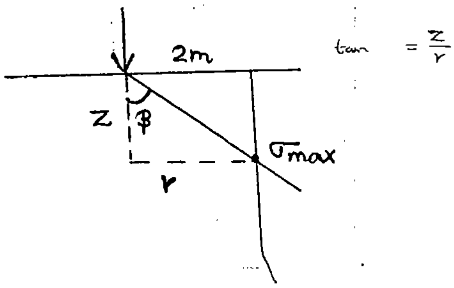


* σ_z variation on a Vertical Plane Passing through Load





Q If a vertical plane is drawn at a radial distance of 2 m away from a vertical load at what depth max σ_z occurs.



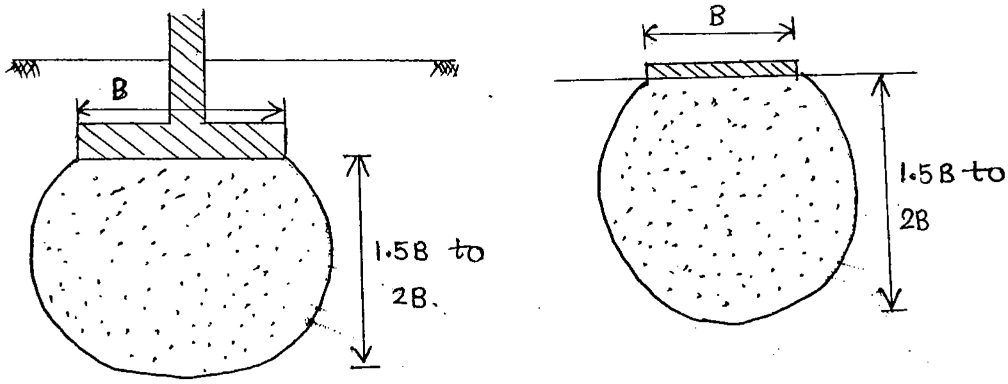
$$\tan\beta = \frac{r}{z}$$

$$z = \frac{r}{\tan\beta} = 1.224\ r$$

$$= 1.224 \times 2 = 2.447$$

→ Pressure Bulb.

It is the zone of the soil in which there is significant stress. Beyond the pressure bulb, stress in the soil is negligible. In the case of footings, the depth of the pressure bulb is taken as 1.5 B to 2B (as shown in the fig) below the footing

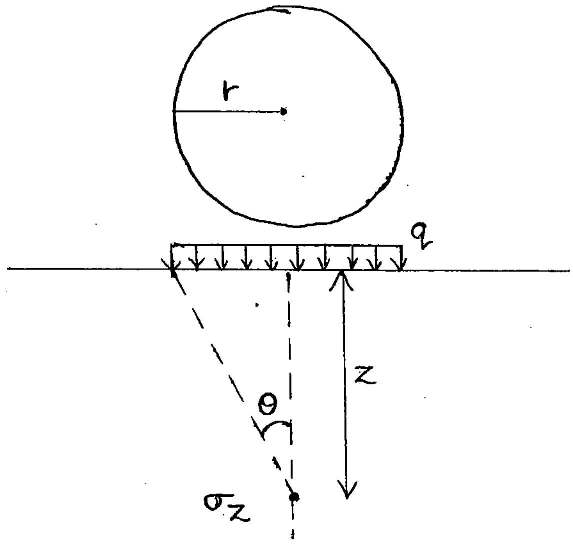


→ Circular Loaded Areas.

$$\sigma_z = q\left[1 - \left\{\frac{1}{1+\left(\frac{r}{z}\right)^2}\right\}^{3/2}\right]$$

OR

$$\sigma_z = q\,(1-\cos^3\theta)$$

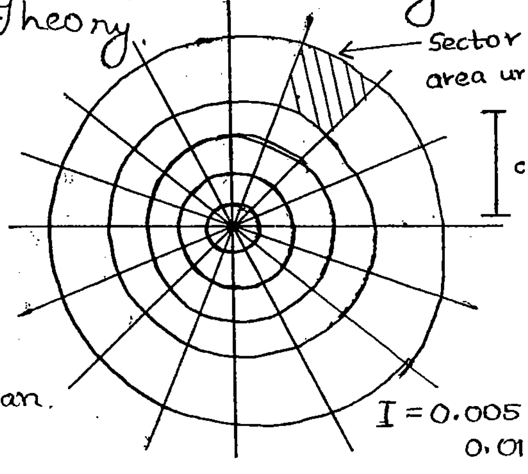


Sept, DAY → Newmark's Influence Chart

- to find σ_z at any point under any shape of loading.
- prepared based on Boussinesq's Theory.
- Each sector causes equal stress at the centre of the chart.

$$\boxed{\sigma_z = Inq}$$

I → influence coeffecient of chart.
n → no. of sectors covered by footing plan.
q → load intensity of footing.



Q. In a Newmarks influence chart depth line is 5cm. If the stress is required at a depth of 10m, what scale is to be used to draw the fig on the tracing paper?

Scale: Depth line = Z.

$$5\text{ cm} = 10\text{ m}.$$

$$1\text{ cm} = 2\text{ m}$$

$$1\text{ cm} = 200\text{ cm}.$$

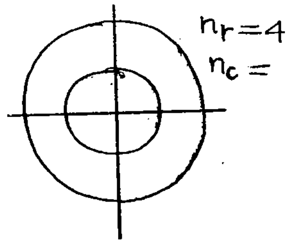
or $\underline{\underline{1 : 200}}$

$$* \quad I = \frac{1}{\text{Total no. of sectors of chart}}$$

Total no. of sectors of chart = No. of concentric circles × no. of radial lines.

If no. of circles = 10 & no. of radial lines = 20

$$I = \frac{1}{10 \times 20} = \underline{\underline{0.005}}$$



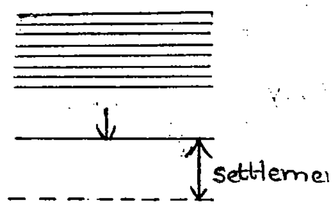
→ Westergaard's Method:

* Assumptions :-

(i) Point Load.

(ii) Soil consists of no. of thin layers.

(iii) Applicable for stratified soils (or) sedimentary soils. or varved clay



$$\boxed{\sigma_z = \frac{Q}{z^2} \cdot \frac{1}{\pi} \left[\frac{1}{1 + 2\left(\frac{r}{z}\right)^2} \right]^{3/2}}$$

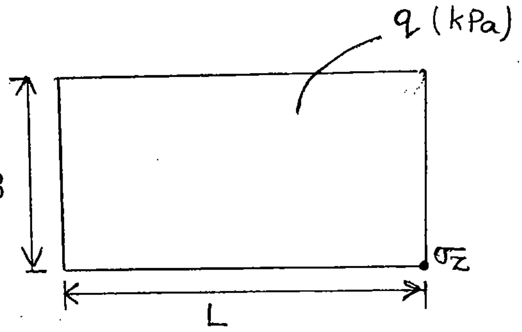
- For $\frac{r}{z} < 1.5$, Boussinesq's eqn gives higher stresses compar to Westergaard's eqn.

- For $\frac{r}{z} = 1.5$ both equations give the same stress value
- For $\frac{r}{z} > 1.5$, Westergaard's eqn gives slightly higher values compared to Bousinesq eqn.

→ Newmark's Method.

- to find σ_z at corner of rectangular loaded area

$$\boxed{\sigma_z = I q}$$

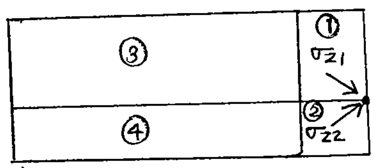


I → influence coeffecient which depends on m & n coeffecients.

$m = \frac{L}{z}$ & $n = \frac{B}{z}$.

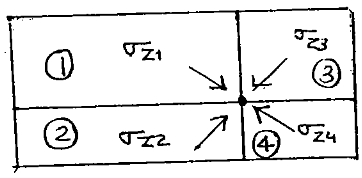
m & n values are available in the form of charts or tables

Pt. outside footing:



$\sigma_z = \sigma_{z3} + \sigma_{z4} - \sigma_{z1} - \sigma_{z2}$

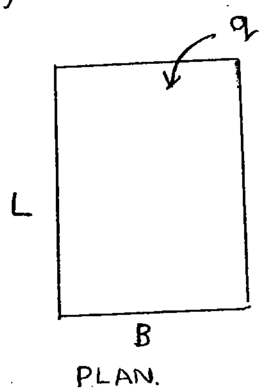
Pt. inside footing:

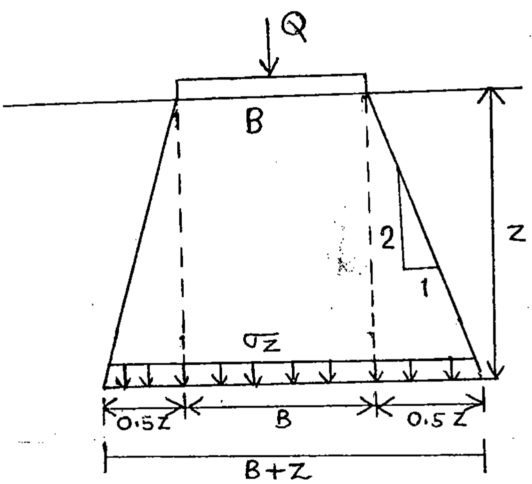


$\sigma_z = \sigma_{z1} + \sigma_{z2} + \sigma_{z3} + \sigma_{z4}$.

→ Approximate Method:

- Load dispersion angle is assumed to be as 2V to 1H or 1:2 (H:V) load dispersion.





$Q = LBq$.

$$\sigma_z = \frac{Q}{(B+z)(L+z)};$$ for rectangular footing

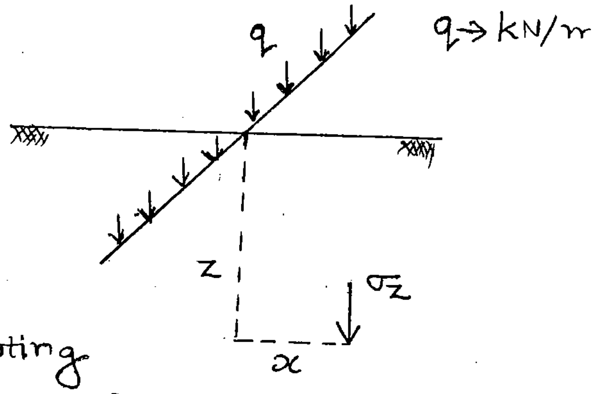
$$\sigma_z = \frac{Q}{(B+z)^2};$$ for square footing

$$\sigma_z = \frac{Q}{(B+z)\,1} \quad ; \text{ for continuous footing}$$

$$Q = (B \times 1)q \quad ; \quad Q \rightarrow \text{load per unit length.}$$

→ Vertical Stress due to Line Load.

$$\sigma_z = \frac{q}{z}\,\frac{2}{\pi}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

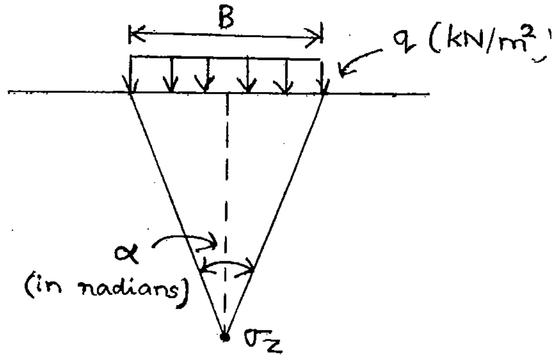


Eg :- Railway lines, sewer pipes etc.

→ Vertical Stress due to Strip footing (Continuous footing).

$$\sigma_z = \frac{q}{\pi}(\alpha + \sin\alpha)$$

(below CG of loading)

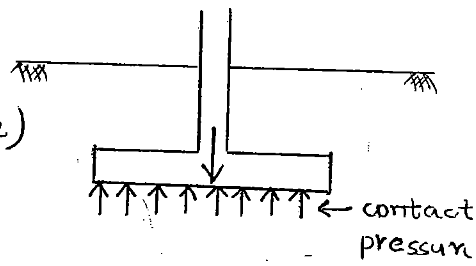


→ Contact Pressure

Variation depends upon :-

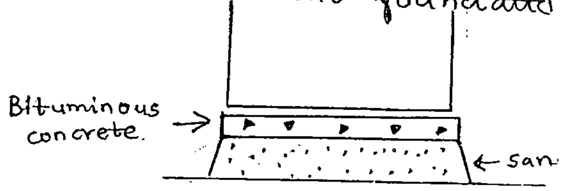
(i) Type of footing (rigid or flexible)

(ii) Type of soil.



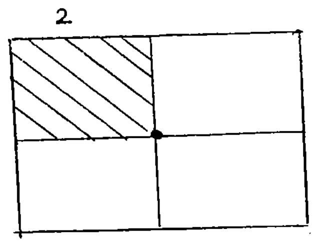
Rigid Footing (Eg: RCC footing)	Flexible Footing
(i) uniform settlement.	(i) Non uniform settlement.
(ii) non-uniform contact pressure	(ii) Uniform contact pressure.

Flexible footing :- oil tank foundation, embankment foundatio



2\. For the hatched rectangle,

$l = 2m$, $b = 1m$.



$$\left.\begin{aligned} m &= \frac{l}{z} = 0.4 \\ n &= \frac{B}{z} = 0.2 \end{aligned}\right\} I = 0.0328$$

$$\sigma_z = 4 . Iq = 4 \times 0.0328 \times 8 = \underline{\underline{1.05}} \ t/m^2$$

To find σ_z below the corner,

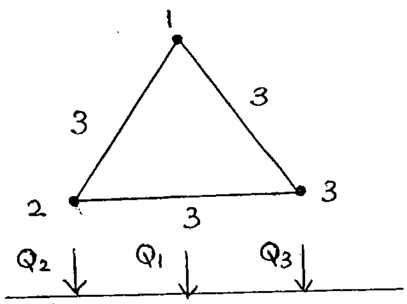
$$\left.\begin{aligned} m &= \frac{L}{z} = \frac{4}{5} = 0.8 \\ n &= \frac{2}{5} = 0.4 \end{aligned}\right\} I = 0.0931.$$

$$\sigma_z = Iq = 0.0931 \times 8 = \underline{\underline{0.74}} \ t/m^2$$

3\. σ_{z1} = stress under column (1). due to q_1

σ_{z2} = stress under column (1) due to q_2

σ_{z3} = " due to q_3.



$$Q_1 = \frac{200}{3} = 66.67 = Q_2 = Q_3.$$

$z = 2\ m$, $r = 0$. (for σ_{z1}).

$$\sigma_{z1} = \frac{66.67}{4} . \frac{3}{2\pi} = 7.96 \ t/m^2$$

$$\sigma_{z2} = \frac{Q_2}{z^2} . \frac{3}{2\pi} \left(\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right)^{5/2}$$

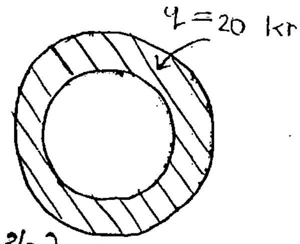
$$= \frac{66.67}{4} . \frac{3}{2\pi} \left(\frac{1}{1 + \left(\frac{3}{2}\right)^2} \right)^{5/2}$$

$$= \underline{\underline{0.418}} \ t/m^2 = \sigma_{z3}$$

$$\sigma_z = \sigma_{z1} + \sigma_{z2} + \sigma_{z3} = 7.96 + 0.418 \times 2$$

$$= \underline{\underline{8.796}} \ t/m^2$$

4.

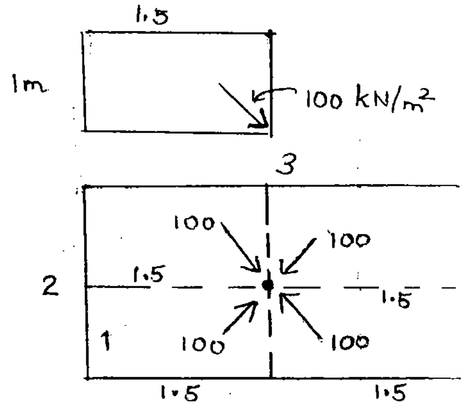


$$\sigma_z = q\left\{1-\left(\frac{1}{1+\left(\frac{r}{z}\right)^2}\right)^{3/2}\right\}$$

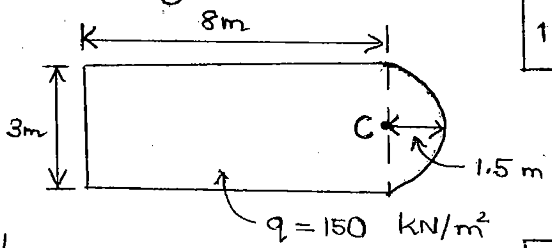
$$\sigma_z = 20\left\{1-\left(\frac{1}{\left(1+\frac{4}{10}\right)^2}\right)^{3/2}\right\} - \left[20\left\{1-\left(\frac{1}{1+\left(\frac{3}{10}\right)^2}\right)^{3/2}\right\}\right.$$

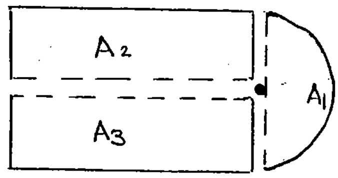
$$= \underline{\underline{1.56}} \quad kN/m^2$$

5. $\sigma_z = 4 \times 100 = \underline{\underline{400}} \; kN/m^2$



Q. A footing is shown in fig below, Determine the vertical stress at the point c shown in the fig at a depth of 3 m. Use the following coefficients.





$m = 0.5 \qquad n = 2.67 \qquad I = 0.1365$

$m = 1 \qquad n = 2.67 \qquad I = 0.2028.$

Stress due to area A_1: (semicircular area)

$$\sigma_z = \frac{1}{2}\cdot q\left(1-\left\{\frac{1}{1+\left(\frac{r}{z}\right)^2}\right\}^{3/2}\right) = \frac{1}{2}\times 150\left(1-\left(\frac{1}{1+\left(\frac{1.5}{3}\right)^2}\right)^{3/2}\right.$$

$$= 21.33 \; kN/m^2$$

Stress due to A_2:

$$\left.\begin{aligned} m &= \frac{L}{z} = \frac{8}{3} = 2.67 \\ n &= \frac{B}{z} = \frac{1.5}{3} = 0.5 \end{aligned}\right\} \quad I = 0.1365$$

$$\sigma_{z2} = Iq = 20.47 \; kN/m^2 = \sigma_{z3}$$

$$\sigma_z = \sigma_{z1} + \sigma_{z2} + \sigma_{z3} = \underline{\underline{62.28}} \; kN/m^2$$