Inverse Trigonometric Functions

Chapter at a Glance

1. **Inverse trigonometric function :** We know that the function $x = \sin\theta$, means that θ is an angle whose sine is x or x is the sine of θ. Hence,

	$\theta = \sin^{-1} x$	if	$\sin\theta = x$
Similarly	$\theta = \cos^{-1} x$	if	$\cos\theta = x$
and	$\theta = \tan^{-1} x$	if	$\tan\theta = x$

Then functions $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\sec^{-1}x$, $\text{cosec}^{-1}x$ and $\cot^{-1}x$ are called *inverse trigonometric functions.*

It is important to note that,

(i) $\sin\theta$ is a number whereas $\sin^{-1}x$ is an angle.

(ii) $\sin^{-1}x \neq (\sin x)^{-1}$ or $1/\sin x$

2. Inverses of trigonometric functions are all relations and not functions. eg. for $y = \sin x = 1/2$, the function $\{(x, y) : y = \sin x\}$ will be an infinite set of ordered pairs $\{(\pi/6 + 2n\pi, 1/2), (5\pi/6 + 2n\pi, 1/2), n \in I\}$, then to find inverse of $\sin x$, y is replaced by 1/2. Thus the inverse of this function is the set of ordered pairs $\{(1/2, \pi/6 + 2n\pi), (1/2, 5\pi/6 + 2n\pi, n \in I\}$, which is obviously not a function, because corresponding to a value of the independent variable (domain), there are more than one value of the dependent variable (range). ***Hence, we have to place certain restrictions on either the domain or range so that inverse of t-functions may be made a function.*** In the above example, since the domain is already restricted to $-1 \le x \le 1$, we consider restricting the range from $-\pi/2$ to $\pi/2$. Then defining the $\sin^{-1}x$ as $\{\theta : \pi - 2 \le \theta \le \pi/2\}$ denoted by $\sin^{-1}x = \theta$. Now we find that all values of x in domain are associated with one and only one value in the restricted range.

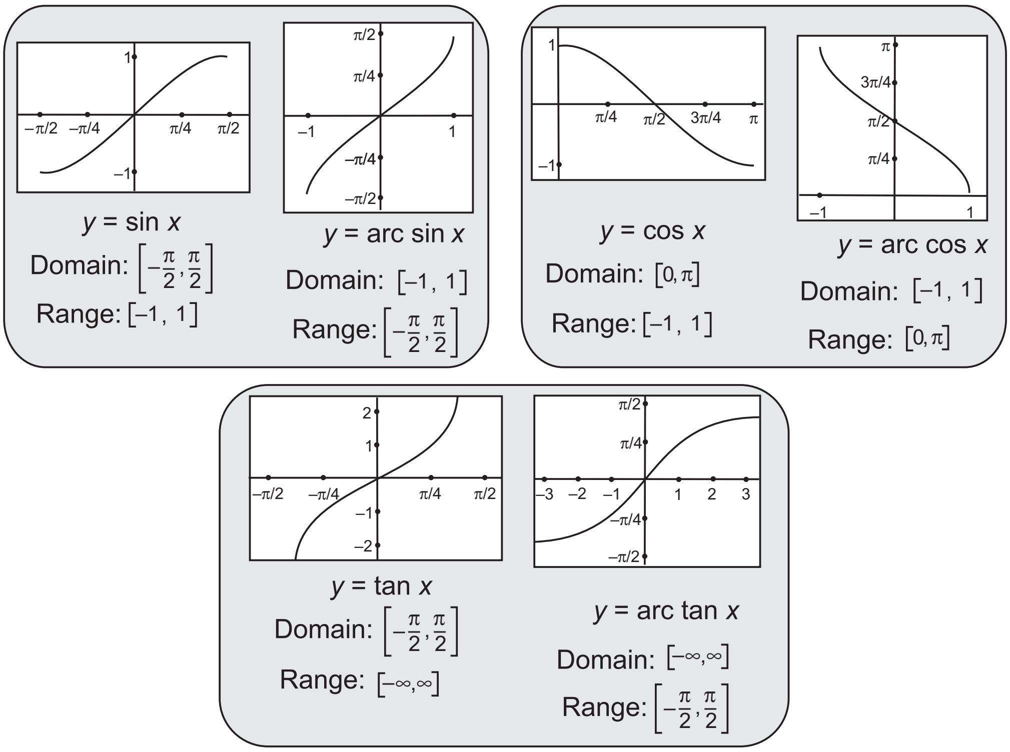
3. **Principal value :** The principal value of an inverse trigonometric function is the smallest numerical value, either positive or negative of the function.

Limitations for principal values of inverse circular functions can be placed in a table given below :

Function	Domain (x)	Range (θ)
$\sin^{-1}x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1) \cup (1, \infty)$	$[(0, \pi/2) \cup (\pi/2, \pi)]$
$\text{cosec}^{-1}x$	$(-\infty, -1) \cup (1, \infty)$	$[-\pi/2, 0) \cup (0, \pi/2]$

4. Graphs of inverse trigonometric functions :

(**Note : 'arc sin** x**'** is also used for $\sin^{-1}x$)



Inverse Trig Graphs

5. Properties of inverse trigonometric functions :

(I) Same angle can be expressed by different inverse trigonometric function.

$$30° = \sin^{-1}(1/2)$$

$$= \cos^{-1}(\sqrt{3}/2) = \tan^{-1}(1/\sqrt{3})$$

(II) Inverse property :

(A) (i) $\sin^{-1}(\sin\theta) = \theta,\quad -\pi/2 \le \theta \le \pi/2$

(ii) $\cos^{-1}(\cos\theta) = \theta,\quad 0 \le \theta \le \pi$

(iii) $\tan^{-1}(\tan\theta) = \theta,\quad -\pi/2 < \theta < \pi/2$

(B) (i) $\sin(\sin^{-1}x) = x,\quad -1 \le x \le 1$

(ii) $\cos(\cos^{-1}x) = x,\quad -1 \le x \le 1$

(iii) $\tan(\tan^{-1}x) = x,\quad x \in R$

(III) Principal of reciprocality : Following reciprocal relations exist between the inverse trigonometric functions.

(i) $\sin^{-1}x = \operatorname{cosec}^{-1}(1/x), -1 \le x \le 1$

(ii) $\cos^{-1}x = \sec^{-1}(1/x), -1 \le x \le 1$

(iii) $\tan^{-1}x = \cot^{-1}(1/x), x > 0$

(iv) $\cot^{-1}x = \tan^{-1}(1/x), x > 0$

or

$\cot^{-1}x = \pi + \tan^{-1}\dfrac{1}{x}\quad x < 0$

(v) $\sec^{-1}x = \cos^{-1}(1/x), x \le -1, x \ge 1$

(vi) $\operatorname{cosec}^{-1}x = \sin^{-1}(1/x), x \le -1, x \ge 1.$

(IV) Inverse trigonometric functions are odd functions within the principal values :

(i) $\sin^{-1}(-x) = -\sin^{-1}(x),\quad -1 \le x \le 1$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}(x),\quad -1 \le x \le 1$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x,\quad x \in R$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x.\quad x \in R$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x, \quad x \in R - [-1, 1]$

(vi) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x, \quad x \in R - [-1, 1]$

(V) Some fundamental formulae :

(i) $\sin^{-1} x + \cos^{-1} x = \pi/2, \quad -1 \le x \le 1$

(ii) $\tan^{-1} x + \cot^{-1} x = \pi/2, \quad x \in R$

(iii) $\sec^{-1} x + \text{cosec}^{-1} x = \pi/2, \quad x \in R - [-1, 1]$

(VI) Addition and subtraction formulae :

(i) $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} \pm y\sqrt{1-x^2}] \quad -1 \le x, y \le 1, x^2 + y^2 \le 1.$

(ii) $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1} [xy \mp \sqrt{1-x^2}.\sqrt{1-y^2}] \quad -1 \le x, y \le 1, \mp x \le y$

(iii) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left[\dfrac{x+y}{1-xy}\right] \quad x, y > 0, xy < 1$

(iv) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left[\dfrac{x-y}{1+xy}\right] \quad x, y > 0, xy > -1$

(v) $\cot^{-1} x + \cot^{-1} y = \cot^{-1} \left[\dfrac{xy-1}{y+x}\right]$

(vi) $\cot^{-1} x - \cot^{-1} y = \cot^{-1} \left[\dfrac{xy+1}{y-x}\right]$

(vii) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \left[\dfrac{x+y+z-xyz}{1-yz-zx-xy}\right]$

(VII) Some important formulae :

(i) $2\tan^{-1} x = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

(ii) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$

(iii) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$

(iv) $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$

(v) $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$

(vi) $3\tan^{-1} x = \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$

(vii) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right) = \text{cosec}^{-1}\left(\dfrac{1}{x}\right)$

(viii) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\dfrac{1}{x}\right) = \text{cosec}^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$

(ix) $\tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\dfrac{1}{x}\right) = \sec^{-1}\sqrt{1+x^2} = \text{cosec}^{-1}\left(\dfrac{\sqrt{1+x^2}}{x}\right)$

Multiple choice questions

1. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ will be :

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) π

2. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to :

(a) π (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

3. If $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, then x is equal to :

(a) $0, \frac{1}{2}$ (b) $1, \frac{1}{2}$

(c) 0 (d) $\frac{1}{2}$

4. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is :

(a) $\frac{5}{17}$ (b) $\frac{6}{17}$

(c) $\frac{3}{17}$ (d) $\frac{4}{17}$

5. $\cot^{-1}\frac{ab+1}{a-b} + \cot^{-1}\frac{bc+1}{b-c} + \cot^{-1}\frac{ca+1}{c-a} =$

(a) 0 (b) 1

(c) $\pi/4$ (d) -1

6. The solution of

$$\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2} \text{ is :}$$

(a) $\frac{a-b}{1-ab}$ (b) $\frac{1+ab}{a-b}$

(c) $\frac{ab-1}{ab+1}$ (d) $\frac{a-b}{1+ab}$

7. $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3) =$

(a) 5 (b) 10

(c) 15 (d) 20

8. If $\tan^{-1} 2$, $\tan^{-1} 3$ are two angles of a triangle, then the third angle will be :

(a) $\pi/4$ (b) $3\pi/4$

(c) $\pi/2$ (d) $\pi/6$

9. The formula $\cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1} x$, holds only for :

(a) $x \in R$ (b) $|x| > 1$

(c) $x \in [-1, 1]$ (d) $x \in [1, \infty)$

10. The sum of $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} + \ldots\ldots$ is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

11. $\cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right) =$

(a) 7 (b) 6

(c) 5 (d) none of these

12. $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] =$

(a) $\sqrt{\frac{x^2+1}{x^2+2}}$ (b) $\sqrt{\frac{x^2-1}{x^2-2}}$

(c) $\sqrt{\frac{x-1}{x-2}}$ (d) $\sqrt{\frac{x+1}{x+2}}$

13. The value of $\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$ is =

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$

(c) $\frac{16}{7}$ (d) none of these

14. If tan – 1 x + tan – 1 y + tan – 1 $z = \pi$ then $x + y + z =$

(a) xyz (b) 1

(c) 0 (d) $\frac{1}{xyz}$

15. Which of the following is the principal value branch of $\cos^{-1} x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $[0, \pi]$

(c) $\pi - [0, \pi]$ (d) $(0, \pi) - \frac{\pi}{2}$

16. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y =$

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$

(c) $\frac{3\pi}{5}$ (d) π

17. The value of $\sin^{-1}(\sin 12°) + \cos^{-1}(\cos 12°)$ is equal to :

(a) zero (b) $24 - 2\pi$

(c) $4\pi - 24°$ (d) none of these

18. $\tan^{-1}\frac{\cos x}{1+\sin x} =$

(a) $\frac{\pi}{4} - \frac{x}{2}$ (b) $\frac{\pi}{4} + \frac{x}{2}$

(c) $\frac{x}{2}$ (d) $\frac{\pi}{4} - x$

19. **If $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{-1}x$, then x is equal to :**

(a) 0 (b) $\frac{\sqrt{5}-4\sqrt{2}}{9}$

(c) $\frac{\sqrt{5}+4\sqrt{2}}{9}$ (d) $\frac{\pi}{2}$

20. **The value of $\sin^{-1}[\sin 10]$ is :**

(a) 10° (b) $10-3\pi$

(c) $3\pi-10°$ (d) none of these

21. $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=$

(a) $\tan^{-1}\frac{27}{11}$ (b) $\sin^{-1}\frac{11}{27}$

(c) $\cos^{-1}\frac{11}{27}$ (d) none of these

22. **If $\tan^{-1}\frac{x-1}{x+2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then $x=$**

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\pm\sqrt{\frac{5}{2}}$ (d) $\pm\frac{1}{2}$

23. **The value of expression $2\sec^{-1}2+\sin^{-1}\frac{1}{2}$:**

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{7\pi}{6}$ (d) 1

24. **The equation $2\cos^{-1}x+\sin^{-1}x=\frac{11\pi}{6}$ has :**

(a) no solution (b) only one solution

(c) two solutions (d) three solutions

25. **If $\alpha=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{1}{3}$ and $\beta=\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{1}{3}$ then**

(a) $\alpha<\beta$ (b) $\alpha=\beta$

(c) $\alpha>\beta$ (d) none of these

26. **If $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$, then x is equal to :**

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(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) 0 (d) 1

27. **Which of the following corresponds to the principal value branch of $\tan^{-1}x$?**

(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(c) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)-\{0\}$ (d) $(0,\pi)$

28. **The principal value branch of $\sec^{-1}x$ is:**

(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ (b) $[0,-\pi]-\left\{\frac{\pi}{2}\right\}$

(c) $(0,\pi)$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

29. **The principal value of the expression $\cos^{-1}[\cos(-680°)]$ is :**

(a) $\frac{2\pi}{9}$ (b) $\frac{-2\pi}{9}$

(c) $\frac{34\pi}{9}$ (d) $\frac{\pi}{9}$

30. **The value of $\cot(\sin^{-1}x)$ is:**

(a) $\frac{\sqrt{1+x^2}}{x}$ (b) $\frac{x}{\sqrt{1+x^2}}$

(c) $\frac{1}{x}$ (d) $\frac{\sqrt{1-x^2}}{x}$

31. **The domain of $\sin^{-1}2x$ is:**

(a) $[0,1]$ (b) $[-1,1]$

(c) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (d) $[-2,2]$

32. **The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is:**

(a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{3}$

(c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$

33. **The domain of $y=\cos^{-1}(x^2-4)$ is:**

(a) $[3,5]$

(b) $[0,\pi]$

(c) $[-\sqrt{5},-\sqrt{3}]\cap[-\sqrt{5},\sqrt{3}]$

(d) $[-\sqrt{5},-\sqrt{3}]\cup[\sqrt{3},\sqrt{5}]$

34. **The domain of the function defined by $f(x)=\sin^{-1}x+\cos x$ is:**

(a) $[-1,1]$ (b) $[-1,\pi+1]$

(c) $(-\infty,\infty)$ (d) ϕ

35. **The value of $\sin[2\sin^{-1}(0.6)]$ is:**

(a) ·48 (b) ·96

(c) 1·2 (d) sin 1·2

36. **The value of $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right\}$ is :**

(a) $\frac{\sqrt{29}}{3}$ (b) $\frac{29}{3}$

(c) $\frac{\sqrt{3}}{29}$ (d) $\frac{3}{29}$

37. **If $\cot^{-1}\left(-\frac{1}{5}\right) = \theta$, the value of $\sin\theta$ is :**

(a) $\frac{\sqrt{26}}{5}$ (b) $\frac{-5}{\sqrt{26}}$

(c) $\frac{\sqrt{5}}{\sqrt{26}}$ (d) $\frac{5}{\sqrt{26}}$

38. **If $\alpha \le 2\sin^{-1}x + \cos^{-1}x \le \beta$, then:**

(a) $\alpha = \frac{-\pi}{2}, \beta = \frac{\pi}{2}$ (b) $\alpha = 0, \beta = \pi$

(c) $\alpha = \frac{-\pi}{2}, \beta = \frac{3\pi}{2}$ (d) $\alpha = 0, \beta = 2\pi$

39. **The value of $\tan^2(\sec^{-1}2) + \cot^2(\text{cosec}^{-1}3)$ is:**

(a) 5 (b) 11

(c) 13 (d) 15

40. **The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is :**

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(a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3-\sqrt{5}}{2}$

(c) $\frac{-3+\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$

41. **The principal value of $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is:**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) π

42. **The principal value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is :**

(a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$ (d) None of these

43. **The inverse of cosine function is defined in the intervals :**

(a) $[-\pi, 0]$ (b) $\left[\frac{-\pi}{2}, 0\right]$

(c) $\left[0, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}, \pi\right]$

44. **If $\sin^{-1}x = y$, then :**

(a) $0 \le y \le x$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

(c) $0 < y < \pi$ (d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

45. **$\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to :**

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1

46. **The value of**

$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$

(c) $-\frac{\pi}{12}$ (d) $\frac{-}{1}$

47. **The value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ is :**

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{6}$

48. **The value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is :**

(a) 0 (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$

49. **The domain of the function $\cos^{-1}(2x-1)$ is :**

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(a) $[0, 1]$ (b) $[-1, 1]$

(c) $(-1, 1)$ (d) $[0, \pi]$

50. **The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is :**

[NCERT Exemplar]

(a) $[1, 2]$ (b) $[-1, 1]$

(c) $[0, 1]$ (d) none of these

51. **The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is equal to :**

[NCERT Exemplar]

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$

(c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

52. **Solve $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to :** **[NCERT]**

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

53. **$\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a$, then x equals.**

(a) a (b) b

(c) ab (d) 1

54. $2\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{24}{25}=$

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{3}$ (d) none of these

55. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$, then x =......

(a) 4 (b) 5

(c) 1 (d) 3

56. $\sin^{-1}(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2})$ = ...

(a) $\sin^{-1}x+\sin^{-1}\sqrt{x}$ (b) $\sin^{-1}x-\sin^{-1}\sqrt{x}$

(c) $\sin^{-1}\sqrt{x}-\sin^{-1}x$ (d) none of these

57. $\cos\left[2\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right]$ = ...

(a) $\frac{2\sqrt{6}}{5}$ (b) $-\frac{2\sqrt{6}}{5}$

(c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

58. $\sin\left\{\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$ **is equal to**

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

59. If $2\cos^{-1}\sqrt{\frac{1+x}{2}}=\frac{\pi}{2}$, then x =

(a) 1 (b) 0

(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

60. If $\frac{\pi}{2}\le x\le\frac{3\pi}{2}$, then $\sin^{-1}(\sin x)$ is equal to ....

(a) x (b) $-x$

(c) $(\pi+x)$ (d) $\pi-x$

61. If $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$, then x =

(a) $\sqrt{2}$ (b) 3

(c) $\sqrt{3}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

62. If $A=\tan^{-1}x$, then $\sin 2A$ =

(a) $\frac{2x}{\sqrt{1-x^2}}$ (b) $\frac{2x}{1-x^2}$

(c) $\frac{2x}{1+x^2}$ (d) none of these

63. $\cos^{-1}\left[\frac{3+5\cos x}{5+3\cos x}\right]$ **is equal to :**

(a) $\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$ (b) $2\tan^{-1}\left(2\tan\frac{x}{2}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(2\tan^{-1}\frac{x}{2}\right)$ (d) $2\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$

Assertion and Reason based questions

Choose the correct option :
(a) Both (A) and (R) are true and R is the correct explanation A.
(b) Both (A) and (R) are true but R is not correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

64. Assertion (A) : $\sin^{-1}(\sin 3)=3$

Reason (R) : For principal values $\sin^{-1}(\sin x)=-x$

65. Assertion (A) : The solution of system of equations

$$\cos^{-1}x+(\sin^{-1}y)^2=\frac{p\pi^2}{4}$$

and $(\cos^{-1}x)(\sin^{-1}y)^2=\frac{\pi^4}{16}$ is $x=\cos\frac{\pi^2}{4}$ and $y=\pm 1$, $\forall\, p\in I$.

Reason (R) : AM ≥ GM

66. Assertion (A) : If $\sum_{i=1}^{2n}\sin^{-1}x_i=n\pi,\ n\in N$

Then, $\sum_{i=1}^{n}x_i=\sum_{i=1}^{n}x_i^2=\sum_{i=1}^{n}x_i^3$

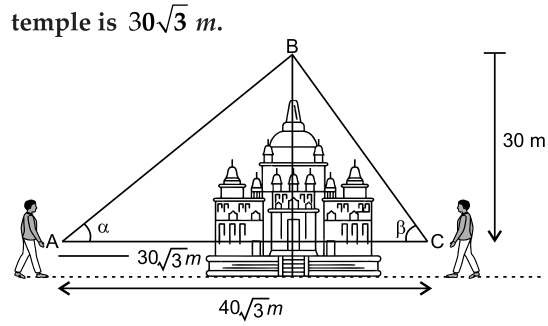
Reason (R) : $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}\ \forall\, x\in[-1,1]$

67. Assertion : The equation $2(\sin^{-1}x)^2-5(\sin^{-1}x+2)=0$.

Reason : $\sin^{-1}(\sin x)=x$ if $x\in[-1.57, 1.57]$.

Competency based questions

68. Two men on either side of temple of 30m height observe its top at the angle of elevation α and β respectively. The distance between the two men is $40\sqrt{3}\,m$ and distance between men A and the temple is $30\sqrt{3}\,m$.



Based on above information answer the following questions:

(i) Find ∠CAB = α =

(a) $\sin^{-1}\frac{1}{2}$ (b) $\sin^{-1}\frac{2}{\sqrt{3}}$

(c) $\sin^{-1}\frac{\sqrt{3}}{2}$ (d) $\sin^{-1} 2$

(ii) $\angle CAB = \alpha = ?$

(a) $\cos^{-1}\frac{1}{5}$ (b) $\cos^{-1}\frac{2}{5}$

(c) $\cos^{-1}\frac{\sqrt{3}}{2}$ (d) $\cos^{-1}\frac{4}{5}$

(iii) $\angle BCA = \beta = ?$

(a) $\tan^{-1}\frac{1}{2}$ (b) $\tan^{-1} 2$

(c) $\tan^{-1}\frac{1}{\sqrt{3}}$ (d) $\tan^{-1}\sqrt{3}$

(iv) $\angle ABC = ?$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(v) Domain and range of $\cos^{-1} x$?

(a) $(-1, 1), (0\ \pi)$

(b) $[-1, 1], (0, \pi)$

(c) $[0, \pi], [-1, 1]$

(d) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solutions

1. (b) $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y = \frac{2\pi}{3}$

$\left(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right)$

$\Rightarrow \pi - \frac{2\pi}{3} = \cos^{-1} x + \cos^{-1} y$

$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{3}$

2. (b) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \tan^{-1}\sqrt{3} - (\pi - \sec^{-1} 2)$

$= \frac{\pi}{3} - \pi + \sec^{-1} 2$

$= \frac{\pi}{3} - \pi + \frac{\pi}{3}$

$= \frac{2\pi}{3} - \pi$

$= -\frac{\pi}{3}$

3. (a) $\sin^{-1}(1 - x) - 2 - \sin^{-1} x = \frac{\pi}{2}$

Let $x = \sin y$

$\Rightarrow \sin^{-1}(1 - \sin y) - 2y = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1 - \sin y) = \frac{\pi}{2} + 2y$

$\Rightarrow 1 - \sin y = \sin\left[\frac{\pi}{2} + 2y\right]$

$\Rightarrow 1 - \sin y = \cos 2y$

$\Rightarrow 1 - \sin y = 1 - 2\sin^2 y$

$\Rightarrow 2\sin^2 y - \sin y = 0$

$\Rightarrow 2x^2 - x = 0$

$\Rightarrow x = 0, \frac{1}{2}$

$x = \frac{1}{2}$ does not sarisfy the given equation.

Hence, $x = 0$

4. (b) $\cot\left[\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right]$

$= \cot\left[\cot^{-1}\sqrt{\frac{(5)^2}{(3)^2} - 1} + \tan^{-1}\frac{2}{3}\right]$

$= \cot\left[\cot^{-1}\sqrt{\frac{25-9}{3^2}} + \tan^{-1}\frac{2}{3}\right]$

$= \cot\left[\cot^{-1}\sqrt{\frac{16}{9}} + \tan^{-1}\frac{2}{3}\right]$

$= \cot\left[\cot^{-1}\frac{4}{3} + \tan^{-1}\frac{2}{3}\right]$

$= \cot\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right]$

$= \cot\left[\frac{\tan^{-1}\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \times \frac{2}{3}}\right]$

$= \cot\left[\tan^{-1}\frac{\frac{9+18}{12}}{\frac{12-6}{12}}\right]$

$$= \cot\left[\tan^{-1}\frac{17}{6}\right]$$

$$= \cot\left(\cot^{-1}\frac{6}{17}\right)$$

$$= \frac{6}{17}$$

5. (a) $\cot^{-1}\frac{ab+1}{a-b}+\cos^{-1}\frac{bc+1}{b-c}+\cot^{-1}\frac{ca+1}{c-a}$

$$= \tan^{-1}\frac{a-b}{ab+1}+\tan^{-1}\frac{b-c}{1+bc}+\tan^{-1}\frac{c-a}{1+ca}$$

$$= \tan^{-1} a - \tan^{-1} b + \tan^{-1} b - \tan^{-1} c + \tan^{-1} c - \tan^{-1} a$$

$$= 0$$

6. (d) $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1} a - 2\tan^{-1} b = 2\tan^{-1} x$

$\Rightarrow 2[\tan^{-1} a - \tan^{-1} b] = 2\tan^{-1} x$

$\Rightarrow \tan^{-1}\frac{a-b}{1+ab} = \tan^{-1} x$

$\Rightarrow x = \frac{a-b}{1+ab}$

7. (c) $\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3)$

$= [\sec(\tan^{-1} 2)]^2 + [\text{cosec}(\cot^{-1} 3)]^2$

$= \left[\sec\left(\sec^{-1}\sqrt{4+1}\right)\right]^2+\left[\text{cosec}\left(\text{cosec}^{-1}\sqrt{3^2+1}\right)\right]^2$

$= 5 + 10$

$= 15$

8. (a) We know that sum of angles of a triangle is 180°.

$\therefore \tan^{-1} 2 + \tan^{-1} 3 + x = 180°$

$\Rightarrow \tan^{-1}\frac{3+2}{1-6} + x = \pi$

$\Rightarrow \tan^{-1}\frac{5}{-5}+x = \pi$

$\Rightarrow \frac{3\pi}{4}+x = \pi$

$\Rightarrow x = \pi - \frac{3\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}$

9. (d) $x \in [1, \infty]$

10. (d) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{18}$

$\Rightarrow \tan^{-1}\frac{1}{2\times 1^2}+\tan^{-1}\frac{1}{2\times 2^2}+\tan^{-1}\frac{1}{2\times 3^2}+.......$

$\therefore a_n = \tan^{-1}\frac{1}{2\times n^2}$

$= \tan^{-1}\frac{2}{4n^2}$

$= \tan^{-1}\frac{2+2n-2n}{1+4n^2-1}$

$= \tan^{-1}\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}$

$a_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$a_1 = \tan^{-1} 3 - \tan^{-1} 1$

$a_2 = \tan^{-1} 5 - \tan^{-1} 3$

$a_3 = \tan^{-1} 7 - \tan^{-1} 5$

$\vdots \quad \vdots \quad \vdots$

$a_\infty = \tan^{-1}\infty - \tan^{-1} 1$

$\sum_{n=1}^{\infty} a_n = \tan^{-1}\infty - \tan^{-1} 1$

$= \frac{\pi}{2}-\frac{\pi}{4}$

$= \frac{\pi}{4}$

11. (a) $\cot\left[\frac{\pi}{4}-2\cot^{-1}3\right]$

$= \cot\left[\frac{\pi}{4}-2\tan^{-1}\frac{1}{3}\right]$

$= \cot\left[\frac{\pi}{4}-\tan^{-1}\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right]$

$= \cot^{-1}\left[\frac{\pi}{4}-\tan^{-1}\frac{\frac{2}{3}}{\frac{8}{9}}\right]$

$= \cot^{-1}\left[\frac{\pi}{4}-\tan^{-1}\frac{3}{4}\right]$

$= \cot\left[\tan^{-1}1-\tan^{-1}\frac{3}{4}\right]$

$= \cot\left[\tan^{-1}\frac{1-\frac{3}{4}}{1+\frac{3}{4}}\right]$

$= \cot\left[\tan^{-1}\frac{\frac{1}{4}}{\frac{7}{4}}\right]$

$= \cot[\cot^{-1} 7]$

$= 7$

12. (a) $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}]$

$= \sin\left[\cot^{-1}\left\{\cos\left(\sec^{-1}\sqrt{1+x^2}\right)\right\}\right]$

$= \sin\left[\cot^{-1}\left\{\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right\}\right]$

$$= \sin\left[\cot^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$$= \sin\left[\operatorname{cosec}^{-1}\sqrt{1+\frac{1}{1+x^2}}\right]$$

$$= \sin\left[\operatorname{cosec}^{-1}\sqrt{\frac{x^2+2}{x^2+1}}\right]$$

$$= \sin\left[\sin^{-1}\sqrt{\frac{x^2+1}{x^2+2}}\right]$$

$$= \sqrt{\frac{x^2+1}{x^2+2}}$$

13. (d) $\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$

$$= \tan\left[\sec^{-1}\frac{5}{4}+\tan^{-1}\frac{2}{3}\right]$$

$$= \tan\left[\tan^{-1}\sqrt{\frac{25}{16}-1}+\tan^{-1}\frac{2}{3}\right]$$

$$= \tan\left[\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right]$$

$$= \tan\left[\tan^{-1}\frac{\left[\frac{3}{4}+\frac{2}{3}\right]}{1-\frac{3}{4}\times\frac{2}{3}}\right]$$

$$= \tan\left[\tan^{-1}\frac{17}{6}\right]$$

$$= \frac{17}{6}$$

14. (a) Given :

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\Rightarrow \tan^{-1}x + \tan^{-1}y = \pi - \tan^{-1}z$

$\Rightarrow \tan^{-1}\frac{x+y}{1-xy} = \pi - \tan^{-1}z$

$\Rightarrow \frac{x+y}{1-xy} = \tan(\pi - \tan^{-1}z)$

$\Rightarrow \frac{x+y}{1-xy} = -\tan(\tan^{-1}z) = -z$

$\Rightarrow x + y = -z + xyz$

$\Rightarrow x + y + z = xyz.$

Hence Proved.

15. (b) $[0, \pi]$

16. (a) $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$

$$\frac{\pi}{2}-\cot^{-1}x+\frac{\pi}{2}-\cot^{-1}y = \frac{4\pi}{5}$$

$$\cot^{-1}x+\cot^{-1}y = \pi-\frac{4\pi}{5}$$

$$\therefore\ \cot^{-1}x+\cot^{-1}y = \frac{\pi}{5}$$

17. (a) $\sin^{-1}12° + \cos^{-1}12°$

We know that, the principal value of

$\sin^{-1}x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right], \forall\, x \in [-1, 1]$

& $\cos^{-1}x \in [0, \pi], \forall\, x \in [-1, 1]$

since $12 \notin \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ or $12 \notin [0, \pi]$

but $12 - 4 \notin \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ & $4\pi - 12 \notin [0, \pi]$

$\sin^{-1}[\sin(12 - 4\pi) + \cos^{-1}(\cos(4\pi - 12))$

$= 12 - 4\pi + 4\pi - 12$

$= 0$

18. (a) $\tan^{-1}\frac{\cos x}{1+\sin x}$

$$= \tan^{-1}\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}.\cos\frac{x}{x}}$$

$$= \tan^{-1}\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}$$

$$= \tan^{-1}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$$= \tan^{-1}\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$$

$$= \frac{\pi}{4}-\frac{x}{2}$$

19. (c) $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3} = \sin^{-1}x$

$$\sin^{-1}\left[\frac{1}{3}\times\sqrt{1-\frac{4}{9}}+\frac{2}{3}\sqrt{1-\frac{1}{9}}\right] = \sin^{-1}x$$

$$\sin^{-1}\left[\frac{1}{3}\times\frac{\sqrt{5}}{3}+\frac{2}{3}\times\frac{2\sqrt{2}}{3}\right] = \sin^{-1}x$$

$$\sin^{-1}\left[\frac{\sqrt{5}}{9}+\frac{4\sqrt{2}}{9}\right] = \sin^{-1}x$$

$$x = \frac{\sqrt{5}}{9}+\frac{4\sqrt{2}}{9}$$

20. (c) $3\pi - 10$

We know that sin lies between $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$

Given that $x = 10$ which not lies between

$$\frac{-\pi}{2} \le x \le \frac{\pi}{2}$$

$3\pi - 10$ lies between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

$$\sin 10 = \sin(3\pi - 10)$$

$$\therefore \quad \sin^{-1}(\sin 10) = \sin^{-1}[\sin(3\pi - 10)] = 3\pi - 10$$

21. (a) $\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5}$

$= \sec^{-1}\frac{5}{4} + \tan^{-1}\frac{3}{5}$

$= \tan^{-1}\sqrt{\left(\frac{5}{4}\right)^2 - 1} + \tan^{-1}\frac{3}{5}$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5}$

$= \tan^{-1}\frac{\frac{3}{4} + \frac{3}{5}}{1 - \frac{9}{20}}$

$= \tan^{-1}\frac{27}{11}$

22. (c) $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{\frac{x-1}{x+2} + \frac{x+1}{x+2}}{1 - \frac{(x-1)(x+1)}{(x+2)^2}} = \frac{\pi}{4}$

$\Rightarrow \frac{(x-1+x+1)(x+2)}{(x+2)^2 - (x^2-1)} = 1$

$\Rightarrow \frac{(2x)\times(x+2)}{x^2+4x+4-x^2+1} = 1$

$\Rightarrow 2x^2 + 4x = 4x + 5$

$\Rightarrow 2x^2 = 5$

$\Rightarrow x^2 = \frac{5}{2}$

$\Rightarrow x = \pm\sqrt{\frac{5}{2}}$

23. (b) $2\sec^{-1}2 + \sin^{-1}\frac{1}{2}$

$= 2\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2}$

$= \cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2}$

$= \frac{\pi}{2} + \cos^{-1}\frac{1}{2}$

$= \frac{\pi}{2} + \frac{\pi}{3}$

$= \frac{3\pi + 2\pi}{6}$

$= \frac{5\pi}{6}$

24. (a) $2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x + \cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x + \frac{\pi}{2} = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x = \frac{11\pi}{6} - \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{11\pi - 3\pi}{6}$

$= \frac{8\neq}{6}$

$= \frac{4\pi}{3} \notin [0, \pi]$

So, function has no solution :

25. (b) $\alpha = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}$

$= \sin^{-1}\left[\frac{4}{5}\times\sqrt{1-\left(\frac{1}{3}\right)^2} + \frac{1}{3}\times\sqrt{1-\left(\frac{4}{5}\right)^2}\right]$

$= \sin^{-1}\left[\frac{4}{5}\times\frac{2\sqrt{2}}{3} + \frac{1}{3}\times\frac{3}{5}\right]$

$= \sin^{-1}\left[\frac{8\sqrt{2}+3}{15}\right]$ ...(1)

$\beta = \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{1}{3}$ (Given)

$= \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{2\sqrt{2}}{3}$

$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\frac{8}{9}} + \frac{2\sqrt{2}}{3}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]$

$= \sin^{-1}\left[\frac{3}{5}\times\frac{1}{3} + \frac{2\sqrt{2}}{3}\times\frac{4}{5}\right]$

$= \sin^{-1}\left[\frac{8\sqrt{2}+3}{15}\right]$ ...(2)

From (1) and (2), we get

$\alpha = \beta$

26. (b) We have,

$\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = 0$

$\Rightarrow \sin^{-1}\frac{2}{5} + \cos^{-1}x = \cos^{-1}0$

$\Rightarrow \sin^{-1}\frac{2}{5} + \cos^{-1}x = \frac{\pi}{2}$

$$\Rightarrow \quad \sin^{-1}\frac{2}{5}+\cos^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \quad \cos^{-1}x = \frac{\pi}{2}-\sin^{-1}\frac{2}{5}$$

$$\Rightarrow \quad \cos^{-1}x = \cos^{-1}\frac{2}{5}$$

$$\left(\because \cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}\right)$$

$$\therefore \quad x = \frac{2}{5}$$

27. (a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

28. (b) $[0, -\pi]-\left\{\frac{\pi}{2}\right\}$

29. (a)

$$\cos^{-1}[\cos(-680°)] = \cos^{-1}[\cos(720°-40°)]$$
$$= \cos^{-1}[\cos(-40°)]$$
$$= \cos^{-1}[\cos(40°)]$$
$$= 40° = \frac{2\pi}{9}.$$

30. (d) Let $\sin^{-1}x = \theta$,

then $\sin\theta = x$

$$\Rightarrow \quad \operatorname{cosec}\theta = \frac{1}{x}$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta = \frac{1}{x^2}$$

$$\Rightarrow \quad 1+\cot^2\theta = \frac{1}{x^2}$$

$$\Rightarrow \quad \cot\theta = \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow \quad \cot(\sin^{-1}x) = \frac{\sqrt{1-x^2}}{x}$$

31. (c) Let $\sin^{-1}2x = \theta$

So that $2x = \sin\theta$.

Now, $-1 \le \sin\theta \le 1$, *i.e.*, $-1 \le 2x \le 1$ which gives $-\frac{1}{2} \le x \le \frac{1}{2}$.

32. (b)

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{3}\right)$$
$$= -\sin^{-1}\left(\sin\frac{\pi}{3}\right)$$
$$= -\frac{\pi}{3}.$$

33. (d) $y = \cos^{-1}(x^2-4)$

$\Rightarrow \quad \cos y = x^2-4$

i.e., $\quad -1 \le x^2-4 \le 1$ (since $-1 \le \cos y \le 1$)

$\Rightarrow \quad 3 \le x^2 \le 5$

$\Rightarrow \quad \sqrt{3} \le |x| \le \sqrt{5}$

$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}]\cup[\sqrt{3}, \sqrt{5}]$

34. (a) The domain of cos is R and the domain of $\sin^{-1}$ is $[-1, 1]$.

$\therefore$ The domain of $\cos x + \sin^{-1}x$ is $R \cap [-1, 1]$, *i.e.,* $[-1, 1]$.

35. **(b)** Let $\sin^{-1}(\cdot 6) = \theta$, *i.e.,* $\sin\theta = \cdot 6$.

Now, $\sin(2\theta) = 2\sin\theta\cos\theta = 2(\cdot 6)(\cdot 8) = \cdot 96$.

36. (d) $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right\}$

Let $\quad A = \cos^{-1}\frac{1}{5\sqrt{2}}$

$$\therefore \quad \cos A = \frac{1}{5\sqrt{2}}$$

$$\Rightarrow \quad \cos^2 A = \frac{1}{50}$$

$\Rightarrow \quad \sec^2 A = 50$

$\Rightarrow \quad \tan^2 A = 50-1 = 49$

and $\quad B = \sin^{-1}\frac{4}{\sqrt{17}}$

$$\sin B = \frac{4}{\sqrt{17}}$$

$$\sin^2 B = \frac{16}{17}$$

$$\operatorname{cosec}^2 B = \frac{17}{16}$$

$$\cot^2 B = \frac{17}{16}-1 = \frac{1}{16}$$

$\therefore \quad \tan A = 7$ and $\tan B = 4$

Now, $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right\}$

$$= \tan(A-B)$$
$$= \frac{\tan A-\tan B}{1+\tan A\tan B}$$
$$= \frac{7-4}{1+7\times 4} = \frac{3}{29}.$$

37. (d) Given : $\cot^{-1}\left(\frac{-1}{5}\right) = \theta$, where $\theta \in (0, \pi)$

$$\therefore \quad \cot\theta = \frac{-1}{5}$$

$$\because \quad \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{1+\cot^2\theta}}$$

$$= \frac{1}{\sqrt{1+\left(-\frac{1}{5}\right)^2}} = \frac{1}{\sqrt{1+\frac{1}{25}}} = \frac{1}{\sqrt{\frac{26}{25}}}$$

$$= \frac{5}{\sqrt{26}}$$

38. (d) We have $\frac{-\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \quad \frac{-\pi}{2} + \frac{\pi}{2} \le \sin^{-1} x + \frac{\pi}{2} \le \frac{\pi}{2} + \frac{\pi}{2}$

$\Rightarrow \quad 0 \le \sin^{-1} x + (\sin^{-1} x + \cos^{-1} x) \le \pi$

$\Rightarrow \quad 0 \le 2 \sin^{-1} x + \cos^{-1} x \le \pi.$

39. (b) $\tan^2 (\sec^{-1} 2) + \cot^2 (\text{cosec}^{-1} 3)$

$= \sec^2 (\sec^{-1} 2) - 1 + \text{cosec}^2 (\text{cosec}^{-1} 3) - 1$

$= 2^2 \times 1 + 3^2 - 2 = 11.$

40. (b) Let $\quad y = \tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$

Putting, $\quad x = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$\Rightarrow \quad \cos x = \frac{\sqrt{5}}{3}$ ...(i)

Now, $\quad y = \tan\left(\frac{1}{2} \times x\right)$

$y = \tan\left(\frac{x}{2}\right)$

$\therefore \quad y = \sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$

$\therefore \quad y = \sqrt{\frac{1-\sqrt{5}/3}{1+\sqrt{5}/3}}$ [From (i)]

$\therefore \quad y = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}}$

$\therefore \quad y = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}} = \frac{3-\sqrt{5}}{\sqrt{4}} = \frac{3-\sqrt{5}}{2}$

$\therefore \quad y = \frac{3-\sqrt{5}}{2}$

i.e., $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] = \frac{3-\sqrt{5}}{2}.$

41. (b) $\frac{\pi}{6}$

42.(a) $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

43. (a) Cosine functions respected to any interval $[-\pi, 0]$, $[0, \pi]$, $[\pi, 2\pi]$ etc., is bijective with range $[-1, 1]$.

44. (b) Range of $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \quad \frac{-\pi}{2} \le y \le \frac{\pi}{2}$

45. (d) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] \quad \left[\because \sin^{-1}\left(\frac{-1}{2}\right) = \frac{-\pi}{6}\right]$

$= \sin\frac{\pi}{2} = 1.$

46. (c) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

$= \frac{-\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{-2\pi + 4\pi - 3\pi}{12} = \frac{-\pi}{12}$

$\left[\because \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}, \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}, \sin\left(\frac{-\pi}{2}\right) = -1\right]$

47. (a) Given, $\tan^{-1}\left[2 \sin\left(2 \cos^{-1} \frac{\sqrt{3}}{2}\right)\right]$

$= \tan^{-1}\left[2 \sin\left(2 \times \frac{\pi}{6}\right) = \tan^{-1}\left(2 \sin \frac{\pi}{3}\right)\right]$

$= \tan^{-1}\left(2 \times \frac{\sqrt{3}}{2}\right) = \tan^{-1} \sqrt{3} = \frac{\pi}{3}.$

48. (a) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) = \tan^{-1} \tan\left(\pi - \frac{\pi}{6}\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) = \frac{-\pi}{6}$

and $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$= \cos^{-1} \cos\left(2\pi + \frac{\pi}{6}\right) = \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$

$\frac{-\pi}{6} + \frac{\pi}{6} = 0.$

49. (a) We know that $\cos^{-1} x$ is defined for $x \in [-1, 1]$

$\therefore f(x) = \cos^{-1} (2x - 1)$ is defined if

$-1 \le 2x - 1 \le 1$

$\Rightarrow \quad 0 \le 2x \le 2$

$\Rightarrow \quad 0 \le x \le 1.$

50. (a) We know that $\sin^{-1} x$ is defined for

$x \in [-1, 1]$

$\therefore f(x) = \sin^{-1} \sqrt{x-1}$ is defined if

$\Rightarrow \quad 0 \le \sqrt{x-1} \le 1$

$\Rightarrow \quad 0 \le x - 1 \le 1$

$[\because \sqrt{x-1} \ge 0 \text{ and } -1 \le \sqrt{x-1} \le 1]$

$\Rightarrow \quad 1 \le x \le 2$

$\therefore \quad x \in [1, 2]$

51. (a) $\cos^{-1}\left(\cos\frac{3\pi}{2}\right) \ne \frac{3\pi}{2}$ as $\frac{3\pi}{2} \notin [0, \pi]$

$\therefore \quad \cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1} 0 = \frac{\pi}{2}$

52. (d) $\tan y = x$

$\Rightarrow \quad \sin y = \dfrac{x}{\sqrt{1+x^2}}$

Let $\tan^{-1} x = y$. Then,

$\therefore \quad y = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$

$\Rightarrow \quad \tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$

$\therefore \quad \sin(\tan^{-1} x) = \sin\left(\sin^{-1}\dfrac{x}{1+x^2}\right) = \dfrac{x}{\sqrt{1+x^2}}$

53. (c) $\sec^{-1}\dfrac{x}{a} - \sec^{-1}\dfrac{x}{b} = \sec^{-1} b - \sec^{-1} a$

$\cos^{-1}\dfrac{a}{x} - \cos^{-1}\dfrac{1}{a} = \cos^{-1}\dfrac{1}{b} - \cos^{-1}\dfrac{b}{x}$

$\cos^{-1}\left[\dfrac{a}{x}\times\dfrac{1}{a} - \sqrt{1-\dfrac{a^2}{x^2}}\sqrt{1-\dfrac{1}{a^2}}\right]$

$= \cos^{-1}\left[\dfrac{1}{b}\times\dfrac{b}{x} - \sqrt{1-\dfrac{1}{b^2}}\sqrt{1-\dfrac{b^2}{x^2}}\right]$

$\dfrac{1}{x} - \sqrt{1-\dfrac{a^2}{x^2}}\sqrt{1-\dfrac{1}{a^2}} = \dfrac{1}{x} - \sqrt{1-\dfrac{1}{b^2}}\sqrt{1-\dfrac{b^2}{x^2}}$

$\dfrac{\sqrt{x^2-a^2}}{x}\cdot\dfrac{\sqrt{a^2-1}}{a} = \dfrac{\sqrt{b^2-1}}{b}\cdot\dfrac{\sqrt{x^2-b^2}}{x}$

On squaring both sides

$\dfrac{x^2-a^2}{x^2}\,\dfrac{a^2-1}{a^2} = \dfrac{b^2-1}{b^2}\cdot\dfrac{x^2-b^2}{x^2}$

$b^2(x^2-a^2)(a^2-1) = a^2(b^2-1)(x^2-b^2)$

$b^2x^2a^2 - b^2x^2 - b^2a^4 + a^2b^2$

$= a^2b^2.x^2 - a^2b^4 - a^2x^2 + a^2b^2$

$x^2(a^2-b^2) = a^2b^2(a^2-b^2)$

$x^2 = a^2b^2$

$x = ab$

54. (a) $2\sin^{-1}\dfrac{3}{5} + \cos^{-1}\dfrac{24}{25}$

$= \sin^{-1}\left(2\times\dfrac{3}{5}\sqrt{1-\dfrac{9}{25}}\right) + \cos^{-1}\dfrac{24}{25}$

$= \sin^{-1}\dfrac{6}{5}\times\dfrac{4}{5} + \cos^{-1}\dfrac{24}{25}$

$= \sin^{-1}\dfrac{24}{25} + \cos^{-1}\dfrac{24}{25}$

$= \dfrac{\pi}{2}$

55. (d) $\sin^{-1}\dfrac{x}{5} + \text{cosec}^{-1}\dfrac{5}{4} = \dfrac{\pi}{2}$

$\Rightarrow \sin^{-1}\dfrac{x}{5} + \sin^{-1}\dfrac{4}{5} = \dfrac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}\dfrac{x}{5} = \dfrac{\pi}{2} - \sin^{-1}\dfrac{4}{5}$

$\Rightarrow \quad \sin^{-1}\dfrac{x}{5} = \cos^{-1}\dfrac{4}{5}$

$\Rightarrow \quad \sin^{-1}\dfrac{x}{5} = \sin^{-1}\sqrt{1-\dfrac{16}{25}}$

$\Rightarrow \quad \sin^{-1}\dfrac{x}{5} = \sin^{-1}\dfrac{3}{5}$

$\Rightarrow \quad \dfrac{x}{5} = \dfrac{3}{5}$

$\Rightarrow \quad x = 3$

56. (b) $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

$= \sin^{-1}\left[x\sqrt{1-(\sqrt{x})^2} - \sqrt{x}\sqrt{1-(x)^2}\right]$

$= \sin^{-1} x - \sin^{-1}\sqrt{x}$

57. (b) $\cos\left[\cos^{-1}\dfrac{1}{5} + \cos^{-1}\dfrac{1}{5} + \sin^{-1}\dfrac{1}{5}\right]$

$= \cos\left[\cos^{-1}\dfrac{1}{5} + \dfrac{\pi}{2}\right]$

$= -\sin\left[\cos^{-1}\dfrac{1}{5}\right]$

$= -\sin\left[\sin^{-1}\sqrt{1-\dfrac{1}{25}}\right]$

$= -\dfrac{2\sqrt{6}}{5}$

58. (b) $\sin\left\{\tan^{-1}\left(\dfrac{1-x^2}{2x}\right) + \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)\right\}$

Put $x = \tan\theta$.

$= \sin\left[\tan^{-1}\left(\dfrac{1-\tan^2\theta}{2\tan\theta}\right) + \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right]$

$= \sin[\tan^{-1}(\tan 2\theta) + \cos^{-1}\cos 2\theta]$

$= \sin\left[\tan^{-1}\left\{\tan\left(\dfrac{\pi}{2} - 2\theta\right)\right\} + 2\theta\right]$

$= \sin\left[\dfrac{\pi}{2} - 2\theta + 2\theta\right]$

$\Rightarrow \quad \sin\dfrac{\pi}{2} = 1$

59. (b) $2\cos^{-1}\sqrt{\dfrac{1+x}{2}} = \dfrac{\pi}{2}$

$\Rightarrow \quad \cos^{-1}\sqrt{\dfrac{1+x}{2}} = \dfrac{\pi}{2} - \cos^{-1}\sqrt{\dfrac{1+x}{2}}$

$\Rightarrow \quad \cos^{-1}\sqrt{\dfrac{1+x}{2}} = \sin^{-1}\sqrt{\dfrac{1+x}{2}}$

$= \cos^{-1}\sqrt{1-\dfrac{1+x}{2}}$

$\Rightarrow \quad \cos^{-1}\sqrt{\dfrac{1+x}{2}} = \cos^{-1}\sqrt{\dfrac{1-x}{2}}$

$$\Rightarrow \quad \sqrt{\frac{1+x}{2}} = \sqrt{\frac{1-x}{2}}$$

$$\Rightarrow \quad \frac{1+x}{2} = \frac{1-x}{2}$$

$$\Rightarrow \quad 1+x = 1-x$$

$$\Rightarrow \quad 2x = 0$$

$$\Rightarrow \quad x = 0$$

60. (d) $\frac{\pi}{2} \le x \le \frac{3\pi}{2} \Rightarrow \frac{-\pi}{2} \le x-\pi \le \frac{\pi}{2}$

$$\frac{-\pi}{2} \le \pi - x \le \frac{\pi}{2}$$

$\Rightarrow \sin^{-1}[(\sin(\pi-x)] = \pi - x$

61. (c) $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$

$$\Rightarrow \tan^{-1}x + \cot^{-1}x + \cot^{-1}x = \frac{2\pi}{3}$$

$$\frac{\pi}{2} + \cot^{-1}x = \frac{2\pi}{3}$$

$$\cot^{-1}x = \frac{2\pi}{3} - \frac{\pi}{2}$$

$$= \frac{4\pi - 3\pi}{6}$$

$$\cot^{-1}x = \frac{\neq}{6}$$

$$\Rightarrow \quad x = \cot\frac{\neq}{6}$$

$$x = \sqrt{3}$$

62. (c) $A = \tan^{-1}x$

$$\Rightarrow \quad x = \tan A$$

We know, $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

$$= \frac{2x}{1+x^2}$$

63. (b) $2\cos^{-1}\sqrt{\frac{1+x}{2}} = \frac{\pi}{2}$

$$\Rightarrow \quad \cos^{-1}\sqrt{\frac{1+x}{2}} = \frac{\pi}{4}$$

$$\sqrt{\frac{1+x}{x}} = \cos\frac{\pi}{4}$$

$$\sqrt{\frac{1+x}{x}} = \frac{1}{\sqrt{2}}$$

Squaring on both sides

$$\frac{1+x}{2} = \frac{1}{2}$$

$$1+x = 1$$

$$x = 0$$

64. (d) $\because 3 \approx 171°$ (lies in II quadrant)

$\therefore \quad \sin^{-1}\sin 3 = 3 - \pi \neq 3$

But $\sin^{-1}\sin x = x$ for principal values.

65. (a) $\because \quad AM \ge GM$

$$\therefore \quad \frac{\cos^{-1}x + (\sin^{-1}y)^2}{2} \ge \sqrt{(\cos^{-1}x)(\sin^{-1}y)^2}$$

$$\Rightarrow \quad \frac{p\pi^2}{8} \ge \frac{p\pi^2}{8}$$

$$\Rightarrow \quad p \ge 2$$

Thus, we conclude that the only value of p that satisfies all conditions is $p = 2$.

Then, $\cos^{-1}x = (\sin^{-1}y)^2$

$$\Rightarrow \quad (\cos^{-1}x)^2 = \frac{\pi^4}{16}$$

$$\Rightarrow \quad \cos^{-1}x = \pm\frac{\pi^2}{4}$$

$$\Rightarrow \quad x = \cos\left(\pm\frac{\pi^2}{4}\right)$$

$$\therefore \quad x = \cos\left(\frac{\pi^2}{4}\right)$$

Also, $(\sin^{-1}y)^4 = \frac{\pi^4}{16}$

$$\Rightarrow \quad \sin^{-1}y = \pm\frac{\pi}{2}$$

$$\therefore \quad y = \sin\left(\pm\frac{\pi}{2}\right)$$

$$= \pm 1$$

66. (a) Since, maximum value of $\sin^{-1}x_i$ is $\frac{\pi}{2}$

$\therefore \sum_{i=1}^{2n} \sin^{-1}x_i = n\pi$ is possible, if

$$x_1 = x_2 = x_3 = ... = x_{2n} = 1$$

$\therefore \sum_{i=1}^{n} x_i = 1 + 1 + 1 + ...$ upto n times $= n$

$\therefore \sum_{i=1}^{n} x_i^2 = 1^2 + 1^2 + 1^2 + 1^2 + ...$ upto n times $= n$

and $\sum_{i=1}^{n} x_i^3 = 1^3 + 1^3 + 1^3 + ...$ upto n times $= n$

Hence, $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^3 = n$

67. (d) $2(\sin^{-1}x)^2 - 5(\sin^{-1}x) + 2 = 0$

$$\Rightarrow \quad \sin^{-1}x = \frac{5 \pm \sqrt{25-16}}{4} = 2, \frac{1}{2}$$

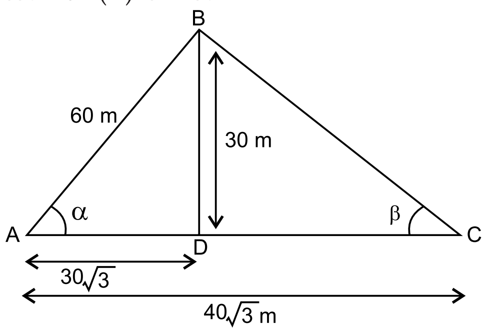
$$\Rightarrow \quad \sin^{-1}x = \frac{1}{2}, \sin^{-1}x = 2$$

$\therefore x = \sin\left(\frac{1}{2}\right)$ and $x = \sin^{-1}2$ is not possible

$\therefore\ x = \sin\left(\frac{1}{2}\right)$ is only solution

$\therefore$ Assertion (A) is false.

68. (i) (a)



$$\sin\alpha = \frac{BD}{AB}$$

$$AB = \sqrt{(30\sqrt{3})^2 + 30^2}$$

$$= \sqrt{2700 + 900}$$

$$= \sqrt{3600} = 60.$$

$$\sin\alpha = \frac{30}{60} = \frac{1}{2}$$

$$\boxed{\alpha = \sin^{-1}\frac{1}{2}}$$

(ii) (c) $\cos\alpha = \frac{AD}{AB} = \frac{30\sqrt{3}}{60} = \frac{\sqrt{3}}{2}$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

(iii) (d) $\tan\beta = \frac{BD}{DC} = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$$\boxed{\beta = \tan^{-1}\sqrt{3}}$$

(iv) (c) $BC = \sqrt{30^2 + (10\sqrt{3})^2}$

$$= \sqrt{900 + 300} = \sqrt{1200} = 20\sqrt{3}\,m$$

$\because$ $AB^2 + BC^2 = AC^2$

$$\sqrt{(60)^2 + 1200} = \sqrt{3600 + 1200}$$

$$AC = \sqrt{4800} = 40\sqrt{3}$$

$\because$ By convex of pythagoras theory

$$\angle ABC = \frac{\pi}{2}$$

(v) (c) $[0, \pi]$, $[-1, 1]$

Word of Advice

1. Several students made errors while applying properties of inverse trigonometric functions, *converting one inverse trigonometric function into another equivalent inverse trigonometric function* and in simplification.
2. Many students made errors in applying the formula of $\sin^{-1} A + \sin^{-1} B$. Also, errors took place in simplifying and solving higher degree algebraic equations. Some students converted all terms into a particular inverse function form, for example $\tan^{-1}$ and could not handle the resulting equations.
3. A few students not only wrote incorrect formula for $\tan^{-1} x + \tan^{-1} y$ but also made errors while simplifying the expression.
4. Several students made errors while converting $\sec^{-1} x$ and $\text{cosec}^{-1} x$ to its correct form.