Part I

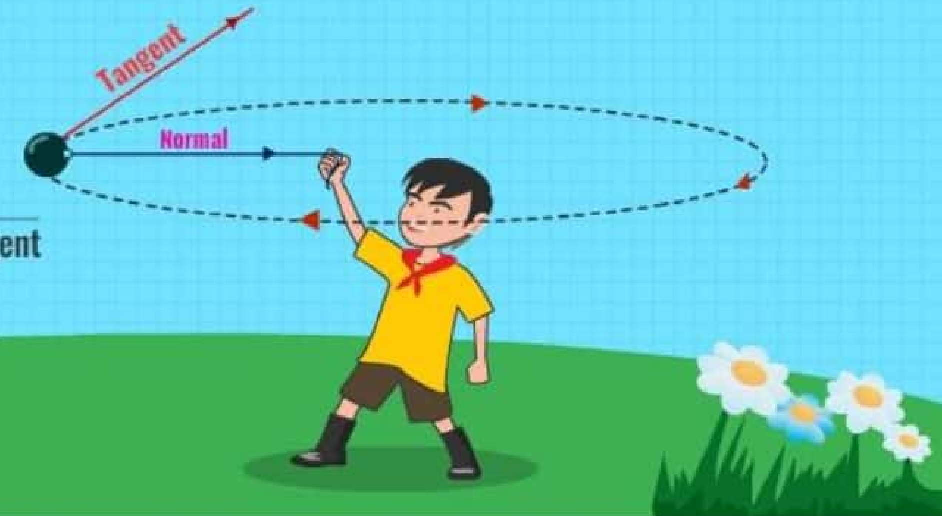
TANGENT & NORMAL

TANGENT

Tangent is a limiting case of a secant

NORMAL

A line that is perpendicular to a tangent line at the point of tangency.



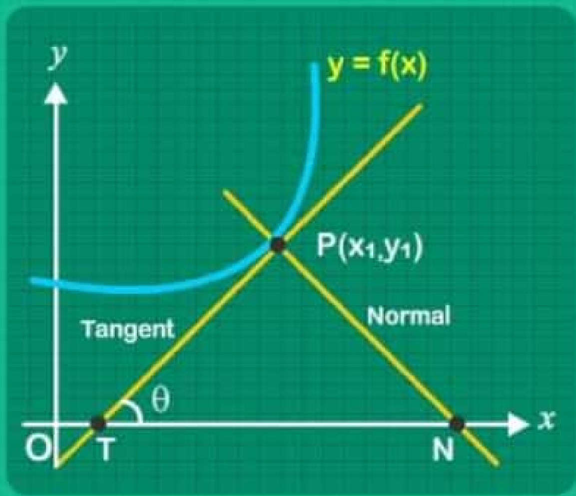
CALCULATING TANGENT LINE & NORMAL LINE TO A CURVE

EQUATION OF TANGENT & ITS LENGTH

Equation: $y - y_1 = m_T (x - x_1)$

Length: $PT = \left| \dfrac{y_1\sqrt{1 + (m_T)^2}}{(m_T)} \right|$

$$m_T = \left(\frac{dy}{dx}\right)_{P(x_1, y_1)} = \tan\theta$$

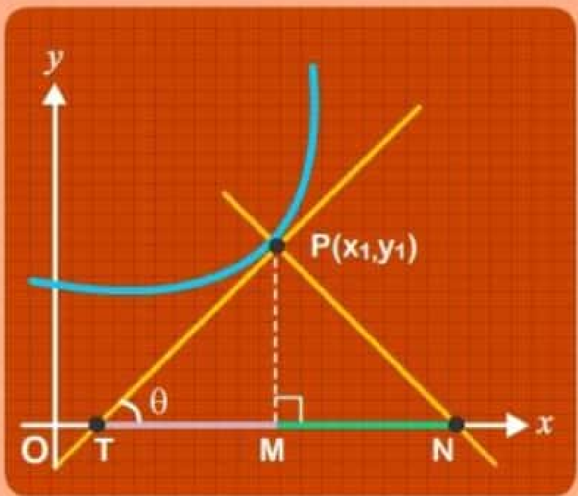


EQUATION OF NORMAL & ITS LENGTH

Equation: $y - y_1 = \dfrac{-1}{m_T} (x - x_1)$

Length: $PN = \left| y_1\sqrt{1 + (m_T)^2} \right|$

SUBTANGENT & SUBNORMAL



TM is the Subtangent and length of $TM = \left| \dfrac{y_1}{m_T} \right|$

MN is the Subnormal and length of $MN = \left| y_1 m_T \right|$

ANGLE BETWEEN TWO INTERSECTING CURVES

Part II

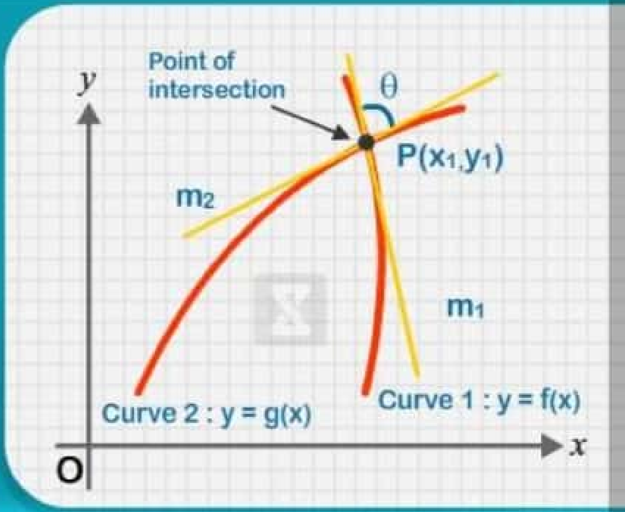
$m_1 \rightarrow$ slope to the curve 1 at point 'P'

$m_2 \rightarrow$ slope to the curve 2 at point 'P'

$$m_1 = \left.\frac{d\,f(x)}{dx}\right|_{(x_1,y_1)} ; \quad m_2 = \left.\frac{d\,g(x)}{dx}\right|_{(x_1,y_1)}$$

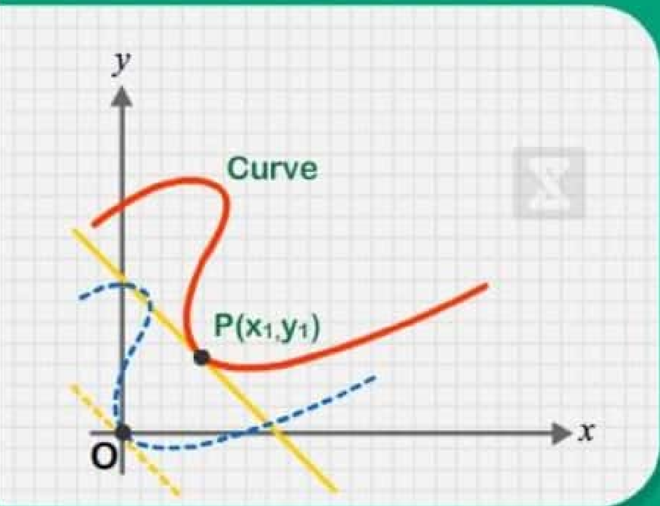
$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

If $\theta = \frac{\pi}{2}$ then the curves are called Orthogonal Curves.



POINTS TO REMEMBER

1



Equation of Tangent at point $P(x_1, y_1)$ to any second degree general curve

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

REPLACE:

$x^2 \rightarrow xx_1$; $y^2 \rightarrow yy_1$; $2x \rightarrow x + x_1$; $2y \rightarrow y + y_1$

$2xy \rightarrow xy_1 + x_1y$; $c \rightarrow c$

If curve passes through the 'O' then the equation of the tangent at 'O' may be directly written by comparing the lowest degree terms equal to 0.

$$2gx + 2fy = 0 \text{ or } gx + fy = 0$$

2

- If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 0 \Rightarrow$ Tangent is parallel to x-axis (Horizontal Tangent).
- If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} \rightarrow \infty$ or $\left(\frac{dx}{dy}\right)_{(x_1,y_1)} = 0 \Rightarrow$ Tangent is parallel to y-axis (Vertical Tangent).
- If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \pm 1 \Rightarrow$ Tangent at $P(x_1,y_1)$ is Equally inclined to the coordinate axis.

3 The shortest distance between two non-intersecting curves is always along to the common normal of the curves.