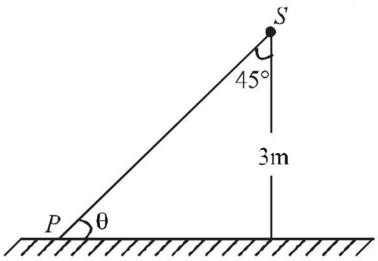
CHAPTER 2

Motion

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. A particle moves in a circle of radius R. In half the period of revolution its displacement is ____________and distance covered is ________. *(1983 - 2 Marks)*
2. Four persons K, L, M, N are initially at the four corners of a square of side d. Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N, and N directly towards K. The four persons will meet at a time .... *(1984- 2 Marks)*
3. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 radian/second. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when $\theta = 45°$ (see fig.) is .... m/s *(1987 - 2 Marks)*

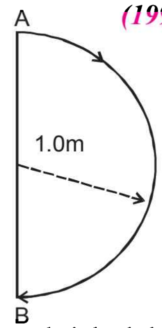


B True/False

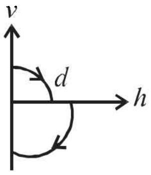
1. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (Neglect air resistance). *(1983 - 2 Marks)*
2. A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path. *(1984 - 2 Marks)*
3. Two identical trains are moving on rails along the equator on the earth in opposite directions with the same speed. They will exert the same pressure on the rails. *(1985 - 3 Marks)*

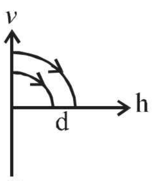
C MCQs with One Correct Answer

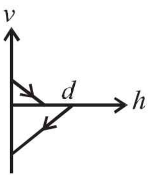
1. A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction *(1983 - 1 Mark)*
   (a) due north (b) 30° east of north
   (c) 30° west of north (d) 60° east of north
2. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is
   (a) 1 (b) 3 *(1988 - 1 Mark)*
   (c) 4 (d) $\sqrt{41}$
3. In 1.0 s, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m (see Figure). The magnitude of the average velocity *(1999S - 2 Marks)*
   (a) 3.14 m/s
   (b) 2.0 m/s
   (c) 1.0 m/s
   (d) Zero

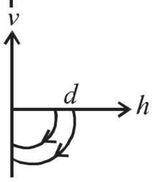


4. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as *(2000S)*

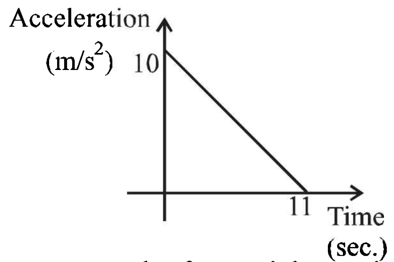
(a) 

(b) 

(c) 

(d) 

5. A particle starts sliding down a frictionless inclined plane. If S_n is the distance travelled by it from time $t = n - 1$ sec to $t = n$ sec, the ratio S_n/S_{n+1} is *(2004S)*
   (a) $\dfrac{2n-1}{2n+1}$ (b) $\dfrac{2n+1}{2n}$
   (c) $\dfrac{2n}{2n+1}$ (d) $\dfrac{2n+1}{2n-1}$
6. A body starts from rest at time $t = 0$, the acceleration time graph is shown in the figure. The maximum velocity attained by the body will be *(2004S)*



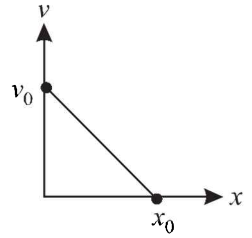
(a) 110 m/s

(b) 55 m/s

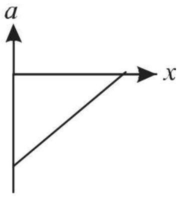
(c) 650 m/s

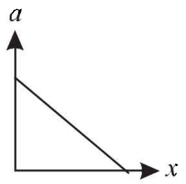
(d) 550 m/s

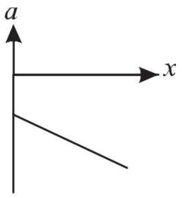
7. The velocity-displacement graph of a particle moving along a straight line is shown *(2005S)*

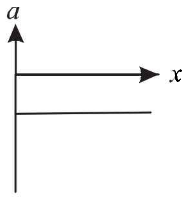


The most suitable acceleration-displacement graph will be

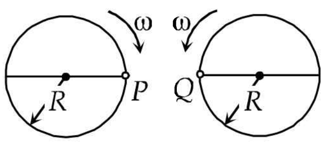
(a) 

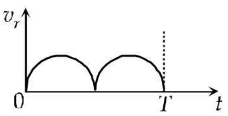
(b) 

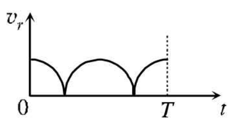
(c) 

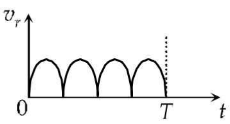
(d) 

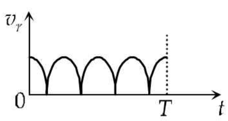
8. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by *(2012)*



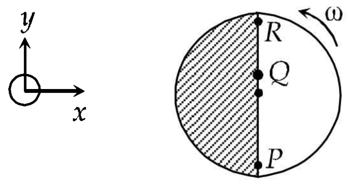
(a) 

(b) 

(c) 

(d) 

9. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then *(2012)*



(a) P lands in the shaded region and Q in the unshaded region.
(b) P lands in the unshaded region and Q in the shaded region.
(c) Both P and Q land in the unshaded region.
(d) Both P and Q land in the shaded region.

D MCQs with One or More than One Correct

1. A particle is moving eastwards with a velocity of 5 m/s. In 10s the velocity changes to 5 m/s northwards. The average acceleration in this time is *(1982 - 3 Marks)*
(a) zero
(b) $1/\sqrt{2}$ m/s^2 towards north-west
(c) $1/\sqrt{2}$ m/s^2 towards north-east
(d) $\frac{1}{2} m/s^2$ towards north-west
(e) $\frac{1}{2} m/s^2$ towards north

2. A particle of mass m moves on the x-axis as follows : it starts from rest at $t = 0$ from the point $x = 0$, and comes to rest at $t = 1$ at the point $x = 1$. NO other information is available about its motion at intermediate times $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle, then: *(1993-2 Marks)*
(a) α cannot remain positive for all t in the interval $0 \le t \le 1$.
(b) $|\alpha|$ cannot exceed 2 at any point in its path.
(c) $|\alpha|$ must be ≥ 4 at some point or points in its path.
(d) α must change sign during the motion, but no other assertion can be made with the information given.

3. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$ where $a, b\ (< a)$ and p are positive constants of appropriate dimensions. Then *(1999S - 3 Marks)*
(a) the path of the particle is an ellipse
(b) the velocity and acceleration of the particle are normal to each other at $t = \pi/(2p)$
(c) the acceleration of the particle is always directed towards a focus
(d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/(2p)$ is a

E Subjective Problems

1. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time lapse is t seconds, evaluate. *(1978)*
(i) maximum velocity reached, and
(ii) the total distance travelled.

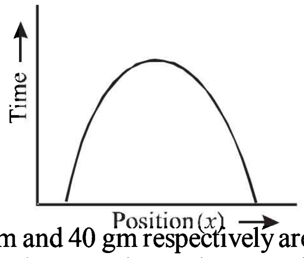
2. The displacement x of particle moving in one dimension, under the action of a constant force is related to the time t

by the equation $t = \sqrt{x} + 3$ *(1979)*
where x is in meters and t in seconds. Find

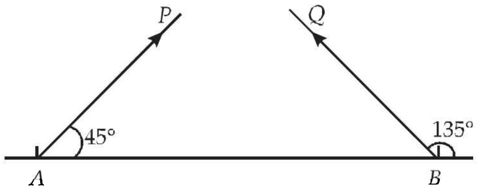
(i) The displacement of the particle when its velocity is zero, and

(ii) The work done by the force in the first 6 seconds.

3. Answer the following giving reasons in brief :
Is the time variation of position, shown in the figure observed in nature? *(1979)*

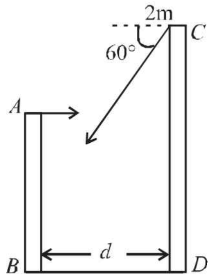


4. Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the figure. Each particle has an initial speed of 49 m/s. The separation AB is 245 m. *(1982 - 8 Marks)*



Both particle travel in the same vertical plane and undergo a collision After the collision, P retraces its path, Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take $g = 9.8\ m/s^2$.

5. Two towers AB and CD are situated a distance d apart as shown in figure.
AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD. *(1994 - 6 Marks)*
Simultaneously another object of mass 2 m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other.

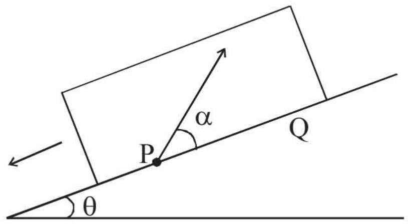


(i) Calculate the distance 'd' between the towers and,

(ii) Find the position where the objects hit the ground.

6. Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}\ m\ s^{-1}$ at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (i) the time-interval between the firings, and (ii) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. *(1996 - 5 Marks)*

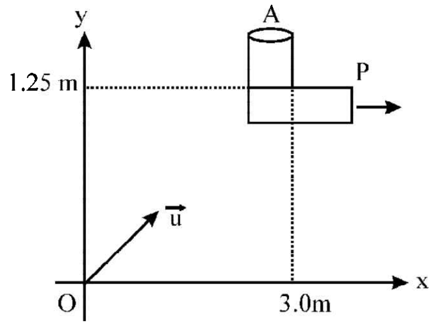
7. A large, heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom as shown in Figure. *(1998 - 8 Marks)*



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when particle was projected.

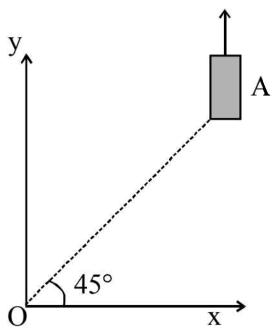
8. An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the $+x$ direction with an acceleration $1.5\ m/s^2$. At the same instant a stone is projected from the origin with a velocity $\vec{u}$ as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the X–Y plane. Find $\vec{u}$ and the time after which the stone hits the object. Take $g = 10$ m/s
(2000 - 10 Marks)



9. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3} - 1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.

(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = 4\theta/4$.
(2002 - 5 Marks)



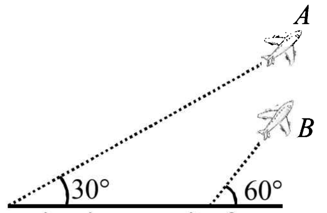
H Assertion & Reson Type Questions

1. **STATEMENT-1 :** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.
STATEMENT-2 : If the observer and the object are moving at velocities $\vec{v}_1$ and $\vec{v}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{v}_2 - \vec{v}_1$. *(2008)*
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement -1 is False, Statement-2 is True

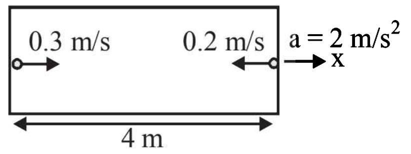
I Integer Value Correct Type

1. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s², is *(2011)*

2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ m/s. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. The observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is *(JEE Adv. 2014)*



3. A rocket is moving in a gravity free space with a constant acceleration of 2 m/s² along +x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in +x direction with a speed of 0.3 m/s relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 m/s from its right end relative to the rocket. The time in seconds when the two balls hit each other is *(JEE Adv. 2014)*



Section-B JEE Main / AIEEE

1. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be **[2002]**
(a) E (b) $E/\sqrt{2}$ (c) $E/2$ (d) zero.

2. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically with the same speed). If v_A and v_B are their respective velocities on reaching the ground, then **[2002]**
(a) $v_B > v_A$ (b) $v_A = v_B$ (c) $v_A > v_B$
(d) their velocities depend on their masses.

3. A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is **[2003]**
(a) 12m (b) 18m (c) 24m (d) 6m

4. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? **[2003]**
$[g = 10\text{m/s}^2, \sin 30^o = \frac{1}{2}, \cos 30^o = \frac{\sqrt{3}}{2}]$
(a) 5.20m (b) 4.33m
(c) 2.60m (d) 8.66m

5. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by **[2003]**
(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

6. A ball is released from the top of a tower of height h meters. It takes T seconds to reach the ground. What is the position of the ball at $\frac{T}{3}$ second **[2004]**
(a) $\frac{8h}{9}$ meters from the ground
(b) $\frac{7h}{9}$ meters from the ground
(c) $\frac{h}{9}$ meters from the ground
(d) $\frac{17h}{18}$ meters from the ground

7. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is **[2004]**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

8. A projectile can have the same range 'R' for two angles of projection. If 'T_1' and 'T_2' to be time of flights in the two cases, then the product of the two time of flights is directly proportional to. **[2004]**

(a) R (b) $\frac{1}{R}$ (c) $\frac{1}{R^2}$ (d) R^2

9. Which of the following statements is **FALSE** for a particle moving in a circle with a constant angular speed ? **[2004]**

(a) The acceleration vector points to the centre of the circle
(b) The acceleration vector is tangent to the circle
(c) The velocity vector is tangent to the circle
(d) The velocity and acceleration vectors are perpendicular to each other.

10. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20m. If the car is going twice as fast i.e., 120 km/h, the stopping distance will be **[2004]**

(a) 60m (b) 40m (c) 20m (d) 80m

11. A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ ? **[2004]**

(a) No (b) Yes, 30° (c) Yes, 60° (d) Yes, 45°

12. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then **[2005]**

(a) $S = \frac{1}{6}ft^2$ (b) $S = ft$

(c) $S = \frac{1}{4}ft^2$ (d) $S = \frac{1}{72}ft^2$

13. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 seconds the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is **[2005]**

(a) $\frac{1}{2} ms^{-2}$ towards north

(b) $\frac{1}{\sqrt{2}} ms^{-2}$ towards north - east

(c) $\frac{1}{\sqrt{2}} ms^{-2}$ towards north - west

(d) zero

14. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is **[2005]**

(a) $2bv^3$ (b) $-2abv^2$ (c) $2av^2$ (d) $-2av^3$

15. A particle located at $x = 0$ at time $t = 0$, starts moving along with the positive x-direction with a velocity 'v' that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[2006]**

(a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

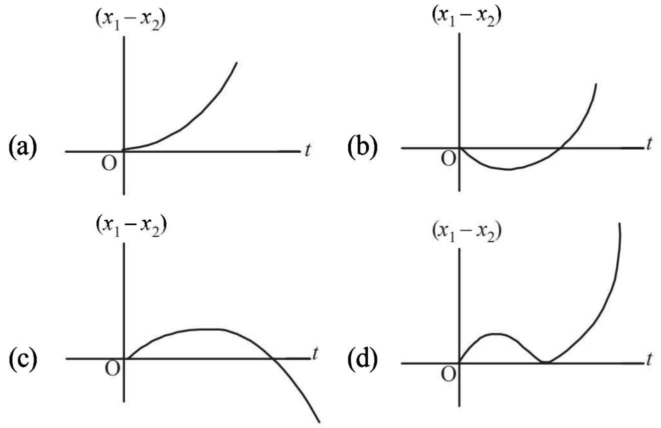
16. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[2007]**

(a) $K/2$ (b) K (c) Zero (d) $K/4$

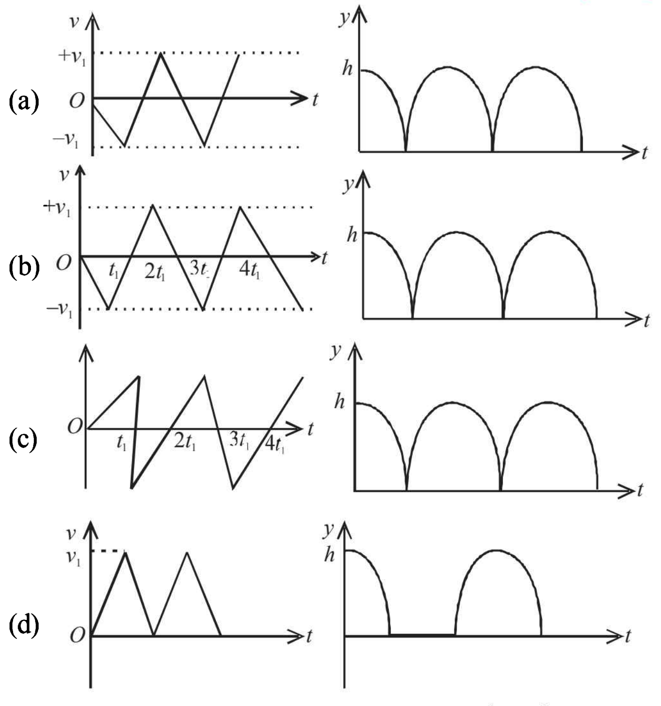
17. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is **[2007]**

(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$
(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$

18. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't'; and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't'? **]2008[**



19. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic.
Then the velocity as a function of time and the height as a function of time will be : **[2009]**

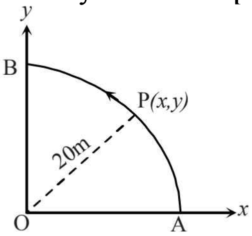


20. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is : **[2009]**

(a) $7\sqrt{2}$ units (b) 7 units (c) 8.5 units (d) 10 units

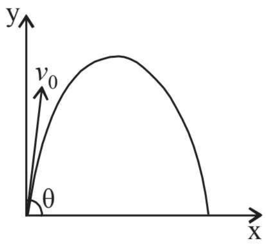
21. A particle is moving with velocity $\vec{v} = k(y\hat{i} + x\hat{j})$, where k is a constant. The general equation for its path is **[2010]**
(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) xy = constant (d) $y^2 = x^2$ + constant

22. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2$ s is nearly. **[2010]**
(a) 13m/s^2
(b) 12 m/s^2
(c) 7.2 ms^2
(d) 14m/s^2



23. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R,θ) on the circle of radius R is (Here θ is measured from the x-axis) **[2010]**
(a) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ (b) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$
(c) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ (d) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

24. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is **[2010]**
(a) $-mg\,v_0 t^2 \cos\theta\,\hat{j}$
(b) $mg\,v_0 t\cos\theta\hat{k}$
(c) $-\frac{1}{2}mg v_0 t^2 \cos\theta\,\hat{k}$
(d) $\frac{1}{2}mg v_0 t^2 \cos\theta\hat{i}$



where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

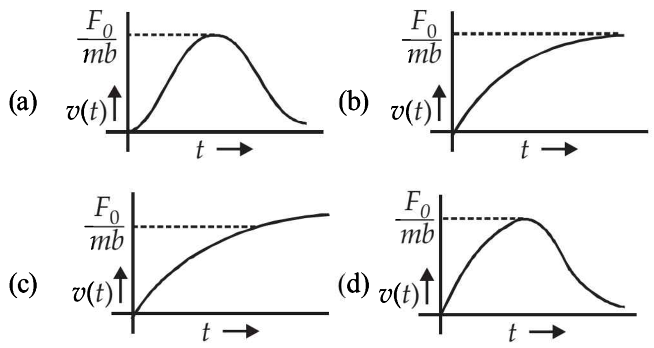
25. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by: **[2011]**
$\frac{dv}{dt} = -2.5\sqrt{v}$ where v is the instantaneous speed. The time taken by the object, to come to rest, would be:
(a) 2 s (b) 4 s (c) 8 s (d) 1 s

26. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is : **[2011]**
(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$ (c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

27. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be : **[2012]**
(a) $20\sqrt{2}$ m (b) 10m (c) $10\sqrt{2}$ m (d) 20m

28. Two cars of mass m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :
(a) $m_1 r_1 : m_2 r_2$ (b) $m_1 : m_2$
(c) $r_1 : r_2$ (d) 1 : 1

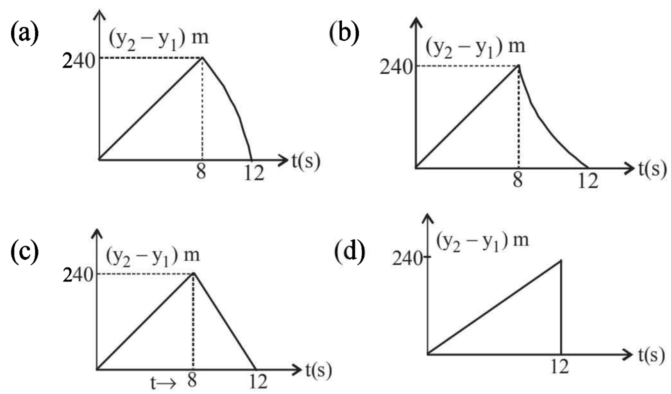
29. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves?



30. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[JEE-Main 2013]**
(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$
(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

31. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is: **[JEE Main 2014]**
(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2 u^2$
(c) $2gH = nu^2(n-2)$ (d) $gH = (n-2)u^2$

32. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/ s^2)
(*The figures are schematic and not drawn to scale*) **[JEE Main 2015]**



2 Motion

Section-A : JEE Advanced/ IIT-JEE

A	**1.**	2R, πR	**2.**	d/v	**3.**	0.6 m/s						
B	**1.**	T	**2.**	T	**3.**	F						
C	**1.**	(a)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(a)		
	6.	(b)	**7.**	(a)	**8.**	(a)	**9.**	(c)				
D	**1.**	(b)	**2.**	(a, c, d)	**3.**	(a, b, c)						
E	**1.**	$\frac{\alpha\beta}{\alpha+\beta}t;\ \frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}t^2$	**2.**	**(i)** 0 ; **(ii)** 0	**3.**	No	**4.**	mid point of AB, 3.53 sec.				
	5.	17.32, 11.547 m from B	**6.**	1 sec, $(5\sqrt{3}, 5)$ in metres								
	7.	**(a)** $\frac{u^2 \sin 2\alpha}{g\cos\theta}$ **(b)** $\frac{u\cos(\alpha+\theta)}{\cos\theta}$	**8.**	$\vec{u} = (3.75\hat{i} + 6.25\hat{j})$ m/s, t = 1 sec.	**9.**	45°, 2 m/sec.						
H	**1.**	**(b)**										
I	**1.**	**5**	**2.**	**5**	**3.**	**8**						

Section-B : JEE Main/ AIEEE

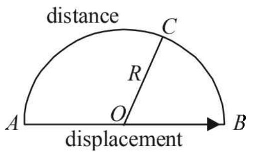
1.	(c)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(b)	**6.**	(a)
7.	(c)	**8.**	(a)	**9.**	(b)	**10.**	(d)	**11.**	(c)	**12.**	(d)
13.	(c)	**14.**	(d)	**15.**	(a)	**16.**	(d)	**17.**	(c)	**18.**	(b)
19.	(b)	**20.**	(a)	**21.**	(d)	**22.**	(d)	**23.**	(c)	**24.**	(c)
25.	(a)	**26.**	(a)	**27.**	(d)	**28.**	(c)	**29.**	(b)	**30.**	(b)
31.	(c)	**32.**	(b)								

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Displacement $= AOB = 2R$

Distance $= ACB = \pi R$

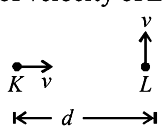


2. The relative velocity of K w.r.t L along the line KL is

$\vec{v}_{KL} = \vec{v}_K - \vec{v}_L = \vec{v}_K + (-\vec{v}_L)$

$= v$

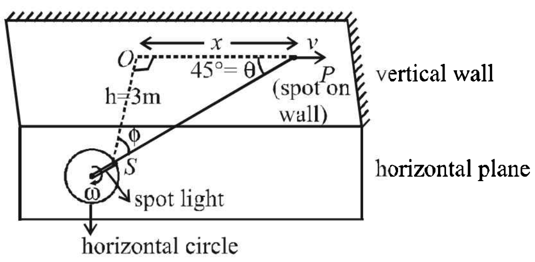
($\because$ the component of velocity of L along KL is zero)



The displacement of K till K and L meet is d.

$\therefore$ Time taken for K and L to meet will be $= \frac{d}{v}$

3.



The velocity (v) of spot $= dx/dt$

and the angular speed (ω) of spot light $= \frac{d\phi}{dt}$

From ΔSOP,

$\tan\phi = \frac{x}{h}$ $\qquad \therefore\ x = h\tan\phi$

$\therefore\ \frac{dx}{dt} = h\sec^2\phi\,\frac{d\phi}{dt}$ $\qquad \therefore\ v = (h\sec^2\phi)\,\omega$

$\therefore\ v = 3\sec^2 45° \times 0.1$ $\qquad [\because \theta + \phi = 90°]$

$\therefore\ v = 3 \times 2 \times 0.1 = 0.6$ m/s

B. True/False

1. **KEY CONCEPT**

When the two balls are thrown vertically upwards with the same speed u then their final speed v at the point of projection is $v^2 - u^2 = 2 \times g \times s$

Here, $s = 0$

$\therefore\ v = u$ for both the cases

2. T.E. = P.E. + K.E.

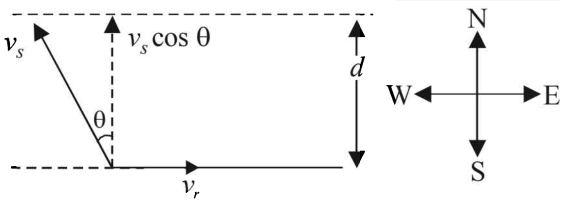
T.E. = Constant

At P, K.E. is minimum and P.E. is maximum. Since K.E. is minimum speed is also minimum.

3. The pressure exerted will be different because one train is moving in the direction of earth's rotation and other in the opposite direction.

C. MCQs with ONE Correct Answer

1. **(a)**



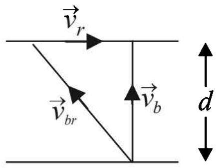
Time taken to cross the river $t = \dfrac{d}{v_s \cos\theta}$

NOTE : For time to be minimum, $\cos\theta$ = maximum

$\Rightarrow \theta = 0°$

The swimmer should swim due north.

2. **(b)** Shortest route corresponds to $\vec{v}_b$ perpendicular to river flow



$\therefore\ t = \dfrac{d}{v_b} = \dfrac{d}{\sqrt{v_{br}^2 - v_r^2}}$

or $\dfrac{1}{4} = \dfrac{1}{\sqrt{25 - v_r^2}}$

$\Rightarrow v_r = 3$ km/h

3. **(b)** $|\text{Average velocity}| = \dfrac{|\text{displacement}|}{\text{time}}$

$= \dfrac{2r}{t} = 2 \times \dfrac{1}{1} = 2$ m/s.

4. **(a)** **KEY CONCEPT**

Before hitting the ground, the velocity v is given by $v^2 = 2gd$ (quadratic equation and hence parabolic path) Downwards direction means **negative** velocity. After collision, the direction become positive and velocity decreases.

Further, $v'^2 = 2g \times \left(\dfrac{d}{2}\right) = gd;$

$\therefore\ \left(\dfrac{v}{v'}\right) = \sqrt{2}$ or $v = v'\sqrt{2} \Rightarrow v' = \dfrac{v}{\sqrt{2}}$

As the direction is reversed and speed is decreased graph (a) represents these conditions correctly.

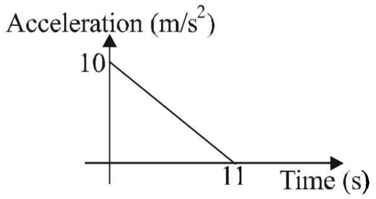
5. **(a)** $s_n = \dfrac{a}{2}(2n-1);$

$s_{n+1} = \dfrac{a}{2}[2(n+1)-1] = \dfrac{a}{2}(2n+1)$

$\dfrac{s_n}{s_{n+1}} = \dfrac{2n-1}{2n+1}$

6. **(b)** Change in velocity = area under the graph

$= \dfrac{1}{2} \times 10 \times 11 = 55\ \text{m/s}$



Since, initial velocity is zero, final velocity is 55 m/s.

7. **(a)** The equation for the given v-x graph is

$v = -\dfrac{v_0}{x_0}x + v_0$ ... (i)

$\dfrac{dv}{dx} = -\dfrac{v_0}{x_0}$

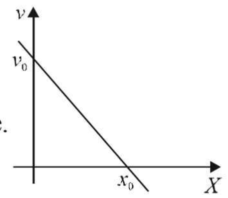
$\therefore\ v\dfrac{dv}{dx} = -\dfrac{v}{x_0} \times v = -\dfrac{v_0}{x_0}\left[-\dfrac{v_0}{x_0}x + v_0\right]$ from (1)

$\therefore\ a = \dfrac{v_0^2}{x_0^2}x - \dfrac{v_0^2}{x_0}$ .... (ii) $\left[\because a = v\dfrac{dv}{dx}\right]$

On comparing the equation (ii) with equation of a straight line

$y = mx + c$

we get $m = \dfrac{v_0^2}{x_0^2} = +\text{ve},$



i.e. $\tan\theta = +$ ve, i.e., θ is acute.

Also $c = -\dfrac{v_0^2}{x_0^2},$

i.e., the y-intercept is negative

The above conditions are satisfied in graph (a).

8. **(a)** At $t = 0$, the relative velocity will be zero.

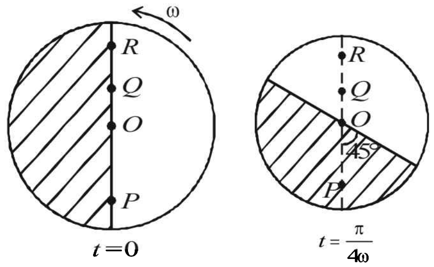
At $t = \dfrac{T}{4}$, the relative velocity will be maximum in magnitude.

At $t = \dfrac{T}{2}$, the relative velocity will be zero.

At $t = \dfrac{3T}{4}$, the relative velocity will be maximum in magnitude

At $t = T$, the relative velocity again becomes zero.

9. **(c)**



The x-coordinate of $P = v_x \times t = \omega R \times \dfrac{\pi}{4\omega} = \dfrac{\pi R}{4}$

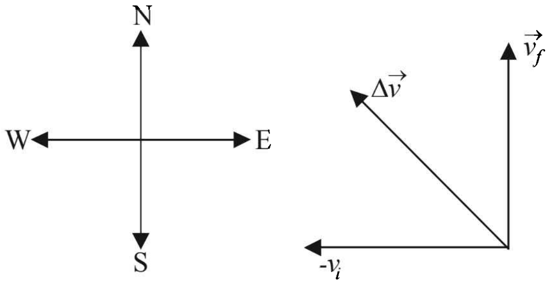
This horizontal distance travelled will be greater than any point on the disc between O and P. Therefore the landing will be in unshaded area. In the same way, the horizontal distance travelled by Q is always less than that of any point between O and R. Therefore the landing will be in unshaded area.

D. MCQs with ONE or MORE THAN ONE Correct

1. **(b)** Average acceleration

$\vec{a} = \dfrac{\vec{v_f} - \vec{v_i}}{t} = \dfrac{\vec{v_f} + (-\vec{v_i})}{t} = \dfrac{\Delta\vec{v}}{t}$

To find the resultant of $\vec{v_f}$ and $-\vec{v_i}$, we draw a diagram

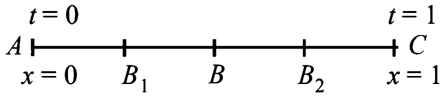


$|\Delta\vec{v}| = \sqrt{v_f^2 + v_i^2} = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ m/s

$|\vec{a}| = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$

Since, $|\vec{v_f}| = |\vec{-v_i}|$, $\therefore\ \vec{v}$ is directed towards N – W.

2. **(a, c, d)** **Note :** α cannot remain positive for all t in the interval $0 \le t \le 1$. This is because since the body starts from rest, it will first accelerate. Finally it stops therefore α will become negative. Therefore α will change its direction. Options (a) and (d) are correct.



Let the particle accelerate uniformly till half the distance (A to B) and then retard uniformly in the remaining half distance (B to C).

The total time is 1 sec. Therefore the time taken from A to B is 0.5 sec.

For A to B

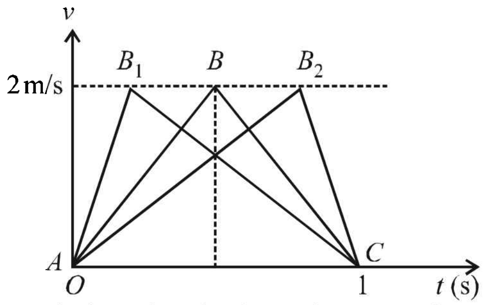
$S = ut + \frac{1}{2}at^2 \qquad 0.5 = 0 + \frac{1}{2} \times a \times (0.5)^2$

$\therefore\quad a = 4\ \text{m/s}^2$

$\therefore\quad V_B = 0 + 4 \times 0.5 = 2\ \text{m/s}^2$

Note : Now, if the particle accelerates till B_2 then for covering the same total distance in same time, acceleration should be less than 4 m/s^2 but |deceleration| should be greater than 4 m/s^2. And if the particle accelerates till B_1, then for covering the same total distance in the same time, the acceleration should be greater than 4 m/s^2 and | deceleration | < 4 m/s^2.

The same is depicted by the graph.



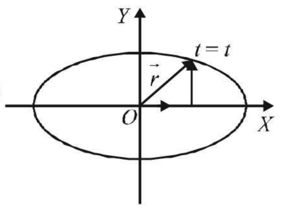
So, the | acceleration | must be greater than or equal to 4 m/s^2 at some point or points in the path.

3. **(a, b, c)** $x = a\cos pt \Rightarrow \cos(pt) = \frac{x}{a}$ ... (1)

$y = b\sin pt \Rightarrow \sin(pt) = \frac{y}{b}$ ... (2)

Squaring and adding (1) and (2), we get,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



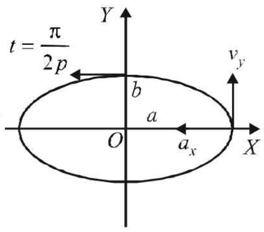
∴ The path of the particle is an ellipse.

From the given equations we can find,

$\frac{dx}{dt} = v_x = -ap\sin pt;\ \frac{d^2x}{dt^2} = a_x = -ap^2\cos pt$

$\frac{dy}{dt} = v_y = \text{pb}\cos pt$

and $\frac{d^2y}{dt^2} = a_y = -bp^2\sin pt$



At time $t = \frac{\pi}{2p}$ or $pt = \frac{\pi}{2}$

a_x and v_y become zero (because $\cos\frac{\pi}{2} = 0$). Only v_x and a_y are left,

or we can say that velocity is along negative x-axis and acceleration along negative y-axis.

Hence, at $t = \frac{\pi}{2p}$, velocity and acceleration of the particle are normal to each other.

At $t = t$, position of the particle $\vec{r}(t) = x\hat{i} + y\hat{j}$ $= a\cos pt\,\hat{i} + b\sin pt\,\hat{j}$ and acceleration of the particle is

$\vec{a}(t) = a_x\hat{i} + a_y\hat{j} = -p^2[a\cos pt\,\hat{i} + b\sin pt\,\hat{j}]$

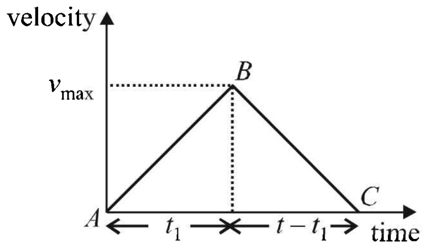
$= -p^2[x\hat{i} + y\hat{j}] = -p^2\vec{r}(t)$

Therefore, acceleration of the particle is always directed towards origin.

At $t = 0$, particle is at $(a, 0)$ and at $t = \frac{\pi}{2p}$, particle is at $(0, b)$. Therefore, the distance covered is one fourth of the elliptical path and not a.

E. Subjective Problems

1.



Distance travelled = area of ΔABC

$= \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times t \times v_{max}$

$= \frac{1}{2} \times t \times \frac{\alpha\beta}{\alpha + \beta}t = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha + \beta}\right)t^2$

2. $\sqrt{x} = t - 3 \Rightarrow x = t^2 + 9 - 6t \ \therefore\ v = \frac{dx}{dt} = 2t - 6$

(i) For velocity to be zero, $2t - 6 = 0 \Rightarrow t = 3$ sec.
The displacement is $x = 9 + 9 - 6 \times 3 = 0$

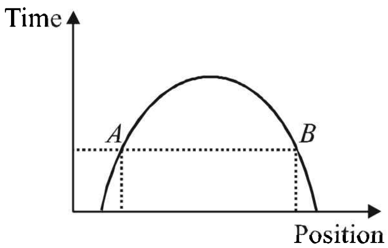
(ii) $a = \frac{dv}{dt} = 2 \quad \therefore$ At $t = 0, v = -6\ \text{ms}^{-1}$

At $t = 6$ sec, $v = 6\ \text{ms}^{-1}$

$\therefore$ Work done = Change in K.E. = $[K.E_f - K.E_i]$

$$= \frac{1}{2}m(6)^2 - \frac{1}{2}m(6)^2 = 0$$

3.



As shown, at a given instant of time, the body is at two different positions A and B which is not possible.

4. If a body drops from a height H above the ground then the time taken by it to reach the ground

$$t = \sqrt{\frac{2H}{g}} \quad \therefore \quad t = \sqrt{\frac{2 \times 61.25}{9.8}} = 3.53\ s$$

5. (i) Let t be the time taken for collision.

For mass m thrown horizontally from A.

For horizontal motion

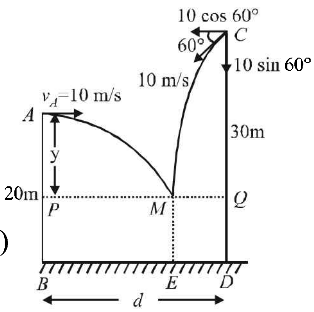
$PM = 10\,t$ ... (i)

For vertical motion

$u_y = 0; s_y = y; a_y = g; t_y = t$

$\therefore\ y = \frac{1}{2}gt^2$ ... (ii)

$v_y = u_y + a_y t = gt$ ... (iii)



For mass 2m thrown from C

For horizontal motion $QM = [10 \cos 60°]\,t$

$QM = 5\,t$ ... (iv)

For vertical motion $v_y = 10 \sin 60° = 5\sqrt{3}$; $a_y = g$

$s_y = y + 10; t_y = t$

Now, $v_y = 5\sqrt{3} + gt$ ... (v)

and $(s_y) = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow\ y + 10 = 5\sqrt{3}\,t + \frac{1}{2}gt^2$ ... (vi)

From (ii) and (vi)

$$\frac{1}{2}gt^2 + 10 = 5\sqrt{3}\,t + \frac{1}{2}gt^2 \Rightarrow t = \frac{2}{\sqrt{3}}\text{sec}$$

$\therefore\ BD = PM + MQ = 10\,t + 5\,t = 15\,t = 15 \times \frac{2}{\sqrt{3}}$

$= 10\sqrt{3} = 17.32$ m

(ii) Applying conservation of linear momentum (during collision of the masses at M) in the horizontal direction

$m \times 10 - 2\,m\,10 \cos 60° = 3\,m \times v_x$

$\Rightarrow\ 10\,m - 10\,m = 3\,m \times v_x \Rightarrow v_x = 0$

Since, the horizontal momentum comes out to be zero, the combination of masses will drop vertically downwards and fall at E.

$$BE = PM = 10\,t = 10 \times \frac{2}{\sqrt{3}} = 11.547\text{ m}$$

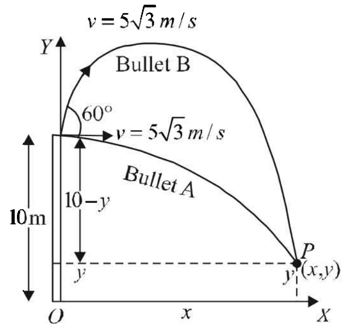
6. **For Bullet A.** Let t be the time taken by bullet A to reach P.

Vertical motion

$u_y = 0; s_y = 10 - y; a_y = 10\text{ m/s}^2; t_y = t$

$s_y = u_y t + \frac{1}{2}a_y t^2$

$10 - y = 5\,t^2$ ... (i)



Horizontal motion

$x = 5\sqrt{3}\,t$ ... (ii)

For bullet B.

Let $(t + t')$ be the time taken by bullet B to reach P

Vertical Motion

Let us consider upward direction negative and downward positive. Then

$u_y = -5\sqrt{3}\sin 60° = -7.5$ m/s, $a_y = +10\text{ m/s}^2$

$s_y = +(10 - y); t_y = t + t',\ s_y = u_y t + \frac{1}{2}a_y t^2$

$10 - y = -7.5\,(t + t') + 5\,(t + t')^2$ ... (iii)

Horizontal motion

$x = (5\sqrt{3}\cos 60°)\,(t + t')$

$\Rightarrow\ 5\sqrt{3}\,t + 5\sqrt{3}\,t' = 2x$ ... (iv)

Substituting the value of x from (ii) in (iv), we get

$5\sqrt{3}\,t + 5\sqrt{3}\,t' = 10\sqrt{3}\,t$

$\Rightarrow\ t = t'$

Putting $t = t'$ in eq. (iii)

$y - 10 = 15\,t - 20\,t^2$ ... (v)

Adding (i) and (v)

$0 = 15\,t - 15\,t^2 \Rightarrow t = 1$ sec.

(ii) Putting $t = 1$ in eq. (ii), we get $x = 5\sqrt{3}$

Putting $t = 1$ in eq. (i), we get $y = 5$

Therefore, the coordinates of point P are $(5\sqrt{3}, 5)$ in metres.

7. **(a)** u is the relative velocity of the particle with respect to the box. Resolve u.

u_x is the relative velocity of particle with respect to the box in x-direction.

u_y is the relative velocity with respect to the box in y-direction.

Since, there is no velocity of the box in the y-direction, therefore this is the vertical velocity of the particle with respect to ground also.

Y-direction motion (Taking relative terms w.r.t. box)

$u_y = +u\sin\alpha$

$a_y = -g\cos\theta$

$s_y = 0$ (activity is taken till the time the particle comes back to the box.)

$t_y = t$

$$s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow 0 = (u\sin\alpha)t - \frac{1}{2}g\cos\theta \times t^2$$

$$\Rightarrow \quad t = 0 \text{ or } t = \frac{2u\sin\alpha}{g\cos\theta}$$

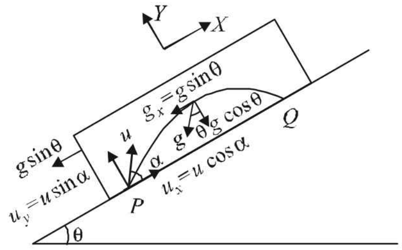
X- direction motion (Taking relative terms w.r.t. box)

$u_x = +u\cos\alpha\,;\ a_x = 0,\ t_x = t,\ s_x = s_x$

$$s_x = u_x t + \frac{1}{2}a_x t^2 \Rightarrow s_x = u\cos\alpha \times \frac{2u\sin\alpha}{g\cos\theta} = \frac{u^2\sin 2\alpha}{g\cos\theta}$$

(b) For the observer (on ground) to see the horizontal displacement to be zero, the distance travelled by the box in time $\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right)$ should be equal to the range of the particle.

Let the speed of the box at the time of projection of particle be U. Then for the motion of box with respect to ground.



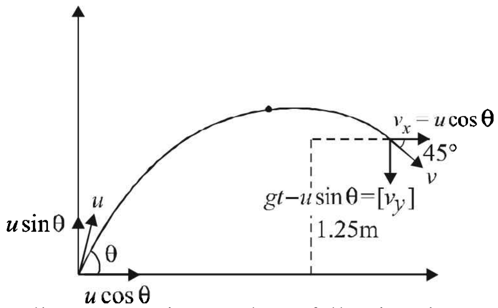
$$u_x = -U;\ a_x = -g\sin\theta\,;\ t_y = \frac{2u\sin\alpha}{g\cos\theta}\,;\ s_x = \frac{-u^2\sin 2\alpha}{g\cos\theta}$$

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$\frac{-u^2\sin 2\alpha}{g\cos\theta} = -U\left(\frac{2u\sin\alpha}{g\cos\theta}\right) - \frac{1}{2}g\sin\theta\left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

On solving we get $U = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

8. Let 't' be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal.



According to question, we have following three conditions.

(i) Vertical displacement of stone is 1.25 m.

Therefore, $1.25 = (u\sin\theta)t - \dfrac{1}{2}gt^2$

where g = 10 m/s^2

or $\quad (u\sin\theta)t = 1.25 + 5t^2 \quad$ ... (i)

(ii) Horizontal displacement of stone = 3 + displacement of object A.

Therefore, $(u\cos\theta)t = 3 + \dfrac{1}{2}at^2$

where $a = 1.5$ m/s^2

or $\quad (u\cos\theta)t = 3 + 0.75\,t^2 \quad$ ... (ii)

(iii) Horizontal component of velocity of stone = vertical component (because velocity vector is inclined at 45° with horizontal.)

Therefore $(u\cos\theta) = gt - (u\sin\theta) \quad$ ... (iii)

(The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion. Therefore, $gt > u\sin\theta$. In upward motion $u\sin\theta > gt$). Multiplying equation (iii) with t we can write,

$(u\cos\theta)t + (u\sin\theta)t = 10t^2 \quad$ ... (iv)

Now, (iv) – (ii) – (i) gives $4.25t^2 - 4.25 = 0$ or $t = 1$s

Substituting $t = 1$s in (i) and (ii), we get,

$u\sin\theta = 6.25$ m/s or $u_y = 6.25$ m/s

and $u\cos\theta = 3.75$ m/s.

or $\quad u_x = 3.75$ m/s therefore $\vec{u} = u_x\hat{i} + u_y\hat{j}$

or $\quad \vec{u} = (3.75\hat{i} + 6.25\hat{j})$ m/s

9. (a) Let the ball strike the trolley at B. Let

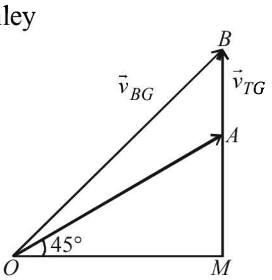
$\vec{v}_{BG}$ = velocity of ball w.r.t. ground

$\vec{v}_{TG}$ = velocity of trolley w.r.t. ground

$\therefore$ Velocity of ball w.r.t. trolley

$\vec{v}_{BT} = \vec{v}_{BG} - \vec{v}_{TG} \quad$ ... (i)

From triangle OAB



$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

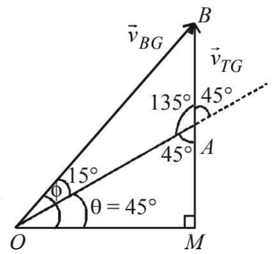
$\therefore \quad \overrightarrow{OA} + \vec{v}_{TG} = \vec{v}_{BG}$

$\therefore \quad \overrightarrow{OA} = \vec{v}_{BG} - \vec{v}_{TG} \quad$ ... (ii)

From (i) and (ii) $\quad \overrightarrow{OA} = \vec{v}_{BT}$

$\Rightarrow$ velocity of ball w.r.t. trolley makes an angle of 45° with the X-axis

(b) Here $\theta = 45°$



$\therefore \quad \phi = \dfrac{4\theta}{3} = \dfrac{4 \times 45}{3} = 60°$

In ΔOMA,

$\theta = 45° \Rightarrow \angle OAM = 45°$

$\therefore \quad \angle OAB = 135°$

Also $\angle BOA = 60° - 45° = 15°$

Using sine law in ΔOBA

$$\frac{v_{BG}}{\sin 135°} = \frac{v_{TG}}{\sin 15°} \Rightarrow v_{BG} = 2 \text{ m/s}$$

H. Assertion & Reason Type Questions

1. (b) Statement-1 is true. For a moving observer, the near by objects appear to move in the opposite direction at a large speed. This is because the angular speed of the near by object w.r.t observer is large. As the object moves away the angular velocity decreases and therefore its speed seems to be less. The distant object almost remains stationary.

Statement-2 is the concept of relative velocity which states that

$\vec{v}_{21} = \vec{v}_{2G} - \vec{v}_{1G}$

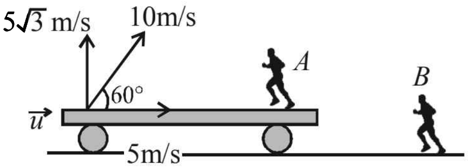
where G is the laboratory frame.

Thus both the statement are true but statement-2 is not the correct explanation of statement-1.

I. Integer Value Correct Type

1. 5 From the perspective of observer A, considering vertical motion of the ball from the point of throw till it reaches back at the initial height.

$U_y = +5\sqrt{3}$ m/s, $S_y = 0$, $a_y = -10 \text{m/s}^2$, $t = ?$



Applying $S = ut + \frac{1}{2}at^2 \Rightarrow 0 = 5\sqrt{3}t - 5t^2$

$\therefore \quad t = \sqrt{3}$ sec

Considering horizontal motion from the perspective of observer B. Let u be the speed of train at the time of throw.

The horizontal distance travelled by the ball $= (u+5)\sqrt{3}$.

The horizonal distance travelled by the boy

$= \left[u\sqrt{3} + \frac{1}{2}a(\sqrt{3})^2\right] + 1.15$

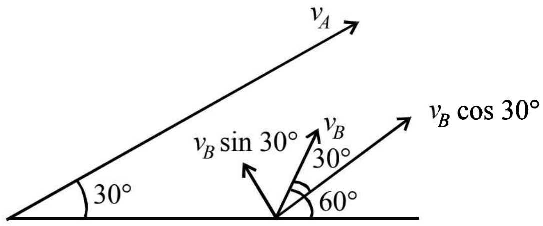
As the boy catches the ball therefore

$(u+5)\sqrt{3} = u\sqrt{3} + \frac{3}{2}a + 1.15$

$\therefore 5\sqrt{3} = 1.5a + 1.15 \quad \therefore 7.51 = 1.5a$

$\therefore a \approx 5 \text{ m/s}^2$

2. 5



Here

$v_A = v_B \cos 30°$

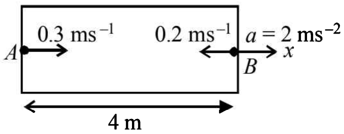
$\therefore 100\sqrt{3} = v_B \times \frac{\sqrt{3}}{2}$

$\therefore v_B = 200 \text{ ms}^{-1}$

$\text{Time} = \frac{\text{displacement}}{\text{velocity}}$

$\therefore t_0 = \frac{500}{v_B \sin 30°} = \frac{500}{200 \times \sin 30°} = 5 \text{ sec}$

3. 8



For ball A

$u_1 = 0.3 \text{ ms}^{-1}, \; a_1 = -2\text{ms}^{-2}, s_1 = x, t_1 = t$

$\therefore \quad s_1 = u_1 t_1 + \frac{1}{2}a_1 t_1^2$

$x = 0.3t - t^2$ ...(1)

For ball B

$u_2 = 0.2 \text{ ms}^{-1}, a_2 = 2\text{ms}^{-2}, s_2 = 4 - x, t_2 = t$

$\therefore \quad s_2 = u_2 t_2 + \frac{1}{2}a_2 t_2^2$

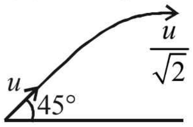
$4 - x = 0.2\,t + t^2$ ...(2)

From (1) and (2) $t = 8$ sec

Section-B JEE Main/ AIEEE

1. (c) Let u be the speed with which the ball of mass m is projected. Then the kinetic energy (E) at the point of projection is

$E = \frac{1}{2}mu^2$ ...(i)

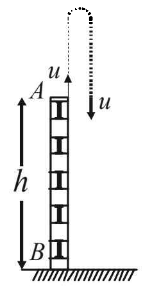


When the ball is at the highest point of its flight, the speed of the ball is $\frac{u}{\sqrt{2}}$ (Remember that the horizontal component of velocity does not change during a projectile motion).

$\therefore$ The kinetic energy at the highest point

$= \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\frac{mu^2}{2} = \frac{E}{2}$ [From (i)]

2. (b) Ball A is thrown upwards from the building. During its downward journey when it comes back to the point of throw, its speed is equal to the speed of throw. So, for the journey of both the balls from point A to B.



We can apply $v^2 - u^2 = 2gh$.

As u, g, h are same for both the balls, $v_A = v_B$

3. (c) **Case-1 :** $u = 50 \times \frac{5}{18}$ m/s, $v = 0, s = 6\text{m}, a = a$

$v^2 - u^2 = 2as \Rightarrow 0^2 - \left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$

$\Rightarrow -\left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$ ....(i)

Case-2 : $u = 100 \times \frac{5}{18}$ m/sec , $v = 0, s = s, a = a$

$\therefore v^2 - u^2 = 2as$

$\Rightarrow 0^2 - \left(100 \times \frac{5}{18}\right)^2 = 2as$

$\Rightarrow -\left(100 \times \frac{5}{18}\right)^2 = 2as$ ... (ii)

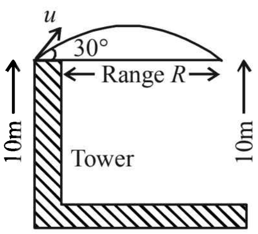
Dividing (i) and (ii) we get

$$\frac{100 \times 100}{50 \times 50} = \frac{2 \times a \times s}{2 \times a \times 6} \Rightarrow s = 24\text{m}$$

4. (d) From the figure it is clear that range is required

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(10)^2 \sin(2 \times 30°)}{10} = 5\sqrt{3}$$



5. (b) $x = \alpha t^3$ and $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4} = 3t^2\sqrt{\alpha^2 + \beta^2}$$

6. (a) We know that $s = ut + \frac{1}{2}gt^2$, or $h = \frac{1}{2}gT^2$ ($\because$ u=0)

now for $T/3$ second, vertical distance moved is given by

$$h' = \frac{1}{2}g\left(\frac{T}{3}\right)^2 \Rightarrow h' = \frac{1}{2} \times \frac{gT^2}{9} = \frac{h}{9}$$

$\therefore$ position of ball from ground $= h - \frac{h}{9} = \frac{8h}{9}$

7. (c) $\vec{A} \times \vec{B} - \vec{B} \times \vec{A} = 0 \Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{B} = 0$

$\therefore \vec{A} \times \vec{B} = 0$

Angle between them is 0, π, or 2 π

from the given options, $\theta = \pi$

8. (a) The angle for which the ranges are same is complementary.

Let one angle be θ, then other is $90° - \theta$

$$T_1 = \frac{2u \sin\theta}{g}, T_2 = \frac{2u\cos\theta}{g}$$

$$T_1 T_2 = \frac{4u^2 \sin\theta\cos\theta}{g} = 2R \left(\because R = \frac{u^2 \sin^2\theta}{g}\right)$$

Hence it is proportional to R.

9. (b) Only option (b) is false since acceleration vector is always radial (i.e . towards the center) for uniform circular motion.

10. (d) Speed, $u = 60 \times \frac{5}{18}\text{m/s} = \frac{50}{3}\text{m/s}$

$d = 20\text{m}, u' = 120 \times \frac{5}{18} = \frac{100}{3}\text{m/s}$

Let declaration be a then $(0)^2 - u^2 = -2ad$

or $u^2 = 2ad$ ... (1)

and $(0)^2 - u'^2 = -2ad'$ or $u'^2 = 2ad'$ ...(2)

(2) divided by (1) gives, $4 = \frac{d'}{d} \Rightarrow d' = 4 \times 20 = 80\text{m}$

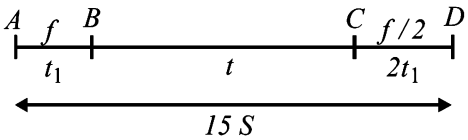
11. (c) Yes, the person can catch the ball when horizontal velocity is equal to the horizontal component of ball's velocity, the motion of ball will be only in vertical direction with respect to person for that,

$$\frac{v_o}{2} = v_o\cos\theta \text{ or } \theta = 60°$$

12. (d) Distance from A to $B = S = \frac{1}{2}ft_1^2 \Rightarrow ft_1^2 = 2S$

Distance from B to $C = (ft_1)t$

Distance from C to $D = \frac{u^2}{2a} = \frac{(ft_1)^2}{2(f/2)} = ft_1^2 = 2S$



$\Rightarrow S + f t_1 t + 2S = 15S \Rightarrow f t_1 t = 12S$

But $\frac{1}{2}f t_1^2 = S$

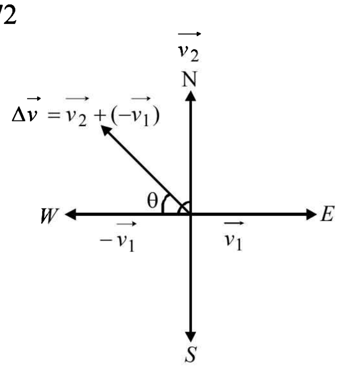
On dividing the above two equations, we get $t_1 = \frac{t}{6}$

$$\Rightarrow S = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$$

13. (c) Average acceleration

$$= \frac{\text{change in velocity}}{\text{time interval}}$$

$$= \frac{\Delta\vec{v}}{t}$$



$\vec{v_1} = 5\hat{i}, \vec{v_2} = 5\hat{j}$

$$\therefore \vec{a} = \frac{5\hat{j} - 5\hat{i}}{10} = \frac{\hat{j} - \hat{i}}{2}$$

$$\therefore a = \frac{\sqrt{1^2 + (-1)^2}}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}\text{ms}^{-2}$$

$\tan\theta = \frac{v_2}{v_1} = \frac{5}{5} = 1 \quad \therefore \theta = 45°$

Therefore the directon is North-west.

14. (d) $t = ax^2 + bx$; Diff. with respect to time (t)

$$\frac{d}{dt}(t) = a\frac{d}{dt}(x^2) + b\frac{dx}{dt} = a.2x\frac{dx}{dt} + b.\frac{dx}{dt}$$

$$1 = 2axv + bv = v(2ax + b) \Rightarrow 2ax + b = \frac{1}{v}.$$

Again differentiating, $2a\frac{dx}{dt} + 0 = -\frac{1}{v^2}\frac{dv}{dt}$

$$\Rightarrow \frac{dv}{dt} = f = -2av^3 \quad \left(\because \frac{dv}{dt} = f = acc\right)$$

15. (a) $v = \alpha\sqrt{x}$, $\frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha\, dt$

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt \Rightarrow \left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t \Rightarrow 2\sqrt{x} = \alpha t \Rightarrow x = \frac{\alpha^2}{4}t^2$$

16. (d) Let u be the velocity with which the particle is thrown and m be the mass of the particle. Then

$$K = \frac{1}{2}mu^2. \qquad ...(1)$$

At the highest point the velocity is u cos 60° (only the horizontal component remains, the vertical component being zero at the top-most point). Therefore kinetic energy at the highest point.

$$K' = \frac{1}{2}m(u\cos 60°)^2 = \frac{1}{2}mu^2\cos^2 60° = \frac{K}{4} \quad \text{[From 1]}$$

17. (c) We know that, $v = \frac{dx}{dt} \Rightarrow dx = v\,dt$

Integrating, $\int_0^x dx = \int_0^t v\,dt$

or $x = \int_0^t (v_0 + gt + ft^2)\,dt = \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$

or, $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$, $x = v_0 + \frac{g}{2} + \frac{f}{3}$.

18. (b) **For the body starting from rest**

$x_1 = 0 + \frac{1}{2}at^2 \Rightarrow x_1 = \frac{1}{2}at^2$

For the body moving with constant speed

$x_2 = vt$

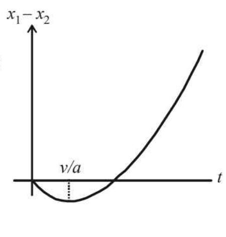
$\therefore\ x_1 - x_2 = \frac{1}{2}at^2 - vt \Rightarrow \frac{d(x_1 - x_2)}{dt} = at - v$

at $t = 0$, $x_1 - x_2 = 0$

For $t < \frac{v}{a}$; the slope is negative

For $t = \frac{v}{a}$; the slope is zero

For $t > \frac{v}{a}$; the slope is positive



These characteristics are represented by graph (b).

19. (b) For downward motion $v = -gt$

The velocity of the rubber ball increases in downward direction and we get a straight line between v and t with a negative slope.

Also applying $y - y_0 = ut + \frac{1}{2}at^2$

We get $y - h = -\frac{1}{2}gt^2 \Rightarrow y = h - \frac{1}{2}gt^2$

The graph between y and t is a parabola with $y = h$ at $t = 0$. As time increases y decreases.

For upward motion.

The ball suffer elastic collision with the horizontal elastic plate therefore the direction of velocity is reversed and the magnitude remains the same.

Here $v = u - gt$ where u is the velocity just after collision.

As t increases, v decreases. We get a straight line between v and t with negative slope.

Also $y = ut - \frac{1}{2}gt^2$

All these characteristics are represented by graph (b).

20. (a) Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, $t = 10$ s

$$\vec{v} = \vec{u} + \vec{a}t = 3\hat{i} + 4\hat{j} + (0.4\hat{i} + 0.3\hat{j}) \times 10 = 7\hat{i} + 7\hat{j}$$

$\therefore |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

21. (d) $\vec{v} = k(y\hat{i} + x\hat{j}) = v_x\hat{i} + v_y\hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

$\therefore \frac{dx}{dt} = ky$ and $\therefore \frac{dy}{dt} = kx$

$\therefore \frac{dy}{dx} = \frac{x}{y} \Rightarrow y\,dy = x\,dx \Rightarrow y^2 = x^2 + \text{constant}$

22. (d) $s = t^3 + 5 \Rightarrow$ velocity, $v = \frac{ds}{dt} = 3t^2$

Tangential acceleration $a_t = \frac{dv}{dt} = 6t$

Radial acceleration $a_c = \frac{v^2}{R} = \frac{9t^4}{R}$

At $t = 2s$, $a_t = 6 \times 2 = 12$ m/s²

$a_c = \frac{9 \times 16}{20} = 7.2$ m/s²

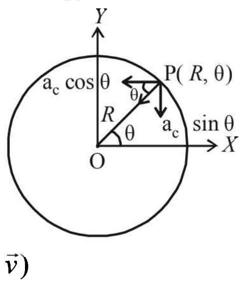
$\therefore$ Resultant acceleration

$= \sqrt{a_t^2 + a_c^2} = \sqrt{(12)^2 + (7.2)^2} = \sqrt{144 + 51.84}$

$= \sqrt{195.84} = 14$ m/s²

23. (c) Clearly $\vec{a} = a_c\cos\theta(-\hat{i}) + a_c\sin\theta(-\hat{j})$

$$= \frac{-v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$



24. (c) $\vec{L} = m(\vec{r} \times \vec{v})$

$$\vec{L} = m\left[v_0\cos\theta t\,\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}\right] \times \left[v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\hat{j}\right]$$

$$= mv_0\cos\theta t\left[-\frac{1}{2}gt\right]\hat{k} = -\frac{1}{2}mgv_0 t^2\cos\theta\,\hat{k}$$

25. (a) $\frac{dv}{dt} = -2.5\sqrt{v} \Rightarrow \frac{dv}{\sqrt{v}} = -2.5\,dt$

Integrating, $\int_{6.25}^{0} v^{-1/2}dv = -2.5\int_{0}^{t} dt$

$\Rightarrow \left[\frac{v^{+1/2}}{(1/2)}\right]_{6.25}^{0} = -2.5[t]_0^t$

$\Rightarrow -2(6.25)^{1/2} = -2.5t \Rightarrow t = 2$ sec

26. (a) Total area around fountain

$A = \pi R_{max}^2 = \pi\frac{v^4}{g^2}$

$[\because R_{max} = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 \sin 90°}{g} = \frac{v^2}{g}]$

27. (d) $R = \frac{u^2 \sin^2 \theta}{g}, H = \frac{u^2 \sin^2 \theta}{2g}$; H_{max} at $2\theta = 90$

$H_{max} = \frac{u^2}{2g}; = 10 \Rightarrow u^2 = 10g \times 2$

$R = \frac{u^2 \sin 2\theta}{(g)} \Rightarrow R_{max} = \frac{u^2}{g}$

$R_{max} = \frac{10 \times g \times 2}{g} = 20$ meter

28. (c) $a = rw^2 = r \times \left(\frac{2\pi}{T}\right)^2$

$\therefore \frac{a_1}{a_2} = \frac{r_1}{r_2}$ $[\because$ T is same$]$

29. (b) Given that $F(t) = F_0 e^{-bt} \Rightarrow m\frac{dv}{dt} = F_0 e^{-bt}$

$\frac{dv}{dt} = \frac{F_0}{m}e^{-bt} \Rightarrow \int_0^v dv = \frac{F_0}{m}\int_0^t e^{-bt}dt$

$v = \frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t = \frac{F_0}{mb}\left[-\left(e^{-bt} - e^{-0}\right)\right]$

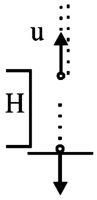
$\Rightarrow v = \frac{F_0}{mb}\left[1 - e^{-bt}\right]$

30. (b) $\vec{u} = \hat{i} + 2\hat{j} = u_x\hat{i} + u_y\hat{j} \Rightarrow u\cos\theta = 1,\ u\sin\theta = 2$

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u_x^2}$

$\therefore y = 2x - \frac{1}{2}gx^2 = 2x - 5x^2$

31. (c) Speed on reaching ground $v = \sqrt{u^2 + 2gh}$



Now, $v = u + at$

$\Rightarrow \sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is $t = \frac{u}{g}$,

$\Rightarrow t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g}$ (from question)

$\Rightarrow 2gH = n(n-2)u^2$

32. (b) $y_1 = 10t - 5t^2$; $y_2 = 40t - 5t^2$

for $y_1 = -240$m, $t = 8$s

$\therefore$ $y_2 - y_1 = 30t$ for $t \le 8$s.

for $t > 8$s,

$y_2 - y_1 = 240 - 40t - \frac{1}{2}gt^2$