Chapter 7

Time and Work

CHAPTER HIGHLIGHTS

- ☞ *Work and Time*
- ☞ *Unitary Method*
- ☞ *Pipes and Cisterns*

Work

Work to be done is usually considered as one unit. It may be constructing a wall or a road, filling up or emptying a tank or cistern, or eating certain amount of food.

There are some basic assumptions that are made in the problems on Time and Work. These are taken for granted and are not specified in every problem.

1. If a person (or one member of the workforce) does some work in a certain number of days, then we assume (unless otherwise explicitly stated in the problem) that he does the work uniformly, i.e. he does the SAME amount of work everyday.
   For example, if a person can do some work in 15 days, he does 1/15th of the work in one day.
   If a person completes the work in 4 days, he does 1/4th of the work on each day, and conversely, if a person can complete 1/4th of the work in one day, he can complete the work in 4 days.
   If a tap can fill a tank in 20 minutes, then in one minute, it can fill 1/20th part of the tank.
2. If there is more than one person (or members of 'workforce') carrying out the work, it is assumed that each person (or members of the workforce), unless otherwise specified, does the same amount of work each day. This means they share the work equally.

If two people together can do the work in 8, days, it means that one man can do it in 16 days. This, in turn means, each person can do 1/16th of the work per day.

If a man works three times as fast as a boy does, the man takes one-third of the time the boy takes to complete the work. If the boy takes 12 days to complete the work, then the man takes 4 days to complete the work.

This method is known as '**UNITARY METHOD**', i.e. the time taken per '**Unit Work**' or number of persons required to complete 'Unit Work' or work completed by 'Unit Person' in 'Unit Time', etc. is what is first calculated.

We should recollect the fundamentals on variation (direct and inverse) here.

1. Time remaining constant, Work and Men are directly proportional to each other, i.e. if the work increases the number of men required to complete the work in the same number of days increases proportionately and viceversa.
2. Work remaining constant, Men and Days are inversely proportional, i.e. if the number of men increases, the number of days required to complete the same work decreases in inverse proportion, and viceversa.
3. The number of workingmen remaining constant, Work and Days are directly proportional, i.e. if the work increases, the number of days required to complete the work with the same number of working men also proportionately increases and viceversa.

The concept of MANDAYS is very important and useful here. The number of men multiplied by the number of days that they take to complete the work will give the number of mandays required to do the work. The total number of mandays required to complete a specific task will remain

a constant. So, if we change one of the variables—men or days—the other will change accordingly so that their product will remain constant (remember from our knowledge of VARIATION, two variables whose product is a constant are said to be inversely proportional to each other). The two variables—men and days—are inversely proportional to each other, when work is constant.

Solved Examples

Example 1

If 15 men take 60 days to complete a job, find the time taken by 45 men to complete it.

Solution

Number of mandays required to complete the job = 900 mandays. Time taken by 45 men to complete it = $\frac{900}{45}$, i.e. 20 days.

Example 2

18 men take 20 days to complete a job working 12 hours a day. Find the number of days that 15 men will take to complete it if they work 9 hours a day.

Solution

Total time for which 18 men work = 240 hours. Number of man hours required to complete the job = (18) (240) man hours.

Number of days taken by 15 men working 9 hours a day to complete it = $\frac{(18)(240)}{(15)(9)} = 32$.

Hence, in general we can say that

If M_1 men can do W_1 work in D_1 days working H_1 hours per day and M_2 men can do W_2 work in D_2 days working H_2 hours per day (where all men work at the same rate), then

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

Example 3

20 men take 10 days to complete a job working 12 hours a day. Find the number of men required to complete a job, twice as large, in 30 days working 8 hours a day.

Solution

Number of man hours required to complete the job = (20) (10) (12) = 2400

Number of men required to complete a job twice as large by 240 hours = $\frac{2400 \times 2}{240}$, i.e. 20 days.

Alternative method:

$M_1 = 20, D_1 = 10, H_1 = 12$

$D_2 = 30, H_2 = 8$

$D_2 = 2W_1$

$$M_2 = \frac{M_1 D_1 H_1 W_2}{W_1 D_2 H_2} = \frac{(20)(10)(12)(2W_1)}{W_1 (30)(8)} = 20.$$

If two persons A and B can individually do some work in p and q days, respectively, we can find out how much work can be done by them together in one day. Since A can do $1/p$th part of the work in one day and B can do $1/q$th part of the work in one day, the two of them together do $(1/p + 1/q)$ th part of the work in one day.

From this, we can find out the number of days that they take to complete the work.

If A can do a piece of work in p days and B can do it in q days then A and B together can complete the same in $\frac{pq}{p+q}$ days.

Example 4

A and B can complete a job in 10 days and 12 days, respectively. Find the time taken to complete it, if both A and B work together.

Solution

Time taken by them to complete it

$$= \frac{(10)(12)}{10+12} = \frac{60}{11} \text{ days.}$$

Example 5

A and B together can complete a job in 12 days. A alone can complete it in 24 days. Find the time taken by B to complete it.

Solution

Part of the job that A and B can complete in a day = $\frac{₹1320}{6}$

Part of the job that A can complete in a day = $\frac{1}{24}$

Part of the job that B can complete in a day

$$= \frac{1}{12} - \frac{1}{24} = \frac{1}{24}$$

$\therefore$ B can complete it in 24 days.

Example 6

Ajay and Bala working together can complete a job in 16 days. Ajay alone can complete it in 18 days. Both work together for 4 days and then Bala leaves. Find the time taken by Ajay to complete the remaining work.

Solution

Part of the job that can be done by both in a day $= \frac{1}{16}$

Part of the job that can be done by them in 4 days $= 4 \times \frac{1}{16} = \frac{1}{4}$

Remaining part of the job $= \frac{3}{4}$

Time taken by Ajay to complete it

$$= \frac{3}{4}(18) = 13.5 \text{ days.}$$

Example 7

A can complete a job in 16 days. He started the work and after 4 days, *B* joined him. They completed the job in 4 more days. Find the number of days in which *B* alone can complete it.

Solution

Part of the job done by *A* in a day $= \frac{1}{16}$

A worked for a total of 8 days

$\therefore$ *A* completed $(8)\left(\frac{1}{16}\right) = \frac{1}{2}$ of the job.

Hence, *B* can complete the remaining $\frac{1}{2}$ of the job in 4 days.

$\therefore$ *B* alone can complete the entire job in 8 days.

Example 8

P and *Q* together can complete a job in $14\frac{2}{5}$ days. *Q* and *R* together can complete it in $20\frac{4}{7}$ days. *P* and *R* together can complete it in 16 days. Find the time taken by each of them to complete the job.

Solution

Part of the job that *P* and *Q* can do in a day $= \frac{5}{72}$

Part of the job that *Q* and *R* can do in a day $= \frac{7}{144}$

Part of the job *P* and *R* can do in a day $= \frac{1}{16}$

Let the time taken by *P*, *Q*, and *R* to complete the job be *p* days, *q* days, and *r* days, respectively.

$$\frac{1}{p} + \frac{1}{q} = \frac{5}{72} \quad (5)$$

$$\frac{1}{q} + \frac{1}{r} = \frac{7}{144} \quad (6)$$

$$\frac{1}{p} + \frac{1}{r} = \frac{1}{16} \quad (7)$$

Adding (5) and (6) and subtracting (7),

$$\Rightarrow \quad \frac{2}{q} = \frac{5}{72} + \frac{7}{144} - \frac{1}{16} = \frac{8}{144}$$

$$q = 36$$

substitute $q = 36$, in (5),

we get $p = 24$

substi $q = 36$ in (6),

we get $r = 48$.

Example 9

A contractor decided to complete a job in 30 days for which he employed 20 men in the beginning. After 10 days, he released that the job could not be completed on time. Hence, he employed 15 more men and thus completed the job on time. Find the number of extra days, it would have taken to complete the job if the additional men were not employed.

Solution

Number of mandays required to complete the job = (20) (10) + (20 + 15) (20) = 900 mandays.

If the additional men were not employed, number of extra days $= \frac{900}{20} - 30 = 15$ days.

Example 10

A and *B* together can complete a job in 18 days and 36 days, respectively. They work on alternate days with A starting the job. In how many days will the job be completed?

Solution

Part of the job completed in the first 2 days

$$= \frac{1}{18} + \frac{1}{36} = \frac{1}{12}$$

$\therefore$ To complete the job, 12 cycles of 2 days, i.e. a total of 24 days will be required.

Example 11

P and *Q* together can complete a job in 8 days and 16 days respectively. They work on alternate days with *Q* starting the job. In how many days will the job be completed?

Solution

Part of the job completed by *P* and *Q* in the first 2 days $= \frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

After 5 cycles of 2 days, i.e. after 10 days, $\frac{15}{16}$th of the job will be completed.

Remaining part $= \frac{1}{16}$th. *Q* will work on the 11th day and he takes exactly one day to complete the remaining part.

$\therefore$ The job will be completed in 11 days.

In general, money earned should be shared by people doing the work together in the ratio of the **SHARE OF WORK** done by each of them.

For example, if A does two-fifth of the work, then he should get two-fifth of the total earnings for the work. If the remaining three-fifth of the work is done by B and C in the ratio of 1 : 2, then the remaining three-fifth of the earnings (after paying A) should be shared by B, and C in the ratio of 1 : 2. Suppose ₹500 is paid to A, B, and C together for doing the work, then A will get ₹200 (which is 2/5 of ₹500), B will get ₹100, and C, ₹200 (because the remaining ₹300 after paying A is to be divided in the ratio 1 : 2 between B and C).

When people work for the same number of days each, then the ratio of the total work done will be the same as the work done by each of them PER DAY. Hence, if all the people involved work for the same number of days, then the earnings can directly be divided in the ratio of work done per day by each of them.

Example 12

P, Q, and R can together earn ₹3100 in 10 days. Q and R together can earn ₹1320 in 6 days. P and R together can earn ₹1050 in 5 days. Find R's daily earning.

Solution

Total daily wage of P, Q, and $R = \dfrac{₹3100}{10} = ₹310$

Total daily wage of Q and $R = \dfrac{₹1320}{6} = ₹220$

Total daily wage of P and $R = \dfrac{₹1050}{5} = ₹210$

Total daily wage of P, Q, and $2R = ₹430$

$\therefore$ R's daily wage = ₹120.

Example 13

Two men under take a job for ₹960. They can complete it in 16 days and 24 days. They work along with a third man and take 8 days to complete it. Find the share that the third man should get.

Solution

The amount payable should be proportional to the fraction of work done.

Part of the job done by the third man

$$= 1 - \left(\frac{8}{16} + \frac{8}{24}\right) = \frac{1}{6}.$$

$\therefore$ The third man should get $\dfrac{₹960}{6} = ₹160.$

Pipes and Cisterns

There can be pipes (or taps) filling (or emptying) tanks with water. The time taken by different taps (to fill or empty the tank) may be different. Problems related to these can also be dealt with in the same manner as the foregoing problems on Work have been dealt with.

There is only one difference between the problems on regular Work (of the type seen earlier on in the chapter) and those in Pipes and Cisterns. In Pipes and Cisterns, a filling pipe or tap does positive work and an emptying pipe or a leak does negative work.

Example 14

Pipes P and Q take 24 minutes and 36 minutes respectively to fill an empty tank. If both take 18 minutes to fill a tank along with an outlet pipe R, find the time R would take to empty the full tank.

Solution

Let the time taken by R to empty the tank be r minutes.

$$\frac{1}{24} + \frac{1}{36} - \frac{1}{r} = \frac{1}{18};\ r = 72.$$

Example 15

Pipes X and Y can fill a tank in 30 minutes and 60 minutes, respectively. Both pipes are opened simultaneously. After how much time should X be closed so that the tank is filled in 30 minutes?

Solution

Let us say pipe X should be closed after n minutes.

i.e. pipe X is in operation for n minutes and pipe Y for all the 30 minutes.

So, $\quad \dfrac{n}{30} + \dfrac{30}{60} = 1$

$\Rightarrow \qquad n = 15.$

Example 16

Pipes P, Q, and R together can empty a full tank in 6 hours. All the three pipes are opened simultaneously and after 2 hours, P is closed. The tank is emptied in another 6 hours. Find the time in which P can empty the tank.

Solution

Part of the tank that can be emptied by P, Q, and R per hour $= \dfrac{1}{6}$.

Part of the tank that was emptied by P, Q, and R in 2 hours $= \dfrac{1}{3}$

Part of the tank which was emptied by Q and R per hour $= \dfrac{1 - \frac{1}{3}}{6} = \dfrac{1}{9}$

Time in which P can empty the tank

$= \dfrac{1}{\frac{1}{6} - \frac{1}{9}}$, i.e. 18 hours.

Example 17

A tank has a leak at its bottom which empties it at 6 litres/minutes. It also has a filling tap which can fill the tank in 6 hours. The tank takes 18 hours to become full. Find the capacity of the tank.

Solution

Let the time that would be taken by the leak to empty the full tank be x hours.

$\therefore \quad \frac{1}{6} - \frac{1}{x} = \frac{1}{18}; x = 9$

$\therefore$ Capacity of the tank = (6) (9) (60)

= 3240 litres.

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. X men can complete a work in 120 days. If there were 10 men more, the work would be completed in 20 days less. Find the value of X.
(A) 75 (B) 50 (C) 90 (D) 60

2. Nine men can complete a job in 15 days. If a man works thrice as fast as a woman, find the number of days taken by 15 women to complete the job.
(A) 20 (B) 24 (C) 27 (D) 36

3. The ratio of the time taken by A, B, and C to complete a job is 3 : 4 : 6. Find the ratio of the work they can complete in an hour.
(A) 6 : 4 : 3 (B) 4 : 3 : 2
(C) 2 : 3 : 4 (D) 3 : 4 : 6

4. Amar, Bharat, and Charu can complete a job in 12, 24, and 24 days, respectively. If they all work together, how long will they take to complete the same work?
(A) 18 days (B) 6 days
(C) 20 days (D) 16 days

5. Adam can complete a job in 25 days. Adam and Chris together can complete it in $9\frac{3}{8}$ days. In how many days can Chris alone complete the job?
(A) $12^5/_8$ (B) 10 (C) 25 (D) 15

6. P and Q can complete a job in 10 days. Q and R can complete it in 12 days. P and R can complete it in 20 days. Who is the slowest of the three workers?
(A) P
(B) Q
(C) R
(D) Cannot be determined

7. Ten men can do a piece of work in 15 days. How many men are needed to complete a work which is five times as large as the first one, in 10 days?
(A) 60 (B) 75 (C) 70 (D) 85

8. Tap X can fill a tank in 10 hours. Tap Y can fill it in 15 hours. If the two taps fill the tank together, what fraction of the tank is filled by X?
(A) 1/10 (B) 1/6 (C) 2/3 (D) 3/5

9. Pipe A can fill an empty tank in 9 hours. Pipe B can empty a full thank in 18 hours. If both pipes are opened simultaneously when the tank is empty, find the time taken to fill the tank (in hours).
(A) 24 (B) 27 (C) 18 (D) 36

10. Raj can build a wall in 18 days and Kiran can do the same in 30 days. After Raj had built half the wall, Kiran joins him. What is the total number of days taken to build the wall?
(A) 24 (B) $14\frac{5}{8}$ (C) $15\frac{1}{2}$ (D) $16\frac{1}{2}$

11. Kaushik is one and a half times more efficient than Ravi. Kaushik can do a piece of work in 20 days. What portion of the total work can both of them together complete in 10 days?
(A) 3/10 (B) 4/5 (C) 9/10 (D) 7/10

12. Had then been one men less, then the number of days required to do a piece of work would have been one more. If the number of mandays required to complete the work is 56, how many workers were there?
(A) 6 (B) 8 (C) 9 (D) 14

13. In 8 days, Peter can do as much work as Pan can do in 12 days. To do a certain job both together take 36 days. In how many days can Pan, working alone, complete the job?
(A) 60 days (B) 80 days
(C) 108 days (D) 90 days

14. X can complete a job in 36 days and Y can complete it in 45 days. Z can complete the job in z days. Z started the job. After 28 days, X and Y joined. The job was completed in 4 more days. Find z.
(A) 40 (B) 35 (C) 30 (D) 50

15. Working in pairs, PQ, QR, and RP can complete a job in 24 days, 20 days, and 30 days, respectively. Find the respective times taken by P, Q, and R individually to complete the same job (in days).
(A) $48, 80, \frac{240}{7}$ (B) $80, 48, \frac{240}{7}$
(C) $80, \frac{240}{7}, 48$ (D) $48, \frac{240}{7}, 80$

16. A frog was at the bottom of a 80 m deep well. It attempted to come out of it by jumping. In each jump, it covered 1.15 m but slipped down by 0.75 m. Find the number of jumps after which it would out of the well.
(A) 198 (B) 201 (C) 200 (D) 199

17. A man, a woman and a boy can do a piece of work in 2, 4, and 8 days, respectively. How many boys must work together with one man and one woman to complete the work in one day?
(A) 5 (B) 4 (C) 2 (D) 1

18. A machine of type *A* which has to produce a set of 1500 bolts, can do so in 30 days. The machine breaks down after 10 days. A machine of type *B* completes the remaining work in 10 days. In 30 days how many bolts can both of them together produce?
(A) 3000 (B) 4500 (C) 6000 (D) 2500

19. In a farm, each cow eats twice as much grass as each sheep. The cost of grass for 10 cows and 40 sheep for 20 days is ₹ 900. Find the cost of grass for 20 cows and 10 sheep for 18 days (in ₹).
(A) 600 (B) 675 (C) 750 (D) 800

20. The cost of grass for 20 cows and 30 sheep for 30 days is ₹720. If the 30 sheep eat double the grass eaten by the 20 cows, then what is the cost of grass eaten by 20 sheep in 15 days?
(A) ₹200 (B) ₹160 (C) ₹240 (D) ₹100

21. George and Gagan together repair a bridge in 45 days and receive ₹13500. If Gagan is three times as efficient as George, what is the amount of money he earns in 10 days?
(A) ₹2000 (B) ₹2250 (C) ₹2500 (D) ₹2750

22. Two pipes *A* and *B* which can fill a tank in 20 and 30 hours, respectively, were opened simultaneously. But there was a leak and it took 3 hours more to fill the tank. In how many hours can the leak empty the tank?
(A) 60 (B) 50 (C) 30 (D) 40

23. Gokul, Govardhan, and Ganesh can do a piece of work in 10, 20, and 30 days, respectively. They begin a new job of similar nature and each of them works on it for one third of the total period of work. If they get ₹6600 for the new job, how much should Govardhan get, given that the amounts distributed are in proportion to the work done by them?
(A) ₹1800 (B) ₹2200 (C) ₹3300 (D) ₹2400

24. Rakesh and Ramesh take 30 days and 60 days, respectively to complete a job. They work on alternate days to complete it with Rakesh starting the job. Find the time in which the job is completed (in days).
(A) 60 (B) 80 (C) 40 (D) 90

25. If Rakesh and Ramesh had instead taken 10 days and 12 days, respectively, to complete the job, find the time in which the job would have been completed (in days).

(A) $10\frac{1}{3}$ (B) $10\frac{5}{6}$

(C) 11 (D) $10\frac{1}{2}$

Answer Keys

1. B	**2.** C	**3.** B	**4.** B	**5.** D	**6.** C	**7.** B	**8.** D	**9.** C	**10.** B
11. D	**12.** B	**13.** D	**14.** A	**15.** C	**16.** D	**17.** C	**18.** B	**19.** B	**20.** B
21. B	**22.** A	**23.** A	**24.** C	**25.** B					