Chapter 2

Inverse Trigonometric Functions

Solutions (Set-1)

Very Short Answer Type Questions :

1. Find the value of $\text{cosec}(\cot^{-1}(-\sqrt{3}))$.

Sol. $\text{cosec}\,(\cot^{-1}(-\sqrt{3}))$

Let $\cot^{-1}(-\sqrt{3}) = \theta$

$\Rightarrow \quad \cot\theta = -\sqrt{3}$

$\Rightarrow \quad \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\therefore \quad \text{cosec}\frac{5\pi}{6} = 2$

2. Find the value of $\tan^{-1}\left(\tan\frac{8\pi}{3}\right)$.

Sol. $\tan^{-1}\tan\left(\frac{8\pi}{3}\right) = \tan^{-1}\left(\tan\left(3\pi - \frac{\pi}{3}\right)\right) = \frac{-\pi}{3}$

3. Find the value of $\cot(\sin^{-1}x + \cos^{-1}x)$.

Sol. $\cot(\sin^{-1}x + \cos^{-1}x) = \cot\frac{\pi}{2} = 0$

4. Find the value of $\tan^{-1}(-1) + \cot^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$.

Sol. $\tan^{-1}(-1) + \cot^{-1}\tan\left(\frac{3\pi}{4}\right)$

$= -\tan^{-1}(1) + \cot^{-1}(-1)$

$= -\tan^{-1} + \pi - \cot^{-1}(1)$

$= \pi - (\tan^{-1}1 + \cot^{-1}1)$

$= \pi - \frac{\pi}{2} = \frac{\pi}{2}$

5. Simplify $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$.

Sol. $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$x = \tan\theta$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4} \quad (\because\ 0 < x < 1)$

$\sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}\cos 2\theta$

$= \sin^{-1}\sin\left(\frac{\pi}{2}-2\theta\right) \qquad \because \quad 0 < \theta < \frac{\pi}{4}$

$= \frac{\pi}{2}-2\theta \qquad 0 < \frac{\pi}{2}-2\theta < \frac{\pi}{2}$

$= \frac{\pi}{2}-2\tan^{-1}x$

6. Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$.

Sol. $\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

$x = \cos\theta$

$\Rightarrow \quad \frac{\pi}{4} < \theta < \frac{\pi}{2}$

$\sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right)$

$= \sec^{-1}\sec 2\theta \qquad \frac{\pi}{2} < 2\theta < \pi$

$= 2\theta$

$= 2\cos^{-1}x$

7. Find the values of x satisfying $\cot^{-1}\left(\frac{1}{x}\right)-\tan^{-1}x = \pi$.

Sol. $\cot^{-1}\left(\frac{1}{x}\right)-\tan^{-1}x = \pi$

Let $x > 0$

$\tan^{-1}x - \tan^{-1}x = \pi$

$0 = \pi$, which is not possible

$\Rightarrow \quad x \neq 0$

Let $x < 0$

$\Rightarrow \quad -x > 0$

$$\Rightarrow \quad \cot^{-1}\left(\frac{1}{x}\right) - \tan^{-1} x = \pi$$

$$\Rightarrow \quad \cot^{-1}\left\{-\left(-\frac{1}{x}\right)\right\} + \tan^{-1}(-x) = \pi$$

$$\Rightarrow \quad \pi - \cot^{-1}\left(\frac{-1}{x}\right) + \cot^{-1}\left(-\frac{1}{x}\right) = \pi$$

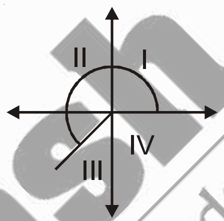
$\Rightarrow \quad \pi = \pi$

$\Rightarrow$ True for $\forall\ x < 0$ *i.e.*, $(-\infty, 0)$

8. Find the value of $\sin^{-1}(\sin 4)$.

Sol. $\sin^{-1}(\sin 4)$

4 radians lie in IIIrd quadrant



$$\Rightarrow \quad \sin^{-1}(\sin(\pi + (4 - \pi))$$

$$= \sin^{-1}(-\sin(4 - \pi))$$

$$= \pi - 4$$

9. If $\cos^{-1}x = y$ then find y.

Sol. $\cos^{-1}x = y$

$\Rightarrow \quad y \in [0, \pi]$

10. If $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{13}{x}\right) = \frac{\pi}{2}$, then find x.

Sol. $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{13}{x}\right) = \frac{\pi}{2}$

$$\Rightarrow \quad \frac{12}{13} = \frac{13}{x}$$

$$\Rightarrow \quad x = \frac{169}{12}$$

Short Answer Type Questions :

11. Prove that $\tan^{-1}\left(\frac{x + \sqrt{x}}{1 - x\sqrt{x}}\right) = \tan^{-1} x + \tan^{-1}\sqrt{x}$, $x\sqrt{x} < 1, x > 0$.

Sol. R.H.S. = $\tan^{-1} x + \tan^{-1}\sqrt{x}$

$\tan^{-1} x = \theta, \tan^{-1}\sqrt{x} = \phi$

$\Rightarrow$ $x = \tan\theta,\ \sqrt{x} = \tan\phi$

$\Rightarrow$ $\tan(\theta+\phi) = \dfrac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi}$

$= \dfrac{x+\sqrt{x}}{1-x\sqrt{x}}, \qquad x\sqrt{x} < 1$

= R.H.S.

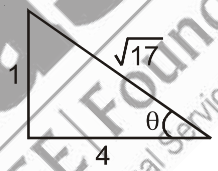
12. Prove that $\sin^{-1}\left(\dfrac{1}{\sqrt{17}}\right) + \cot^{-1}\left(\dfrac{9}{2}\right) = \cot^{-1}(2)$.

Sol. L.H.S. $\sin^{-1}\left(\dfrac{1}{\sqrt{17}}\right) + \cot^{-1}\left(\dfrac{9}{2}\right)$

$\sin^{-1}\dfrac{1}{\sqrt{17}} = \theta$

$\Rightarrow$ $\sin\theta = \dfrac{1}{\sqrt{17}}$

$\Rightarrow$ $\tan\theta = \dfrac{1}{4}$



$\Rightarrow$ $\theta = \tan^{-1}\dfrac{1}{4}$

$\Rightarrow$ L.H.S. $\tan^{-1}\left(\dfrac{1}{4}\right) + \tan^{-1}\left(\dfrac{2}{9}\right)$

$= \tan^{-1}\left(\dfrac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right) \qquad \left(\because \dfrac{1}{4}\times\dfrac{2}{9} < 1\right)$

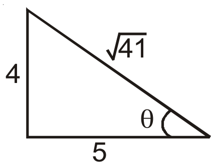
$= \tan^{-1}\left(\dfrac{17}{34}\right) = \tan^{-1}\left(\dfrac{1}{2}\right)$

$= \cot^{-1}(2)$ = RHS

13. Prove that $\cot^{-1}(9) + \mathrm{cosec}^{-1}\left(\dfrac{\sqrt{41}}{4}\right) = \dfrac{\pi}{4}$.

Sol. L.H.S. $\cot^{-1}(9) + \mathrm{cosec}^{-1}\left(\dfrac{\sqrt{41}}{4}\right)$

Let $\mathrm{cosec}^{-1}\left(\dfrac{\sqrt{41}}{4}\right) = \theta$



$\mathrm{cosec}\theta = \dfrac{\sqrt{41}}{4}$

$\Rightarrow \quad \tan\theta = \frac{4}{5}$

$\Rightarrow$ L.H.S. $\tan^{-1}\left(\frac{1}{9}\right) + \tan^{-1}\left(\frac{4}{5}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}.\frac{4}{5}}\right) = \tan^{-1}\left(\frac{41}{41}\right)$

$= \frac{\pi}{4} = \text{R.H.S.}$

14. Find the value of $\sin^{-1}\left(\sin\frac{47\pi}{7}\right)+\cos^{-1}\left(\cos\frac{60\pi}{7}\right)+\tan^{-1}\left(-\tan\frac{29\pi}{8}\right)+\cot^{-1}\left(\cot\left(-\frac{35\pi}{8}\right)\right)$.

Sol. $\sin^{-1}\sin\left(\frac{47\pi}{7}\right)+\cos^{-1}\left(\cos\frac{60\pi}{7}\right)+\tan^{-1}\left(-\tan\frac{29\pi}{8}\right)+\cot^{-1}\cot\left(\frac{-35\pi}{8}\right)$

$= \sin^{-1}\sin\left(7\pi-\frac{2\pi}{7}\right)+\cos^{-1}\cos\left(9\pi-\frac{3\pi}{7}\right)-\tan^{-1}\tan\left(4\pi-\frac{3\pi}{8}\right)+\cot^{-1}\left\{-\left(\cot\left(4\pi+\frac{3\pi}{8}\right)\right)\right\}$

$= \frac{2\pi}{7}+\cos^{-1}\left(-\cos\frac{3\pi}{7}\right)+\frac{3\pi}{8}+\cot^{-1}\left(-\cot\frac{3\pi}{8}\right)$

$= \frac{2\pi}{7}+\pi-\frac{3\pi}{7}+\frac{3\pi}{8}+\pi-\frac{3\pi}{8} = \frac{13\pi}{7}$

15. Prove that $2\tan^{-1}(-3) = \frac{-\pi}{2}+\tan^{-1}\left(-\frac{4}{3}\right)$.

Sol. L.H.S. $= 2\tan^{-1}(-3)$

$= -2\tan^{-1}3$

$= -2\cot^{-1}\left(\frac{1}{3}\right)$

$= -2\left\{\frac{\pi}{2}-\tan^{-1}\frac{1}{3}\right\}$

$= -\pi+2\tan^{-1}\frac{1}{3}$

$= -\pi+\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}$

$= -\pi+\tan^{-1}\left(\frac{3}{4}\right)$

$= -\pi+\left(\tan^{-1}\frac{3}{4}+\cot^{-1}\frac{3}{4}\right)-\cot^{-1}\frac{3}{4}$

$= -\frac{\pi}{2}-\tan^{-1}\left(\frac{4}{3}\right)$

= R.H.S.

16. Simplify $\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right), x \in \left(0, \frac{\pi}{2}\right)$.

Sol. $\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

$$= \tan^{-1}\left(\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)$$

$$= \tan^{-1}\left\{\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right\}$$

$$= \tan^{-1}\left\{\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$$

$\left(\because\ 0 < x < \frac{\pi}{2} \Rightarrow \frac{\pi}{4} < \frac{\pi}{4}+\frac{x}{2} < \frac{\pi}{2}\right)$

$$= \frac{\pi}{4}+\frac{x}{2}$$

17. Show that $\sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\frac{8}{17} = \cos^{-1}\left(\frac{84}{85}\right)$.

Sol. L.H.S. = $\sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{8}{17}\right)$

Let $\sin^{-1}\left(\frac{3}{5}\right) = \theta$, $\sin^{-1}\left(\frac{8}{17}\right) = \phi$

$\Rightarrow \quad \sin\theta = \frac{3}{5}, \sin\phi = \frac{8}{17}$

$\Rightarrow \quad \cos\theta = \frac{4}{5}, \cos\phi = \frac{15}{17}$

$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$= \frac{4}{5}\times\frac{15}{17}+\frac{3}{5}\times\frac{8}{17}$

$= \frac{84}{85}$

$\Rightarrow \quad \theta-\phi = \cos^{-1}\left(\frac{84}{85}\right)$

$\Rightarrow \quad \sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$

= R.H.S.

18. If 4 $\sin^{-1}x + \cos^{-1}x = \pi$ then find the value of x.

Sol. $4\sin^{-1}x + \cos^{-1}x = \pi$

$$\Rightarrow \quad 3\sin^{-1}x + \frac{\pi}{2} = \pi$$

$$\Rightarrow \quad 3\sin^{-1}x = \frac{\pi}{2}$$

$$\sin^{-1} = \frac{\pi}{6}$$

$$x = \frac{1}{2}$$

19. Prove that $\sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) = \sqrt{1-x^2}, -1 \le x < 1.$

Sol. L.H.S. $= \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)$

$x = \cos 2\theta$

$$= \sin\left(2\tan^{-1}\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}\right)$$

$$= \sin\left(2\tan^{-1}\cot\theta\right)$$

$$= \sin 2\tan^{-1}\tan\left(\frac{\pi}{2}-\theta\right)$$

$$= \sin\left(2\left(\frac{\pi}{2}-\theta\right)\right)$$

$= \sin 2\theta$

$$= \sqrt{1-\cos^2 2\theta}$$

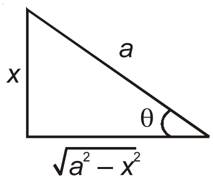
$$= \sqrt{1-x^2}$$

= R.H.S.

20. Simplify $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x| < a$.

Sol. $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

Let $x = a\sin\theta$



$$\Rightarrow \quad \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$= \theta$

$$= \sin^{-1}\left(\frac{x}{a}\right)$$

21. Find x, if $\tan^{-1}\left(\frac{1}{x}\right)-\tan^{-1}\left(\frac{1}{x+2}\right)=\frac{\pi}{12},\ x>0$.

Sol. $\tan^{-1}\left(\frac{1}{x}\right)-\tan^{-1}\left(\frac{1}{x+2}\right)=\frac{\pi}{12}$

$$\tan^{-1}\left\{\frac{\frac{1}{x}-\frac{1}{x+2}}{1+\frac{1}{x}\cdot\frac{1}{x+2}}\right\}=\frac{\pi}{12} \quad \text{Only if } \left(\frac{1}{x}\right)\left(\frac{1}{x+2}\right)>-1$$

$$\frac{x+2-x}{x^2+2x+1}=\tan\frac{\pi}{12} = 2-\sqrt{3}$$

$$\Rightarrow \quad (x+1)^2 = \frac{2}{2-\sqrt{3}}=2(2+\sqrt{3})$$

$$(x+1)^2=4+2\sqrt{3} = (\sqrt{3}+1)^2$$

$$x=\pm(\sqrt{3}+1)-1$$

$$\sqrt{3},-\sqrt{3}-2$$

22. Prove that $\cos^{-1}x=2\sin^{-1}\sqrt{\frac{1-x}{2}},\ |x|\le 1$

Sol. Let $x = \cos\theta$

$\therefore \quad 2\sin^{-1}\sqrt{\frac{1-x}{2}}$

$= 2\sin^{-1}\sqrt{\left(\frac{1-\cos\theta}{2}\right)}$

$= 2\sin^{-1}\sin\frac{\theta}{2} = \theta$

$= \cos^{-1}x$ = L.H.S.

23. Prove that $\cos^{-1}x=2\cos^{-1}\sqrt{\frac{1+x}{2}},\ |x|\le 1$.

Sol. Let $x = \cos\theta$

$2\cos^{-1}\sqrt{\frac{1+x}{2}}$

$= 2\cos^{-1}\sqrt{\frac{1+\cos\theta}{2}}$

$= 2\cos^{-1}\left(\cos\frac{\theta}{2}\right)$

$= \theta$

$= 2\cos^{-1}x$

= L.H.S.

24. Prove that $\cos\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right) = x,\ -1 < x \le 1$.

Sol. L.H.S. $\cos\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)$

Let $x = \cos 2\theta$

$\Rightarrow$ L.H.S. = $\cos\left(2\tan^{-1}\sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}\right)$

= $\cos\left(2\tan^{-1}\sqrt{\dfrac{2\sin^2\theta}{2\cos^2\theta}}\right)$

= $\cos 2\tan^{-1}(\tan\theta)$

= $\cos 2\theta = x$

25. Find x, if $3\tan^{-1}(2-\sqrt{3}) - \tan^{-1}\left(\dfrac{1}{x}\right) = \cot^{-1}(3)$.

Sol. $3\tan^{-1}(2-\sqrt{3}) - \tan^{-1}\left(\dfrac{1}{x}\right) = \tan^{-1}\left(\dfrac{1}{3}\right)$

$\because\ \tan^{-1}(2-\sqrt{3}) = \dfrac{\pi}{12}$

$\Rightarrow\ 3\tan^{-1}(2-\sqrt{3}) = \dfrac{\pi}{4}$

$\Rightarrow\ \dfrac{\pi}{4} - \tan^{-1}\left(\dfrac{1}{3}\right) = \tan^{-1}\left(\dfrac{1}{x}\right)$

$\Rightarrow\ \tan^{-1}\left(\dfrac{1}{x}\right) = \tan^{-1}\left(\dfrac{1-\dfrac{1}{3}}{1+\dfrac{1}{3}}\right) = \tan^{-1}\left(\dfrac{1}{2}\right)$

$\Rightarrow\ x = 2$

26. Find x, if $\tan^{-1}(x+1) + \tan^{-1}x + \tan^{-1}(x-1) - \tan^{-1}(3x) = 0$.

Sol. $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}(x)$

$\tan^{-1}\left(\dfrac{x-1+x+1}{1-(x^2-1)}\right) = \tan^{-1}\left(\dfrac{3x-x}{1+3x^2}\right)$

$\dfrac{2x}{2-x^2} = \dfrac{2x}{1+3x^2}$

$x(1 + 3x^2) - x(2 - x^2) = 0$

$x(-1 + 4x^2) = 0$

$x = 0, \pm\dfrac{1}{2}$

27. Find the value of $\sin^{-1}(\sin 5)+\cos^{-1}(\cos 5)+\tan^{-1}(\tan 5)$.

Sol. $\sin^{-1}(\sin 5)\ \cos^{-1}(\cos 5) + \tan^{-1}(\tan 5) = ?$

$\because$ 5 radian lies in the fourth quadrant,

$\therefore$ $\sin^{-1}\sin(2\pi - (2\pi - 5)) + \cos^{-1}\cos(2\pi - (2\pi - 5)) + \tan^{-1}\tan(2\pi - (2\pi - 5))$

$= \sin^{-1}\sin(5-2\pi)+\cos^{-1}\cos(2\pi-5)+\tan^{-1}\tan(5-2\pi)$

$= 5 - 2\pi + 2\pi - 5 + 5 - 2\pi$

$= 5 - 2\pi$

28. Prove that $\tan^{-1}\left(\dfrac{1}{21}\right)+\tan^{-1}\left(\dfrac{1}{13}\right)-\tan^{-1}\left(\dfrac{1}{8}\right)=0$.

Sol. L.H.S. = $\tan^{-1}\left(\dfrac{1}{21}\right)+\tan^{-1}\left(\dfrac{1}{13}\right)-\tan^{-1}\left(\dfrac{1}{8}\right)$

$$= \tan^{-1}\left\{\frac{\frac{1}{21}+\frac{1}{13}}{1-\frac{1}{21}\times\frac{1}{13}}\right\}-\tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left\{\frac{34}{272}\right\}-\tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\frac{1}{8}-\tan^{-1}\frac{1}{8}$$

$= 0$

= R.H.S.

29. If $f : [-1, 1] \to B$ and $f(x) = \sin^{-1}x$, if $f(x)$ is one-one onto function, then find the value of B.

Sol. In one-one onto function,

Range of f = Co-domain of f (which is B)

So, Range of function for $x \in [-1, 1]$ is $\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$.

So, $B=\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$

Long Answer Type Questions :

30. Solve the equation $\sin(2\cos^{-1}(\cot(2\tan^{-1}x)))=0$.

Sol. $\sin(2\cos^{-1}(\cot(2\tan^{-1}x))) = 0$

$\Rightarrow$ $2\cos^{-1}(\cot(2\tan^{-1}x)) = n\pi,\ n\in z$

But $\cos^{-1}t \in [0, \pi]$

$\Rightarrow$ $\cos^{-1}(\cot(2\tan^{-1}x)) = 0, \dfrac{\pi}{2}, \pi$

$\Rightarrow$ $\cot(2\tan^{-1}x) = 0, \pm 1$

(i) $\cot(2\tan^{-1}x) = 0$

$\Rightarrow \quad 2\tan^{-1}x = \pm\frac{\pi}{2}$

$x = \pm 1,$

(ii) $\cot(2\tan^{-1}x) = 1$

$\Rightarrow \quad 2\tan^{-1}x = \frac{\pi}{4}, -\frac{3\pi}{4}$

$\Rightarrow \quad \tan^{-1}x = \frac{\pi}{8}, -\frac{3\pi}{8}$

$x = (\sqrt{2}-1), -(\sqrt{2}+1)$

$\cot(2\tan^{-1}x) = -1$

$\Rightarrow \quad 2\tan^{-1}x = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\Rightarrow \quad x = -(\sqrt{2}-1), (\sqrt{2}+1)$

$\Rightarrow \quad x = \pm 1, \pm(\sqrt{2}-1), \pm(\sqrt{2}+1)$

31. If $x > y > z > 0$, then prove that.

$$\cot^{-1}\left(\frac{1+xy}{x-y}\right) + \cot^{-1}\left(\frac{1+yz}{y-z}\right) + \cot^{-1}\left(\frac{1+zx}{z-x}\right) = \pi.$$

Sol. $\cot^{-1}\left(\frac{1+xy}{x-y}\right) + \cot^{-1}\left(\frac{1+yz}{y-z}\right) + \cot^{-1}\left(\frac{1+zx}{z-x}\right)$

$= \tan^{-1}\left(\frac{x-y}{1+xy}\right) + \tan^{-1}\left(\frac{y-z}{1+yz}\right) + \pi - \cot^{-1}\left(\frac{1+zx}{x-z}\right), (\because x > y > z > 0)$

$= \tan^{-1}x - \tan^{-1}y + \tan^{-1}y - \tan^{-1}z + \pi - \tan^{-1}\left(\frac{x-z}{1+xz}\right)$

$= \tan^{-1}x - \tan^{-1}z + \pi - (\tan^{-1}x - \tan^{-1}z)$

$= \pi$ = R.H.S.

32. Find x, if $\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$.

Sol. Let $\sin^{-1}x = \theta$

$\Rightarrow \quad x = \sin\theta$

$\Rightarrow \quad \sin^{-1}(1-\sin\theta) = \frac{\pi}{2} + 2\theta$

$1 - \sin\theta = \sin\left(\frac{\pi}{2} + 2\theta\right)$

$= \cos 2\theta$

$= 1 - 2\sin^2\theta$

$\Rightarrow \quad 2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0, \frac{1}{2}$

$\Rightarrow \quad x = 0, \frac{1}{2}$

But $x = \frac{1}{2}$ does not satisfy the given equation

$\therefore \quad x = 0$

33. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle then find the third angle.

Sol. $\alpha = \cot^{-1}2$, $\beta = \cot^{-1}3$ and 3rd angle be γ

$\because \quad \alpha + \beta + \gamma = \pi$

$\Rightarrow \quad \alpha + \beta = \pi - \gamma$

$\Rightarrow \quad \tan(\alpha + \beta) = \tan(\pi - \gamma)$

$\Rightarrow \quad \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = -\tan\gamma$

$\Rightarrow \quad \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = -\tan\gamma$

$\Rightarrow \quad \tan\gamma = -1$

$\Rightarrow \quad \gamma = \frac{3\pi}{4}$

34. Find the value of $\sin\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)\right)$

Sol. $\sin\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)\right)$

Let $\cot^{-1}\left(-\frac{3}{4}\right) = \theta$

$\Rightarrow \quad \cot\theta = -\frac{3}{4}\left(\pi > \theta > \frac{\pi}{2}\right)$

$\therefore \quad \tan\theta = -\frac{4}{3}$

$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

$\therefore \quad -\frac{4}{3} = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

$\therefore \quad 2\tan^2\frac{\theta}{2} - 3\tan\frac{\theta}{2} - 2 = 0$

$\left(2\tan\frac{\theta}{2}+1\right)\left(\tan\frac{\theta}{2}-2\right)=0$

$\tan\frac{\theta}{2} = -\frac{1}{2}$ or 2

$\pi > \theta > \frac{\pi}{2}$

$\frac{\pi}{2} > \frac{\theta}{2} > \frac{\pi}{4}$

$\therefore \quad \tan\frac{\theta}{2} = 2$

$\therefore \quad \sin\frac{\theta}{2} = \frac{2}{\sqrt{5}}$

35. If $x = \cot^{-1}7$, $y = \cot^{-1}3$ then prove that $\cos 2x = \sin 4y$.

Sol. $x = \cot^{-1}(7)$, $y = \cot^{-1}(3)$

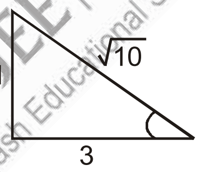
$\Rightarrow \quad \cot x = 7$, $\cot y = 3$

$\Rightarrow \quad \cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1-\frac{1}{49}}{1+\frac{1}{49}} = \frac{48}{49}\times\frac{49}{50} = \frac{24}{25}$

$\sin 4y = 4\sin y\cos y \, . \, (1 - 2\sin^2 y)$

$= 4\times\frac{1}{\sqrt{10}}\frac{3}{\sqrt{10}}\left(1-\frac{2}{10}\right)$

$= \frac{24}{25}$



$\Rightarrow \quad \cos 2x = \sin 4y$

36. Simplify $\sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$, where $-1 \le x \le 1$.

Sol. $\sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$

Let $x = \sin\theta$

$\Rightarrow \quad \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$

$= \theta + \sin^{-1}(\cos\theta)$

$= \theta + \sin^{-1}\left(\sin\left(\frac{\pi}{2}-\theta\right)\right)$

$= \theta + \frac{\pi}{2} - \theta$

$= \frac{\pi}{2}$

37. Find the value of $\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A),\ 0<A<\frac{\pi}{4}$.

Sol. $\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A),\ 0<A<\frac{\pi}{4}$

Let $\tan^{-1}(\cot A) = \theta$

$\tan^{-1}(\cot^3 A) = \phi$

$$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta.\tan\phi} = \frac{\cot A+\cot^3 A}{1-\cot^4 A} = \frac{\cot A}{1-\cot^2 A} = \frac{\tan A}{\tan^2 A-1} = -\frac{\tan A}{1-\tan^2 A}$$

$$\Rightarrow \frac{\tan A}{1-\tan^2 A}=\tan\{\pi-(\theta+\phi)\} \qquad \left(\because \theta \,\&\, \phi \text{ are cotangent functions which are } >1 \text{ for } 0<A<\frac{\pi}{4}\right)$$

$$\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)=\pi-(\theta+\phi)$$

$$\Rightarrow \theta+\phi=\pi-\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)$$

$$\Rightarrow \tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A)=\pi-\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A)=\pi$$

38. Find the value of $\tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)+\cot^{-1}\left(2\sin\left(2\cos^{-1}\frac{1}{2}\right)\right)$

Sol. $\tan^{-1}\left(2\cos 2\left(\sin^{-1}\frac{1}{2}\right)\right)+\cot^{-1}\left(2\sin\left(2\cos^{-1}\frac{1}{2}\right)\right)$

$$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right)+\cot^{-1}\left(2\sin\frac{2\pi}{3}\right)$$

$$= \tan^{-1}(1)+\cot^{-1}\left(\sqrt{3}\right)$$

$$= \frac{\pi}{4}+\frac{\pi}{6} = \frac{5\pi}{12}$$

39. Prove that $2\tan^{-1}\frac{1}{3}+\sin^{-1}\frac{4}{5}=\frac{\pi}{2}$.

Sol. $2\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}=\tan^{-1}\frac{3}{4}$

and, $\sin^{-1}\frac{4}{5}=\tan^{-1}\left(\frac{4}{3}\right)$

Hence, L.H.S. = $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{4}{3} = \tan^{-1}\frac{3}{4}+\cot^{-1}\frac{3}{4} = \frac{\pi}{2}$

40. Let $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ be the angles of a triangle *ABC*, then prove that

$$\frac{x}{1+x^2}+\frac{y}{1+y^2}+\frac{z}{1+z^2}=\frac{2xyz}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}}$$

Sol. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

Let $\tan^{-1}x = A \Rightarrow \tan A = x$

$\tan^{-1}y = B \Rightarrow \tan B = y$

$\tan^{-1}z = C \Rightarrow \tan C = z$

$\Rightarrow A + B + C = \pi$

$2A + 2B + 2C = 2\pi$

$(2A + 2B) = 2\pi - 2C$

$$\frac{\tan 2A+\tan 2B}{1-\tan 2A\tan 2B}=-\tan 2C$$

$\tan 2A + \tan 2B = -\tan 2C + \tan 2A \tan 2B \tan 2C$

$\Rightarrow \tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

$$\Rightarrow \frac{2\tan A}{1-\tan^2 A}+\frac{2\tan B}{1-\tan^2 B}+\frac{2\tan C}{1-\tan^2 C}=\frac{8\tan A\tan B\tan C}{\sqrt{(1-\tan^2 A)(1-\tan^2 B)(1-\tan^2 C)}}$$

$$\Rightarrow \frac{x}{1-x^2}+\frac{y}{1-y^2}+\frac{z}{1-z^2}=\frac{4xyz}{\sqrt{(1-x^2)(1-y^2)(1-z^2)}}$$

❑ ❑ ❑

Chapter 2

Inverse Trigonometric Functions

Solutions (Set-2)

[Principal Value and Graphical Representation]

1. The value of $\tan^{-1}(\sqrt{3})+\cot^{-1}(-1)+\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is

(1) $-\frac{\pi}{12}$ (2) $\frac{11\pi}{12}$ (3) $\frac{5\pi}{4}$ (4) $\frac{23\pi}{12}$

Sol. Answer (4)

$\tan^{-1}\sqrt{3}+\cot^{-1}(-1)+\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)=?$

$\tan^{-1}\sqrt{3}=\alpha,\ \alpha\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\Rightarrow\ \tan\alpha=\sqrt{3}$

$\Rightarrow\ \alpha=\frac{\pi}{3}$

$\cot^{-1}(-1)=\beta,\ \beta\in(0,\pi)$

$\Rightarrow\ \cot\beta=-1$

$\Rightarrow\ \beta=\frac{3\pi}{4}$

$\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)=\gamma,\ \gamma\in[0,\pi]-\left\{\frac{\gamma}{2}\right\}$

$\Rightarrow\ \sec\gamma=\frac{-2}{\sqrt{3}}$

$\Rightarrow\ \gamma=\frac{5\pi}{6}$

$\Rightarrow\ \alpha+\beta+\gamma=\frac{\pi}{3}+\frac{3\pi}{4}+\frac{5\pi}{6}=\frac{23\pi}{12}$

2. The value of $\cot^{-1}(-\sqrt{3})+\text{cosec}^{-1}(2)+\tan^{-1}(\sqrt{3})$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) $\frac{4\pi}{3}$

Sol. Answer (4)

$\cot^{-1}(-\sqrt{3})+\text{cosec}^{-1}(2)+\tan^{-1}(\sqrt{3}) = ?$

$\cot^{-1}(-\sqrt{3}) = \alpha,\ \alpha\in(0,\pi)$

$\Rightarrow \quad \cot\alpha = -\sqrt{3}$

$\Rightarrow \quad \alpha = \dfrac{5\pi}{6}$

$\text{cosec}^{-1}(2) = \beta,\ \beta\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]-\{0\}$

$\Rightarrow \quad \text{cosec}\beta = 2$

$\Rightarrow \quad \beta = \dfrac{\pi}{6}$

$\tan^{-1}(\sqrt{3}) = \gamma,\ \gamma\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

$\Rightarrow \quad \tan\gamma = \sqrt{3}$

$\Rightarrow \quad \gamma = \dfrac{\pi}{3}$

$\therefore \quad \alpha+\beta+\gamma = \dfrac{5\pi}{6}+\dfrac{\pi}{6}+\dfrac{\pi}{3} = \dfrac{4\pi}{3}$

3. The maximum and minimum values of $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ respectively is

(1) $\dfrac{3\pi}{4},\dfrac{\pi}{2}$ (2) $\dfrac{3\pi}{4},\dfrac{\pi}{4}$ (3) $\dfrac{\pi}{4},\dfrac{-\pi}{4}$ (4) π, 0

Sol. Answer (2)

$f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

Common domain of $f(x)$ is [–1, 1]

$\Rightarrow \quad f(x) = \dfrac{\pi}{2} + \tan^{-1}x$

$(f(x))_{min} = \dfrac{\pi}{2} + \tan^{-1}(-1) = \dfrac{\pi}{2}-\dfrac{\pi}{4}=\dfrac{\pi}{4}$

$(f(x))_{max} = \dfrac{\pi}{2}+\dfrac{\pi}{4}=\dfrac{3\pi}{4}$

4. Select the wrong option.

(1) $-1\le\sin^{-1}x\le1\Rightarrow-\sin1<x\le\sin1$

(2) $\dfrac{\pi}{3}\le\cos^{-1}x\le\dfrac{2\pi}{3}\Rightarrow-\dfrac{1}{2}\le x\le\dfrac{1}{2}$

(3) $\dfrac{\pi}{4}\le\cot^{-1}x\le\dfrac{5\pi}{6}\Rightarrow-\sqrt{3}\le x\le1$

(4) $\sec^{-1}x\ge\dfrac{\pi}{4}\Rightarrow x\le\sqrt{2}$

Sol. Answer (4)

Option (1), (2), (3) are true.

But (4) is not true as

$\sec^{-1}x \ge \dfrac{\pi}{4}$

$\Rightarrow \quad \dfrac{\pi}{4} \le \sec^{-1}x \le \pi$

$\Rightarrow \quad x \in (-\infty, -1] \cup [1, \sqrt{2}\,]$

5. If x_1 and x_2 are the roots of $15x^2 + 28x + 12 = 0$, then

(1) Both $\cos^{-1}x_1$ and $\cos^{-1}x_2$ are real
(2) Both $\sin^{-1}x_1$ and $\sin^{-1}x_2$ are real
(3) Both $\sec^{-1}x_1$ and $\sec^{-1}x_2$ are real
(4) Both $\cot^{-1}x_1$ and $\cot^{-1}x_2$ are real

Sol. Answer (4)

On solving, $15x^2 + 28x + 12 = 0$

$$x = \frac{-28 \pm \sqrt{520}}{30}$$

One root is greater than –1 and other root is less than –1.

Hence, option (4) is the only possibility.

6. The expression

$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x\right)$ equals

(1) $\frac{1}{x}$ (2) x (3) $\frac{2}{x}$ (4) $2x$

Sol. Answer (3)

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right)$$

$$= \cot\left[\frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right)\right]$$

$$= \cot\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x\right)$$

Let $\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x = \theta$

Then L.H.S. = $\cot\theta + \tan\theta = \frac{1}{\tan\theta} + \tan\theta$

$$= 2\left(\frac{1+\tan^2\theta}{2\tan\theta}\right) = \frac{2}{\sin 2\theta}$$

$$= \frac{2}{\sin\left(\frac{\pi}{2} - \cos^{-1}x\right)}$$

$$= \frac{2}{\cos\left(\cos^{-1}x\right)}$$

$$= \frac{2}{x}$$

7. If $p > q > 0$ and $pr < -1 < qr$, then $\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right)$ is equal to

(1) 0 (2) $-\pi$ (3) π (4) $\frac{\pi}{2}$

Sol. Answer (3)

$p > q > 0 \Rightarrow pq > 0$

$\therefore \tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1}p - \tan^{-1}q$ ...(i)

Since, $qr > -1$, then

$$\tan^{-1}\frac{q-r}{1+qr} = \tan^{-1}q - \tan^{-1}r \quad \ldots\text{(ii)}$$

and $pr < -1$, then

$$\tan^{-1}\left(\frac{r-p}{1+rp}\right) = \pi + \tan^{-1}r - \tan^{-1}p \quad \ldots\text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right) = \pi$$

8. If k satisfies $k^2 - k - 6 > 0$, then a value exists for

(1) $\sec^{-1}k$ (2) $\sin^{-1}k$ (3) $\cos^{-1}k$ (4) $\sec^{-1}\left(\frac{1}{k}\right)$

Sol. Answer (1)

$k^2 - k - 6 > 0$

$\Rightarrow (k-3)(k+2) > 0$

$\Rightarrow k < -2, \ k > 3$

$\therefore$ A value exist for $\sec^{-1}k$

9. The domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right)$ is

(1) $[-6, 6]$ (2) $(-\infty, 2) \cup (2, 3)$ (3) $(2, 3)$ (4) $[-6, 2) \cup (2, 3)$

Sol. Answer (1)

$f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right)$

$\Rightarrow -1 \le \frac{2-|x|}{4} \le 1$

$-4 \le 2 - |x| \le 4$

$-6 \le -|x| \le 2$

$-2 \le |x| \le 6$

$\Rightarrow -6 \le x \le 6$

10. The domain of the function given by $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ is

(1) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (2) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (3) $[-1, 1]$ (4) $\left[-1, \frac{1}{2}\right]$

Sol. Answer (2)

$f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$

$-1 \le 2x \le 1$

$\Rightarrow \frac{-1}{2} \le x \le \frac{1}{2}$ ... (i)

$\sin^{-1}2x + \frac{\pi}{6} \ge 0$

$\sin^{-1} 2x \geq -\frac{\pi}{6}$

$2x \geq -\frac{1}{2}$

$\Rightarrow \quad x \geq -\frac{1}{4}$ ... (ii)

From (i) & (ii), we have

$\Rightarrow \quad x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

11. The domain and range of $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x + \cot^{-1}x + \sec^{-1}x + \text{cosec}^{-1}x$ respectively are

(1) $\{-1, 1\}, \frac{3\pi}{2}$ (2) $\{-1, 1\}, \frac{\pi}{2}$ (3) $(-1, 1), \frac{\pi}{2}$ (4) $(-1, 1), 2\pi$

Sol. Answer (1)

$f(x) = (\sin^{-1}x + \cos^{-1}x) + (\tan^{-1}x + \cot^{-1}x) + (\sec^{-1}x + \text{cosec}^{-1}x)$

Common domain = [–1, 1]

But range = $\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{2}$

12. Let $f(x) = \sec^{-1}(x-10) + \cos^{-1}(10-x)$. The range of $f(x)$ is

(1) $\left\{0, \frac{\pi}{2}, \pi\right\}$ (2) $\left\{0, \frac{\pi}{2}\right\}$ (3) $\left\{\frac{\pi}{2}\right\}$ (4) $\{\pi\}$

Sol. Answer (4)

$\sec^{-1}(x-10)$ is defined, if

$x - 10 \leq -1$ and $x - 10 \geq 1$

$\Rightarrow x \leq 9$ and $x \geq 11$

$\cos^{-1}(10-x)$ is defined, if

$-1 \leq 10 - x \leq 1$

$\Rightarrow 9 \leq x \leq 11$

Hence, $f(x)$ is defined, when $x = 9$ or 11.

$f(9) = \sec^{-1}(-1) + \cos^{-1}(1) = \pi + 0 = \pi$

$f(11) = \sec^{-1}(1) + \cos^{-1}(-1) = 0 + \pi = \pi$

$\therefore$ Range of $f(x)$ is $\{\pi\}$.

13. $\cos^{-1}(-x)$, $|x| \leq 1$, is equal to

(1) $-\cos^{-1}x$ (2) $\cos^{-1}x$ (3) $\frac{\pi}{2} - \cos^{-1}x$ (4) $\frac{\pi}{2} + \sin^{-1}x$

Sol. Answer (4)

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$= \pi - \left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= \frac{\pi}{2} + \sin^{-1}x$

14. $\text{cosec}^{-1}(-x)$, $x \in R-(-1, 1)$, is equal to

(1) $\text{cosec}^{-1}x$ (2) $-\sin^{-1}x$ (3) $-\sin^{-1}\left(\frac{1}{x}\right)$ (4) $\pi - \text{cosec}^{-1}x$

Sol. Answer (3)

$$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$$
$$= -\sin^{-1}\left(\frac{1}{x}\right)$$

15. $\cot^{-1}(-2)$ is equal to

(1) $\pi - \tan^{-1}\left(\frac{1}{2}\right)$ (2) $\tan^{-1}\left(\frac{1}{2}\right)$ (3) $-\tan^{-1}\frac{1}{2}$ (4) $\cot^{-1}2$

Sol. Answer (1)

$$\cot^{-1}(-2) = \pi - \cot^{-1}(2)$$
$$= \pi - \tan^{-1}\left(\frac{1}{2}\right)$$

16. The value of $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}$ is

(1) $\frac{3\pi}{2}$ (2) $\frac{\pi}{2}$ (3) $\pi - \sin^{-1}\frac{3713}{4225}$ (4) $\pi - \tan^{-1}\frac{3713}{2016}$

Sol. Answer (2)

$$\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right) + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{48}{65} + \frac{15}{65}\right) + \sin^{-1}\left(\frac{16}{65}\right)$$

$$= \sin^{-1}\left(\frac{63}{65}\right) + \sin^{-1}\left(\frac{16}{65}\right)$$

$$= \cos^{-1}\left(\frac{16}{65}\right) + \sin^{-1}\left(\frac{16}{65}\right) \quad \left(\because \sin^{-1}x = \cos^{-1}\sqrt{1-x^2}\right)$$

$$= \frac{\pi}{2}$$

17. If $-1 \le x \le -\frac{1}{2}$, then $\sin^{-1}(3x - 4x^3)$ equals

(1) $3\sin^{-1}x$ (2) $\pi - 3\sin^{-1}x$ (3) $-\pi - 3\sin^{-1}x$ (4) $\pi + 3\sin^{-1}x$

Sol. Answer (3)

$$3\sin^{-1}x = -\pi - \sin^{-1}(3x - 4x^3), \text{ when } -1 \le x \le \frac{-1}{2}$$

$$\Rightarrow \sin^{-1}(3x - 4x^3) = -\pi - 3\sin^{-1}(x)$$

18. $\sin^{-1}\left(-\left(\frac{1}{2}\right)\right)+\cos^{-1}\left(-\left(\frac{1}{2}\right)\right)+\cot^{-1}(-\sqrt{3})+\text{cosec}^{-1}(\sqrt{2})\ +\tan^{-1}(-1)+\sec^{-1}(\sqrt{2})$ equals

(1) $\frac{9\pi}{4}$ (2) $\frac{19\pi}{12}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

Sol. Answer (2)

$$\text{Sum} = \frac{-\pi}{6}+\frac{2\pi}{3}+\frac{5\pi}{6}+\frac{\pi}{4}-\frac{\pi}{4}+\frac{\pi}{4}=\frac{19\pi}{12}$$

19. The value of $\sin^{-1}\sin(16) + \cos^{-1}\cos(10)$ is

(1) 26 (2) –26 (3) $6 + \pi$ (4) $9\pi - 26$

Sol. Answer (4)

$\sin^{-1} \sin16 = \sin^{-1} \sin(16 - 5\pi + 5\pi)$

$= 5\pi - 16$

$\cos^{-1} (\cos 10) = \cos^{-1} \cos (10 - 3\pi + 3\pi)$

$= \cos^{-1} \cos \{3\pi + (10 - 3\pi)\}$

$= \cos^{-1} \{-\cos (10 - 3\pi)\}$

$= \pi - \cos^{-1} \cos (10 - 3\pi)$

$= \pi - (10 - 3\pi) = 4\pi - 10$

Sum $= 5\pi - 16 + 4\pi - 10 = 9\pi - 26$

20. $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right) =$

(1) 0 (2) $\frac{\pi}{2}$ (3) $10\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

Sol. Answer (1)

$\cos^{-1}\left(\cos\frac{5\pi}{3}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

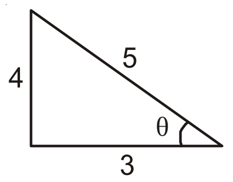
Adding, $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)=0$

21. $\sin^{-1}\left(\frac{4}{5}\right)+2\tan^{-1}\left(\frac{1}{3}\right)$ is equal to

(1) $\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\tan^{-1}\left(\frac{3}{\sqrt{10}}\right)$ (4) $\frac{\pi}{2}$

Sol. Answer (4)

$\sin^{-1}\frac{4}{5}+2\tan^{-1}\left(\frac{1}{3}\right)$



Let $\sin^{-1}\left(\frac{4}{5}\right)=\theta$

$\Rightarrow \quad \sin\theta=\frac{4}{5}$

$\Rightarrow \quad \tan\theta = \frac{4}{3}$

$\Rightarrow \quad \sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right)$

$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)$

$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right)$

$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

$= \tan^{-1}\frac{4}{3} + \cot^{-1}\left(\frac{4}{3}\right)$

$= \tan^{-1}\frac{4}{3} + \cot^{-1}\left(\frac{4}{3}\right)$

$= \frac{\pi}{2}$

22. $\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) + \cos\left(\tan^{-1}(2\sqrt{2})\right)$ is equal to

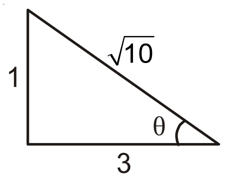
(1) $\frac{14}{15}$ (2) $\frac{3}{4}$ (3) $\frac{2\sqrt{2}}{7}$ (4) $\frac{2\sqrt{2}}{15}$

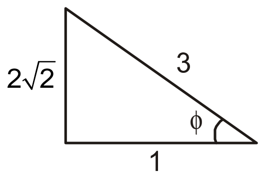
Sol. Answer (1)

$\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) + \cos\left(\tan^{-1}(2\sqrt{2})\right) = ?$

$\tan^{-1}\left(\frac{1}{3}\right) = \theta,\ \tan^{-1}(2\sqrt{2}) = \phi$

$\Rightarrow \quad \sin 2\theta + \cos\phi \quad = \frac{2\tan\theta}{1+\tan^2\theta} + \cos\phi$





$= \frac{2\left(\frac{1}{3}\right)}{1+\left(\frac{1}{3}\right)^2} + \frac{1}{3} = \frac{3}{5} + \frac{1}{3}$

$= \frac{14}{15}$

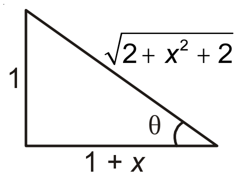
23. If $\sin(\cot^{-1}(1 + x)) = \cos(\tan^{-1}x)$, then x is

(1) 1 (2) $\frac{1}{2}$ (3) 0 (4) $-\frac{1}{2}$

Sol. Answer (4)

$\sin(\cot^{-1}(1 + x)) = \cos(\tan^{-1}(x))$

$\cot^{-1}(1 + x) = \theta$



$\Rightarrow \quad \cot\theta = (1 + x)$

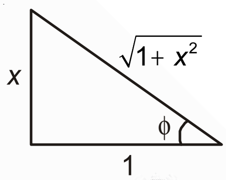
$\Rightarrow \quad \sin\theta = \dfrac{1}{\sqrt{2+2x+x^2}}$

$\Rightarrow \quad \tan^{-1} x = \phi$

$\Rightarrow \quad \tan\phi = x$

$\cos\phi = \dfrac{1}{\sqrt{1+x^2}}$

$\therefore \quad \sin\theta = \cos\phi$



$\Rightarrow \quad \dfrac{1}{\sqrt{2+2x+x^2}} = \dfrac{1}{\sqrt{1+x^2}}$

$\Rightarrow \quad 1 + x^2 = 2 + 2x + x^2$

$\Rightarrow \quad x = -\dfrac{1}{2}$

24. If $x_1 = \cos^{-1}\left(\dfrac{3}{5}\right) + \cos^{-1}\left(\dfrac{2\sqrt{2}}{3}\right)$ and $x_2 = \sin^{-1}\left(\dfrac{3}{5}\right) + \sin^{-1}\left(\dfrac{2\sqrt{2}}{3}\right)$, then

(1) $x_1 < x_2$ (2) $x_1 = x_2$ (3) $x_1 > x_2$ (4) Can't be determined

Sol. Answer (1)

$x_1 = \cos^{-1}\left(\dfrac{3}{5}\right) + \cos^{-1}\left(\dfrac{2\sqrt{2}}{3}\right)$

$= \tan^{-1}\left(\dfrac{4}{3}\right) + \tan^{-1}\left(\dfrac{1}{2\sqrt{2}}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{4}{3} + \dfrac{1}{2\sqrt{2}}}{1 - \dfrac{4}{3} \times \dfrac{1}{2\sqrt{2}}}\right)$

$= \tan^{-1}\left(\dfrac{8\sqrt{2}+3}{6\sqrt{2}-4}\right) > 1$

$\Rightarrow \quad x_1 \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

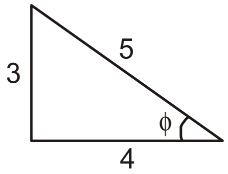
$x_2 = \sin^{-1}\left(\dfrac{3}{5}\right) + \sin^{-1}\left(\dfrac{2\sqrt{2}}{3}\right)$

$\sin^{-1}\left(\dfrac{3}{5}\right) = \theta$

$\sin\theta = \dfrac{3}{5}$

$\Rightarrow \quad \tan\theta = \frac{3}{4}$

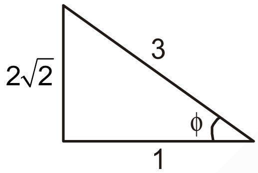
$\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \phi$



$\Rightarrow \quad \sin\phi = \frac{2\sqrt{2}}{3}$

$\Rightarrow \quad \tan\phi = 2\sqrt{2}$

$\Rightarrow \quad \tan\theta\tan\phi = \frac{3}{4} \times 2\sqrt{2} > 1$



$\Rightarrow \quad x_2 \in \left(\frac{\pi}{2}, \pi\right)$

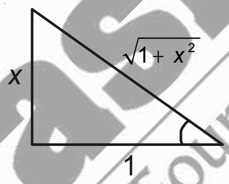
$\Rightarrow \quad x_2 > x_1$

25. $\cos\left(\tan^{-1}\frac{3}{4}\right) + \cos(\tan^{-1} x)$ is equal to

(1) $\frac{4}{5} + \frac{x}{\sqrt{1+x^2}}$ (2) $\frac{3}{5} + \frac{1}{\sqrt{1+x^2}}$ (3) $\frac{4}{5} + \frac{1}{\sqrt{1+x^2}}$ (4) $\frac{3}{5} + \frac{x}{\sqrt{1+x^2}}$

Sol. Answer (3)

$\cos\left(\tan^{-1}\frac{3}{4}\right) + \cos(\tan^{-1} x)$



$= \cos\cos^{-1}\left(\frac{4}{5}\right) + \cos\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$= \frac{4}{5} + \frac{1}{\sqrt{1+x^2}}$

26. The value of sin cot^{-1} tan cos^{-1}x is equal to

(1) x (2) $\frac{x}{2}$ (3) $-x$ (4) x^2

Sol. Answer (1)

sin cot^{-1} tan cos^{-1}x

$= \sin\left\{\frac{\pi}{2} - \tan^{-1}\tan\cos^{-1} x\right\}$

$= \sin\left(\frac{\pi}{2} - \cos^{-1} x\right)$

$= \sin\left(\sin^{-1} x\right)$

$= x$

Second Method

We have,

$\sin\cot^{-1}\tan\cos^{-1} x$

$= \sin\cot^{-1}\tan\theta$, where $\cos^{-1} x = \theta$

$= \sin\cot^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)$

$= \sin\phi$, where $\cot^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right) = \varphi$

$= x$

27. The value $\tan\left\{\sin^{-1}(\cos(\sin^{-1}x))\right\}\tan\left\{\cos^{-1}(\sin(\cos^{-1}x))\right\}$ is equal to (where $0 < x < 1$)

(1) 0 (2) 1 (3) – 1 (4) 2

Sol. Answer (2)

$\tan\left\{\sin^{-1}\left(\cos\sin^{-1}x\right)\right\}$

$= \tan\left\{\dfrac{\pi}{2} - \cos^{-1}\left(\cos\sin^{-1}x\right)\right\}$

$= \tan\left(\dfrac{\pi}{2} - \sin^{-1}x\right)$

$= \cot(\sin^{-1}x)$

Similarly,

$\tan\left\{\cos^{-1}\left(\sin\cos^{-1}x\right)\right\} = \tan\left\{\dfrac{\pi}{2} - \sin^{-1}(\sin\cos^{-1})\right\}$

$= \tan\left(\dfrac{\pi}{2} - \cos^{-1}x\right)$

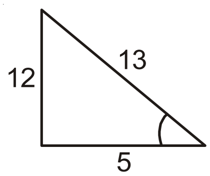
$= \tan(\sin^{-1}x)$

$\therefore\ \tan\left\{\sin^{-1}\cos\sin^{-1}x\right\}.\ \tan\left\{\cos^{-1}\sin\cos^{-1}x\right\} = 1$

28. $\dfrac{1}{2}\tan^{-1}\left(\dfrac{12}{5}\right)$ is equal to

(1) $\tan^{-1}\left(\dfrac{3}{2}\right)$ (2) $\tan^{-1}\left(\dfrac{2}{3}\right)$ (3) $\tan^{-1}\left(\dfrac{3}{4}\right)$ (4) $\tan^{-1}\left(\dfrac{7}{17}\right)$

Sol. Answer (2)



$\dfrac{1}{2}\tan^{-1}\left(\dfrac{12}{5}\right) = ?$

Let $\tan^{-1}\left(\dfrac{12}{5}\right) = \theta$

$\Rightarrow\ \tan\theta = \dfrac{12}{5}$

$$\Rightarrow \quad \cos\theta = \frac{5}{13} = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$\Rightarrow \quad \frac{1-\tan^2\frac{\theta}{2}+1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}-1-\tan^2\frac{\theta}{2}} = \frac{5+13}{5-13}$$

$$\tan^2\frac{\theta}{2} = \frac{8}{18}$$

$$\Rightarrow \quad \tan\frac{\theta}{2} = \pm\frac{2}{3}$$

But $\tan\frac{\theta}{2} \neq -\frac{2}{3}$ as it is positive

$$\therefore \quad \tan\frac{\theta}{2} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{\theta}{2} = \tan^{-1}\left(\frac{2}{3}\right)$$

[Solution of Inverse Trigonometric Equations]

29. If $\sin^{-1}\left(\frac{3}{x}\right)+\sin^{-1}\left(\frac{4}{x}\right)=\frac{\pi}{2}$, then x equals

(1) –5 (2) 5 (3) 7 (4) 12

Sol. Answer (2)

Given, $\sin^{-1}\left(\frac{3}{x}\right)+\sin^{-1}\left(\frac{4}{x}\right)=\frac{\pi}{2}$ ...(i)

Also, $\sin^{-1}\left(\frac{3}{x}\right)+\cos^{-1}\left(\frac{3}{x}\right)=\frac{\pi}{2}$ ...(ii)

From (i) and (ii),

$$\sin^{-1}\left(\frac{4}{x}\right)=\cos^{-1}\left(\frac{3}{x}\right)$$

$$\sin^{-1}\left(\frac{4}{x}\right)=\sin^{-1}\sqrt{1-\left(\frac{3}{x}\right)^2}$$

$$\Rightarrow \quad \frac{4}{x}=\sqrt{1-\left(\frac{3}{x}\right)^2}$$

$$\Rightarrow \quad \frac{4^2}{x^2}+\frac{3^2}{x^2}=1$$

$\Rightarrow$ x = ±5

$\Rightarrow$ x = 5 is a solution

30. $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$ holds good for

(1) All $x, y \in R$ (2) $|x| < 1, |y| < 1$ (3) $|x| > 1, |y| > 1$ (4) $xy > -1$

Sol. Answer (4)

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$, if $xy > -1$

31. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $\dfrac{1}{xy}+\dfrac{1}{yz}+\dfrac{1}{zx} =$

(1) 0 (2) 1 (3) $\dfrac{1}{xyz}$ (4) xyz

Sol. Answer (2)

Let $\tan^{-1}x = A, \tan^{-1}y = B, \tan^{-1}z = C$

Then $A + B + C = \pi$

$$\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}=\frac{x+y+z}{xyz}$$

$$=\frac{\tan A+\tan B+\tan C}{\tan A\tan B\tan C}$$

$= 1$ $(\because A+B+C=\pi \Rightarrow \tan A+\tan B+\tan C=\tan A\tan B\tan C)$

32. If $x + y + z = xyz$, then $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

(1) π (2) $\dfrac{\pi}{2}$ (3) 1 (4) $\dfrac{\pi}{3}$

Sol. Answer (1)

Let $x = \tan A, y = \tan B, z = \tan C$

Then, $x + y + z = xyz$

$\Rightarrow \tan A+\tan B+\tan C=\tan A\tan B\tan C$

$\Rightarrow \tan A+\tan B+\tan C-\tan A\tan B\tan C=0$

$\Rightarrow \tan(A+B+C)=0$

$\Rightarrow A+B+C=n\pi$

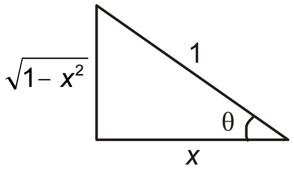
$\therefore A+B+C=\pi$

33. If $\tan(\sin^{-1}\sqrt{1-x^2})=\sin(\tan^{-1}2)$, then x is

(1) $-\dfrac{\sqrt{5}}{3}$ (2) $\dfrac{3}{\sqrt{10}}$ (3) $\dfrac{\sqrt{5}}{3}$ (4) Both (1) & (3)

Sol. Answer (4)

$\tan\left(\sin^{-1}\left(\sqrt{1-x^2}\right)\right)=\sin(\tan^{-1}2)$



$\sin^{-1}(\sqrt{1-x^2})=\theta$

$\Rightarrow \sin\theta=\sqrt{1-x^2}$

$$\tan\theta=\frac{\sqrt{1-x^2}}{x}$$

Let $\tan^{-1}(2) = \phi$

$\tan\phi = 2$

$\Rightarrow \quad \sin\phi = \frac{2}{\sqrt{5}}$

$\therefore \quad \tan\theta = \sin\phi$

$\Rightarrow \quad \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow \quad 5(1 - x^2) = 4x^2$

$\Rightarrow \quad x = \pm\frac{\sqrt{5}}{3}$

34. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ equals

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

Sol. Answer (2)

$$\cos^{-1} x + \cos^{-1} y = \left(\frac{\pi}{2} - \sin^{-1} x\right) + \left(\frac{\pi}{2} - \sin^{-1} y\right)$$

$$= \pi - \left(\sin^{-1} x + \sin^{-1} y\right)$$

$$= \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

35. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} .....\infty\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - .....\infty\right) = \frac{\pi}{2}$ there $0 < |x| < \sqrt{2}$, then x equals

(1) –1 (2) $\frac{-1}{2}$ (3) $\frac{1}{2}$ (4) 1

Sol. Answer (4)

$$\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^4}{4} - .....\infty\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - ....\infty\right) = \frac{\pi}{2}$$

$$\sin^{-1} x + \cot^{-1} y = \frac{\pi}{2}$$

$\Rightarrow \quad x = y$

$\therefore \quad x - \frac{x^2}{2} + \frac{x^4}{4} - ....\infty = x^2 - \frac{x^4}{2} + \frac{x^6}{4} - ....\infty$

$\Rightarrow \quad \frac{x}{1-\left(-\frac{x}{2}\right)} = \frac{x^2}{1-\left(-\frac{x^2}{2}\right)}$

$\Rightarrow \quad \frac{2x}{2+x} = \frac{2x^2}{2+x^2}$

$\Rightarrow \quad x \neq 0$

$\Rightarrow \quad 2 + x^2 = 2x + x^2$

$\Rightarrow \quad x = 1$

36. If $\sin\left(\sin^{-1}\frac{3\pi}{11}+\cos^{-1}x\right)=1$, then x is

(1) $-\frac{3\pi}{11}$ (2) $\frac{\pi}{2}-\frac{3\pi}{11}$ (3) $\frac{3\pi}{11}$ (4) $\frac{8\pi}{11}$

Sol. Answer (3)

$$\sin\left(\sin^{-1}\frac{3\pi}{11}+\cos^{-1}x\right)=1$$

$$\Rightarrow \sin^{-1}\frac{3\pi}{11}+\cos^{-1}x=\sin^{-1}1=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\frac{3\pi}{11}=\frac{\pi}{2}-\cos^{-1}x=\sin^{-1}x$$

$$\Rightarrow x=\frac{3\pi}{11}$$

37. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$ is

(1) 0 (2) 1 (3) 2 (4) 3

Sol. Answer (1)

Since $\frac{-\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$, etc.

Hence, $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, only when

$$\sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$$

$\Rightarrow$ $x = y = z = 1$

Hence,

$$x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$$

$$=1+1+1-\frac{9}{1+1+1}$$

$$=3-\frac{9}{3}$$

$$=0$$

38. Let $x_i \in [-1, 1]$ for $i = 1, 2, 3, ...24$, such that $\sin^{-1}x_1 + \sin^{-1}x_2 + ... + \sin^{-1}x_{24} = 12\pi$ then the value of $x_1 + 2x_2 + 3x_3 + ...+ 24x_{24}$ is

(1) 276 (2) 300 (3) 325 (4) 351

Sol. Answer (2)

Since, $-\frac{\pi}{2}\le\sin^{-1}x_i\le\frac{\pi}{2}$

Therefore, $\sin^{-1}x_1+\sin^{-1}x_2+...+\sin^{-1}x_{24}\le 12\pi$

Equality holds, when $\sin^{-1}x_1=\sin^{-1}x_2=....=\sin^{-1}x_{24}=\frac{\pi}{2}$

$\Rightarrow \; x_1 = x_2 = \dots = x_{24} = 1$

$\therefore \; x_1 + 2x_2 + 3x_3 + ... + 24x_{24}$

$= 1 + 2 + 3 \dots . + 24$

$= \frac{24}{2}(24+1)$

$= 300$

39. How many solutions does the equation $5\tan^{-1}x + 3\cot^{-1}x = 2\pi$ have?

(1) Zero (2) Exactly one (3) Exactly two (4) Infinite

Sol. Answer (2)

$5\tan^{-1}x + 3\cot^{-1}x = 2\pi$

$\Rightarrow \; 2\tan^{-1}x + 3\left(\tan^{-1}x + \cot^{-1}x\right) = 2\pi$

$\Rightarrow \; 2\tan^{-1}x = 2\pi - \frac{3\pi}{2} = \frac{\pi}{2}$

$\Rightarrow \; \tan^{-1}x = \frac{\pi}{4}$

$\Rightarrow \; x = 1$

$\therefore$ Exactly one solution

40. The equation $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has

(1) No solutions (2) Unique solution

(3) Infinite number of solution (4) Two solutions

Sol. Answer (2)

$\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \; 2\sin^{-1}x - \frac{\pi}{2} = \frac{\pi}{6}$

$\Rightarrow \; \sin^{-1}x = \frac{\pi}{3}$

$\Rightarrow \; x = \frac{\sqrt{3}}{2}$

$\therefore$ Unique solution exists.

41. Solution of $\sin^{-1}x = \cos^{-1}x$ is $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

(1) Less than $\frac{1}{2}$ (2) Less than $\frac{1}{3}$

(3) Greater than $\frac{1}{3}$ (4) Greater than $\frac{4}{5}$

Sol. Answer (3)

$\sin^{-1}x = \cos^{-1}x$

$\Rightarrow \; \sin^{-1}x = \sin^{-1}\sqrt{1-x^2}$

$\Rightarrow \quad x = \sqrt{1-x^2}$

$\Rightarrow \quad 2x^2 = 1 \;\Rightarrow\; x^2 = \dfrac{1}{2}$

$\Rightarrow \quad x = \pm\dfrac{1}{\sqrt{2}}$

$\therefore \quad x = \dfrac{1}{\sqrt{2}}$ is a solution.

42. Number of solution of the equation $\sin\left(\dfrac{1}{5}\cos^{-1}x\right) = 1$ is

(1) 1 (2) 0 (3) 2 (4) Infinitely many

Sol. Answer (2)

Given equation is

$\sin\left(\dfrac{1}{5}\cos^{-1}x\right) = 1$

i.e., $\dfrac{1}{5}\cos^{-1}x = (4n+1).\dfrac{\pi}{2} \qquad n \in I$

i.e., $\cos^{-1}x = \dfrac{5}{2}.(4n+1)\pi$

which is not possible as $0 \le \cos^{-1}x \le \pi$.

$\therefore$ No solution of given equation is possible.

[Miscellaneous]

43. The set of values of k for which $x^2 - kx + \sin^{-1}(\sin 4) > 0$ for all values of x

(1) No values of k is possible (2) $[-4(\pi - 4), 4(\pi - 4)]$

(3) [0, 1] (4) All real values of k

Sol. Answer (1)

We know that

$\sin^{-1}(\sin 4) = \sin^{-1}\sin(\pi - 4) = \pi - 4$

We have, $x^2 - kx + \pi - 4 > 0 \; x \in R$

$\Rightarrow \; D < 0 \Rightarrow k^2 - 4(\pi - 4) < 0$

$\Rightarrow \; k^2 + 4(4 - \pi) < 0$

which is not true for any real value of k.

44. Number of solutions of the equation $2(\sin^{-1}x)^2 - \sin^{-1}x - 6 = 0$ is

(1) 2 (2) 1 (3) 0 (4) 3

Sol. Answer (2)

Solving $2.(\sin^{-1}x)^2 - \sin^{-1}x - 6 = 0$

either $\sin^{-1}x = 2$ or $\sin^{-1}x = -1.5$

$\sin^{-1}x$ cannot be greater then $\frac{\pi}{2}$

$\therefore \quad \sin^{-1}x \neq 2$

Only one solution is there

$x = \sin(-1.5)$

45. $\cos^{-1}(2x^2 - 1)$, $0 \le x \le 1$, is equal to

(1) $2\cos^{-1}x$ (2) $2\sin^{-1}x$ (3) $\pi - 2\cos^{-1}x$ (4) $\pi + 2\cos^{-1}x$

Sol. Answer (1)

Let $x = \cos\theta$

$\cos^{-1}(2x^2 - 1) = \cos^{-1}(\cos 2\theta) = 2\theta$

$= 2\cos^{-1}x$

if $0 \le 2\theta \le \pi$

$0 \le \theta \le \frac{\pi}{2}$

$0 \le x \le 1$

46. $\sin^{-1}(2x\sqrt{1-x^2})$, $x \in \left[\frac{1}{\sqrt{2}}, 1\right]$ is equal to

(1) $2\sin^{-1}x$ (2) $2\cos^{-1}x$ (3) $-2\sin^{-1}x$ (4) $-2\cos^{-1}x$

Sol. Answer (2)

Let $x = \cos\theta$

$\sin^{-1}(2\cos\theta\sin\theta)$

$= \sin^{-1}(\sin 2\theta) = 2\theta,$

$= 2\cos^{-1}x$

if $-\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$\frac{1}{\sqrt{2}} \le x \le 1$

47. $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), |x| \le \frac{1}{\sqrt{2}}$, is equal to

(1) $\frac{1}{2}\cos^{-1}x^2$ (2) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

(3) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$ (4) $\frac{\pi}{2}-\frac{1}{2}\cos^{-1}x^2$

Sol. Answer (3)

Let $x^2 = \cos 2\theta$

$$\Rightarrow \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right) \qquad \left(\because x^2 \le \frac{1}{2} \Rightarrow \cos 2\theta \le \frac{1}{2}\, \frac{\pi}{4} \le 2\theta \le \frac{\pi}{2}\right)$$

$$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right) \qquad \left(\because \frac{\pi}{8} \le \theta \le \frac{\pi}{4} \therefore \frac{3\pi}{8} \le \theta + \frac{\pi}{4} \le \frac{\pi}{2}\right)$$

$$= \frac{\pi}{4}+\theta$$

$$= \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

48. The value of $\cos(2\cos^{-1} 0.8)$ is

(1) 0.48 (2) 0.96 (3) 0.6 (4) 0.28

Sol. Answer (4)

$$\cos\left(2\cos^{-1}0.8\right) = \cos\left(\cos^{-1}\left(2(0.8)^2-1\right)\right)$$

$$= 2(0.8)^2 - 1$$

$$= 2\,(0.64) - 1$$

$$= 1.28 - 1$$

$$= 0.28$$

49. The sum $\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{2^n+2^{1-n}}\right)$ equals

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Sol. Answer (2)

$$\frac{1}{2^n+2^{1-n}} = \frac{2^{n-1}}{1+2^n.2^{n-1}} = \frac{(2-1).2^{n-1}}{1+2^n.2^{n-1}} = \frac{2^n-2^{n-1}}{1+2^n.2^{n-1}}$$

$$\therefore \sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{2^n+2^{1-n}}\right)$$

$$= \sum_{n=1}^{\infty}\left\{\tan^{-1}(2^n)-\tan^{-1}(2^{n-1})\right\}$$

$$= \lim_{n\to\infty}\sum_{n=1}^{n}\left\{\tan^{1}(2^n)-\tan^{-1}(2^{n-1})\right\}$$

$$= \lim_{n\to\infty}\left\{\tan^{-1}(2^n)-\tan^{-1}(1)\right\}$$

$$= \frac{\pi}{2}-\frac{\pi}{4}$$

$$= \frac{\pi}{4}$$

50. The sum of the maximum and minimum values of the function $f(x) = \sin^{-1}4x + \cos^{-1}4x + \sec^{-1}4x$ is

(1) 2π (2) π (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

Sol. Answer (1)

Domain of $f(x) \Rightarrow x \in \left\{-\frac{1}{4}, \frac{1}{4}\right\}$ only

$\therefore$ $f(x)$ is min when $x = \frac{1}{4}$ *i.e.*, $f_{\min}\left(\frac{1}{4}\right) = \frac{\pi}{2}$.

and $f(x)$ is max when $x = -\frac{1}{4}$ *i.e.*, $f_{\max}\left(-\frac{1}{4}\right) = \frac{3\pi}{2}$

$\therefore$ Sum of maximum and minimum value of function is 2π.

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