Areas Related to Circles

Very Short Answer Type Questions (1 mark each)

Q. 1. In fig. 4 is a sector of circle of radius 10.5 cm. Find the perimeter of the sector. $\left(\text{Take } \pi = \frac{22}{7}\right)$

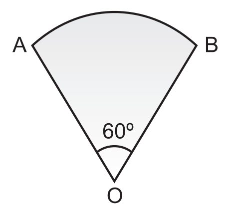


Fig. - 4

[CBSE OD, Set 1, 2020]

Solution : Given : OA = OB = 10.5 cm

and $\angle AOB = 60°$

Perimeter of the sector = $OA + OB + AB$

$$= 10.5 + 10.5 + \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 10.5$$

$$\left[\because l = \frac{\theta}{360°} \times 2\pi r\right]$$

= 21 + 11 = 32 cm **Ans.**

Q. 2. Find the area of the sector of a circle of radius 6 cm whose central angle is 30°. (Take π = 3.14) [CBSE OD, Set 3, 2020]

Ans. Given : Radius of sector (r) = 6 cm and central angle (θ) = 30°.

Area of sector of a circle = $\frac{\theta}{360°} \times \pi r^2$

$$= \frac{30°}{360°} \times 3.14 \times 6^2$$

$= 3 \times 3.14 = 9.42 \text{ cm}^2$

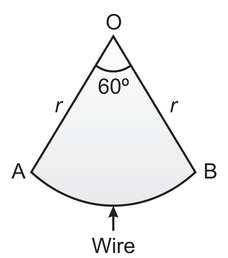
Ans.

Short Answer Type Questions-I (3 marks each)

Q. 1. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[CBSE Delhi, Set 1, 2020]

Ans. Let the radius of the arc of the circle be r cm



We have

Length of wire (l) = 22 cm

and, $\theta = 60°$

We know,

$$\text{Length of arc/ wire} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times r$$

$$= \frac{22}{21} r$$

$$\therefore \quad \frac{22}{21} r = 22$$

$$\Rightarrow \quad r = 22 \times \frac{21}{22}$$

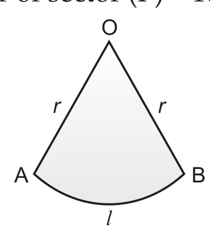
$\Rightarrow \quad r = 21$ cm **Ans.**

Q. 2. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector. [CBSE Delhi, Set 2, 2020]

Ans. We have,

Radius (r) = 5.2 cm.

Perimeter of sector (P) = 16.4 cm



We know that,

$$P = l + 2r$$

$\Rightarrow \quad 16.4 = l + 2(5.2)$

$\Rightarrow \quad 16.4 = l + 10.4$

$\Rightarrow \quad l = 16.4 - 10.4$

$l = 6$ cm.

Now, Area of sector $= \frac{1}{2} lr$

$= \frac{1}{2} \times 6 \times 5.2$

$= 3 \times 5.2$

$= 15.6 \text{ cm}^2$ **Ans.**

Q. 3. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes. [CBSE Delhi, Set 3, 2020]

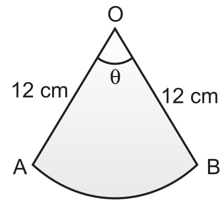
Ans. Given, $r = 12$ cm

Angle swept by minute hand in 35 minutes

$= 35 \times 6°$

$(\because 1 \text{ minute} = 6°)$

$\Rightarrow \quad \theta = 210°$



Area swept by minute hand

$= \text{Area of sector}$

$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{210}{360} \times \frac{22}{7} \times 12 \times 12$

$= 22 \times 12 \text{ cm}^2$

$= 264 \text{ cm}^2$ **Ans.**

Short Answer Type Questions-II (3 marks each)

Q. 1. In Figure 4, a square OABC is inscribed in a quadrant OPBQ. If OA = 15 cm, find the area of the shaded region. (Use $\pi = 3.14$)

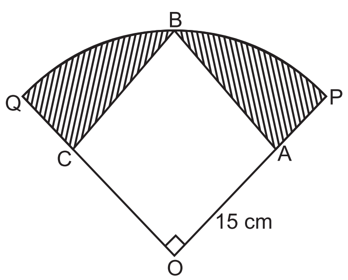
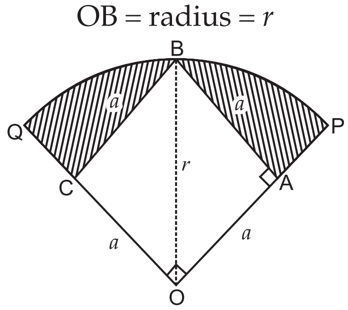


Figure 4

[CBSE OD, Set 1, 2019]

Ans. Given, OABC is a square with OA = 15 cm

OB = radius = r



Let side of square be a then,

$a^2 + a^2 = r^2$

$2a^2 = r^2$

$r = \sqrt{2}\, a$

$r = 15\sqrt{2}$ cm $\quad (\because a = 15 \text{ cm})$

Area of square = Side × Side

$= 15 \times 15$

$= 225 \text{ cm}^2$

Area of quadrant OPBQ $= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 15\sqrt{2} \times 15\sqrt{2}$

$= \frac{225 \times 2 \times 3.14}{4}$

$= 225 \times 1.57$

$= 353.25 \text{ cm}^2$

Area of shaded region

= Area of quadrant OPBQ – Area of square

$= 353.25 - 225$

$= 128.25 \text{ cm}^2$

Q. 2. In Figure 5, ABCD is a square with side $2\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region. (Use $\pi = 3.14$)

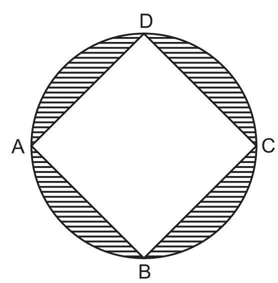
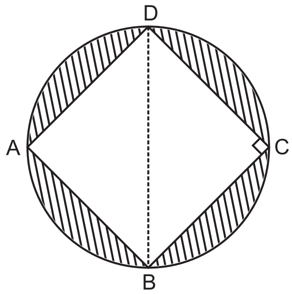


Figure 5

[CBSE OD, Set 1, 2019]

Ans. Given, ABCD is a square with side $2\sqrt{2}$ cm

$\because \quad BD = 2r$



In ΔBDC

$$BD^2 = DC^2 + BC^2$$

$$4r^2 = 2(DC)^2 \quad (\because DC = CB = \text{Side} = 2\sqrt{2})$$

$$4r^2 = 2 \times 2\sqrt{2} \times 2\sqrt{2}$$

$$4r^2 = 8 \times 2$$

$$4r^2 = 16$$

$$\Rightarrow \quad r^2 = 4$$

$$r = 2 \text{ cm}$$

Area of square BCDA = Side × Side

$$= DC \times BC$$

$$= 2\sqrt{2} \times 2\sqrt{2}$$

$$= 8 \text{ cm}^2$$

Area of circle $= \pi r^2$

$$= 3.14 \times 2 \times 2$$

$$= 12.56 \text{ cm}^2$$

Area of shaded region

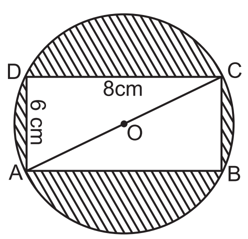
= Area of circle – Area of square.

$$= 12.56 - 8$$

$$= 4.56 \text{ cm}^2$$

Q. 3. Find the area of the shaded region in Fig. if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle.

[Take $\pi = 3.14$]



[CBSE Delhi, Set 1, 2019]

Ans. Given, ABCD is a rectangle with sides AB = 8 cm and BC = 6 cm

In ΔABC

$$AC^2 = 8^2 + 6^2 \quad \text{(By Pythagoras Theorem)}$$

$$= 64 + 36$$

$$\Rightarrow \quad AC^2 = 100$$

$$\Rightarrow \quad AC = 10 \text{ cm}$$

The diagonal of the rectangle will be the diameter of the circle

$\therefore$ radius of the circle $= \dfrac{10}{2} = 5$ cm

Area of shaded portion

= Area of circle – Area of Rectangle

$$= \pi r^2 - l \times b$$

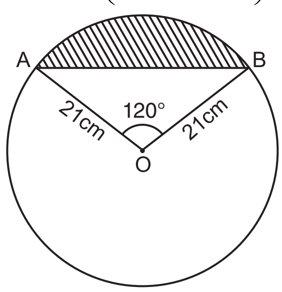
$$= 3.14 \times 5 \times 5 - 8 \times 6$$

$$= 78.50 - 48$$

$$= 30.50 \text{ cm}^2$$

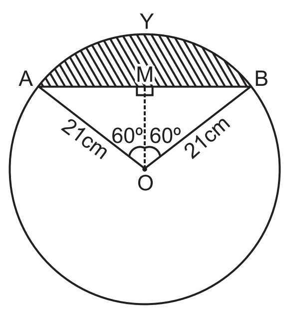
Hence, Area of shaded portion = 30.5 cm^2

Q. 4. Find the area of the segment shown in Fig. if radius of the circle is 21 cm and $\angle AOB = 120°$ $\left(\text{Use } \pi = \dfrac{22}{7}\right)$



[CBSE Delhi, Set 2, 2019]

Ans. Given, Radius of the circle = 21 cm and $\angle AOB = 120°$



Area of the segment AYB

= Area of sector AOB – Area of ΔAOB

Area of sector AOB $= \dfrac{120°}{360°} \times \dfrac{22}{7} \times 21 \times 21$

$$= 462 \text{ cm}^2$$

To find the area of ΔOAB, draw $OM \perp AB$

$\Delta AMO \cong \Delta BMO$ (by R.H.S.)

$\therefore \angle AOM = \angle BOM = \dfrac{1}{2} \times 120° = 60°$

From ΔOMA, $\dfrac{OM}{OA} = \cos 60°$

$$\frac{OM}{21} = \frac{1}{2}$$

$$OM = \frac{21}{2} \text{ cm}$$

Also, $$\frac{AM}{OA} = \sin 60°$$

$$AM = 21 \times \frac{\sqrt{3}}{2}$$

or $$AB = 2 \times AM = 21\sqrt{3} \text{ cm}$$

So, $$\text{area of } \Delta OAB = \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$$

$$= \frac{441}{4}\sqrt{3} \text{ cm}^2$$

$$\therefore \text{ Area of segment} = \left(462 - \frac{441}{4}\sqrt{3}\right) \text{cm}^2$$

$$= \frac{21}{4}(88 - 21\sqrt{3}) \text{ cm}^2$$

$$= 271.04 \text{ cm}^2$$

Ans.

Q. 5. Find the area of the shaded region in Fig. 2, where arcs drawn with centres *A*, *B*, *C* and *D* intersect in pairs at mid-points *P*, *Q*, *R* and *S* of the sidess *AB*, *BC*, *CD* and *DA* respectively of a square *ABCD* of side 12 cm. [Use $\pi = 3.14$]

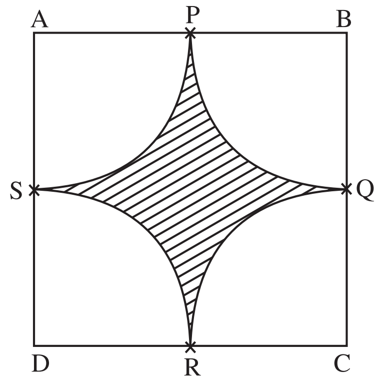


Fig. 2

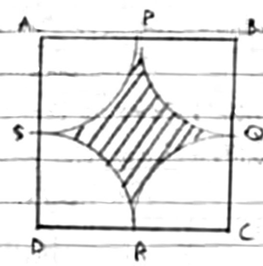
[CBSE, 2018]

Topper's Answers

20) Given: side of square ABCD = 12 cm.

To find: shaded area.

Shaded area + Area of 4 quadrants = Area of square.



Area of square = s^2 sq. units

$= 12^2 = 144 \text{ cm}^2.$

Area of quadrant = $\frac{1}{4} \times \pi r^2$ sq. units

$= \frac{1}{4} \times 3.14 \times \frac{12}{2} \times \frac{12}{2}$

$= 9 \times 3.14 =$ ~~28.26~~ $28.26 \text{ cm}^2.$

=> Shaded area = Area of square − 4 × (Area of quadrant) sq. units

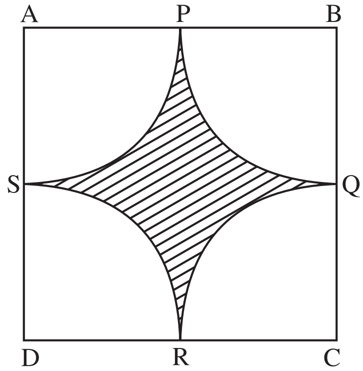
$= 144 - 4(28.26)$ sq. cm

$= 144 - 113.04$

$= 30.96 \text{ cm}^2.$

The area of the shaded region is 30.96 cm^2.

Given, *ABCD* is a square of side 12 cm.



P, *Q*, *R* and *S* are the mid points of sides *AB*, *BC*, *CD* and *AD* respectively.

Area of shaded region

= Area of square – 4 × Area of quadrant

$= a^2 - 4 \times \frac{1}{4}\pi r^2$

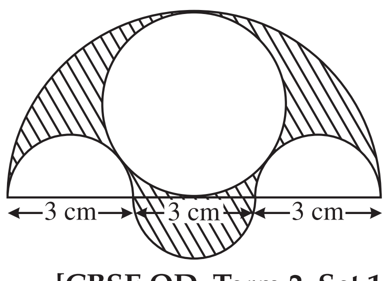
$= (12)^2 - 3.14 \times (6)^2$

$= 144 - 3.14 \times 36$

$= 144 - 113.04$

$= 30.96 \text{ cm}^2$

Q. 6. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semi-circle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



[CBSE OD, Term 2, Set 1, 2017]

Ans. Given, radius of large semi-circle = 4.5 cm

Area of large semi-circle $= \frac{1}{2}\pi R^2$

$= \frac{1}{2} \times \frac{22}{7} \times 4.5 \times 4.5$

Diameter of inner circle = 4.5 cm

$\Rightarrow \quad r = \frac{4.5}{2}$ cm

Area of inner circle $= \pi r^2$

$= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2}$

Diameter of small semi-circle = 3 cm

$\Rightarrow \quad r = \frac{3}{2}$ cm

Area of small semi-circle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$

Area of shaded region

= Area of large semi-circle + Area of 1 small semi-circle – Area of inner circle – Area of 2 small semi-circle

$= \frac{1}{2} \times \frac{22}{7} \times 4.5 \times 4.5 + \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$

$- \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} - 2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$

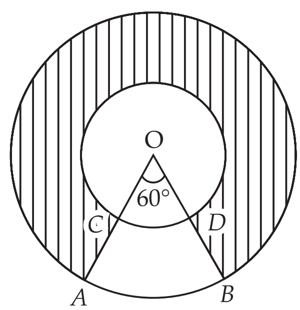
$= \frac{1}{2} \times \frac{22}{7}\left[20.25 + \frac{9}{4}\right] - \frac{22}{7}\left[\frac{20.25}{4} + \frac{9}{4}\right]$

$= \frac{11}{7} \times \frac{90}{4} - \frac{22}{7} \times \frac{29.25}{4}$

$= \frac{900 - 643.5}{28} = \frac{346.5}{28}$

$= 12.37 \text{ cm}^2$

Q. 7. In the given figure, two concentric circles with centre *O* have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



[CBSE OD, Term 2, Set 1, 2017]

Ans. Angle for shaded region

$= 360° - 60°$

$= 300°$

Area of shaded region

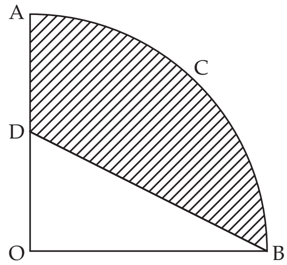
$= \frac{\theta}{360°}\pi(R^2 - r^2)$

$= \frac{300°}{360°} \times \frac{22}{7}[42^2 - 21^2]$

$= \frac{5}{6} \times \frac{22}{7} \times 63 \times 21$

$= 3465 \text{ cm}^2$

Q. 8. In the given figure, *OACB* is a quadrant of a circle with centre *O* and radius 3.5 cm. If *OD* = 2 cm, find the area of the shaded region.



[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Area of shaded region = Area of quadrant *OACB* – Area of Δ *DOB*

$$= \frac{90}{360} \times \pi \times (3.5)^2 - \frac{1}{2} \times 2 \times 3.5$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} - 3.5$$

$$= \frac{1925}{200} - 3.5$$

$$= 9.625 - 3.5$$

$$= 6.125 \text{ cm}^2$$

Hence, area of shaded region is 6.125 cm^2

Q. 9. In fig. 4, *O* is the centre of a circle such that diameter *AB* = 13 cm and *AC* = 12 cm. *BC* is joined. Find the area of the shaded region. (Take π = 3.14)

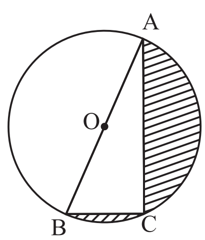


Figure 4

[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, *AB* is a diameter of length 13 cm and *AC* = 12 cm.

Then, by Pythagoras theorem,

$$(BC)^2 = (AB)^2 - (AC)^2$$

$$\Rightarrow \quad (BC)^2 = (13)^2 - (12)^2$$

$$\Rightarrow \quad BC = \sqrt{169 - 144}$$

$$\Rightarrow \quad BC = \sqrt{25}$$

$$\because \quad BC = 5 \text{ cm}$$

Now, Area of shaded region

= Area of semi-circle – Area of Δ*ABC*

$$= \frac{\pi r^2}{2} - \frac{1}{2} \times BC \times AC$$

$$= \frac{1}{2} \times 3.14 \times \left(\frac{13}{2}\right)^2 - \frac{1}{2} \times 5 \times 12$$

$$= \frac{1.57 \times 169}{4} - 30$$

$$= 66.33 - 30$$

$$= 36.33 \text{ cm}^2$$

So, area of shaded region is 36.33 cm^2

Q. 10. In fig. 6, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm, where ∠*AOC* = 40°. (Use $\pi = \frac{22}{7}$)

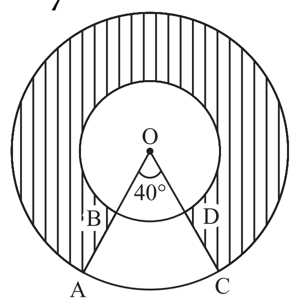


Figure 6

[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, $r = 7$ cm and $R = 14$ cm.

Area of shaded region $= \pi(R^2 - r^2)\,\frac{\theta}{360°}$

$$= \frac{22}{7}(14^2 - 7^2) \times \frac{(360° - 40°)}{360°}$$

$$= \frac{22}{7} \times 7 \times 21 \times \frac{320°}{360°}$$

$$= 410.67 \text{ cm}^2$$

Q. 11. In Fig. 4, *ABCD* is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. $\left(\text{use } \pi = \frac{22}{7}\right)$

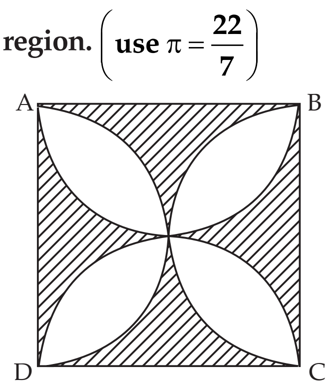


Fig. 4

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Given, a square $ABCD$ of side 14 cm

Then, Area of square $= (\text{side})^2$

$= (14)^2 = 196 \text{ cm}^2$

2 [Area of semi-circle] $= \pi r^2$

$= \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2}$

$= 154 \text{ cm}^2$

Now, Area of shaded region

= 2[Area of square – 2 (Area of semi-circle)]

$= 2\,[196 - 154] = 2 \times 42 = 84 \text{ cm}^2$

Q. 12. In Fig. 7, are shown two arcs *PAQ* and *PBQ*. Arc *PAQ* is a part of circle with centre *O* and radius *OP* while arc *PBQ* is a semi-circle drawn on *PQ* as diameter with centre *M*. If *OP* = *PQ* = 10 cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm².

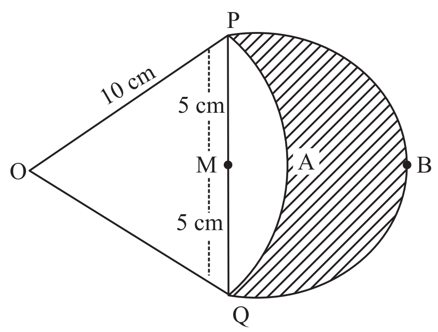


Fig. 7

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Given, $OP = PQ = 10$ cm

Since, OP and OQ are radii of circle with centre O.

$\therefore$ ΔOPQ is equilateral.

$\Rightarrow \angle POQ = 60°$

Now, Area of segment $PAQM$

= (Area of sector $OPAQO$ – Area of ΔPOQ)

$= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin\theta$

$= \frac{\pi \times (10)^2 \times 60°}{360°} - \frac{1}{2}(10)^2 \sin 60°$

$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{cm}^2$

and, area of semi-circle $PBQ = \frac{\pi r^2}{2} = \frac{\pi}{2}(5)^2$

$= \frac{25}{2}\pi \text{ cm}^2$

$\therefore$ Area of shaded region

= Area of semi-circle

– Area of segment $PAQM$

$= \frac{25}{2}\pi - \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)$

$= \frac{25}{2}\pi - \frac{50\pi}{3} + 25\sqrt{3}$

$= \frac{75\pi - 100\pi}{6} + 25\sqrt{3}$

$= \frac{-25\pi}{6} + 25\sqrt{3}$

$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2$

Hence Proved.

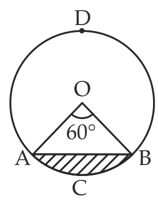
Q. 13. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment.

[Use $\pi = \frac{22}{7}$]

[CBSE OD, Term 2, Set 1, 2015]

Ans. Let ACB be the given arc subtending an angle of 60° at the centre.

Here, $r = 14$ cm and $\theta = 60°$.



Area of the minor segment $ACBA$

= (Area of the sector $OACBO$)

– (Area of ΔOAB)

$= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin\theta$

$= \frac{22}{7} \times 14 \times 14 \times \frac{60°}{360°} - \frac{1}{2} \times 14 \times 14 \times \sin 60°$

$= \frac{308}{3} - 7 \times 14 \times \frac{\sqrt{3}}{2}$

$= \frac{308}{3} - 49\sqrt{3}$

$= 17.79 \text{ cm}^2$

Area of the major segment $BDAB$

= Area of circle

– Area of minor segment $ACBA$

$= \pi r^2 - 17.79$

$= \frac{22}{7} \times 14 \times 14 - 17.79$

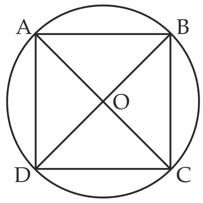
$= 616 - 17.79$

$= 598.21 \approx 598 \text{ cm}^2$

Q. 14. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if the area of the circle is 1256 cm^2. [Use $\pi = 3.14$]

[CBSE OD, Set 2, 2015]

Ans.



Given that the area of the circle is 1256 cm^2.

$\because$ Area of the circle $= \pi r^2$

$\Rightarrow \quad 1256 = 3.14 \times r^2$

$\Rightarrow \quad r^2 = \frac{1256 \times 100}{314}$

$\Rightarrow \quad r = \sqrt{400}$

$\therefore \quad r = 20$ cm

Now, A, B, C, D are the vertices of a rhombus.

$\therefore \quad \angle A = \angle C \quad$ ...(i)

[opposite angles of rhombus]

But $ABCD$ lie on the circle.

So, $ABCD$ is called cyclic quadrilateral

$\therefore \quad \angle A + \angle C = 180° \quad$ ...(ii)

On using equation (i), we get

$\angle A + \angle A = 180°$

$\Rightarrow \quad 2\angle A = 180°$

$\Rightarrow \quad \angle A = 90°$

so, $\quad \angle C = 90°$ [From *eq.* (i)]

$\therefore$ $ABCD$ is square.

So, BD is a diameter of circle.

[$\because$ The angle in a semi-circle is a right angle triangle]

Now, Area of rhombus

$= \frac{1}{2} \times$ Product diagonals

$= \frac{1}{2} \times 40 \times 40$

$= 800 \text{ cm}^2$

Hence, area of rhombus is 800 cm^2.

Q. 15. In Fig. 3, *APB* and *AQO* are semi-circles, and *AO* = *OB*. If the perimeter of the figure is 40 cm, find the area of the shaded region.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

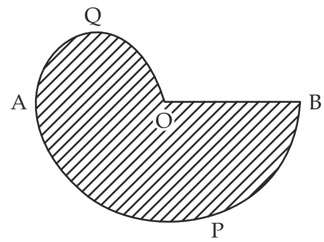


Figure 3

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Given, $OA = OB = r$(say)

We have, perimeter of the figure

$= \pi r + \frac{\pi r}{2} + r$

$\therefore \quad 40 = \frac{22}{7} \times r + \frac{22}{7} \times \frac{r}{2} + r$

$\Rightarrow \quad 280 = 22r + 11r + 7r$

$\Rightarrow \quad 40r = 280$

$\therefore \quad r = 7$

Now, area of the shaded region

$= \frac{\pi r^2}{2} + \frac{\pi}{2}\left(\frac{r}{2}\right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 77 + \frac{77}{4}$

$= \frac{77 \times 5}{4} = \frac{385}{4}$

$= 96\frac{1}{4} \text{ cm}^2$

Q. 16. In Fig. 6, find the area of the shaded region

[Use $\pi = 3.14$]

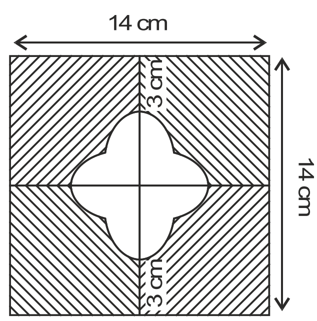
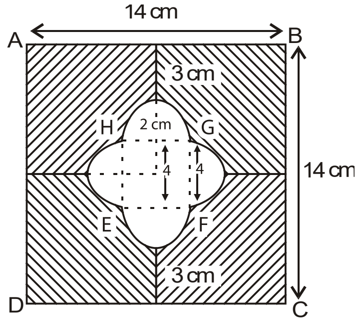


Figure 6

Ans. Area of square (*ABCD*)



$= 14 \times 14$

$= 196 \text{ cm}^2$

Area of small square (*EFGH*)

$= (7 - 3)^2$

$= 4 \times 4 = 16 \text{ cm}^2$

4 (Area of semi-circle) $= 4 \times \dfrac{\pi r^2}{2}$

$= 2 \times 3{\cdot}14 \times 2 \times 2$

$= 25{\cdot}12 \text{ cm}^2$

$\therefore$ Required area (shaded) = Area of square – Area of small square – 4 × Area of semi-circle

$= (196 - 16 - 25{\cdot}12) \text{ cm}^2$

$= 154{\cdot}88 \text{ cm}^2$

Long Answer Type Questions

(4 marks each)

Q. 1. Find the area of the shaded region in fig. 8, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

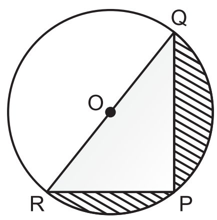


Fig - 8

[CBSE OD, Set 1, 2020]

Ans. Given : PR = 24 cm, PQ = 7 cm.

We have, $\angle RPQ = 90°$

[$\because$ Angle in a semicircle is 90°]

$\therefore$ In right ΔPRQ, we have

$RQ^2 = PR^2 + PQ^2$

[Pythagoras theorem]

$= (24)^2 + 7^2$

$= 576 + 49$

$\Rightarrow \quad RQ^2 = 625$

$\Rightarrow \quad RQ = \sqrt{625} = 25 \text{ cm}$

$\therefore$ Radius of circle $= \dfrac{1}{2} \times 25 \text{ cm} = \dfrac{25}{2} \text{ cm}$

Now, Area of shaded region

= Area of semicircle with radius $\dfrac{25}{2}$ cm

– Area of ΔPQR

$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2 - \frac{1}{2} \times 7 \times 24$$

$$= \left(\frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}\right) - (7 \times 12)$$

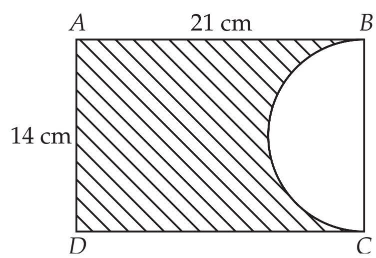
$$= \left(\frac{11 \times 25 \times 25}{28}\right) - 84$$

$$= \frac{6875}{28} - 84$$

$= (245.53 - 84) \text{ cm}^2$

$= 161.54 \text{ cm}^2$. **Ans.**

Q. 2. In the given figure, *ABCD* is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with *BC* as diameter. Find the area and the perimeter of the shaded region in the figure.



[CBSE OD, Term 2, Set 1, 2017]

Ans. Area of shaded region

= Area of rectangle – Area of semi-circle

$= l \times b - \dfrac{1}{2}\pi r^2$

$= 21 \times 14 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 294 - 77$

$= 217 \text{ cm}^2$

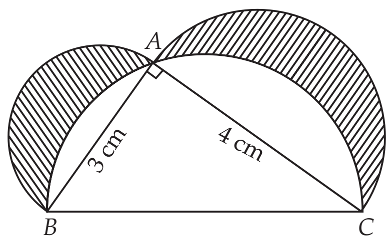
Perimeter of shaded region $= 2l + b + \pi r$

$= 2 \times 21 + 14 + \frac{22}{7} \times 7$

$= 42 + 14 + 22$

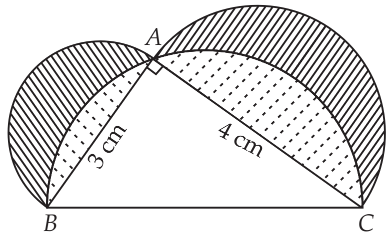
$= 78$ cm

Q. 3. In the given figure, ΔABC is a right-angled triangle in which $\angle A$ is 90°. Semi-circles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



[CBSE OD, Term 2, Set 2, 2017]

Ans. In right ΔBAC, by pythagoras theorem,



$$BC^2 = AB^2 + AC^2$$
$$= (3)^2 + (4)^2$$
$$= 9 + 16 = 25$$
$$BC = \sqrt{25} = 5 \text{ cm}$$

Area of semi-circle with diameter BC

$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \left(\frac{5}{2}\right)^2 = \frac{25}{8}\pi \text{ cm}^2$$

Area of semi-circle with diameter AB

$$= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left(\frac{3}{2}\right)^2 = \frac{9}{8}\pi \text{ cm}^2$$

Area of semi-circle with diameter AC

$$= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left(\frac{4}{2}\right)^2 = \frac{16}{8}\pi \text{ cm}^2$$

Area of rt. $\Delta BAC = \frac{1}{2} \times AB \times AC$

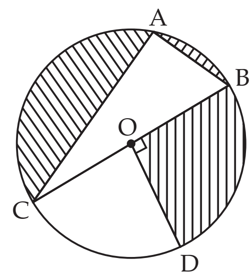
$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

Area of dotted region $= \left(\frac{25}{8}\pi - 6\right) \text{ cm}^2$

Area of shaded region

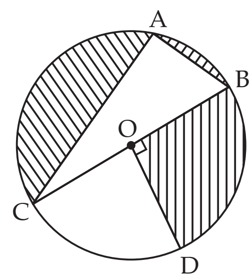
$$= \frac{16}{8}\pi + \frac{9}{8}\pi - \left(\frac{25}{8}\pi - 6\right)$$
$$= \frac{16}{8}\pi + \frac{9}{8}\pi - \frac{25}{8}\pi + 6$$
$$= 6 \text{ cm}^2$$

Q. 4. In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and $\angle BOD$ = 90°. Find the area of the shaded region.



[CBSE OD, Term 2, Set 3, 2017]

Ans. Given, $C(O, OB)$ with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$



$\angle CAB = 90°$ (Angle in semi-circle)

In ΔCAB,

$$BC^2 = AC^2 + AB^2$$
$$= (24)^2 + (7)^2$$
$$= 576 + 49$$
$$= 625$$
$$BC = 25 \text{ cm}$$

Radius of circle $= OB = OD = OC = \frac{25}{2}$ cm

Area of shaded region

= Area of semi-circle with diameter BC – Area of ΔCAB + Area of sector BOD

$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2 - \frac{1}{2}\times 24\times 7 + \frac{90°}{360°}\pi\left(\frac{25}{2}\right)^2$$

$$= \frac{3}{4}\times\frac{22}{7}\times\frac{25}{2}\times\frac{25}{2} - 84$$

$$= \frac{20625}{56} - 84$$

$$= \frac{20625 - 4704}{56}$$

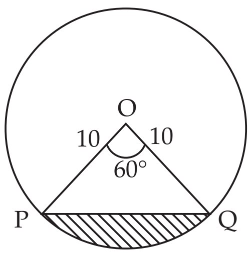
$$= \frac{15921}{56}$$

$$= 284.3 \text{ cm}^2$$

Q. 5. A chord *PQ* of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle.

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. $r = 10$ cm, $\theta = 60°$



Minor Segment

Area of minor segment

$$= \frac{\theta}{360}\times\pi r^2 - \frac{1}{2}r^2\sin\theta$$

$$= \frac{60°}{360°}\times 3.14\times 10\times 10 - \frac{1}{2}\times 10\times 10\times\sin 60°$$

$$= \frac{1}{6}\times 3.14\times 100 - \frac{1}{2}\times 100\times\frac{\sqrt{3}}{2}$$

$$= \frac{314}{6} - \frac{100}{4}\times 1.73$$

$$= \frac{314}{6} - \frac{173}{4} = \frac{628 - 519}{12} = \frac{109}{12}\text{ cm}^2$$

Area of major segment

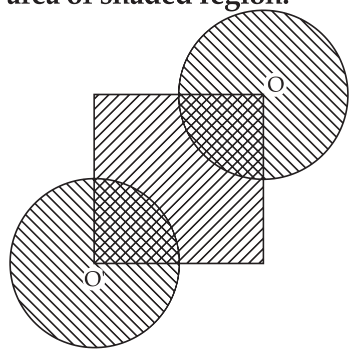
$$= \text{Area of circle} - \text{Area of minor segment}$$

$$= \pi r^2 - \frac{109}{12}$$

$$= 3.14\times 10\times 10 - \frac{109}{12}$$

$$= 314 - \frac{109}{12} = \frac{3768 - 109}{12} = \frac{3659}{12}\text{ cm}^2$$

Q. 6. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where *O* and *O'* are centres of the circles. Find the area of shaded region.



[CBSE Delhi, Term 2, Set 1, 2017]

Ans. $r = \frac{1}{2}$ (side) = 14 cm, [∵ side = 28 cm]

Area of shaded region

$$= 2\times(\text{area of circle}) + \text{area of square} - 2\times(\text{area of quadrant})$$

$$= 2\times\pi r^2 + (\text{side})^2 - 2\left(\frac{1}{4}\times\pi r^2\right)$$

$$= 2\pi r^2 - \frac{1}{2}\pi r^2 + (\text{side})^2$$

$$= \frac{3}{2}\pi r^2 + (\text{side})^2$$

$$= \frac{3}{2}\times\frac{22}{7}\times 14\times 14 + 28\times 28$$

$$= 924 + 784$$

$$= 1708\text{ cm}^2$$

Q. 7. In Fig. 9, is shown a sector *OAP* of a circle with centre *O*, containing ∠θ. *AB* is perpendicular to the radius *OA* and meets *OP* produced at *B*. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right]$.

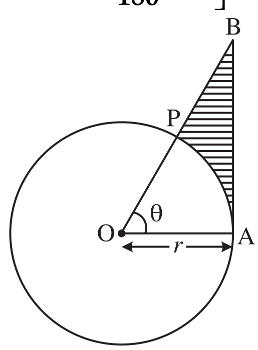


Figure 9

[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, the radius of circle with centre O is r.

$\angle POA = \theta$.

then, length of the arc $\widehat{PA} = \frac{2\pi r\theta}{360°} = \frac{\pi r\theta}{180°}$

And $\tan\theta = \frac{AB}{r}$

$\Rightarrow \quad AB = r\tan\theta$

And $\sec\theta = \frac{OB}{r}$

$\Rightarrow \quad OB = r\sec\theta$

Now, $PB = OB - OP$

$= r\sec\theta - r$

$\therefore$ Perimeter of shaded region

$= AB + PB + \widehat{PA}$

$= r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180°}$

$$= r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right]$$

Hence Proved.

Q. 8. An elastic belt is placed around the rim of a pulley of radius 5 cm. (Fig. 10) From one point *C* on the belt, the elastic belt is pulled directly away from the centre *O* of the pulley until it is at *P*, 10 cm from the point *O*. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. (use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

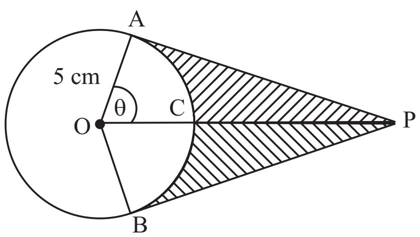


Fig. 10

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Given, a circular pulley of radius 5 cm with centre O.

$\therefore \quad AO = OB = OC = 5$ cm

and $\quad OP = 10$ cm

Now, in right ΔAOP,

$$\cos\theta = \frac{AO}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

$\therefore \angle AOB = 2\theta = 120°$

$\Rightarrow$ Reflex $\angle AOB = 360° - 120° = 240°$

Length of major arc $\widehat{AB} = \frac{2\pi r}{360°}$ reflex $\angle AOB$

$$= \frac{2 \times 3.14 \times 5 \times 240°}{360°}$$

$= 20{\cdot}93$ cm

Hence, length of the belt that is still in contact with pulley = 20·93 cm

Now, by Pythagoras theorem,

$(AP)^2 = (OP)^2 - (AO)^2$

$\Rightarrow \quad (AP)^2 = (10)^2 - (5)^2$

$\Rightarrow \quad AP = \sqrt{100 - 25}$

$= \sqrt{75} = 5\sqrt{3}$ cm

$\therefore \quad$ Area of $\Delta AOP = \frac{1}{2} \times 5 \times 5\sqrt{3}$

$= \frac{25\sqrt{3}}{2}$ cm^2

Also, Area of ΔBOP = Area of ΔAOP

and, Area of quad. $AOBP$ = 2(Area of ΔAOP)

$= 2 \times \frac{25\sqrt{3}}{2} = 25\sqrt{3}$ cm^2

$= 43.25$ cm^2

Area of sector $ACBO = \frac{\pi r^2 \angle AOB}{360°}$

$$= \frac{3.14 \times 5 \times 5 \times 120°}{360°}$$

$= 26{\cdot}16$ cm^2

$\therefore$ Area of shaded region = Area of quad. $AOBP$ – Area of sector $ACBO$

$= (43.25 - 26.16)$ cm^2

$= 17.09$ cm^2

Q. 9. In figure 5, *PQRS* is a square lawn with side *PQ* = 42 m. Two circular flower beds are there on the sides *PS* and *QR* with center at *O*, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).

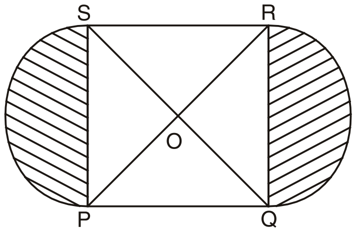
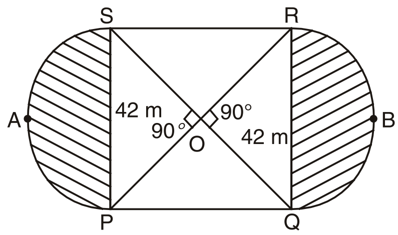


Figure 5
[CBSE OD, Term 2, Set 1, 2015]

Ans.



Given, *PQRS* is a square with side 42 m.

Its diagonals intersect at *O*.

Then, $OP = OQ = OR = OS$

and $\angle POS = \angle QOR = 90°$

$$PR^2 = PQ^2 + QR^2$$

$\because$ $PR = (\sqrt{2} \times 42)\text{m}$

Now, $OP = \frac{1}{2} \times \text{(diagonal)} = 21\sqrt{2}$ m

$\because$ Area of flower bed *PAS* = Area of flower bed *QBR*

$\therefore$ Total area of the two flower beds = Area of flower bed *PAS* + Area of flower bed *QBR*

= 2 × [Area of sector *OPAS* – Area of Δ *POS*]

$$= 2 \times \left[\pi r^2 \frac{\theta}{360°} - \frac{1}{2} r^2 \sin\theta\right] \text{ [Where, } \theta = 90°]$$

$$= 2 \times \left[\frac{22}{7} \times (21\sqrt{2})^2 \frac{90°}{360°} - \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2}\right]$$

[$\because$ sin 90° = 1]

$$= 2 \times \left[\frac{22}{7} \times 21 \times 21 \times 2 \times \frac{1}{4} - \frac{1}{2} \times 21 \times 21 \times 2\right]$$

= 2[33 × 21 – 441]

= 2[693 – 441]

= 504 m^2

Hence area of flower beds is 504 m^2.