CHAPTER -03

MATRICES

One mark questions:

1. Define a scalar matrix. **(K)**
2. Define an Identity matrix. **(K)**
3. Define a diagonal matrix. **(K)**

4. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write: (i) the order of the matrix

(ii) The number of elements (iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$. **(U)**

5. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements? **(U)**

6. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements? **(U)**

7. Find the number of all possible matrices of order 3 ×3 with each entry 0 or 1 ? **(U)**

8. If a matrix has 8 elements, what are the possible orders it can have? **(U)**

9. Construct a 2 × 2 matrix, A = [a_{ij}], whose elements are given by; **(A)**

(i) $a_{ij} = \frac{(i+j)^2}{2}$ (ii) $a_{ij} = \frac{i}{j}$ (iii) $a_{ij} = \frac{(i+2j)^2}{2}$

10. Construct a 3 × 3 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i-3j|$ **(A)**

11. Construct a 3 × 4 matrix, whose elements are given by: **(A)**

(i) $a_{ij} = \frac{1}{2}|-3j+j|$ (ii) $a_{ij} = 2i - j$

12. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ **(U)** (ii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ **(A)**

13. Find x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ **(U)**

14. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y. **(U)**

15. Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ **(A)**

16. Find the values of x and y from the following equation

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 16 \end{bmatrix}$$ **(U)**

17. Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$ **(A)**

18. Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix. **(U)**

19. Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix. **(U)**

20. Let A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, C= $\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following

(i) A + B (ii) A – B (iii) 3A – C (iv) AB (v) BA **(U)**

21. Consider the following information regarding the number of men and women workers in three factories I, II and III

	Men Workers	Women workers
I	30	25
II	25	31
III	27	26

Represent the above information in the form of a 3 × 2 matrix. What does the entry in the third row and second column represent? **(U)**

22. Given $A=\begin{bmatrix}\sqrt{3} & 1 & 1\\ 2 & 3 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & \sqrt{5} & 1\\ -2 & 3 & \frac{1}{2}\end{bmatrix}$, find A + B **(U)**

23. If A = $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{bmatrix}$ and B = $\begin{bmatrix}3 & -1 & 3\\ -1 & 0 & 2\end{bmatrix}$, then find 2A – B **(U)**

24. Find AB, if A = $\begin{bmatrix}6 & 9\\ 2 & 3\end{bmatrix}$ and B = $\begin{bmatrix}2 & 6 & 0\\ 7 & 9 & 8\end{bmatrix}$ **(U)**

25. If A = $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ and B = $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then prove that i) AB = $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ and ii)BA = $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$. **(U)**

26. Find AB, if A = $\begin{bmatrix}0 & -1\\ 0 & 2\end{bmatrix}$ and B = $\begin{bmatrix}3 & 5\\ 0 & 0\end{bmatrix}$ **(U)**

27. Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$ **(A)**

28. Find P^{-1}, if it exists, given P = $\begin{bmatrix}10 & -2\\ -5 & 1\end{bmatrix}$ **(A)**

29. Find the transpose of each of the following matrices: A = $\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}$ and B = $\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix}$.

(U)

30. If A = $\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}$ and B = $\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix}$, verify that **(A)**

(i) $(A')' = A$ (ii) $(A+B)' = A' + B'$

31. Compute the following **(A)**

(i) $\begin{bmatrix}a & b\\ -b & a\end{bmatrix}+\begin{bmatrix}a & b\\ b & a\end{bmatrix}$ (ii) $\begin{bmatrix}a^2+b^2 & b^2+c^2\\ a^2+c^2 & a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab & 2bc\\ -2ac & -2ab\end{bmatrix}$

(iii) $\begin{bmatrix}-1 & 4 & -6\\ 8 & 5 & 16\\ 2 & 8 & 5\end{bmatrix}+\begin{bmatrix}12 & 7 & 6\\ 8 & 0 & 5\\ 3 & 2 & 4\end{bmatrix}$ (iv) $\begin{bmatrix}\cos^2 x & \sin^2 x\\ \sin^2 x & \cos^2 x\end{bmatrix}+\begin{bmatrix}\sin^2 x & \cos^2 x\\ \cos^2 x & \sin^2 x\end{bmatrix}$

32. Compute the indicated products: **(U)**

(i) $\begin{bmatrix} a & b \\ -b & 1 \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ (v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ (vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Three marks questions:

1. Find the values of x, y and z from the following equations: $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$. **(S)**

2. Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ **(U)**

3. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w. **(S)**

4. Find the values of a, b, c and d from the following equation

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$ **(A)**

5. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$ **(S)**

Find the values of a, b, c, x, y and z.

6. Using elementary transformations, find the inverse of each of the matrices **(A)**

(i) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ iii. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ iv. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ v. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

vi. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ vii. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ viii. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ ix. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ xi. $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

xii. $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$ xiii. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ xiv. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

7. By using elementary operations, find the inverse of the matrix A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ **(U)**

8. Find X and Y, if X + Y = $\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and X – Y = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ **(U)**

9. Find X and Y, if 2X + 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3X + 2Y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ **(U)**

10. If A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, such that 2A + 3X = 5B. **(A)**

11. Find X and Y, if X + Y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and X – Y = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ **(U)**

12. If A = $\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and B = $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute 3A – 5B **(A)**

13. If F(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)\cdot F(y) = F(x+y)$ **(A)**

14. If (i) A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that A′ A = I **(A)**

(ii) If A = $\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that A′ A = I

15. Show that $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ **(U)**

16. Show that $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ **(A)**

17. If A = $\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$ **(U)**

18. If A′ = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that **(U)**

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$

19. If A′ = $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$ **(U)**

20. If A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ then find AB, BA. Show that $AB \neq BA$. **(U)**

21. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways: telephone, house calls, and letters. The cost per contact (in paise) is given in matrix A as

Cost per contact

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Telephone \\ Hou\sec all \\ Letter \end{matrix}$$

The number of contacts of each type made in two cities X and Y is given by

$$\begin{matrix} Telephone & Hous\sec all & Letter \end{matrix}$$

$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{matrix} \to X \\ \to Y \end{matrix}$$

. Find the total amount spent by the group in the two cities X and Y. **(A)**

22. A trust fund has RS. 30,000 that must be invested in two different types of bonds. The first bond pays5 % interest per year, and the second bond pays 7 % interest per year. Using matrix multiplication, determine how to divide Rs. 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of :

(a) Rs. 1800 (b) Rs. 2000 **(A)**

23. A book shop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra. **(A)**

24. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute, that is AB = BA. **(K)**

25. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$ **(A)**

26. For the matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that **(U)**

(i) (A + A′) is a symmetric matrix (ii) (A - A′) is a skew symmetric matrix

27. Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when A = $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ **(U)**

28. Express the following matrices as the sum of a symmetric and skew symmetric matrix: **(U)**

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

29. Express the matrix B = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **(U)**

30. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if AB = BA. **(K)**

31. If A and B are invertible matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$ **(K)**

32. Prove that for any square matrix A with real number entries, $A+A'$ is a symmetric matrix and $A-A'$ is a skew symmetric matrix. **(K)**

33. Prove that any square matrix can be expressed as the sum of symmetric and skew symmetric matrix. **(K)**

34. Prove inverse of a square matrix, if it exist, is unique. **(K)**

Five marks questions:

1. If $A=\begin{bmatrix}-2\\4\\5\end{bmatrix}, B=\begin{bmatrix}1 & 3 & -6\end{bmatrix}$, verify that $(AB)' = B'A'$. **(U)**

4. If A = $\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}$, B = $\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$ and C = $\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$ **(U)**

   Then compute (A + B) and (B – C).Also, verify that A + (B – C) = (A + B) – C

5. If A = $\begin{bmatrix}1 & 1 & -1\\2 & 0 & 3\\3 & -1 & 2\end{bmatrix}$, B = $\begin{bmatrix}1 & 3\\0 & 2\\-1 & 4\end{bmatrix}$ and C = $\begin{bmatrix}1 & 2 & 3 & -4\\2 & 0 & -2 & 1\end{bmatrix}$, find A(BC), (AB)C and show that (AB)C = A(BC). **(U)**

6. Find $A^2-5A+6I$, if $A=\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$ **(U)**

7. If $A=\begin{bmatrix}0 & 6 & 7\\-6 & 0 & 8\\7 & -8 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 1 & 1\\1 & 0 & 2\\1 & 2 & 0\end{bmatrix}, C=\begin{bmatrix}2\\-2\\3\end{bmatrix}$

   Calculate AC, BC and (A + B) C. Also, verify that (A + B)C = AC + BC. **(U)**

8. If $A=\begin{bmatrix}1 & 1 & 1\\1 & 2 & -3\\2 & -1 & 3\end{bmatrix}$, verify that $A^3-6A^2+5A+11I=O$,

   where O is zero matrix of order 3 x 3. **(A)**

9. If A = $A=\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}$, then show that $A^3-23A-40I=O$. **(A)**

10. If $A=\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=0$. **(A)**

11. Let A = $A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B=\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, C=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that $CD-AB=O$ **(U)**

12. If $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. **(S)**

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