SURFACE AREAS AND VOLUMES

IMPORTANT FORMULAE

Name of the Solid	Curved Surface Area	Total Surface Area	Volume
Cuboid	$2h(l+b)$	$2(lb+bh+hl)$	lbh
Cube	$4a^2$	$6a^2$	a^3
Right Circular Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Right Circular Cone	πrl	$2\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$
Sphere	–	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Frustum of a Cone	$\pi(r_1+r_2)l$ where $l=\sqrt{h^2+(r_1-r_2)^2}$	$\pi(r_1+r_2)l + \pi r_1^2 + \pi r_2^2$	$\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$

COMBINATIONAL FIGURE BASED QUESTIONS

IMPORTANT QUESTIONS

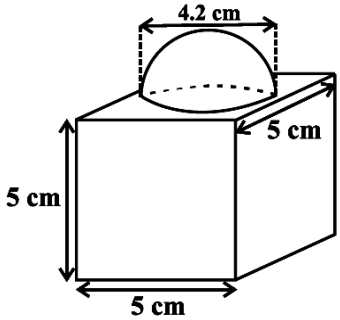
The decorative block is shown in below left figure made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block.

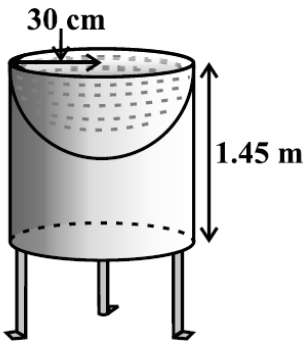
Solution: The total surface area of the cube = 6 × (edge)^2 = 6 × 5 × 5 cm^2 = 150 cm^2.

So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere

= 150 – πr^2 + 2πr^2 = (150 + πr^2) cm2

$$=150+\left(\frac{22}{7}\times\frac{4.2}{2}\times\frac{4.2}{2}\right)cm^2=150+13.86cm^2=163.86cm^2$$





Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.

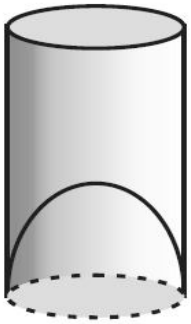
Solution : Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere. (See above right sided figure)

Total surface area of the bird-bath = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r2 = 2\pi\ r\ (h + r) = 2\times\frac{22}{7}\times 30(145+30) = 2\times\frac{22}{7}\times 30\times 175 = 33000cm^2 = 3.3m^2$

A juice seller was serving his customers using glasses as shown in below figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14.)

Solution: Here, inner diameter = 5 cm. height, h = 10 cm



So, radius, r = $\frac{5}{2}$ cm

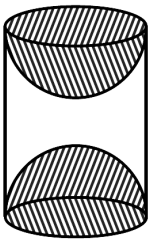
Apparent capacity of the glass = Volume of cylinder – Volume of hemisphere

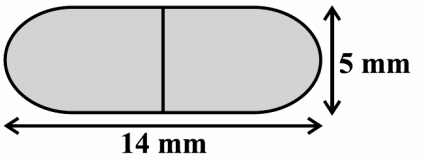
$= \pi r^2 h - \frac{2}{3}\pi r^3 = \pi r^2\left(h - \frac{2}{3}r\right) = 3.14\times\frac{5}{2}\times\frac{5}{2}\times\left(10 - \frac{2}{3}\times\frac{5}{2}\right)$

$= 3.14\times\frac{25}{4}\times\frac{25}{3} = \frac{19625}{12} = 163.54cm^3$

Questions for Practice

1. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder (see below left figure). If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.





2. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see above right sided figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

3. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2.

4. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

5. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

6. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take π = 3.14)

7. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm

8. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8g mass. (Use π = 3.14)

9. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it

touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

MCQ QUESTIONS (1 mark)

1. A cylindrical pencil sharpened at one edge is the combination of
(a) a cone and a cylinder (b) frustum of a cone and a cylinder
(c) a hemisphere and a cylinder (d) two cylinders.

2. A surahi is the combination of
(a) a sphere and a cylinder (b) a hemisphere and a cylinder
(c) two hemispheres (d) a cylinder and a cone.

3. The shape of a gilli, in the gilli-danda game (see below figure), is a combination of
(a) two cylinders (b) a cone and a cylinder
(c) two cones and a cylinder (d) two cylinders and a cone

4. A shuttle cock used for playing badminton has the shape of the combination of
(a) a cylinder and a sphere (b) a cylinder and a hemisphere
(c) a sphere and a cone (d) frustum of a cone and a hemisphere

5. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called
(a) a frustum of a cone (b) cone (c) cylinder (d) sphere

6. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is
(a) $4\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

7. A right circular cylinder of radius r cm and height h cm (h>2r) just encloses a sphere of diameter
(a) r cm (b) 2r cm (c) h cm (d) 2h cm

8. During conversion of a solid from one shape to another, the volume of the new shape will
(a) increase (b) decrease (c) remain unaltered (d) be doubled

9. In a right circular cone, the cross-section made by a plane parallel to the base is a
(a) circle (b) frustum of a cone (c) sphere (d) hemisphere

10. The volume of a cube is 2744 cm^3. Its surface area is
(a) 196 cm^2 (b) 1176 cm^2 (c) 784 cm^2 (d) 588 cm^2

11. The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is
(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 5 : 1

12. The height of a cylinder is 14 cm and its curved surface area is 264 cm^2. The volume of the cylinder is
(a) 296 cm^3 (b) 396 cm^3 (c) 369 cm^3 (d) 503 cm^3

13. The ratio of the volumes of two spheres is 8 : 27. The ratio between their surface areas is
(a) 2 : 3 (b) 4 : 27 (c) 8 : 9 (d) 4 : 9

14. The radii of the base of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3, then ratio of their volumes is
(a) 9 : 8 (b) 9 : 4 (c) 3 : 1 (d) 27 : 64

15. If two cubes, each of edge 4 cm are joined end to end, then the surface area of the resulting cuboid is
(a) 100 cm^2 (b) 160 cm^2 (c) 200 cm^2 (d) 80 cm^2

16. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. The ratio of its diameter to its height is
(a) 3 : 7 (b) 7 : 3 (c) 6 : 7 (d) 7 : 6

17. The radius of spherical balloon increases from 8 cm to 12 cm. The ratio of the surface areas of the balloon in two cases is
(a) 2 : 3 (b) 3 : 2 (c) 8 : 27 (d) 4 : 9

18. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9