EQUATION

PRACTICE SHEET

1. If a, b and c are real numbers, then roots of the equation $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$ are always:
(a) Real (b) Imaginary
(c) Positive (d) Negative

2. If $x = 2 + 2^{1/3} + 2^{2/3}$, then what is the value of $x^3 - 6x^2 + 6x$?
(a) 1 (b) 2
(c) 3 (d) –2

3. For the two equations $x^2 + mx + 1 = 0$ and $x^2 + x + m = 0$, what is/are the value/values of m for which these equations have at least one common root?
(a) Only –2 (b) Only 1
(c) –2 and 1 (d) –2 and –1

4. If $(x+a)$ is a factor of both the quadratic polynomials $x^2 + px + q$ and $x^2 + lx + m$, where p,q, l and m are constants, then which one of the following is correct?
(a) $a=\frac{(m-q)}{(1-p)},(1\neq p)$ (b) $a=\frac{(m+q)}{(1+p)},(1\neq p)$
(c) $1=\frac{(m-q)}{(a-p)},(a\neq p)$ (d) $p=\frac{(m-q)}{(a-1)},(a\neq 1)$

5. If the roots of the equation $4\beta^2 + \lambda\beta - 2 = 0$ are of the from $\frac{k}{k+1}$ and $\frac{k+1}{k+2}$ then what is the value of λ?
(a) 2k (b) 7
(c) 2 (d) k + 1

6. If the sum of the squares of the roots of $x^2 - (p-2)x - (p+1) = 0$ $(p \in R)$ is 5, then p is equal to:
(a) 0 (b) –1
(c) 1 (d) 3/2

7. If the roots of $x^2 - 2mx + m^2 - 1 = 0$ lie between –2 and 4, then which one of the following is correct?
(a) $-1 \le m \le 3$ (b) $-3 \le m \le 3$
(c) $3 \le m \le 5$ (d) $-1 \le m \le 5$

8. For what values of a does the equation $\cos 2x + a \sin x = 2a - 7$ posses a real solution?
(a) $a< 2$ (b) $a \ge 8$
(c) $a > 8$ (d) a is integer < – 2

9. If $\sin\theta$ and $\cos\theta$ are the roots of $ax^2 + bx + c = 0$, then constants a, b, c will satisfy which one of the following conditions?
(a) $a^2 + b^2 + 2ac = 0$ (b) $a^2 + b^2 - 2ac = 0$
(c) $a^2 - b^2 + 2ac = 0$ (d) $-a^2 + b^2 + 2ac = 0$

10. Let $a, b \in \{1, 2, 3\}$. What is the number of equations of the form $ax^2 + bx + 1 = 0$ having real roots?
(a) 1 (b) 2
(c) 5 (d) 3

11. If $px^2 + qx + r = p(x-\alpha)(x-\beta)$, and $p^3 + pq + r = 0$; p, q and r being real numbers, then which of the following is possible:
(a) $\alpha = \beta = p$ (b) $\alpha \neq \beta = p$
(c) $\alpha = \beta \neq p$ (d) $\beta \neq \alpha = p$

12. If the equations $x^2 + k^2 = 2(k+1)x$ has equal roots, then what is the value of k?
(a) $-\frac{1}{3}$ (b) $-\frac{1}{2}$
(c) 0 (d) 1

13. If α, β are the roots of the equations $x^2 - 2x - 1 = 0$, then what is the value of $\alpha^2\beta^{-2} + \alpha^{-2}\beta^2$
(a) –2 (b) 0
(c) 30 (d) 34

14. Which one of the following values of x, y satisfies the equation $2x + 3y \le 6$; $x \ge 0$, $y \ge 0$?
(a) x = 0, y = 3 (b) x = 1, y = 2
(c) x = 1, y = 1 (d) x = 4, y = 0

15. What is the value of x at the intersection of $y=\frac{8}{(x^2+4)}=$ and x + y = 2?
(a) 0 (b) 1
(c) 2 (d) –1

16. If α and β are the roots of the equation $x^2 + 6x + 1 = 0$, then what is $|\alpha - \beta|$ equal to?
(a) 6 (b) $3\sqrt{2}$
(c) $4\sqrt{2}$ (d) 12

17. If $r^{1/3}+\frac{1}{r^{1/3}}=3$ for a real number $r\neq 0$, then what $r+\frac{1}{r}$ equal to?
(a) 27 (b) 36
(c) 9 (d) 18

18. The number of rows in a lecture hall equals the number of seas in a row. If the number of rows is doubled and the number of seats in every row is reduced by 10, the number of seats is increased by 300. If x denotes the number of rows in the lecture hall, then what is the value of x?
(a) 10 (b) 15
(c) 20 (d) 30

19. For what value of k, are the rots of the quadratic equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ real and equal?
(a) k = 0 only (b) k = –3 only
(c) k = 0 or k = 3 (d) k = 0 or k = –3

20. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then what are the roots of the equation $cx^2 + bx + a = 0$?
(a) $\beta, \frac{1}{\alpha}$ (b) $\alpha, \frac{1}{\beta}$
(c) $-\alpha, -\beta$ (d) $\frac{1}{\alpha}, \frac{1}{\beta}$

21. If the roots of the equation: $(a^2 + b^2)x^2 - 2b(a+c)x + (b^2 + c^2) = 0$ are equal, then which one of the following is correct?
(a) 2b = a + c (b) $b^2 = ac$
(c) b + c = 2a (d) b = ac

22. Which of the following are the two roots of the equation $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$?
(a) $1 \pm i$ (b) $2 \pm i$
(c) $1 \pm \sqrt{2}$ (d) $2 \pm I\sqrt{2}$

23. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then what is the value of $\alpha^3 + \beta^3$?

(a) 16 (b) –16
(c) 8 (d) –8

24. If α and β are the roots of the equation $x^2 + x + 1 = 0$, then which of the following are the roots of the equation $x^2 - x + 1 = 0$?

(a) α^7 and β^{13} (b) α^{13} and β^7
(c) α^{20} and β^{20} (d) None of these

25. If the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ have real roots, then what is the value of k?

(a) 4 (b) 8
(c) 12 (d) 16

26. If the product of the roots of the equation $x^2 - 5x + k = 15$ is –3, then what is the value of k?

(a) 12 (b) 15
(c) 16 (d) 18

27. If $\frac{1}{2-\sqrt{-2}}$ is one of the roots of $ax^2 + bx + c = 0$, where a, b and c are real, then what are the values of a, b, c respectively?

(a) 6, –4, 1 (b) 4, 6, –1
(c) 3, –2, 1 (d) 6, 4, 1

28. If p and q are positive integers, then which one of the following equations has $p-\sqrt{q}$ as one of its roots?

(a) $x^2 - 2px - (p^2 - q) = 0$
(b) $x^2 - 2px + (p^2 - q) = 0$
(c) $x^2 + 2px - (p^2 - q) = 0$
(d) $x2 + 2px + (p^2 - q) = 0$

29. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then which one of the following is correct?

(a) $-3 < b < 3$ (b) $-2 < b < 2$
(c) $b < 2$ (d) $b < -2$

30. If p and q are the roots of the equation $x^2 - px + q = 0$, then what are the values of p and q respectively?

(a) 1, 0 (b) 0, 1
(c) –2, 0 (d) –2, 1

31. If α, β are the roots of the quadratic equation $x^2 - x + 1 = 0$, then which one of the following is correct?

(a) $(\alpha^4 - \beta^4)$ is real (b) $2(\alpha^5 + \beta^5) = (\alpha\beta)^5$
(c) $(\alpha^6 - \beta^6) = 0$ (d) $(\alpha^8 + \beta^8) = (\alpha\beta)^8$

32. Consider the equation (x–p) (x–6) + 1 = 0 having integral coefficients. If the equation has integral roots, then what values can p have?

(a) 4 or 8 (b) 5 or 10
(c) 6 or 12 (d) 3 or 6

ANSWER KEY																			
1.	a	**2.**	b	**3.**	c	**4.**	a	**5.**	b	**6.**	c	**7.**	b	**8.**	b	**9.**	c	**10.**	d
11.	a	**12.**	b	**13.**	d	**14.**	c	**15.**	a	**16.**	c	**17.**	d	**18.**	d	**19.**	c	**20.**	d
21.	b	**22.**	b	**23.**	b	**24.**	d	**25.**	d	**26.**	a	**27.**	a	**28.**	b	**29.**	b	**30.**	a
31.	c	**32.**	a																

Solutions

Sol.1. (a)

Given,

(x–a) (x–b) + (x–b) (x–c) + (x–c) (x–a) = 0

$\Rightarrow 3x^2 - 2(b+a+c)x + ab + bx + ca = 0$

Now, D =

$\sqrt{4(a+b+c)^2 - 12(ab+bc+ca)}$

$= 2\sqrt{a^2+b^2+c^2-ab-bc-ca}$

$= 2\sqrt{\frac{1}{2}\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}} \geq 0$

=2

Sol.2. (b)

$x = 2 + 2^{1/3} + 2^{2/3}$

$(x - 2) = 2^{1/3} + 2^{2/3}$

$(x - 2)^3 = (2^{1/3} + 2^{2/3})^3$

$x^3 - 8 - 6x^2 + 12x = 2 + 4 + 6(x - 2)$

$x^3 - 6x^2 + 6x = 2$

Sol.3. (c)

Suppose the given equations have a common root α.

Then, $\alpha^2 + m\alpha + 1 = 0$ and $\alpha^2 + \alpha + m = 0$

$\Rightarrow \frac{\alpha}{1-m} = \frac{1}{1-m} = \frac{1}{1-m}$

$\Rightarrow \frac{\alpha}{1-m} = \frac{1}{1-m} \Rightarrow \alpha = 1$

Also, $\frac{\alpha}{m2-1} = \frac{1}{1-m}$

$\Rightarrow 1-m = m^2 - 1$

$\Rightarrow m^2 + m - 2 - 1$ $(\because \alpha=1)$

$M^2 + m - 2 = 0$

$\Rightarrow (m+2)(m-1) = 0$

$\therefore$ m=1, –2

Sol.4. (a)

Since, (x+a) is a factor of $x^2 + px + q$ and $x^2 + lx + m$.

$\therefore a^2 - ap + q = 0$ …(i)

and $a^2 - la + m = 0$ …(ii)

From Eqs. (i) and (ii), we get

$-ap + q + la - m = 0 \Rightarrow (l-p)a = m - q$

$\therefore a = \frac{m-q}{l-p}$ $(l \neq p)$

Sol.5. (b)

Let $\frac{k}{k+1}$ and $\frac{k+1}{k+2}$ are the roots of the equation $4\beta^2 + \lambda\beta - 2 = 0$, then

Sum of the roots

$= \frac{k}{k+1} + \frac{k+1}{k+2} = -\frac{\lambda}{4}$

and product of the roots, $\frac{k}{k+1} \times \frac{k+1}{k+2} = -\frac{2}{4}$

$\Rightarrow \frac{k}{k+2} = -\frac{1}{2} \Rightarrow 2k = -k - 2 \Rightarrow k = -\frac{2}{3}$

Putting the value of k in (i), we get

$\frac{-\frac{2}{3}}{-\frac{2}{3}+1} + \frac{-\frac{2}{3}+1}{-\frac{2}{3}+2} = \frac{\lambda}{4}$

$\Rightarrow \frac{-\frac{2}{3}}{\frac{1}{3}} + \frac{\frac{1}{3}}{\frac{4}{3}} = -\frac{\lambda}{4} \Rightarrow -2 + \frac{1}{4} = -\frac{\lambda}{4}$

$\Rightarrow \lambda = 7$

Sol.6. (c)

Let α and β be the roots of

$x^2 - (p-2)x - (p+1) = 0$,
Then $\alpha + \beta = (p-2)$ and $\alpha\beta = -(p+1)$
$\because \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 5$
$\Rightarrow (p-2)^2 + 2(p+1) = 5$
$\Rightarrow p^2 - 4p + 4 + 2p + 2 = 5$
$\Rightarrow p^2 - 2p + 1 = 0$
$\Rightarrow (p-1)^2 = 0 \Rightarrow p = 1$

Sol.7. (b)
Since, the roots of $x^2 - 2mx + m^2 - 1 = 0$ lie between -2 and 4 i.e., $b2 - 4ac \geq 0$ and
$-2 < \frac{-ba}{2a} < 4$
$\therefore (2m)^2 - 4(m^2 - 1) \geq 0$
and $-2 < \frac{2m}{2} < 4$
$\Rightarrow -2 < m < 4$
From (i)
$4m^2 - 4m^2 + 4 \geq 0$
$\Rightarrow m \in R$
and $f(-2) > 0$, also $f(4) > 0$
$4 + 4m + m^2 - 1 > 0$, $16 - 8m + m^2 - 1 > 0$
$\Rightarrow m^2 + 4m + 3 > 0$, $m^2 - 8m + 15 > 0$
$\Rightarrow (m+1)(m+3) > 0$, $(m-3)(m-5) > 0$
$\Rightarrow -3 < m < -1$ and $3 < m < 5$

Sol.8. (b)
Given equations $\cos 2x + a \sin x = 2a - 7$ can be written as
$\cos^2 x - \sin^2 x + a \sin x = 2a - 7$
$\Rightarrow 1 - \sin^2 x - \sin^2 x + a \sin x = 2a - 7$
$\Rightarrow 2\sin^2 x - a \sin x + (2a - 7 - 1) = 0$
$\Rightarrow 2\sin^2 x - a\sin x + 2a - 8 = 0$
This is a quadratic equation in sin x and its discriminate ≥ 0
Here, $a = 2$, $b = -a$, $c = 2a - 8$
$\Rightarrow a^2 - 4.2.(2a - 8) \geq 0$
$\Rightarrow a^2 - 16a + 64 \geq 0$
$\Rightarrow (a-8)^2 \geq 0 \Rightarrow a \geq 8$

Sol.9. (c)
As given $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$
So, sum of roots
$= \sin\theta + \cos\theta = -\frac{b}{a}$ ...(1)
and product of roots
$= \sin\theta\cos\theta = \frac{c}{a}$ .... (2)
On squaring both sides in Eq. (1), we get
$(\sin\theta + \cos\theta)^2 = \frac{b^2}{a^2}$
$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$
$\Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2}$
$\Rightarrow a^2 + 2ca = b^2$ (using Equations 02)
$\Rightarrow a^2 - b^2 + 2ac = 0$

Sol.10. (d)
The given equations is $ax^2 + bx + 1 = 0$
This equation has real roots
When discriminate ≥ 0
$\therefore b^2 - 4a \geq 0$
$\Rightarrow b^2 \geq 4a$
a, b has to be selected from, three numbers, so total 3 selections are possible when (a, b) are (1, 2), (1, 3) and (2, 3).
Thus, the number of equations of the form $ax^2 + bx + 1 = 0$ having real root is 3.

Sol.11. (a)
Given equations is
$px^2 + qx + r = p(x - \alpha)(x - \beta)$
$\Rightarrow px^2 + qx + r = px^2 - p(\alpha + \beta)x + \alpha\beta p$
$\Rightarrow \alpha\beta p = r$ and $q = -(\alpha + \beta)p$
Also given that
$p^3 + pq + r = 0$
Putting value of q and r from (1)
$\Rightarrow p^3 - p^2(\alpha + \beta) + \alpha\beta p = 0$
$\Rightarrow p^2 - p(\alpha + \beta) + \alpha\beta = 0$
$\Rightarrow (p - \alpha)(p - \beta) = 0 \Rightarrow \alpha = \beta = p$

Sol.12. (b)
$x^2 + k^2 = 2(k+1)x$
$\Rightarrow x^2 - 2(k+1)x + k^2 = 0$
For roots to be equal discriminate = 0
So, $\{-2(k+1)\}^2 - 4k^2 = 0$
or, $4(k+1)^2 - 4k^2 = 0$
or, $(k+1)^2 - k^2 = 0$
or, $2k + 1 = 0$ $\Rightarrow k = -\frac{1}{2}$

Sol.13. (d)
Since, α and β are the roots of the equation $x^2 - 2x - 1 = 0$, then
Sum of roots, $\alpha + \beta = 2$ and
Product of the roots $\alpha\beta = -1$
Since, $(\alpha + \beta) = \alpha^2 + \beta^2 + 2\alpha\beta$
$\Rightarrow 4 = \alpha^2 + \beta^2 - 2$
$\Rightarrow \alpha^2 + \beta^2 = 6$
Now, $\alpha^2\beta^{-2} + \alpha^{-2}\beta^2 = \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{(\alpha\beta)^2}$
$\Rightarrow (\alpha^2 + \beta^2)^2 = 6^2$
$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 36$
$\Rightarrow \alpha^4 + \beta^4 + 2 = 36$ $(\because \alpha\beta = -1)$
$\Rightarrow \alpha^4 + \beta^4 = 34$ .....(i)
$\Rightarrow \frac{\alpha^4 + \beta^4}{(\alpha\beta)^2} = \frac{34}{(-1)^2} = 34$
Putting value of $\alpha^4 + \beta^4 = 34$ from Equation (i)]

Sol.14. (c)
There can be many values of x and y for this in equation. In the given options only x =1, y = 1 satisfy the given equation.

Sol.15. (a)
Given equations are
$y = \frac{8}{x^2 + 4}$ and $x + y = 2$
Putting value of y from 1st equation into second equations
$x + \frac{8}{x^2 + 4} = 2$
$\Rightarrow x^3 + 4x + 8 = 2x^2 + 8$
$\Rightarrow x^3 - 2x^2 + 4x = 0$
$\Rightarrow x(x^2 - 2x + 4) = 0$
$\Rightarrow x = 0$
[The other value of x is not real]

Sol.16. (c)
α and β are the roots of the equation $x^2 + 6x + 1 = 0$
$\Rightarrow \alpha + \beta = -6$ and $\alpha\beta = 1$
Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$= (-6)^2 - 4 = 36 - 4 = 32$
$\Rightarrow |\alpha - \beta| = \sqrt{32} = 4\sqrt{2}$

Sol.17. (d)
Given equation is:
$r^{1/3} + \frac{1}{r^{1/3}} = 3$
Cubing both sides, we get
$\left(r^{1/3} + \frac{1}{r^{1/3}}\right)^3 = 3^3$
$[(a+b)^3 = a^3 + b^3 + 3ab(a+b)]$
$\Rightarrow r + \frac{1}{r} + 3\left(r^{1/3} + \frac{1}{r^{1/3}}\right)^3 = 27$
$\Rightarrow r + \frac{1}{r} + 3.3 = 27$ $r + \frac{1}{r} + 27 - 9 = 18$

Sol.18. (d)
As given
$\Rightarrow$ Number of rows = x
$\Rightarrow$ Number of seats in each row = x
Total number of seats in the hall = x^2
Revised number of rows = 2x
Revised number of rows = 2x
Revised number in each row = x – 10
Thus Revised number of seats = $2x(x-10) = 2x^2 - 20x$
According to questions
$2x^2 - 20x = 300 + x^2$
$\Rightarrow x^2 - 20x - 300 = 0$
$\Rightarrow x^2 - 30x + 10x - 300 = 0$
$\Rightarrow (x - 30)(x + 10) = 0$
$\Rightarrow x = 30$ $(\because x \neq 0)$

Sol.19. (c)
As given:
Roots of the quadratic equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ are real and equal, Its discriminate
$\{-2(k-1)\}^2 - 4(k+1) = 0$
$\Rightarrow 4(k^2 - 2k + 1) - 4(k+1) = 0$
$\Rightarrow k^2 - 2k + 1 - k - 1 = 0$
$\Rightarrow k^2 - 3k = 0$
$\Rightarrow k = 0, k = 3$

Sol.20. (d)
As given, α, and β are the roots of $ax^2 + bx + c = 0$, then the roots of $cx2 + bx + a = 0$ will be reciprocal of α and β, i.e.,
$\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Sol.21. (b)
Since, roots of the given equation are equal.
$\therefore D = 0$
On solving, we get $b^2 = ac$

Sol.22. (b)
$(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$

$\Rightarrow \left(\frac{x^2+2}{x}\right)^2 - 6\left(\frac{x^2+2}{x}\right) + 8 = 0$

$\Rightarrow \left(\frac{x^2+2}{x} - 2\right)\left(\frac{x^2+2}{x} - 4\right) = 0$

$\Rightarrow \frac{x^2+2}{x} = 2 \Rightarrow x^2 + 2 = 2x$

$\Rightarrow x^2 - 2x + 2 = 0 \Rightarrow D \le 0$

$\Rightarrow \frac{x^2+2}{x} = 4 \Rightarrow x^2 + 2 = 4x$

$\Rightarrow x^2 - 4x + 2 = 0 \Rightarrow D \ge 0$

$\Rightarrow x^2 - 2x + 2 = 0$

$\Rightarrow 2 \pm i$

Sol.23. (b)

Use $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

Sol.24. (d)

Since, α and β be the roots of the equation $x^2 + x + 1 = 0$

$\therefore \alpha+\beta = 1$ and $\alpha\beta = 1$

$\Rightarrow \alpha = \omega$ and $\beta = \omega^2$

Since none of the options (a), (b), (c) satisfy the given equation, hence option (d) is correct.

Sol.25. (d)

Since, equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ have real roots

$\therefore k^2 \ge 4 \times 64 \quad \Rightarrow k \ge 16 \quad \ldots$(i)

and $64 \ge 4k \quad \Rightarrow k \le 16 \quad \ldots$(ii)

From eqs. (i) and (ii),

$\therefore k = 16$

Sol.26. (a)

$k - 15 = -3 \Rightarrow k = 12$

Sol.27. (a)

he given root is $= \frac{1}{2-\sqrt{-2}} = \frac{2+\sqrt{2}i}{6}$

$\therefore$ Another root $= \frac{2-\sqrt{2}i}{6}$

$\therefore$ equation is $- x^2 - \frac{2}{3}x + \frac{1}{6} = 0$ or $6x^2 - 4x + 1 = 0$

Thus, a = 6, b = –4 and c = 1

Sol.28. (b)

sum of roots = 2p

product of roots = $p^2 - q$

Sol.29. (b)

Since, given equation does not possess real root.

$\therefore D < 0 \Rightarrow b^2 - 4 < 0$

$\Rightarrow b^2 < 4 \Rightarrow -2 < b < 2$

Sol.30. (a)

$x^2 - px + q = 0$

p is root of this equation

$p^2 - p^2 + q = 0 \Rightarrow q = 0$

and $pq = q \Rightarrow p = 1$

Sol.31. (c)

Since, α and β be the roots of the equation $x^2 - x + 1 = 0$

$\therefore \alpha + \beta = 1$ and $\alpha\beta = 1$

Now, $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{3}i$

$\Rightarrow \alpha = \frac{1+i\sqrt{3}}{2}$ and $\beta = \frac{1-i\sqrt{3}}{2}$

$\Rightarrow \alpha = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

(a) $\alpha^4 - \beta^4 = \cos \frac{4\pi}{3} + i\sin\frac{4\pi}{3} - \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= 2i\sin\frac{4\pi}{3}$

$\Rightarrow \alpha^4 - \beta^4$ is not real

(b) $2(\alpha^5+\beta^5)$

$= 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} + \cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}\right)$

$= 2.2\cos\frac{5\pi}{3} = 4.\frac{1}{2} = 2$

Now, $(\alpha\beta)^5 = 1$

$\Rightarrow 2(\alpha^5 + \beta^5) \ne (\alpha\beta)^5$

(c) $\alpha^6 - \beta^6 = \cos \frac{6\pi}{3} + i\sin\frac{6\pi}{3} - \cos\frac{6\pi}{3} + i\sin\frac{6\pi}{3}$

$= 2$ is in $2\pi = 0$

Hence, option (c) is correct.

Sol.32. (a)

The given equation can be rewritten as

$x^2 - (p+6)x + (6p+1) = 0$

Now, $b^2 - 4ac = 0$ ($\because$ equation has integral roots)

$\Rightarrow (p+6)^2 - 4(6p+1) = 0$

Now $p^2 - 12p + 32 = 0$

$\Rightarrow (p-4)(p-8) = 0$

$\therefore p = 4, 8$

NDA PYQ

1. One of the root of the quadratic equations $ax^2 + bx + c = 0$, $a \neq 0$ is positive and the other is negative. The condition for this to happen is:

(a) $a > 0, b > 0, c > 0$ (b) $a > 0, b < 0, c > 0$
(c) $a < 0, b > 0, c < 0$ (d) $a < 0, c > 0$

[NDA (I) 2011]

2. What is the sum of the roots of the equation $(2-\sqrt{3})x^2 - (7 - 4\sqrt{3})x + (2+\sqrt{3}) = 0$?

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $7-4\sqrt{3}$ (d) 4

[NDA (I) 2011]

3. If one root of the equation $ax^2 + bx + c = 0$, $a \neq 0$ is reciprocal of the other root, then which one of the following is correct?

(a) $a = c$ (b) $b = c$
(c) $a = -c$ (d) $b = 0$

[NDA (I) 2011]

4. If sum of squares of the roots of the equation $x^2 + kx - b = 0$ is 2b, then what is k equal to?

(a) 1 (b) b
(c) – b (d) 0

[NDA (I) 2011]

5. If 3 is the root of the equation $x^2 - 8x + k = 0$, then what is the value of k?

(a) –15 (b) 9
(c) 15 (d) 24

[NDA (I) 2011]

6. What is the condition that one root of the equation $ax^2 + bx + c = 0$, $a \neq 0$ should be double the other?

(a) $2a^2 = 9bc$ (b) $2b^2 = 9ac$
(c) $2c^2 = 9ab$ (d) None of these

[NDA (I) 2011]

7. What is the solution set for the equation $x^4 - 26x^2 + 25 = 0$?

(a) $\{-5, -1, 1, 5\}$ (b) $\{-5, -1\}$
(c) $\{1,5\}$ (d) $\{-5,0,1,5\}$

[NDA (I) 2011]

8. If p, q, r are rational numbers, then the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are:

(a) Complex (b) Pure imaginary
(c) Irrational (d) Rational

[NDA (I) 2011]

9. If $x + y \leq 4$, then the how many non-zero positive integer ordered pair (x,y) ?

(a) 4 (b) 5
(c) 6 (d) 8

[NDA (I) 2011]

10. If α and β are the roots of equation $4x^2 + 3x + 7 = 0$, then what is the value of $\alpha^{-2} + \beta^{-2}$?

(a) 47/49 (b) 49/47
(c) – 47/49 (d) –49/47

[NDA (I) 2011]

11. What is the value of $y = \sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8+\infty}}}}$

(a) 10 (b) 8
(c) 6 (d) 4

[NDA (II) 2011]

12. If α and β be the roots of the equation $(x-a)(x-b) = c$, $c \neq 0$. Then, the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are:

(a) a,c (b) b,c
(c) a, b (d) a+b, a+c

[NDA (II) 2011]

13. The equation $x^2 - 4x + 29 = 0$ has one root $2 + 5i$. What is the other root? $(i=\sqrt{-1})$

(a) 2 (b) 5
(c) 2 + 5i (d) 2 – 5i

[NDA (II) 2011]

14. One of the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ is 1, then what is the second root?

(a) $\frac{b(c-a)}{a(b-c)}$ (b) $\frac{b(c-a)}{a(b-c)}$
(c) $\frac{c(a-b)}{a(b-c)}$ (d) $-\frac{c(a-b)}{a(b-c)}$

[NDA (II) 2011]

15. What are the roots of the equation:
$2(y + 2)^2 - 5(y + 2) = 12$?

(a) –7/2,2 (b) –3/2,4
(c) –5/3,3 (d) 3/2,4

[NDA (II) 2011]

16. If the roots of the equation $3x^2 - 5x + q = 0$ are equal, then what is the value of q?

(a) 2 (b) 5/12
(c) 12/25 (d) 25/12

[NDA (II) 2011]

17. If the equations $x^2 - px + q = 0$ and $x^2 - ax + b = 0$ have a common root and the roots of the second equation are equal, then which one of the following is correct?

(a) aq = 2 (b+p) (b) aq = b+p
(c) ap = 2 (b+q) (d) ap = b+q

[NDA (II) 2011]

18. If the roots of equation $x^2 - 4x - \log_3 N = 0$, are real , then what is the minimum value of N?

(a) 1/256 (b) 1/27
(c) 1/64 (d) 1/81

[NDA (II) 2011]

19. The equation $\tan^4 x - 2\sec^2 x + a^2 = 0$ will have at-least one real solution if

(a) $|a| \leq 4$ (b) $|a| \leq 4$
(c) $|a| \leq \sqrt{3}$ (d) none of above

[NDA (II) 2011]

20. If $\sin\theta = x + \frac{a}{x}$ for all $x \in R - \{0\}$, then which one of the following is correct ?

(a) $a \geq 4$ (b) $a \geq \frac{1}{2}$
(c) $a \leq \frac{1}{4}$ (d) $a \leq \frac{1}{2}$

[NDA (II) 2011]

21. What is the sum of the squares of the roots of the equation $x^2 + 2x - 143 = 0$?

(a) 170 (b) 180
(c) 190 (d) 290

[NDA (I) 2012]

22. If one of the roots of the equation $x^2 + ax - b = 0$ is, 1, then what is the value of (a–b)?

(a) –1 (b) 1

(c) 2 (d) –2

[NDA (I) 2012]

23. If α and β are the roots of the equation. $x^2 - q(1 + x) - r = 0$ then what is the value of $(1 + \alpha)(1 + \beta)$?

(a) $1 - r$ (b) $q - r$

(c) $1 + r$ (d) $q + r$

[NDA (I) 2012]

24. The solution of the simultaneous linear equations $2x + y = 6$ and $3y = 8 + 4x$ will also be satisfied by which one of the following linear equation?

(a) $x + y = 5$ (b) $2x + y = 5$

(c) $2x-3y = 10$ (d) $2x + 3y = 6$

[NDA (I) 2012]

25. If the difference between the roots of $ax^2 + bx + c = 0$ is 1, then which one of the following is correct?

(a) $b^2 = a(a+4c)$ (b) $a^2 = b(b+4c)$

(c) $a^2 = c(a+4c)$ (d) $b^2 = a(b+4c)$

[NDA (I) 2012]

Direction (for next two): The equation formed by multiplying each root of $ax^2 + bx + c = 0$ by 2 is $x^2 + 36x + 24 = 0$.

26. What is the value of b:c?

(a) 3:1 (b) 1:2

(c) 1:3 (d) 3:2

[NDA (I) 2012]

27. Which one of the following correct?

(a) $bc = a^2$ (b) $bc = 36a^2$

(c) $bc = 72a^2$ (d) $bc = 108a^2$

[NDA (I) 2012]

28. If the roots of the quadratic equation $3x^2 - 5x + p = 0$ are real and unequal, then which one of the following is correct?

(a) $p = 25/12$ (b) $P < 25/12$

(c) $p > 25/12$ (d) $p \le 25/12$

[NDA (II) 2012]

29. If the roots of a quadratic equation are m + n and m – n, then the quadratic equation will be:

(a) $x^2 + 2mx + m^2 - mn + n^2 = 0$

(b) $x^2 + 2mx + (m-n)^2 = 0$

(c) $x^2 - 2mx + m^2 - n^2 = 0$

(d) $x^2 + 2mx + m^2 - n^2 = 0$

[NDA (II) 2012]

30. If α, β are the roots of $x^2 + px - q = 0$ and γ, δ are the roots of $x^2 - px + r = 0$, then what is the value of $(\beta+\gamma)(\beta+\delta)$?

(a) $p + r$ (b) $p + q$

(c) $q + r$ (d) $p - q$

[NDA (II) 2012]

31. If α and β are roots of the equation $x^2 + bx + c = 0$, then what is the value of $\alpha^{-1} + \beta^{-1}$?

(a) $-\frac{b}{c}$ (b) $\frac{b}{c}$

(c) $\frac{c}{b}$ (d) $-\frac{c}{b}$

[NDA (I) 2013]

32. $(x+1)^2 - 1 = 0$ has

(a) One real root (b) Two real roots

(c) Two imaginary roots (d) Four real roots

[NDA (I) 2013]

33. If $4^x - 6.2^x + 8 = 0$, then the values of x are:

(a) 1,2 (b) 1,1

(c) 1,0 (d) 2,2

[NDA (I) 2013]

34. If the roots of a quadratic equation $ax^2 + bx + c = 0$ are α and β, then the quadratic equation having roots α^2 and β^2 is:

(a) $x^2 - (b^2 - 2ac)x + c = 0$

(b) $a^2x^2 - (b^2 - 2ac)x + c = 0$

(c) $ax^2 - (b^2 - 2ac)x + c^2 = 0$

(d) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

[NDA (I) 2013]

35. If the sum of the roots of a quadratic equation is 3 and the product is 2, then the equation is:

(a) $2x^2 - x + 3 = 0$ (b) $x^2 - 3x + 2 = 0$

(c) $x^2 + 3x + 2 = 0$ (d) $x^2 + 3x - 2 = 0$

[NDA (I) 2013]

36. What is the degree of the equation $\frac{1}{x-3} = \frac{1}{x+2} - \frac{1}{2}$?

(a) 0 (b) 1

(c) 2 (d) 3

[NDA (I) 2013]

37. If the roots of the equation $3ax^2 + 2bx + c = 0$ are in the ratio 2: 3, then which one of the following is correct?

(a) $8ac = 25b^2$ (b) $8ac = 9b^2$

(c) $8b^2 = 9ac$ (d) $8b^2 = 25ac$

[NDA (I) 2013]

38. The area of a rectangle whose length is five more than twice of its width is 75 square unit. The length is

(a) 5 unit (b) 10 unit

(c) 15 unit (d) 20 unit

[NDA (I) 2013]

39. The quadratic equation $x^2 + bx + 4 = 0$ will have real roots, if:

(a) Only $b \le -4$ (b) Only $b \ge 4$

(c) $-4 < b < 4$ (d) $b \le -4, b \ge 4$

[NDA (II) 2013]

40. If α and β are the roots of the equation $x^2 + x + 2 = 0$, then what is $\frac{\alpha^{10}+\beta^{10}}{\alpha^{-10}+\beta^{-10}}$ equal to?

(a) 4096 (b) 2048

(c) 1024 (d) 512

[NDA (II) 2013]

41. What is the difference in the roots of the equation $x^2 - 10x + 9 = 0$.

(a) 2 (b) 3

(c) 5 (d) 8

[NDA (II) 2013]

42. If α and β are the roots of the equation $ax^2 + bx + b = 0$, then what is the value of $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = ?$

(a) –10 (b) 0

(c) 1 (d) 2

[NDA (II) 2013]

43. The roots of the equation $x^2 - 8x + 16 = 0$:

(a) Are imaginary (b) Are distinct and real

(c) Are equal and real (d) Cannot be determined

[NDA (II) 2013]

44. If a and b are rational and b is not perfect square, then the quadratic equation with rational coefficients whose one root is $3a + \sqrt{b}$ is:

(a) $x^2 - 6ax + 9a^2 - b = 0$ (b) $3ax^2 + x - \sqrt{b}=0$

(c) $x^2 + 3ax + \sqrt{b}=0$ (d) $\sqrt{b}\,x^2 + x - 3a = 0$

[NDA (II) 2013]

45. How many real roots does the quadratic equation $f(x) = x^2 + 3|x| + 2 = 0$ have?
(a) One (b) Two
(c) Four (d) No real root
[NDA (II) 2013]

46. If $8x - 9y = 20$, and $7x - 10y = 9$, then what is $2x-y$ equal to?
(a) 10 (b) 11
(c) 12 (d) 13
[NDA (II) 2013]

47. What is the positive square root of $7+4\sqrt{3}$?
(a) $\sqrt{3}-1$ (b) $\sqrt{3}+1$
(c) $\sqrt{3}-2$ (d) $\sqrt{3}+2$
[NDA-2013(2)]

48. If α and β are the roots the equation $ax^2 + bx + c = 0$, where $a \neq 0$, then $(a\alpha + b)(a\beta + b)$ is equal to:
(a) ab (b) bc
(c) ca (d) abc
[NDA (I) 2014]

49. The roots of the equation $2a^2x^2 - 2abx + b^2 = 0$, when $a < 0$ and $b > 0$ are:
(a) Sometimes complex (b) Always irrational
(c) Always complex (d) Always real
[NDA (I) 2014]

50. Every quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$, $a \neq 0$ has:
(a) Exactly one real root (b) At least one real root
(c) At least two real roots (d) At most two real roots
[NDA (II) 2014]

51. If α, β are roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$, then what is h equal to?
(a) $\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$ (b) $\frac{1}{2}\left(-\frac{b}{a}+\frac{q}{p}\right)$
(c) $\frac{1}{2}\left(\frac{b}{p}+\frac{q}{a}\right)$ (d) $\frac{1}{2}\left(-\frac{b}{a}+\frac{q}{a}\right)$
[NDA (II) 2014]

52. If $2p + 3q = 18$ and $4p^2 + 4pq - 3q^2 - 36 = 0$, then what is $(2p + q)$ equal to?
(a) 6 (b) 7
(c) 10 (d) 20
[NDA (I) 2015]

53. In solving a problem that reduces to a quadratic equation, one student makes a mistake in the constant term and obtain 8 and 2 for roots. Another student makes a mistake only in the coefficient of first degree term and finds –9 and –1 for roots. The correct equation is:
(a) $x^2 - 10x + 9 = 0$ (b) $x^2 + 10x + 9 = 0$
(c) $x^2 - 10x + 6 = 0$ (d) $x^2 - 8x - 9 = 0$
[NDA (I) 2015]

54. If m and n are roots of the equation $(x + p)(x + q) - k = 0$, then roots of the equation: $(x-m)(x-n) + k = 0$ are
(a) p and q (b) 1/p and 1/q
(c) – p and – q (d) p + q and p – q
[NDA (I) 2015]

55. Consider the following statements in respect of the given equation.
$(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$
I. All the roots of the equation are complex.
II. The sum of all the roots of the equation is 6.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
[NDA (I) 2015]

56. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of their squares, then:
(a) $a^2 + b^2 = c^2$ (b) $a^2 + b^2 = a + b$
(c) $ab + b^2 = 2ac$ (d) $ab - b^2 = 2ac$
[NDA (II) 2015]

57. If the roots of the equation $x^2 - nx + m = 0$ differ by 1, then:
(a) $n^2 - 4m - 1 = 0$ (b) $n^2 + 4m - 1 = 0$
(c) $m^2 + 4n + 1 = 0$ (d) $m^2 - 4n - 1 = 0$
[NDA (II) 2015]

58. The number of real roots of the equation $x^2 - 3|x| + 2 = 0$ is:
(a) 4 (b) 3
(c) 2 (d) 1
[NDA (II) 2015]

Direction (for next two):
Let α and β ($\alpha < \beta$) be the roots of the equation $x^2 + bx + c = 0$, where $b > 0$ and $c < 0$.

59. Consider the following:
I. $\beta < -\alpha$ II. $\beta < |\alpha|$
Which of the above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
NDA (I) 2016]

60. Consider the following:
I. $\alpha + \beta + \alpha\beta > 0$ II. $\alpha^2\beta + \beta^2\alpha > 0$
Which of the above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
[NDA (I) 2016]

61. If one root of the equation: $(l-m)x^2 + lx + 1 = 0$ is double the other and l is real, then what is the greatest value of m?
(a) –9/8 (b) 9/8
(c) –8/9 (d) 8/9
[NDA (I) 2016]

Direction (for next two):
Given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$

62. What is $\tan(\alpha + \beta)$ equal to?
(a) $b(c - 1)$ (b) $c(b - 1)$
(c) $c(b - 1)^{-1}$ (d) $b(c - 1)^{-1}$
[NDA (I) 2016]

63. What is $\sin(\alpha+\beta)\sec\alpha\sec\beta$ equal to?
(a) b (b) –b
(c) c (d) –c
[NDA (I) 2016]

64. If $x^2 - px + 4 > 0$ for all real values of x, then which one of the following is correct?
(a) $|p| < 4$ (b) $|p| \leq 4$
(c) $|p| > 4$ (d) $|p| \geq 4$
[NDA (I) 2016]

65. If both the roots of the equation $x^2 - 2kx + k^2 - 4 = 0$ lie between –3 and 5, then which one of the following is correct?
(a) $-2 < k < 2$ (b) $-5 < k < 3$
(c) $-3 < k < 5$ (d) $-1 < k < 3$
[NDA (II) 2016]

Direction (for next two): Consider the following the next two items that follow: Let α and β be the roots of the equation. $x^2 - (1-2a^2)x + (1-2a^2) = 0$

66. Under what condition does the above equation have real roots?
(a) $a^2 < 1/2$ (b) $a^2 > 1/2$

(c) $a^2 \le 1/2$ (d) $a^2 \ge 1/2$

[NDA (II) 2016]

67. Under what condition is $\frac{1}{\alpha^2}+\frac{1}{\beta^2}<1$?

(a) $a^2 < 1/2$ (b) $a^2 > 1/2$

(c) $a^2 > 1$ (d) $a^2 \in \left(\frac{1}{3},\frac{1}{2}\right)$

[NDA (II) 2016]

Direction (for next two): Consider the following for the next two items that follow:

$2x^2 + 3x - \alpha = 0$ has roots -2 and β while the equation $x^2 - 3mx + 2m^2 = 0$ has both roots positive, where $\alpha > 0$ and $\beta > 0$.

68. What is the value of α?

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 4

[NDA (II) 2016]

69. If β, 2, 2m are in GP, then what is the value of $\beta\sqrt{m}$?

(a) 1 (b) 2

(c) 4 (d) 6

[NDA (II) 2016]

70. If $c > 0$ and $4a + c < 2b$, then $ax^2 - bx + c = 0$ has a root in which one of the following intervals?

(a) (0,2) (b) (2,3)

(c) (3,4) (d) (−2,0)

[NDA (II) 2016]

71. If the difference between the roots of the equation $x^2 + kx + 1 = 0$ is strictly less than $\sqrt{5}$, where $|k| \ge 2$, then k can be any element of the interval:

(a) $(-3, -2) \cup (2,3)$ (b) $(-3, 3)$

(c) $(-3, -2] \cup [2,3)$ (d) None of these

[NDA (I) 2017]

72. If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, then which one of the following is correct?

(a) $p^2m = l^2 q$ (b) $m^2p = l^2 q$

(c) $m^2p = q^2 l$ (d) $m^2 p^2 = l^2 q$

[NDA (I) 2017]

73. If cot α and cot β are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$, then the value of $\cot(\alpha+\beta)$ is:

(a) $\frac{c-1}{b}$ (b) $\frac{1-c}{b}$

(c) $\frac{c}{c-1}$ (d) $\frac{b}{1-c}$

[NDA (I) 2017]

74. If the graph a quadratic polynomial lies entirely above x-axis, which one of the following is correct?

(a) Both the roots are real

(b) One root is real and the other is complex

(c) Both the roots are complex

(d) Cannot say

[NDA (I) 2017]

75. The roots of the equation $(q - r)x^2 + (r - p)x + (p - q) = 0$ are:

(a) $\frac{(r-p)}{(q-r)},\frac{1}{2}$ (b) $\frac{(p-q)}{(q-r)},1$

(c) $\frac{(q-r)}{(p-q)},1$ (d) $\frac{(r-p)}{(p-q)},\frac{1}{2}$

[NDA (II) 2017]

76. The sum of al real roots of the equation $|x - 3|^2 + |x - 3| - 2 = 0$ is:

(a) 2 (b) 3

(c) 4 (d) 6

[NDA (II) 2017]

77. If α and β are the roots of the equation $3x^2 + 2x + 1 = 0$, then the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$ is:

(a) $3x^2 + 8x + 16 = 0$ (b) $3x^2 - 8x - 16 = 0$

(c) $3x^2 + 8x - 16 = 0$ (d) $x^2 + 8x + 16 = 0$

[NDA (II) 2017]

78. In ΔPQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

(a) $a = b + c$ (b) $b = c + a$

(c) $c = a + b$ (d) $b = c$

[NDA (II) 2017]

79. It is given that the roots of the equation $x^2 - 4x - \log_3 P = 0$ are real. For this, the minimum value of P is:

(a) 1/27 (b) 1/64

(c) 1/81 (d) 1

[NDA-2017(2)]

80. The equation $|1 - x| + x^2 = 5$ has:

(a) A rational root and an irrational root

(b) Two rational roots

(c) Two irrational roots

(d) No real roots

[NDA (I) 2018]

81. Suppose f(x) is such a quadratic expression that it is positive for all real x.

If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x.

(a) $g(x) < 0$ (b) $g(x) > 0$

(c) $g(x) = 0$ (d) $g(x) \ge 0$

[NDA-2018(2)]

82. The ratio of roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ are equal. If D_1 and D_2 are respective discriminates, then what is $\frac{D_1}{D_2}$ equal to?

(a) $\frac{a^2}{p^2}$ (b) $\frac{b^2}{q^2}$

(c) $\frac{c^2}{r^2}$ (d) None of these

[NDA (II) 2018]

83. If $\cos\alpha$ and $\cos\beta$ $(0 < a < \beta < \pi)$ are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of $\sec\alpha \times \sec\beta$?

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $\left(-\frac{3}{4}\right)$

[NDA (II) 2018]

84. If α and β $(\neq 0)$ are the roots of the quadratic equation $x^2 + \alpha x - \beta = 0$, then the quadratic expression $-x^2 + \alpha x + \beta$ where $x \in R$ has:

(a) Least value $-\frac{1}{4}$ (b) Least value $\frac{9}{4}$

(c) Greatest value $\frac{1}{4}$ (d) Greatest value $\frac{9}{4}$

[NDA (II) 2018]

85. Consider the following expression:

1. $x + x^2 - \frac{1}{x}$

2. $\sqrt{ax^2 + bx + x - c + \frac{d}{x} - \frac{e}{x^2}}$

3. $3x^2 - 5x + ab$

4. $\frac{2}{x^2 - ax + b^3}$

5. $\frac{1}{x} - \frac{2}{x+5}$

Which of the above are rational expressions.
(a) 1, 4 and 5 only (b) 1, 3, 4 and 5 only
(c) 2, 4 and 5 only (d) 1 and 2 only
[NDA (II) 2018]

86. What are the roots of the equation $|x^2 - x - 6| = x + 2$?
(a) - 2,1,4 (b) 0,2,4
(c) 0,1,4 (d) - 2,2,4
[NDA (I) 2019]

87. If a and b are the different real roots of the equation $px^2 + qx + r = 0$ (where p,q,r are positive),then which of the following is correct?
(a) $a > 0, b > 0$ (b) $a < 0, b < 0$
(c) $a > 0, b < 0$ (d) $a < 0, b > 0$
[NDA (I) 2019]

88. If roots of the equation $x^2 + px + q = 0$ are tan 19^0 and tan 26^0, then which of the following is correct?
(a) $q - p = 1$ (b) $p - q = 1$
(c) $p + q = 2$ (d) $p + q = 3$
[NDA (I) - 2019]

89. Number of real roots of the equation $x^2 + 9|x| + 20 = 0$ is/are
(a) zero (b) one
(c) two (d) three
[NDA (I) 2019]

90. What is the solution of $x \le 4$, $y \ge 0$ and $x \le -4$, $y \le 0$?
(a) $x \ge -4$, $y \le 0$ (b) $x \le 4$, $y \ge 0$
(c) $x \le -4$, $y = 0$ (d) $x \ge -4$, $y = 0$
[NDA (II) 2019]

91. How many real roots of the equation $x^2 + 3|x| + 2 = 0$ have
(a) zero (b) one
(c) two (d) four
[NDA (II) 2019]

92. Consider the following statements in respect of the quadratic equation $4(x - p)(x - q) - r^2 = 0$ where p, q, r are real numbers
1. The roots are real
2. The roots are equal if p = q and r = 0
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2
[NDA (II) 2019]

93. If both p and q belongs to set {1,2,3,4}, then how many equations of the form $px^2 + qx + 1 = 0$ will have real roots?
(a) 12 (b) 10
(c) 7 (d) 6
[NDA (II) 2019]

94. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then which one of the following is correct?
(a) a, b, c are in AP
(b) a, b, c are in GP
(c) a, b, c are in HP
(d) a, b, c do not follow any regular pattern
[NDA (II) 2019]

95. If $|x^2 -3x + 2| > x^2 -3x + 2$, then which one of the following is correct?
(a) $x \le 1$ or $x \ge 2$
(b) $1 \le x \le 2$
(c) $1 < x < 2$
(d) x is any real value except 3 and 4
[NDA (II) 2019]

96. Under which one of the following conditions will the quadratic equation $x^2 + mx + 2 = 0$ always have real roots?
(a) $2\sqrt{3} \le m^2 < 8$ (b) $\sqrt{3} \le m^2 < 4$
(c) $m^2 \ge 8$ (d) $m^2 \le \sqrt{3}$
[NDA (II) 2019]

97. What is the value of
$2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + ...}}}$?
(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$
(c) 3 (d) 4
[NDA (II) 2019]

98. Let $A \cup B = \{x|(x - a)(x - b) > 0$, where $a < b$ }, what are A and B equal to ?
(a) A = { x|x > a} and B = { x|x > b}
(b) A = { x|x < a} and B = { x|x > b}
(c) A = { x|x < a} and B = { x|x < b}
(d) A = { x|x > a} and B = { x|x < b}
[NDA (II) 2019]

99. If p and q are the roots of the equation $x^2 - 30x + 221 = 0$, what is the value of $p^3 + q^3$?
(a) 7010 (b) 7110
(c) 7210 (d) 7240
[NDA (II) 2019]

100. The roots α and β of a quadratic equation satisfy the relations $\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2$. What is number of such quadratic equations?
(a) 0 (b) 2
(c) 3 (d) 4
[NDA 2020]

101. If $1.5 \le x \le 4.5$, then which one of the following is correct?
(a) $(2x-3)(2x-9) > 0$ (b) $(2x-3)(2x-9) < 0$
(c) $(2x-3)(2x-9) \ge 0$ (d) $(2x-3)(2x-9) \le 0$
[NDA 2020]

102. If α and β are the roots of the equation $4x^2 + 2x - 1 = 0$, then which one of the following is correct ?
(a) $\beta = -2\alpha^2 -2\alpha$ (b) $\beta = 4\alpha^2 - 3\alpha$
(c) $\beta = \alpha^2 - 3\alpha$ (d) $\beta = -2\alpha^2 +2\alpha$
[NDA (I) 2021]

103. If the roots of the quadratic equation $x^2 + 2x +k = 0$ are real, then
(a) $k < 0$ (b) $k \le 0$
(c) $k < 1$ (d) $k \le 1$
[NDA (I) 2021]

104. If one root of $5x^2 + 26x + k = 0$ is reciprocal is other then what is the value of k?
(a) 2 (b) 3
(c) 5 (d) 8
[NDA (I) 2021]

105. If the roots of the equation $4x^2 - (5k + 1) x +5k = 0$ differ by unity, then which one of the following is a possible value of k?
(a) –3 (b) –1

(c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

[NDA (II) 2021]

106. The quadratic equation $3x^2 - (k^2 + 5k)x + 3k^2 - 5k = 0$ has real roots of equal magnitude and opposite sign. Which one of the following is correct?

(a) $0 < k < \frac{5}{3}$ (b) $0 < k < \frac{3}{5}$

(c) $\frac{3}{5} < k < \frac{5}{3}$ (d) No such value of k exists

[NDA (II) 2021]

107. If p and q are the non-zero roots of the equation $x^2 + px + q = 0$, then how many possible values can q have?

(a) Nil (b) One

(c) Two (d) Three

[NDA (II) 2021]

108. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$. What is the sum of the absolute values of the roots?

(a) 4 (b) 6

(c) 8 (d) 10

[NDA (II) 2021]

109. Consider the in equations $5x - 4y + 12 < 0$, $x + y < 2$, $x < 0$ and $y > 0$. Which one of the following points lies in the common region?

(a) (0, 0) (b) (–2, 4)

(c) (–1, 4) (d) (–1, 2)

[NDA (I) 2022]

110. Let α and β be the roots of the equation $x^2 + px + q = 0$. If α^3 and β^3 are the roots of the equation $x^2 + mx + n = 0$, then what is the value of m + n?

(a) $p^3 + q^3 + pq$ (b) $p^3 + q^3 - pq$

(c) $p^3 + q^3 + 3pq$ (d) $p^3 + q^3 - 3pq$

[NDA (I) 2022]

111. Let α and β be the roots of the equation $x^2 - ax - bx + ab - c = 0$. What is the quadratic equation whose roots are a and b?

(a) $x^2 - \alpha x - \beta x + \alpha\beta + c = 0$

(b) $x^2 - \alpha x - \beta x + \alpha\beta - c = 0$

(c) $x^2 + \alpha x + \beta x + \alpha\beta + c = 0$

(d) $x^2 + \alpha x + \beta x + \alpha\beta - c = 0$

[NDA (I) 2022]

112. If the roots of the equation $x^2 - ax - bx - cx + bc + ca = 0$ are equal, then which one of the following is correct?

(a) a + b + c = 0 (b) a – b + c = 0

(c) a + b – c = 0 (d) – a + b +c=0

[NDA (I) 2022]

113. Let α and β ($\alpha > \beta$) be the roots of the equation $x^2 - 8x + q = 0$. If $\alpha^2 - \beta^2 = 16$, then what is the value of q?

(a) –15 (b) –10

(c) 10 (d) 15

[NDA (I) 2022]

114. For how many quadratic equations, the sum of roots is equal to the product of roots?

(a) 0 (b) 1

(c) 3 (d) Infinitely many

[NDA 2022 (II)]

115. Let p, q (p > q) be the roots of the quadratic equation $x^2 + bx + c = 0$ where c > 0. If $p^2 + q^2 - 11pq = 0$, then what is p – q equal to?

(a) $3\sqrt{c}$ (b) 3c

(c) $9\sqrt{c}$ (d) 9c

[NDA 2022 (II)]

116. How many real numbers satisfy the equation $|x - 4| + |x - 7| = 15$?

(a) only one (b) only two

(c) only three (d) infinitely many

[NDA – 2023 (1)]

117. α and β are distinct real roots of the quadratic equation $x^2 + ax + b = 0$. Which of the following statements is/are sufficient to find α?

1. $\alpha + \beta = 0$, $\alpha^2 + \beta^2 = 2$
2. $\alpha\beta^2 = -1$, a = 0

Select the correct answer using the code given below:

(a) 1 only (b) 2 only

(c) both 1 and 2 (d) Neither 1 nor 2

[NDA – 2023 (1)]

118. If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then what is the value of (a + b)?

(a) –11 (b) –3

(c) 0 (d) 3

[NDA – 2023 (1)]

119. For how many integral values of k, the equation $x^2 - 4x + k = 0$, where k is an integer has real roots and both of them lie in the interval (0, 5)?

(a) 3 (b) 4

(c) 5 (d) 6

[NDA – 2023 (1)]

Consider the following for the next (02) items that follow:

Consider the equation $(1-x)^4 + (5-x)^4 = 82$

120. What is the number of real roots of the equation?

(a) 0 (b) 2

(c) 4 (d) 8

[NDA-2023 (2)]

121. What is the sum of all the roots of the equation?

(a) 24 (b) 12

(c) 10 (d) 6

[NDA-2023 (2)]

Consider the following for the next (02) items that follow:

A quadratic equation is given by $(a + b)x^2 - (a + b + c)x + k = 0$, where a,b,c are real.

122. If $k = \frac{c}{2}$, ($c \neq 0$), then the roots of the equation are:

(a) Real and equal (b) Real and unequal

(c) Real iff a > c (d) Complex but not real

[NDA-2023 (2)]

123. If k = c, then the roots of the equation are:

(a) $\frac{a+c}{a+b}$ and $\frac{b}{a+b}$ (b) $\frac{a+c}{a+b}$ and $-\frac{b}{a+b}$

(c) 1 and $\frac{c}{a+b}$ (d) –1 and $-\frac{c}{a+b}$

[NDA-2023 (2)]

124. If $-\sqrt{2}$ and $\sqrt{3}$ are roots of the equation $a_0 + a_1x + a_2x^2 + a_3x^3 + x^4 = 0$ where a_0, a_1, a_2, a_3 are integers, then which one of the following is correct?

(a) $a_2 = a_3 = 0$ (b) $a_2 = 0$ and $a_3 = -5$

(c) $a_0 = 6$, $a_3 = 0$ (d) $a_1 = 0$ and $a_2 = 5$

[NDA-2024 (1)]

125. Under which one of the following conditions does the equation $(\cos\beta - 1)x^2 + (\cos\beta)x + \sin\beta = 0$ in x have a real root for $\beta \in [0, \pi]$?

(a) $1 - \cos\beta < 0$ (b) $1 - \cos\beta \le 0$

(c) $1 - \cos\beta > 0$ (d) $1 - \cos\beta \ge 0$

[NDA-2024 (1)]

126. If a, b and c ($a > 0$, $c > 0$) are in GP, then consider the following in respect of the equation $ax^2 + bx + c = 0$:

1. The equation has imaginary roots
2. The ratio of the roots of the equation is $1 : \omega$ where ω is a cube root of unity
3. The product of roots of the equation is $\left(\frac{b^2}{a^2}\right)$.

Which of the statements given above are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

[NDA-2024 (1)]

127. If $x^2 + mx + n$ is an integer for all integral values of x, then which of the following is/are correct?

1. m must be an integer
2. n must be an integer

Select the correct answer using the code given below:

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

[NDA-2024 (1)]

Direction: Consider the following for the **two items** given below

The roots of the quadratic equation $a^2(b^2 - c^2)x^2 + b^2(c^2 - a^2)x + c^2(a^2 - b^2) = 0$ are equal. Given that $(a^2 \neq b^2 \neq c^2)$

128. Which one of the following statement is correct?

(a) a^2, b^2, c^2 are in AP
(b) a^2, b^2, c^2 are in GP
(c) a^2, b^2, c^2 are in HP
(d) a^2, b^2, c^2 are neither in AP nor in GP nor in HP

[NDA-2024 (2)]

129. Which one of the following is a root of the equation ?

(a) $\frac{b^2(c^2 - a^2)}{a^2(c^2 - b^2)}$ (b) $\frac{b^2(c^2 - a^2)}{a^2(b^2 - c^2)}$
(c) $\frac{b^2(c^2 - a^2)}{2a^2(c^2 - b^2)}$ (d) $\frac{b^2(c^2 - a^2)}{2a^2(b^2 - c^2)}$

[NDA-2024 (2)]

130. What is the number of real roots of the equation $(x - 1)^2 + (x - 3)^2 + (x - 5)^2 = 0$

(a) none (b) only one
(c) only two (d) Three

[NDA-2024 (2)]

131. If n is a root of the equation $x^2 + px + m = 0$ and m is a root of the equation $x^2 + px + n = 0$, where $m \neq n$, then what is the value of $p + m + n$?

(a) – 1 (b) 0
(c) 1 (d) 2

[NDA-2024 (2)]

132. If $f(x) = 9x - 8\sqrt{x}$ such that $g(x) = f(x) - 1$, then which one of the following is correct ?

(a) g(x) has no real roots
(b) g(x) = 0 has only one real root which is an integer
(c) g(x) = 0 has two real roots which are integers.
(d) g(x) = 0 has only one real root which is not an integer.

[NDA-2024 (2)]

ANSWER KEY

1.	d	**2.**	a	**3.**	a	**4.**	d	**5.**	c	**6.**	b	**7.**	a	**8.**	d	**9.**	c	**10.**	c
11.	d	**12.**	c	**13.**	d	**14.**	c	**15.**	a	**16.**	d	**17.**	c	**18.**	d	**19.**	c	**20.**	c
21.	d	**22**	a	**23.**	a	**24.**	a	**25.**	a	**26.**	a	**27.**	d	**28.**	b	**29.**	c	**30.**	c
31.	a	**32.**	b	**33.**	a	**34.**	d	**35.**	b	**36.**	c	**37.**	d	**38.**	c	**39.**	d	**40.**	c
41.	d	**42.**	b	**43.**	c	**44.**	a	**45.**	d	**46.**	a	**47.**	d	**48.**	c	**49.**	c	**50.**	d
51.	a	**52.**	c	**53.**	a	**54.**	c	**55.**	b	**56.**	c	**57.**	a	**58.**	a	**59.**	c	**60.**	b
61.	b	**62.**	d	**63.**	b	**64.**	d	**65.**	d	**66.**	d	**67.**	a	**68.**	c	**69.**	a	**70.**	a
71.	c	**72.**	a	**73.**	b	**74.**	c	**75.**	b	**76.**	d	**77.**	a	**78.**	c	**79.**	c	**80.**	a
81.	b	**82.**	b	**83.**	a	**84.**	d	**85.**	b	**86.**	d	**87.**	b	**88.**	a	**89.**	a	**90.**	c
91.	a	**92.**	c	**93.**	c	**94.**	c	**95.**	c	**96.**	c	**97.**	b	**98.**	b	**99.**	b	**100**	d
101.	d	**102.**	a	**103.**	d	**104.**	c	**105.**	c	**106.**	d	**107.**	b	**108.**	c	**109.**	d	**110.**	d
111.	a	**112.**	c	**113.**	d	**114.**	d	**115.**	a	**116.**	b	**117.**	c	**118.**	d	**119.**	b	**120.**	b
121.	b	**122.**	b	**123.**	c	**124.**	c	**125.**	d	**126.**	d	**127.**	c	**128.**	c	**129.**	c	**130.**	a
131.	c	**132.**	b																

Solutions

Sol. 1. (d)
$\alpha\beta < 0$

$\frac{c}{a} < 0$

The condition that both roots are of opposite sign is, a < 0, c > 0.

Sol. 2. (a)
$(2-\sqrt{3})x^2 - (7-4\sqrt{3})x + (2+\sqrt{3}) = 0$

$\alpha + \beta = \frac{7-4\sqrt{3}}{2-\sqrt{3}} = (7-4\sqrt{3})(2+\sqrt{3})$

$= 14 - 12 - 8\sqrt{3} + 7\sqrt{3} = 2 - \sqrt{3}$

Sol. 3. (a)
If one root of the equation $ax^2 + bx + c = 0$, $a \neq 0$ is reciprocal of the other root

Then $\alpha = \frac{1}{\beta}$

We know that $\alpha\beta = \frac{c}{a}$

$\Rightarrow \frac{1}{\beta}.\beta = \frac{c}{a} \Rightarrow c = a$

Sol. 4. (d)
Let the root of the quadratic equation is α, β
$\therefore \alpha+\beta = -k$, $\alpha\beta = -b$
Now, $\alpha^2 + \beta^2 = 2b$ (given)
$(\alpha+\beta)^2 - 2\alpha\beta = 2b \Rightarrow k^2 + 2b = 2b$
$\therefore k = 0$

Sol. 5. (c)
$9 - 24 + k = 0$
$k = 15$

Sol. 6. (b)
Given quadratic equation
$ax^2 + bx + c = 0$, $a \neq 0$
Let the roots of equation be (α,β),
Where $\beta = 2a$
Then, sum of roots $= -b/a$
$\Rightarrow \alpha + 2\alpha = -b/a$
$\Rightarrow \alpha = -b/3a$ …(i)
Product of roots $= c/a$
$\Rightarrow 2a^2 = c/a$

$\Rightarrow 2\left(-\frac{b}{3a}\right)^2 = \frac{c}{a}$ [from Eq. (i)]

$\Rightarrow 2b^2 = 9a^2. c/a$
$\therefore 2b^2 = 9ac$

Sol. 7. (a)
The given equation is, $x^4 - 26x^2 + 25 = 0$
$\Rightarrow x^2 - 25x^2 - x^2 + 25 = 0$
$\Rightarrow x^2(x^2 - 25) - 1(x^2 - 25) = 0$
$\Rightarrow (x^2 - 25)(x^2 - 1) = 0$
$\Rightarrow (x - 5)(x + 5)(x - 1)(x + 1) = 0$
$\therefore x = -5, -1, 1, 5$
So, the solution set is $\{-5, -1, 1, 5\}$

Sol. 8. (d)
The given equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ and p,q,r are rational numbers.
Now, $D = B^2 - 4AC$
$D = 4p^2 - 4\{p^2 - (q-r)^2\}$
$D = 4p^2 - 4p^2 + 4(q-r)^2$
$D = 4(q-r)^2$ = rational and positive
So, the roots of the equation will always be rational.

Sol. 9. (c)
$x + y \leq 4$
(x,y) can be
$\{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$
required number of ordered pair = 6

Sol. 10. (c)
given α and β are the roots of equation $4x^2 + 3x + 7 = 0$, so $\alpha + \beta = -3/4$ and $\alpha\beta = 7/4$.

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$\frac{\frac{9}{16} - \frac{7}{2}}{\frac{49}{16}} = -\frac{47}{49}$

Sol . 11 (d) .

$y = \sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8+.....}}}}$ …(i)

Squaring both the sides, we get

$y^2 = 8+2\sqrt{8+2\sqrt{8+2\sqrt{8+.....}}}$

or $y^2 = 8 + 2y$ [from (i)]
or $y^2 - 2y - 8 = 0$
$\Rightarrow y = 4$ or -2
Since the value of y cannot be –ve, therefore y = 4

Sol. 12.(c)
Given, quadratic equation is;
$(x-a)(x-b) = c$, $c \neq 0$
$\Rightarrow x^2 - (a+b)x + (ab-c) = 0$
The roots of this equation are (α,β)
Then, $\alpha+\beta = -\{-(a+b)\} = a + b$ …(i)
and $\alpha\beta = ab - c$ …(ii)

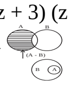
Sol. 13 (d)
We know, that, if one root of the quadratic equation is complex, then its other root is its conjugate.
Now, $x^2 - 4x + 29 = 0$ has one root $\alpha = 2+5i$ so, $2-5i$ is the second root of this equation.

Sol. 14 (c)
Given quadratic equation is:
$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$
one root of above equation is 1

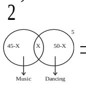
$\therefore$ Product of the roots $= \frac{c(a-b)}{a(b-c)}$

so other root will be $\Rightarrow \alpha = \frac{c(a-b)}{a(b-c)}$

Sol. 15(a)
The given quadratic equation is
$2(y+2)^2 - 5(y+2) = 12$
$\Rightarrow 2(y+2)^2 - 5(y+2) - 12 = 0$
Let $z = y + 2$ …(i)
$\Rightarrow 2z^2 - 5x - 12 = 0$
$\Rightarrow 2z^2 - 8z + 3z - 12 = 0$
$\Rightarrow (2z + 3)(z - 4) = 0$

z = – 

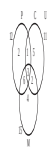
$\Rightarrow y + 2 = -\frac{3}{2}, 4$ [from Eq. (i)]

$\Rightarrow y = -2 -$ 

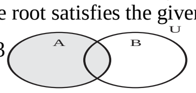
$\Rightarrow y = -\frac{7}{2}, 2$

Sol. 16(d) Let the roots of the equation, $3x^2 + 5x + q = 0$ is (α, α)

Then, sum of the roots $= -\left(\frac{-5}{3}\right)$

$\Rightarrow \alpha + \alpha =$  $\Rightarrow \alpha =$ 

The root satisfies the given equation

$\Rightarrow 3$ 

$\Rightarrow 36q = 75 \Rightarrow q = 25/12$

Sol. 17(c)
Let the roots of the equation $x^2 - px + q = 0$ is (α,β) and the roots of the equation $x^2 - ax + b = 0$ is (α, α)
Then, $\alpha + \beta = p$ and $\alpha\beta = q$ …(i)
$\alpha+\alpha = a$ and $\alpha.\alpha = b$

$\Rightarrow \alpha = \frac{a}{2}$ and $\alpha^2 = b$

$\Rightarrow \left(\frac{a}{2}\right)^2 = b \Rightarrow a = a^2 = 4b$

From Eq. (i), $\alpha\beta = q$

$\Rightarrow \frac{a}{2}.\beta = q \Rightarrow \beta = \frac{2q}{a}$

Now, putting the value of α and β in the following equation

$\alpha+\beta = p \Rightarrow \frac{a}{2} + \frac{2q}{a} = p \Rightarrow a^2 + 4q = 2ap$

$\Rightarrow 4b + 4q = 2ap$ [from Eq. (ii)]
$\therefore 2(b+q) = ap$

Sol. 18(d)
for real root $b^2 - 4ac \geq 0$
$\Rightarrow 16 + 4\log_3 P \geq 0$
$\Rightarrow 4\log_3 P \geq -16$
$\Rightarrow \log_3 P \geq -4$
$\Rightarrow P \leq 1/81$

Sol. 19(c)
Given equation is
$\tan^4 x - 2\sec^2 x + a^2 = 0$
$(\tan^2 x)^2 - 2(1+\tan^2 x) + a^2 = 0$
$(\tan^2 x)^2 - 2\tan^2 x + a^2 - 2 = 0$
This is the quadratic equation in tanx
so roots of this will be real
$\therefore b^2 - 4ac \geq 0$
$(-2)^2 - 4(a^2 - 2) \geq 0$
$a^2 \leq 3 \Rightarrow |a| \leq \sqrt{3}$

Sol. 20(c)
$x^2 - x\sin\theta + a = 0$
discriminant ≥ 0
$\sin^2\theta - 4a \geq 0$

$a \leq \frac{\sin^2\theta}{4} \Rightarrow a \leq 1/4$

Sol. 21(d)
Given, quadratic equation $x^2 + 2x - 143 = 0$
Let (α, β) be the roots of this equation
Then, $\alpha + \beta = -2$ and $\alpha.\beta = -143$
We have,
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-2)^2 - 2(-143)$
$= 4 + 286 = 290$

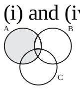
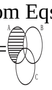
Sol. 22 (b)
Since, one root of $x^2 + ax - b = 0$ is 1
$\therefore 1^2 + 1.a - b = 0 \Rightarrow 1 + a - b = 0$
$\Rightarrow a - b = -1$

Sol. 23 (a)

If α and β are the roots of the equation
$x^2 - q(1+x) - r = 0$
then
$\alpha + \beta = q$ and $\alpha\beta = -q - r$
$(1+\alpha)(1+\beta) = (1+\alpha+\beta+\alpha\beta)$
$= 1 + q - q - r = 1 - r$

Sol. 24(a)
The solution of the simultaneous linear equations $2x + y = 6$ and $3y = 8 + 4x$ is (1,4) which satisfy our first option
$x + y = 5$

Sol. 25(a)
Let the roots of the equation $ax^2 + bx + c = 0$ are α and $(\alpha-1)$ by given condition.
Then,
Sum of the roots $= -b/a \Rightarrow \alpha + (\alpha-1) = -b/a$
$\Rightarrow 2\alpha = 1 - \frac{b}{a} \Rightarrow \alpha = \frac{a-b}{2a}$
And product of roots = c/a
$\Rightarrow \alpha(\alpha-1) = \frac{c}{a} \Rightarrow \frac{(a-b)}{2a}\left\{\frac{2-b}{2a} - 1\right\} = \frac{c}{a}$
$\Rightarrow \frac{(a-b)}{2a} \cdot \frac{(-b-a)}{2a} = \frac{c}{a}$
$\Rightarrow -(a^2-b^2) = 4ac$
$\Rightarrow b^2 - a^2 = 4ac \Rightarrow b^2 = a(a+4c)$

Solution (for next two) Let α and β be the roots of the equation $ax^2 + bx + c = 0$
Then $\alpha+\beta = -b/a$
$\Rightarrow 2\alpha + 2\beta = -2b/a$ …(i)
and $\alpha.\beta = c/a$
$\Rightarrow (2\alpha).(2\beta) = 4c/a$ …(ii)
Also given that, the equation $x^2 + 36x + 24 = 0$ is formed by multiplying each root of $ax^2 + bx + c = 0$ by 2
$\therefore 2\alpha + 2\beta = -36$ …(iii)
and $(2\alpha)(2\beta) = 24$ …(iv)
Now, from Eqs. (i) and (iv), we get
$-36 = -2b/a \Rightarrow$  …(v)
From Eqs.(ii) and (iii), we get
$24 =$  $\Rightarrow \frac{c}{a} = \frac{6}{1}$ …(vi)

Sol. 26(a)
Now, dividing Eq. (v) by Eq. (vi), we get
$\frac{b}{c} = \frac{3}{1} \Rightarrow b : c = 3:1$

Sol. 27(d)
Now, dividing Eq. (v) by Eq. (vi), we get
$\frac{b}{a} \times \frac{3}{1} = 18 \times 6 \Rightarrow bc = 108a^2$

Sol. 28(b)
Since, the roots of the quadratic equation $3x^2 - 5x + p = 0$ are real and unequal.
$\therefore$ Discriminate $> 0 \Rightarrow b^2 - 4ac > 0$
$\Rightarrow (-5)^2 - 4(3)(p) > 0$
(here, $b = -5$, $a = 3$, $c = p$)
$\Rightarrow 25 - 12p > 0 \Rightarrow 25 > 12p$
$\Rightarrow 12p < 25 \Rightarrow p < 25/12$

Sol. 29(c)
Sum of roots $= (m+n) + (m-n) = 2m$
Product of roots $= (m+n)(m-n) = m^2 - n^2$
$\therefore$ Quadratic equation
x^2 – (sum of roots) x + Product of roots = 0
$x^2 - 2mx + (m^2-n^2) = 0$

Sol. 30(c)
Since, α and β are the roots of $x^2 + px - q = 0$
$\therefore \alpha + \beta = -p$, $\alpha\beta = -q$
Again, since γ, δ are the roots of
$x^2 - px + r = 0$
$\therefore \gamma+\delta = p$, $\gamma\delta = r$
$(\beta+\gamma)(\beta+\delta) = \beta^2 + \beta\delta + \gamma\beta + \gamma\delta$
$= \beta^2 + \beta(\gamma+\delta) + \gamma\delta$
$= \beta^2 + \beta(p) + \gamma\delta$ $(\because \gamma+\delta = p$ and $\gamma\delta = r)$
$= \beta^2 + \beta(-\alpha-\beta) + r$ $[\because p = -(\alpha+\beta)]$
$= \beta^2 + (-\beta)(\alpha+\beta) + r$
$= \beta^2 - \alpha\beta - \beta^2 + r = -\alpha\beta + r$
$= -(-q) + r = q + r$
Hence, $(\beta+\gamma)(\beta+\delta) = q + r$

Sol. 31(a)
Given that, α and β are the roots of the equation $x^2 + bx + c = 0$.
Then, sum of the roots $= \alpha + \beta = \frac{-b}{1} = -b$ …(i)
and product of roots
$= \alpha.\beta = \frac{c}{1} = c$ …(ii)
$\therefore \alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha \times \beta}{\alpha\beta} = \frac{-b}{c}$

Sol. 32(b) Given, that, $(x+1)^2 - 1 = 0$
$\Rightarrow (x+1)^2 - (1)^2 = 0$
$\Rightarrow (x+1+1)(x+1-1) = 0$
$[\because a^2 - b^2 = (a-b)(a+b)]$
$\Rightarrow (x+2)(x) = 0 \therefore x = 0, -2$
Hence, $(x+1)^2 - 1 = 0$ has two real roots.

Sol. 33(a)
$4^x - 6.2^x + 8 = 0$
$2^{2x} - 6.2^x + 8 = 0$
$(2^x - 2)(2^x - 4) = 0$
$2^x = 2 \Rightarrow x = 1$
$2^x = 4 \Rightarrow x = 2$

Sol. 34(d)
Given quadratic equation is $ax^2 + bx + c = 0$
Since, its root are α and β
$\therefore$ Sum of the roots $= \alpha + \beta = -b/a$
and product of the root $= \alpha.\beta = c/a$
We have, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(-\frac{b}{a}\right)^2 - 2.$ c/a
$= \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$
and $\alpha^2.\beta^2 = (\alpha\beta)^2 = \left(\frac{c}{a}\right)^2 = \frac{c^2}{a^2}$
$\therefore$ Required quadratic equation whose roots are α^2 and β^2 is
$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$
$\Rightarrow x^2 - \frac{(b^2 - 2a)}{a^2}x + \frac{c^2}{a^2} = 0$
$\therefore a^2x^2 - (b^2 - 2ac)a + c^2 = 0$

Sol. 35(b)
Let the required quadratic equation is
$ax^2 + bx + c = 0$ …(i)
Now, sum of the roots $= \frac{-b}{a} = 3 = \frac{-(-3)}{1}$ (given)
and product of the roots = c/a = 2 = 2/1 (given)
$\therefore a = 1$, $b = -3$ and $c = 2$
Hence, required quadratic equation ix $x^2 - 3x + 2 = 0$

Sol. 36(c)
$\frac{1}{x-3} = \frac{1}{x+2} - \frac{1}{2}$
By solving this we will get a quadratic equation so degree is 2

Sol. 37(d)
Given quadratic equation $3ax^2 + 2bx + c = 0$
Let its root are 2α and 3α.
(Since, roots are in the ratio 2:3)
Now, sum of the roots $= 2\alpha + 3\alpha = \frac{-2b}{3a}$
$\Rightarrow 5\alpha = \frac{-2b}{3a} \Rightarrow \alpha = \frac{-2b}{15a}$ …(i)
and product of the roots
$= 2\alpha.3\alpha = \frac{c}{3a} \Rightarrow 6\alpha^2 = \frac{c}{3a} \Rightarrow \left(\frac{-2b}{15a}\right)^2 = \frac{c}{18a}$
$\Rightarrow \frac{4b^2}{225a^2} = \frac{c}{18a} \Rightarrow \frac{4b^2}{25a^2} = \frac{c}{2a}$
$\therefore 8b^2 = 25ac$

Sol. 38(c)
The area of a rectangle whose length is five more than twice of its width is 75 square unit.
L x B = 75
(2B + 5) x (B) = 75
By solving above quadratic equation
B = 5 and L = 15

Sol. 39(d)
Given that, the equation $x^2 + bx + 4 = 0$ have real roots, if discriminant (D) $= B^2 - 4AC \geq 0$
$\Rightarrow b^2 - 4(1)(4) \geq 0 \Rightarrow b^2 - 16 \geq 0$
$\Rightarrow (b-4)(b+4) \geq 0$
$\therefore b \leq -4$, or $b \leq 4$

Sol. 40(c)
Given that, (α, β) are the roots of the equation $x^2 + x + 2 = 0$, then
$\alpha + \beta = -1$ …(i)
and $\alpha.\beta = 2$ …(ii)
Now, we have $\frac{\alpha^{10} + \beta^{10}}{\alpha^{-10} + \beta^{-10}}$ $(\alpha\beta)^{10} = (2)^{10}$
[from Eq. (ii)]
=1024

Sol. 41(d)
Given, equation, $x^2 - 10x + 9 = 0$
Let (α, β) be the root of the given equation
Then, $\alpha+\beta = 10$ …(i)
and $\alpha.\beta = 9$ …(ii)
Now, we use the identity
$(\alpha, \beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (10)^2 - 4(9)$
$= (10)^2 - 4(9)$
$\Rightarrow \alpha - \beta = \pm \Rightarrow |\alpha - \beta| = 8$

Sol. 42(b)
Given quadratic equation is $ax^2 + bx + b = 0$
Let $(\alpha,\beta) = -b/a$ and $\alpha\beta = b/a$
Now, we have
$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{b}{a}} = \frac{-b}{a} \times \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$
$= -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}} = 0$

Sol. 43(c)
Given equation is $x^2 - 8x + 16 = 0$
$\Rightarrow (x-4)^2 = 0 \Rightarrow x = 4, 4$
Also, discriminant $= b^2 - 4ac = 0$
So, the roots of the equation are equal and real.

Sol. 44(a)
If one root of any quadratic equation is in the form $3a + \sqrt{b}$, then other root of this equation should be $3a - \sqrt{b}$.
$\therefore$ Required equation is

x^2 – (sum of roots) x + (product of roots) = 0

$\Rightarrow x^2 - \{(3a+\sqrt{b}) + (3a-\sqrt{b})\}.x$

$+\{(3a+\sqrt{b})(3a-\sqrt{b})\} = 0$

$\therefore x^2 - 6ax + 9a^2 - b = 0$

Sol. 45(d)

Given quadratic equation,

$f(x) \equiv x^2 + 3|x| + 2 = 0$

Case I. $f(x) \equiv x^2 + 3x + 2 = 0$(when, $x > 0$)

$\Rightarrow x^2 + 2x + x + 2 = 0$

$\Rightarrow x(x+2) + 1(x+2) = 0$

$\Rightarrow (x+2)(x+1) = 0$

$\therefore x = -2, -1$ (but $x > 0$)

So, here no real roots exist.

Case II. $f(x) \equiv x^2 - 3x + 2 = 0$ (when, $x < 0$)

$\Rightarrow x = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3+1}{2}$

$\Rightarrow x = 2, 1$ (but $x < 0$)

So, here also no real root exists

Hence, given quadratic equation has no real root.

Shortcut Method: In equation $x^2 + 3|x| + 2 = 0$, taking any value of x the LHS will always give +ve answer which can never be equal to zero. Thus, there are no real roots of the equation.

Sol. 46(a)

by solving both equations

x = 7 and y = 4 then 2x − y = 2(7) − 4 = 10.

Sol. 47(d)

$\sqrt{7+4\sqrt{3}} = \sqrt{7+2\sqrt{12}} = \sqrt{\left(2+\sqrt{3}\right)^2} = 2+\sqrt{3}$

Sol. 48(c) Given that, α and β are the roots of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.

Then, sum of roots = $\alpha + \beta = -b/a$

and product of roots = $\alpha.\beta = c/a$

Now, we have

$(a\alpha + b)(a\beta + b) = a^2(\alpha\beta) + ab(\alpha+\beta) + b^2$

$= a^2\left(\frac{c}{a}\right) + ab\left(-\frac{b}{a}\right) + b^2$

$= ac - b^2 + b^2 = ac.$

Sol. 49(c)

Given equation, $2a^2x^2 - 2abx + b = 0$

When, a < 0 and b > 0

$\therefore x = \frac{-(-2ab) \pm \sqrt{(-2ab)^2 - 4.2a^2.b^2}}{2.2a^2}$

(By Quadratic formula)

$= \frac{2ab \pm \sqrt{-4a^2b^2 - 8a^2b^2}}{4ab^2}$

$= \frac{2ab \pm \sqrt{-ba^2b^2}}{4a^2} = \frac{2ab \pm i\,2ab}{4a^2}$

$= \frac{2ab(1 \pm i)}{4a^2} = \frac{b}{2a}(1 \pm i)$

Which shows that the roots of the given equation is always complex.

Sol. 50(d)

Every quadratic equation

$ax^2 + bx + c = 0$, where a, b, c $\in$ R, $a \neq 0$

has at most two real roots.

Sol. 51(a)

$\alpha + \beta = -b/a$ and $\alpha\beta = c/a$

Also, $\alpha + h + \beta + h = -q/p$

$\Rightarrow \alpha + \beta + 2h = -q/p$

$\Rightarrow 2h = -\frac{q}{p} + \frac{b}{a}\left(\because \alpha + \beta = -\frac{b}{a}\right)$

$\Rightarrow h = \frac{1}{2}\left[\frac{b}{a} - \frac{q}{p}\right]$

Sol. 52(c)

Given, 2p + 3q = 18 …(i)

and $4p^2 + 4pq - 3q^2 - 36 = 0$

$\Rightarrow (2p+3q)^2 - 8pq - 12q^2 = 36$

$\Rightarrow 18^2 - 4q(2p + 3q) = 36$ [from (i)]

$\Rightarrow 18^2 - 4q.18 = 36$

$\Rightarrow$ q = 4 than p = 3

2q + p = 10.

Sol. 53(a)

Given, roots are 8 and 2, then the equation is

x^2 – (sum of roots) x + product of roots = 0

$\Rightarrow x^2 - (8+2)x + 16 = 0$

$\Rightarrow x^2 - 10x + 16 =$ …(i)

(mistake in the constant term)

For roots −9 and −1, we have

$x^2 - (-9x - 1)x + (-9) \times (-1) = 0$

$\Rightarrow x^2 - 9x + 9 = 0$ …(ii)

(mistake in the coefficient of first degree term) So as per given information, we have the right equation as $x^2 - 10x + 9 = 0$.

Sol. 54(c)

$(x + p)(x + q) - k = 0$

$\Rightarrow x^2 + qx + px + pq - k = 0$

$\Rightarrow x^2 + (p+q)x + pq - k = 0$…(i)

For m and n to be roots the equation should be

$x^2 - (m + n)x + m.n = 0$ …(ii)

On comparing Eqs. (i) and (ii), we get

$p + q = -(m + n)$ …(iii)

and $pq - k = m.n$ …(iv)

Also, $(x - m)(x - n) + k = 0$

$\Rightarrow x^2 - nx - mx + mn + k = 0$

$\Rightarrow x^2 - (m+n)x + mn + k = 0$

$\Rightarrow x^2 + (p + q)x + pq - k + k = 0$

(using Eqs. (iii) (iv)

$\Rightarrow x^2 + (p + q)x + pq = 0$

$\Rightarrow x^2 - [(-p) - (-q)] + [(-p)(-q)] = 0$

[using Eqs. (iii) and (iv)]

$\Rightarrow x^2 + (p + q)x + pq = 0$

$\Rightarrow x^2 - [(-p) - (-q)] + [(-p)(-q)] = 0$

Hence, −p and −q are the required roots.

Sol. 55(b)

We have, $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$

$(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$

$\Rightarrow \left(\frac{x^2+2}{x}\right)^2 - 6\left(\frac{x^2+2}{x}\right) + 8 = 0$

$\Rightarrow \left(\frac{x^2+2}{x} - 2\right)\left(\frac{x^2+2}{x} - 4\right) = 0$

$\Rightarrow \frac{x^2+2}{x} = 2 \Rightarrow x^2 + 2 = 2x$

$\Rightarrow x^2 - 2x + 2 = 0 \Rightarrow D \le 0$

$\Rightarrow \frac{x^2+2}{x} = 4 \Rightarrow x^2 + 2 = 4x$

$\Rightarrow x^2 - 4x + 2 = 0 \Rightarrow D \ge 0$

sum of roots of first equation is 2 and second is 4, sum off all roots = 6.

Sol. 56(c)

Equation: $ax^2 + bx + c = 0$

$\alpha + \beta = a^2 + \beta^2$ (given)

Sum of roots, $(\alpha + \beta) = -b/a$

Product of roots, $(\alpha\beta) = c/a$

$(\alpha+\beta) = (\alpha+\beta)^2 - 2\alpha\beta$

$\{\because \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta\}$

or $-\frac{b}{a} = \frac{b^2}{a^2} - \frac{2c}{a}$

or $-ab = b^2 - 2ac$

or $b^2 + ab = 2ac$

Sol. 57(a)

$x^2 - nx + m = 0$

$\alpha - \beta = 1$ …(i)

Sum of roots $(\alpha+\beta) = n$

Product of roots $(\alpha\beta) = m$

Squaring of both sides in Eq. (i)

$(\alpha - \beta)^2 = (1)^2$

or $\alpha^2 + \beta^2 - 2\alpha\beta = 1$

or $(\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta = 1$

$\{\because (\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta\}$

or $n^2 - 4m - 1 = 0$.

Sol. 58(a)

Equation: $x^2 - 3|x| + 2 = 0$

$x > 0 \Rightarrow$ +(Taking positive value)

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

x = 1,2

$x < 0 \Rightarrow$ Taking negative value

$x^2 + 3x + 2 = 0$

$x^2 + 2x + x + 2 = 0$

x = −1, −2

Hence, number of real roots are 4.

Sol. 59(c)

$\alpha+\beta = -b = -ve$ $\{\because b>0\}$ …(i)

and $\alpha\beta = c = -ve$ $\{\because x>0\}$ …(ii)

From (ii), **Case I.** $\alpha > 0$ and $\beta < 0 \Rightarrow \alpha.\beta = -ve$

Now, in (i), if $\alpha > 0$, then $\beta < 0$

$\Rightarrow \alpha > \beta$ but it is given that $\alpha < \beta$

$\therefore \alpha \ngtr 0$

Case II. If α <, then $\beta > 0 \Rightarrow \alpha.\beta = -ve$

Now, in (i), if $\alpha < 0$, $\beta > 0$ and $\alpha < \beta$

Which satisfies the given condition

Thus, $\alpha < 0 \Rightarrow -\alpha > 0$

If $\alpha < \beta$ then $-\alpha > \beta$ (Use number line)

$\therefore$ Statement 1 is correct.

From above, $\alpha<\beta$ and $-\alpha>\beta$

$\Rightarrow +\alpha < \beta < -\alpha$

$\Rightarrow \beta < |\alpha|$

$\therefore$ statement 2 is correct.

Sol. 60(b)

$\alpha + \beta = -(b)$

Since b > 0, then $\alpha + \beta = -(+ve)$

$\alpha+\beta = -ve$

and $\alpha.\beta = c$

Since b > 0, then $\alpha.\beta = -ve$

Case I. $\alpha+\beta+\alpha\beta = (-ve) + (-ve) = -ve$

$\Rightarrow \alpha + \beta + \alpha\beta < 0$

$\therefore$ Statement I is not correct.

Case II. $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta)$

$= (-ve)(-ve) = +ve$

$\alpha^2\beta + \beta^2\alpha > 0$

$\therefore$ Statement 2 is correct.

Sol. 61(b)

Let the roots of the equation are 2α and α

Product = 2α. α = $\frac{1}{l - m}$

$\alpha^2 = \frac{1}{2(l - m)}$

and sum = $3\alpha = \frac{-l}{(l - m)} \Rightarrow \alpha = -\frac{l}{3(l - m)}$

$\frac{2l^2}{9(l - m)^2} = \frac{1}{l - m} \Rightarrow l = \frac{9 \pm \sqrt{81 - 72m}}{4}$

l is real

$D \ge 0$

$81 - 72m \ge 0 \Rightarrow m \le 9/8$

Sol. 62(d)

$x^2 + bx + c = 0$

Roots of the equation are tanα and tanβ

Then,

$\tan\alpha + \tan\beta = -b$
$\tan\alpha.\tan\beta = c$
Now,

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha.\tan\beta}$

or $\tan(\alpha+\beta) = \dfrac{-b}{1-c} \Rightarrow -b(1-c)^{-1} = b(c-1)^{-1}$

Sol. 63(b)

$\sin(\alpha+\beta).\sec\alpha\sec\beta = \dfrac{\sin\alpha.\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$=\tan\alpha + \tan\beta$
$\sin(\alpha+\beta)\cos\alpha\sec\beta = -b$

Sol. 64(d)

If $x^2 - px + 4 > 0$
Then for all real values of x
$b^2 - 4ac \ge 0$
or $(-p)^2 - 4(1)(4) \ge 0$
or $p^2 - 16 \ge 0$
or $|p| \ge 4$

Sol. 65(d)

Roots of the equation,
$x^2 - 2kx + k^2 - 4 = 0$ are

$\dfrac{-(-21) \pm \sqrt{(-2k)^2 - 4(k^2-4)}}{2\times 1}$

$= \dfrac{2k \pm \sqrt{16}}{2} = k \pm 2$

As given roots are lie between –3 and 5, so
$-3 < k + 2 < 5$ and $-3 < k - 2 < 5$
$\Rightarrow -5 < k < 3$ and $-1 < k < 7$
$\therefore -1 < k < 3$

Sol. 66(d)

We have $x^2-(1-2a^2)x + (1-2a^2) = 0$
For real roots, $D \ge 0$
$\therefore (1-2a^2)^2 - 4(1-2a^2) \ge 0$
$\Rightarrow (1-2a^2)(1-2a^2-4) \le 0$
$\Rightarrow (1-2a)(2a^2+3) \le 0$ $[\because 2a^2+3>0]$
$\Rightarrow (1-2a^2) \le 0 \Rightarrow a^2 \ge 1/2$

Sol. 67(a)

We have,
α, β as the roots of the equation,
$x^2 - (1-2a^2)x + (1-2a^2) = 0$
$\therefore \alpha + \beta = 1 - 2a^2$
$\alpha\beta = 1 - 2a^2$

Now, $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

$\Rightarrow \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{(\alpha+\beta)^2}{(\alpha\beta)^2} - \dfrac{2}{\alpha\beta}$

$\Rightarrow \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{(1-2a^2)^2}{(1-2a^2)^2} - \dfrac{2}{1-2a^2}$

$\Rightarrow \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = 1 - \dfrac{2}{1-2a^2}$

Since, $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} < 1$

$\therefore 1 - \dfrac{2}{1-2a^2} < 1$

$\Rightarrow \dfrac{2}{2a^2-1} < 0$

$\Rightarrow 2a^2 < 1$ $\{\because 2>0\}$
$\Rightarrow a^2 < 1/2$

Sol. 68(c)

$\because -2$ and β are the roots of the equation
$2x^2 + 3x - \alpha = 0$
$\therefore 2(-2)^2 + 3(-2) - \alpha = 0$
$\Rightarrow 8 - 6 - \alpha = 0 \Rightarrow \alpha = 2$

Sol. 69(a)

Since, $\alpha = 2$
$\therefore$ Equation becomes
$2x^2 + 3x - 2 = 0$
$\Rightarrow 2x^2 + 4x - x - 2 = 0$
$\Rightarrow (2x-1)(x+2) = 0$
$\Rightarrow x = -2, x = 1/2$
$\therefore \beta = 1/2$
The roots of second equation are given by
$x^2 - 3mx + 2m^2 = 0$
$\Rightarrow x^2 = 2mx - mx + 2m^2 = 0$
$\Rightarrow (x-2m)(x-m) = 0$
$\Rightarrow x = 2m, x = m$
Since, both roots are positive
$\therefore m > 0$
$\because \beta, 2, 2m$ are in P.
$\therefore 4 = 2m\beta \Rightarrow 4 = 2m \times 1/2 \Rightarrow m = 4$

$\therefore \beta\sqrt{m} = \dfrac{1}{2}\sqrt{4} = \dfrac{1}{2}\times 2 = 1$

Sol. 70(a)

We have,
$c > 0$ and $4a + c < 2b$
Let $f(x) = ax^2 - bx + c$
Then, $f(0) = c > 0$
$\Rightarrow f(2) = 4a - 2b + c$
$\Rightarrow f(2) < 0$
$\therefore f(0).f(2) < 0$
Hence, one of the root is lie between (0,2)

Sol. 71(c)

According to the question,
$\alpha + \beta = -k$
$\alpha\beta = 1$
and $\alpha - \beta < \sqrt{5}$
or $(\alpha - \beta)^2 < 5$ …(i)
Now, $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$
or $(\alpha-\beta)^2 = k^2 - 4$ …(ii)
From equations (i) and (ii),
$k^2 - 4 < 5$
or $k^2 < 9 \Rightarrow -3 < k < 3$
Also, it is given that $|k| \ge 2$
$\therefore$ The required interval in which k lies, is
$(-3, -2] \cup [2,3)$

Sol. 72(a)

If α, β are the roots of $x^2 + px + q = 0$
and γ, δ are the roots of $x^2 + lx + m = 0$
then

$\dfrac{(\alpha+\beta)}{(\gamma+\delta)} = \sqrt{\dfrac{\alpha.\beta}{\gamma.\delta}}$ (Remember)

or $\dfrac{(-p)^2}{(-l)^2} = \dfrac{q}{m}$ or $p^2 m = l^2 q$.

Sol. 73(b)

Since, $\cot\alpha$ and $\cot\beta$ are the roots of the equation $x^2 + bx + c$, therefore
$\cot\alpha + \cot\beta = -b$
and $\cot\alpha.\cot\beta = c$
Now, $\cot(\alpha+\beta)=$

$\dfrac{\cot\alpha.\cot\beta - 1}{\cot\alpha + \cot\beta} = \dfrac{c-1}{-b} = \dfrac{1-c}{b}$

Sol. 74(c)

$\Rightarrow$ Both the roots are imaginary i.e. complex

Sol. 75(b)

$(q-r)x^2 + (r-p)x + (p-q) = 0$

The product of the roots = $\dfrac{p-q}{q-r}$

sum of coefficients is zero, so one root is definitely 1.so other must be

$\dfrac{p-q}{q-r}$

Sol. 76(d)

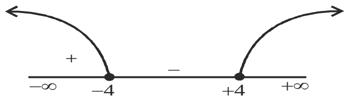
$|x-3|^2 + |x-3| - 2 = 0$
Case I. $(x-3)^2 + (x-3) - 2 = 0$
or $y^2 + y - 2 = 0$ $\{$where $y = (x-3)\}$
or $(y+2)(y-1) = 0$
or $y = -2$ or $y = 1$
or $(x-3) = -2$ or $(x-3) = 1$
or $x = 1$ or $x = 4$
But x=1 does not satisfy the given equation, so x = 4.
Case II. $[-(x-3)]^2 - (x-3) - 2 = 0$
or $(x-3)^2 - (x-3) - 2 = 0$
or $y^2 - y - 2 = 0$ $\{$where $y = (x-3)\}$
or $(y-2)(y+1) = 0$
or $y = 2$ or $y = -1$
or $x-3 = 2$ or $x - 3 = -1$
or $x = 5$ or $x = 2$
But x = 5 does not satisfy the given equation, so x = 2. Thus, the required sum = 4 + 2 = 6

Sol. 77(a)

α and β are the roots of $3x^2 + 2x + 1 = 0$

$\alpha + \beta = -\dfrac{2}{\beta} = -2 = \dfrac{1}{3}$

Now=



$= -8/3 =$ sum

and $= \left(\alpha + \dfrac{1}{\beta}\right).\left(\beta + \dfrac{1}{\alpha}\right) = \alpha\beta + \dfrac{1}{\alpha\beta} + 2$

$= \dfrac{1}{3} + 3 + 2 = \dfrac{16}{3} =$ product

$\therefore$ required equation is: $x^2 - \left(-\dfrac{8}{3}\right).x + 16/3 = 0$

Or $3x^2 + 8x + 16 = 0$

Sol. 78(c)

In ΔPQR. $\angle R = \pi/2 \Rightarrow \angle P + \angle Q = \pi/2$

$\therefore \tan\left(\dfrac{P+Q}{2}\right) = \tan\left(\dfrac{\pi}{4}\right)$

or $\dfrac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{4}\right)}{1-\tan\left(\frac{P}{2}\right).\tan\left(\frac{Q}{2}\right)} = 1$ or $\dfrac{-\frac{b}{a}}{1-\frac{c}{a}} = 1$

$\left\{\because \tan\left(\dfrac{P}{2}\right) \text{and} \tan\left(\dfrac{Q}{2}\right) \text{ are the roots of } ax^2 + bx + c = 0\right\}$

or $\dfrac{-b}{a-c} = 1$ or $a+b = c$

Sol. 79(c)

for real root $b^2 - 4ac \ge 0$
$\Rightarrow 16 + 4\log_3 P \ge 0$
$\Rightarrow 4\log_3 P \ge -16$
$\Rightarrow \log_3 P \ge -4$
$\Rightarrow P \le 1/81$

Sol.80(a)

$|1-x| + x^2 = 5$
$\Rightarrow 1 - x + x^2 = 5, x < 1$
or $-1 + x + x^2 = 5$ $x \ge 1$
$\Rightarrow x^2 - x - 4 = 0, x < 1$ or $x^2 + x - 6 = 0, x \ge 1$

$\Rightarrow x = \dfrac{-1-\sqrt{17}}{2}$ or $x = 2$

Equation has a rational root and an irrational root.

Sol. 81(b)

f(x) > 0

let $f(x) = x^2 + 1$

$g(x) = f(x) + f'(x) + f''(x)$

$g(x) = x^2 + 1 + 2x + 2$

$g(x) = x^2 + 2x + 3 = (x + 1)^2 + 2 > 0$

$g(x) > 0$

Sol. 82(b)

$$\frac{d_1}{d} = \left(\text{ratio of coefficient of } x\right)^2 = \frac{b^2}{q^2}$$

Sol. 83(a)

$\cos\alpha.\cos\beta = -3/4$

$$\Rightarrow \frac{1}{\cos\alpha.\cos\beta} = \sec\alpha.\sec\beta = -\frac{4}{3}$$

Sol. 84(d)

$\alpha + \beta = -\alpha,\ \alpha\beta = -\beta \Rightarrow \alpha\beta + \beta = 0$

$\Rightarrow (\alpha+1)\beta = 0 \Rightarrow \alpha = -1\ (\beta \neq 0)$

$\Rightarrow 2\alpha + \beta = 0$

$\Rightarrow \beta = 2$

$\therefore -x^2 + \alpha x + \beta = -x^2 - x + 2$

Greatest value = $-\frac{1+8}{-4} = \frac{9}{4}$

Sol. 85(b)

By definition of rational function

Option (b) is correct

Sol.86(d)

$|x^2 - x - 6| = x + 2$

$|(x - 3)(x + 2)| = x + 2$

for $x \le -2$ or $x \ge 3$

$\Rightarrow x^2 - x - 6 = x + 2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = 4, -2$

for $-2 < x < 3$

$\Rightarrow -(x^2 - x - 6) = x + 2$

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow x = 2, -2$

so $x = 2, -2, 4$

Sol. 87(b)

If all coefficients are positive than both roots will be negative.

Sol. 88(a)

$x^2 + px + q = 0$

$$\tan 45° = \tan(19° + 26°) = \frac{\tan 19° + \tan 26°}{1 - \tan 19° \tan 26°}$$

$$\Rightarrow 1 = \frac{-p}{1-q} \Rightarrow 1 - q = -p \Rightarrow q - p = 1$$

are $\tan 19^0$ and $\tan 26^0$

Sol. 89(a)

$x^2 + 9|x| + 20 = 0$

$(|x| + 4)(|x| + 5) = 0$

$|x| = -4$ or -5 (not possible)

number of real roots is zero.

Sol. 90(c)

$x \le 4$ and $x \le -4$

common interval $x \le -4$

$y \ge 0$ and $y \le 0$

common interval $y = 0$

Sol.91(a)

$x^2 + 3|x| + 2 = 0$

$(|x| + 2)(|x| + 1) = 0$

$|x| = -2$ or -1 (not possible)

number of real roots is zero.

Sol. 92(c)

$x^2 - (p + q)x + pq - r^2/4 = 0$

for real root $b^2 - 4ac \ge 0$

$$(p + q)^2 - 4\left(pq - \frac{r^2}{4}\right) \ge 0$$

$$(p - q)^2 + \frac{r^2}{4} \ge 0$$

it is always correct.

for equal root $p = q$ and $r = 0$

Sol. 93(c)

for real root $b^2 - 4ac \ge 0$

$q^2 - 4p \ge 0$

$q^2 \ge 4p$

if p =1, then possible value of q ={2,3,4}

if p =2, then possible value of q ={3,4}

if p =3, then possible value of q ={4}

if p =4, then possible value of q ={4}

total 7 solution.

Sol. 94(c)

sum of coefficients is zero so its one root is 1 and another is $\frac{c(a-b)}{a(b-c)}$

both are equal $\frac{c(a-b)}{a(b-c)} = 1$

2ac = ab + bc

this condition is of HP.

Sol. 95(c)

it is only possible when

$x^2 - 3x + 2 < 0$

$(x - 1)(x - 2) < 0$

$1 < x < 2$

Sol. 96(c)

for real root $b^2 - 4ac \ge 0$

$m^2 - 8 \ge 0$

$m^2 \ge 8$

Sol. 97(b)

$$x = 2 + \frac{1}{x}$$

$x^2 - 2x - 1 = 0$

$x = \sqrt{2} + 1$

Sol. 98(b)

$(x - a)(x - b) > 0$

i.e. $x < a$ or $x > b$

Sol. 99(b)

$p + q = 30$ and $pq = 221$

$(p + q)^3 = p^3 + q^3 + 3pq(p + q)$

$(30)^2 = p^3 + q^3 + 3(221)(30)$

$p^3 + q^3 = 7110$

Sol. 100(d)

$\alpha\beta = \alpha^2\beta^2$

$\alpha\beta(\alpha\beta - 1) = 0$

$\alpha\beta = 0$ or 1

Now $\alpha + \beta = \alpha^2 + \beta^2$

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

put $\alpha\beta = 0$ then there are two possible values of $\alpha + \beta$

$\alpha + \beta = (\alpha + \beta)^2$

$\alpha + \beta = 0$ or 1

put $\alpha\beta = 1$ then there are also two possible values of $\alpha + \beta$

$\alpha + \beta = (\alpha + \beta)^2 - 2$

$\alpha + \beta = 2$ or -1

so 4 possible quadratic equations are there.

Sol. 101(d)

given $1.5 \le x$

$3 \le 2x$ or $2x \ge 3 \Rightarrow (2x - 3) \ge 0$ ....(i)

and $x \le 4.5$

$2x \le 9 \Rightarrow (2x - 9) \le 0$ ....(ii)

multiply (i) and (ii)

$(2x - 3)(2x - 9) \le 0$

Sol. 102(a)

If α and β are the roots of the equation

$4x^2 + 2x - 1 = 0$,

$\alpha + \beta = -1/2$

$\beta^2 + 2\beta - 1 = 0$,

$$4\left(-\frac{1}{2} - \alpha\right)^2 + 2\beta - 1 = 0$$

by solving it

$\beta = -2\alpha^2 - 2\alpha$

Sol. 103(d)

Roots of the equation $x^2 + 2x + k = 0$ are real then $D \ge 0$

$\Rightarrow 4 - 4k \ge 0$

$\Rightarrow k \le 1$

Sol. 104(c)

If one root of equation is reciprocal to other then a = c

So k = 5

Sol. 105(c)

$\alpha - \beta = 1$

$$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1$$

$$\left(\frac{5k+1}{4}\right)^2 - 4\left(\frac{5k}{4}\right) = 1$$

$$\frac{25k^2 + 10k + 1}{16} = 5k + 1$$

$25k^2 - 70k - 15 = 0$

K =3, k = $-1/5$

Sol. 106(d)

Roots are real

$b^2 - 4ac > 0$

$(k^2 + 5k)^2 - 4(3)(3k^2 - 5k) > 0$

$k^4 + 10k^3 + 25k^2 - 36k^2 + 60k > 0$

$k^4 + 10k^3 - 11k^2 + 60k > 0$

$k(k^3 + 10k^2 - 11k + 60) > 0$

Both values of k = 0, – 5

Do not satisfy given Inequality

So no such value of k exist

Roots are equal magnitude

But opposite sing so,

$\alpha + \beta = 0$

$k^2 + 5k = 0$

k = 0, k = – 5

Sol. 107(b)

$x^2 + px + q = 0$

$p + q = -p/1$

$\Rightarrow 2p = q$ is sum of roots = $-b/a$

$pq = (q)/1$

$\Rightarrow (p-1) = 0$ product of roots = c/a

$\Rightarrow$ q=0 or p = 1

As p and q are non zero roots so $q \neq 0$

So we are left with p = 1 when p = 1

$\Rightarrow 9 = -2$

So ans is one (b) option

Sol. 108(c)

$x^4 - 10x + 9 = 0$

$(x^2 - 9)(x^2 - 1) = 0$

All values of x are 3, –3, 1, –1

Sum of absolute values=3+3+1+1=8

Sol. 109(d)

$5x - 4y + 12 < 0$

$x + y < 2$

$x < 0,\ y > 0$

By checking option.

Only (–1, 2) satisfies these inequalities.

Sol. 110(d)

$\alpha + \beta = -p$ ………….(1)

$\alpha\beta = q$ ………….(2)

$\alpha^3 + \beta^3 = -m$ ……..(3)

$\alpha^3\beta^3 = n$ ………(4)

By (3) and (4)

$m + n = \alpha^3\beta^3 - (\alpha^3 + \beta^3)$

$= \alpha^3\beta^3 - [(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)]$

$= q^3 - [-p^3 - 3q(-p)]$

$= p^3 + q^3 - 3pq$

Sol. 111(a)

$\alpha + \beta = a + b$

$\alpha\beta = ab - c$

$\Rightarrow a + b = \alpha + \beta$

$ab = \alpha\beta + c$

Equation will be

$x^2 - \alpha x - \beta x + \alpha\beta + c = 0$

Sol. 112(c)

Both roots are equal so B3 – 4AC = 0

$(a + b + c)^2 - 4(bc + ca) = 0$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 4bc - 4ca = 0$

$(a + b - c)^2 = 0$

$\Rightarrow a + b - c = 0$

Sol. 113(d)

$\alpha + \beta = 8$ …(1)

$\alpha\beta = q$ …(b)

$\alpha^2 - \beta^2 = 16$

$(\alpha + \beta)(\alpha - \beta) = 16$

of $(\alpha - \beta) = 16$

$\alpha - \beta = 2$ …(3)

By (1) & (2)

$\alpha = 5, \beta = 3$

$\because \alpha\beta = q$

$\therefore q = 5 \times 3 = 15$

Sol. 114(d)

$x^2 - (\text{sum})x + (\text{product}) = 0$

$x^2 - 2x + 2 = 0.$

$x^2 - 3x + 3 = 0$

$x^2 - 4x + 4 = 0$

infinite equations of this type are possible

Sol. 115(a)

Let p, q (p > q) be the roots of the quadratic equation $x^2 + bx + c = 0$ where c > 0.

If $p^2 + q^2 - 11pq = 0$,

$(p-q)^2 = p^2 + q^2 - 2pq$

$(p - q)^2 = 11pq - 2pq = 9pq = 9c$

$p - q = 3\sqrt{c}$

Sol. 116(b)

How many real numbers satisfy the equation

$|x - 4| + |x - 7| = 15$

If x > 7

$x - 4 + x - 7 = 15$

x = 13

if 4 < x < 7

$x - 4 - x + 7 = 15$

3 = 15 not possible

If x < 4

$-x + 4 - x + 7 = 15$

x = –2

so two values of x are possible 13 and –2

Sol. 117(c)

1. $\alpha + \beta = 0 \Rightarrow \alpha = -\beta$

$\alpha^2 + \beta^2 = 2 \Rightarrow \beta^2 + \beta^2 = 2$

$\beta^2 = 1$

$\beta = 1$ than $\alpha = -1$

$\beta = -1$ than $\alpha = 1$

2. $\alpha\beta^2 = -1$, a = 0

a = 0 then $\alpha + \beta = 0 \Rightarrow \alpha = -\beta$

$\alpha^3 = -1$

$\alpha = -1$

Both statements are individually sufficient to answer the question

Sol. 118(d)

If $\alpha = 2 - i\sqrt{3}$ then $\beta = 2 - i\sqrt{3}$

Then equation will be

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 4x + 7 = 0$

a = – 4 and b = 7

a + b = – 4 + 7 = 3

Sol. 119(b)

roots are real

$b^2 - 4ac \geq 0$

$16 - 4k \geq 0$

$4 \geq k$ ………(i)

both roots lie between 0 and 5

so f(0)f(5) > 0

k(5+k) > 0

i.e. $k \in (-5, 0)$

so k may be 1,2,3,4

Sol. 120(b)

$\Rightarrow (1 - x)^4 + (5 - x)^4 = 82$

$\Rightarrow (x - 1)^4 + (x - 5)^4 = 82$

let x – 3 = y

$\Rightarrow (y + 2)^4 + (y - 2)^4 = 82$

$\Rightarrow (y^4 + 4.y^3.2 + 6.y^2.2^2 + 4.y.2^3 + 2^4) + (y^4 - 4.y^3.2 + 6.y^2.2^2 - 4.y.2^3 + 2^4) = 82$

$\Rightarrow 2y^4 + 48y^2 + 32 = 82$

$\Rightarrow y^4 + 24y^2 - 25 = 0$

$\Rightarrow (y^2 + 25)(y^2 - 1) = 0$

so y = 1 , – 1 , 5i , – 5i

then x = 4, 2, 3 + 5i, 3 – 5i

there is two real roots

Sol. 121(b)

from above solution

sum of all roots = 4 + 2 + 3 + 5i + 3 – 5i = 12

Sol. 122(b)

for nature of roots

$B^2 - 4AC$

$(a + b + c)^2 - 4(a + b)k$

put k = c/2

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 2(a + b)c$

$a^2 + b^2 + c^2 + 2ab$

$(a + b)^2 + c^2$

given that $c \neq 0$

so $(a + b)^2 + c^2 > 0$

$B^2 - 4AC > 0$ so roots will be real and different.

Sol. 123(c)

if sum of coefficients is 0 then one root will be definitely 1 so let $\alpha = 1$

and we know that $\alpha\beta = \dfrac{C}{A} \Rightarrow \beta = \dfrac{c}{a+b}$

Sol. 124(c)

if one root is $-\sqrt{2}$ then other will be $\sqrt{2}$

and if one root is $\sqrt{3}$ then other will be $-\sqrt{3}$

so all four roots are $-\sqrt{2}, \sqrt{2}, \sqrt{3}, -\sqrt{3}$

then equation will be

$(x - \sqrt{2})(x + \sqrt{2})(x + \sqrt{3})(x - \sqrt{3}) = 0$

$(x^2 - 2)(x^2 - 3) = 0$

$x^4 - 5x^2 + 6 = 0$

now compare with given equation in question

$a_0 = 6, a_1 = 0, a_2 = -5, a_3 = 0$

Sol. 125(d)

roots are real

$b^2 - 4ac \geq 0$

$\cos^2\beta - 4(\cos\beta - 1)\sin\beta \geq 0$

$\cos^2\beta + 4(1 - \cos\beta)\sin\beta \geq 0$

we know that $\cos^2\beta$ and $\sin\beta$ is always positive for given interval

and $(1 - \cos\beta)$ should be ≥ 0

Sol. 126(d)

a, b, c are in GP so $b^2 = ac$

we know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{-3b^2}}{2a} = \left(\frac{b}{a}\right)\frac{-1 \pm \sqrt{3}i}{2}$$

equation has imaginary roots $\left(\frac{b}{a}\right)\omega$ and $\left(\frac{b}{a}\right)\omega^2$

ratio of both above roots is $1 : \omega$

product of roots is $\left(\frac{b}{a}\right)^2 \omega^3 = \left(\frac{b}{a}\right)^2$

Sol. 127(c)

$x^2 + mx + n = Z$ for all x

put x = 0 then n = Z so n must be integer.

put x = 1 then 1 + m + n = Z so m must be integer.

both statements are correct.

Sol. 128(c)

if sum of coefficients is 0 then one root will be definitely 1 so let $\alpha = 1$

both roots are equal so $\alpha = \beta = 1$

and we know that

$$\alpha\beta = \frac{C}{A} \Rightarrow \beta = \frac{c^2(a^2 - b^2)}{a^2(b^2 - c^2)} = 1$$

$$c^2(a^2 - b^2) = a^2(b^2 - c^2)$$

$$2a^2c^2 = a^2b^2 + b^2c^2$$

$$\frac{1}{b^2} = \frac{1}{a^2} + \frac{1}{c^2}$$

so a^2, b^2, c^2 are in HP.

Sol. 129(c)

$$\alpha + \beta = -\frac{B}{A}$$

$$\alpha + \alpha = -\frac{b^2(c^2 - a^2)}{a^2(b^2 - c^2)}$$

$$2\alpha = \frac{b^2(c^2 - a^2)}{a^2(c^2 - b^2)}$$

$$\alpha = \frac{b^2(c^2 - a^2)}{2a^2(c^2 - b^2)}$$

Sol. 130(a)

$(x - 1)^2 + (x - 3)^2 + (x - 5)^2 = 0$

$3x^2 - 18x + 35 = 0$

$b^2 - 4ac < 0$

so both roots are imaginary.

Sol. 131(c)

n is a root of the equation $x^2 + px + m = 0$

so $n^2 + pn + m = 0$ ……(i)

m is a root of the equation $x^2 + px + n = 0$

so $m^2 + pm + n = 0$ …..(ii)

subtract above equations

$n^2 - m^2 + p(n - m) + (m - n) = 0$

$(n - m)(n + m + p - 1) = 0$

so m + n + p = 1

Sol. 132(b)

$$g(x) = 9x - 8\sqrt{x} - 1$$

$$g(x) = 9x - 9\sqrt{x} + \sqrt{x} - 1$$

$$g(x) = 9\sqrt{x}(\sqrt{x} - 1) + 1(\sqrt{x} - 1)$$

$$g(x) = (\sqrt{x} - 1)(9\sqrt{x} + 1) = 0$$

so x = 1 is only one real root.