Chapter 20

INTEGRALS - 1
(INDEFINITE INTEGRATION)

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. Antiderivative of $\dfrac{x-1}{(x+1)\sqrt{x^3+x^2+x}}$ is

(a) $\tan^{-1}\left(x+\dfrac{1}{x}+1\right)$ (b) $\tan^{-1}\sqrt{x+\dfrac{1}{x}+1}$

(c) $2\tan^{-1}\sqrt{x+\dfrac{1}{x}+1}$ (d) $\sqrt{x+\dfrac{1}{x}+1}$

2. The integral of $\int e^{\sin x}(x\cos x-\sec x\tan x)\,dx$ is

(a) $xe^{\sin x}-e^{\sin x}\sec x+C$

(b) $(x+\sec x)e^{\sin x}+C$

(c) $e^{\sin x}\cos x+C$

(d) $e^{\sin x}(\cos x-\sec x)+C$

3. $\displaystyle\int\frac{\sin^3 x\,dx}{(\cos^3 x+3\cos^2 x+1)\tan^{-1}(\sec x+\cos x)}=$

(a) $\tan^{-1}(\sec x+\cos x)+c$

(b) $\log\tan^{-1}(\sec x+\cos x)+c$

(c) $\dfrac{1}{(\sec x+\cos x)^2}+c$

(d) None of these

4. $\displaystyle\int e^x\frac{1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}dx=$

(a) $e^x\left(\dfrac{1-x^n}{1+x^n}\right)+C$ (b) $e^x\left(\dfrac{1+x^n}{1-x^n}\right)+C$

(c) $e^x\left(\sqrt{\dfrac{1+x^n}{1-x^n}}\right)+C$ (d) $e^x\left(\sqrt{\dfrac{1-x^n}{1+x^n}}\right)+C$

5. If $f(x)=\lim\limits_{n\to\infty}n^2(x^{1/n}-x^{1/(n+1)})$, $x>0$ then $\int xf(x)\,dx$ is equal to

(a) $\dfrac{x^2}{2}+\ln x+C$ (b) $-\dfrac{x^2}{4}\ln x+\dfrac{x^2}{2}+C$

(c) $\dfrac{x^3}{3}+x\ln x+C$ (d) $\dfrac{x^2}{2}\ln x-\dfrac{x^2}{4}+C$

6. $\displaystyle\int\frac{1+x^4}{(1-x^4)^{3/2}}dx=$

(a) $\dfrac{1}{\sqrt{x^2-\dfrac{1}{x^2}}}+c$ (b) $\dfrac{1}{\sqrt{\dfrac{1}{x^2}-x^2}}+c$

(c) $\dfrac{1}{\sqrt{\dfrac{1}{x^2}+x^2}}+c$ (d) None of these

7. Let $f(x)=\dfrac{x+2}{2x+3}$, $x>0$. If $\displaystyle\int\left(\frac{f(x)}{x^2}\right)^{1/2}dx$

$$=\frac{1}{\sqrt{2}}g\left(\frac{1+\sqrt{2f(x)}}{1-\sqrt{2f(x)}}\right)-\sqrt{\frac{2}{3}}h\left(\frac{\sqrt{3f(x)}+\sqrt{2}}{\sqrt{3f(x)}-\sqrt{2}}\right)+C$$

Where C is the constant of intergation, then

(a) $g(x)=\tan^{-1}(x), h(x)=\ln|x|$

(b) $g(x)=\ln|x|, h(x)=\tan^{-1}(x)$

(c) $g(x)=\tan^{-1}(x), h(x)=\tan^{-1}(x)$

(d) $g(x)=\ln|x|, h(x)=\ell n|x|$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ			

8. $\int \sqrt{1+\operatorname{cosec} x}\, dx =$

(a) $\pm \sin^{-1}(\tan x - \sec x) + c$

(b) $2\sin^{-1}(\cos x) + c$

(c) $2\sin^{-1}\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) + c$

(d) $\pm 2\sin^{-1}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + c$

9. If $f(x) = \lim\limits_{n\to\infty} [2x + 4x^3 + ... + 2nx^{2n-1}]$; $(0 < x < 1)$, then $\int (f(x))\, dx$ is equal to

(a) $-\sqrt{1-x^2} + c$ (b) $\frac{1}{\sqrt{1-x^2}} + c$

(c) $\frac{1}{x^2-1} + c$ (d) $\frac{1}{1-x^2} + c$

10. If $f(x) = \int \frac{dx}{\sin^{1/2} x \cos^{7/2} x}$, then $f\left(\frac{\pi}{4}\right) - f(0) =$

(a) $\frac{7}{5}$ (b) $\frac{5}{2}$

(c) $\frac{12}{5}$ (d) 5

11. If $f\left(\frac{3x-4}{3x+4}\right) = x + 2$, then $\int f(x)dx$ is equal to

(a) $e^{x+2} \ln\left|\frac{3x-4}{3x+4}\right| + c$

(b) $-\frac{8}{3}\ln|(1-x)| + \frac{2}{3}x + c$

(c) $\frac{8}{3}\ln|1-x| + \frac{x}{3} + c$

(d) None of these

12. The integral $\int \frac{\sec^{3/2}\theta - \sec^{1/2}\theta}{2+\tan^2\theta} \tan\theta d\theta$ is equal to

(a) $\sqrt{2}\tan^{-1}\left(\frac{\sec\theta + 1}{\sqrt{2\sec\theta}}\right) + C$

(b) $\frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta - \sqrt{2\sec\theta} + 1}{\sec\theta + \sqrt{2\sec\theta} + 1}\right| + C$

(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left|\frac{\sec\theta + 1}{\sqrt{2\sec\theta}}\right| + C$

(d) $\sqrt{2}\log_e\left|\frac{\sec\theta - \sqrt{2\sec\theta} + 1}{\sec\theta + \sqrt{2\sec\theta} + 1}\right| + C$

13. If $f(x) = \lim\limits_{n\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}, 0 < x < 1,\ n \in N$ then $\int (\sin^{-1} x) f(x) dx$ is equal to

(a) $-[x\sin^{-1} x + \sqrt{1-x^2}] + C$

(b) $x\sin^{-1} x + \sqrt{1-x^2} + C$

(c) $\frac{x^2}{2} + C$

(d) $\frac{1}{2}(\sin^{-1} x)^2 + C$

14. If $f(x) = \lim\limits_{n\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}, x > 1$; then

$$\int \frac{xf(x)\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx \text{ is equal to}$$

(a) $\log(x+\sqrt{1+x^2}) - x + C$

(b) $\frac{1}{2}[x^2\log(x+\sqrt{1+x^2}) - x^2] + C$

(c) $x\log(x+\sqrt{1+x^2}) - \log(x+\sqrt{1+x^2}] + C$

(d) $\sqrt{1+x^2}\log(x+\sqrt{1+x^2}) - x + C$

MARK YOUR RESPONSE	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ
	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ			

15. If $I_n = \int \cot^n x\, dx$, then

$I_0 + I_1 + 2(I_2 + I_3 + ...I_8) + I_9 + I_{10} =$

(a) $-\sum_{k=1}^{9} \frac{\cot^k x}{k}$ (b) $\sum_{k=1}^{9} \frac{\cot^k x}{k!}$

(c) $\sum_{k=1}^{10} \frac{\cot^k x}{10}$ (d) $-\sum_{k=1}^{10} k \cot^k x$

16. The value of integral $\int \frac{(1-\cos\theta)^{2/7}}{(1+\cos\theta)^{9/7}} d\theta$ is

(a) $\frac{7}{11}\left(\tan\frac{\theta}{2}\right)^{\frac{11}{7}} + C$ (b) $\frac{7}{11}\left(\cos\frac{\theta}{2}\right)^{\frac{11}{7}} + C$

(c) $\frac{7}{11}\left(\sin\frac{\theta}{2}\right)^{\frac{11}{7}} + C$ (d) none of these

17. If $l_{m,n} = \int \cos^m x \sin nx\, dx$, then $7I_{4,3} - 4I_{3,2} =$

(a) constant (b) $-\cos^2 x + C$

(c) $-\cos^4 x \cos 3x + C$ (d) $\cos 7x - \cos 4x + C$

18. Anti derivative of the fucntion

$x^{\sin x - 1} \sin x + x^{\sin x}.\cos x.ln\, x$ is

(a) $x^{\sin x}$ (b) $x^{\sin x} + \sin x$

(c) $(\sin x)^{1/x}$ (d) $x^{\sin x} + x^{ln\, x}$

19. Let $f(xy) = f(x).f(y), \forall x > 0, y > 0$, where $f(x)$ is not constant and $f(x+1) = 1 + x\{1 + g(x)\}$ where $\lim_{x \to 0} g(x)$ = 0, then $\int \frac{f(x)}{f'(x)} dx$ is

(a) $\frac{x^2}{2} + c$ (b) $\frac{x^3}{3} + c$

(c) $\frac{x^2}{3} + c$ (d) $\ell n|x| + c$

20. If $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}} = -[f(x)]^{1/n} + C$, then f(x) is

(a) $(1+x^n)$ (b) $1 + x^{-n}$

(c) $x^n + x^{-n}$ (d) none of these

21. If $\int \frac{dx}{(x-1)(-x^2+3x-2)} = -2f(x) + C$, then $f(x) =$

(a) $\sqrt{\frac{2-x}{x-1}}$ (b) $\sqrt{\frac{x-2}{x-1}}$

(c) $\sqrt{\frac{x+2}{x-1}}$ (d) $\sqrt{\frac{x-1}{x-2}}$

22. $\int \frac{(x+\sqrt{1+x^2})^{15}}{\sqrt{1+x^2}} dx = A\{x + \sqrt{1+x^2}\}^n + C$, then

(a) $A = \frac{1}{15}, n = 15$ (b) $A = \frac{1}{14}, n = 14$

(c) $A = \frac{1}{16}, n = 16$ (d) none of these

23. $\int \frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}} =$

(a) $\frac{e^x}{\sqrt{1-x^2}} + c$ (b) $e^x\sqrt{1-x^2} + c$

(c) $\frac{e^x(2-x^2)}{\sqrt{1-x^2}} + c$ (d) $\frac{e^x(1+x)}{\sqrt{1-x^2}} + c$

24. If $\int \frac{ln(1+\sin^2 x)}{\cos^2 x} dx =$

$\frac{1}{\sqrt{2}} f(x)\, ln\, g(x) - 2x + \sqrt{2}\tan^{-1} f(x) + c$, then

(a) $f(x) = \tan x, g(x) = 1 + \sin^2 x$

(b) $f(x) = \sqrt{2}\tan x, g(x) = 1 + \sin^2 x$

(c) $f(x) = \sin x, g(x) = 1 + \tan^2 x$

(d) $f(x) = \sqrt{2}\cos x, g(x) = 1 + \sin^2 x$

MARK YOUR RESPONSE					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. If $\int \frac{(\sin^{3/2}\theta+\cos^{3/2}\theta)d\theta}{\sqrt{\sin^3\theta\cos^3\theta\sin(\theta+\alpha)}} =$

$a\sqrt{\cos\alpha\tan\theta+\sin\alpha}+b\sqrt{\cos\alpha+\sin\alpha\cot\theta}+c$, then

(a) $a = 2\sec\alpha, b = 2\operatorname{cosec}\alpha, c \in R$

(b) $a = 2\sec\alpha, b = -2\operatorname{cosec}\alpha, c \in R$

(c) $a = -2\sec\alpha, b = 2\operatorname{cosec}\alpha, c \in R$

(d) $a = 2\operatorname{cosec}\alpha, b = 2\sec\alpha, c \in R$

26. $\int \frac{x^2 dx}{(x\sin x+\cos x)^2} =$

(a) $\frac{\sin x - x\cos x}{x\sin x+\cos x}+c$ (b) $\frac{x\sin x-\cos x}{x\sin x+\cos x}+c$

(c) $\frac{\sin x + x\cos x}{x\sin x+\cos x}+c$ (d) none of these

27. If $\int \frac{\cos^2 x+\sin 2x}{(2\cos x-\sin x)^2}dx =$

$\frac{\cos x}{2\cos x-\sin x}+ax+b\ln|2\cos x-\sin x|+c$, then

(a) $a=\frac{1}{5}, b=\frac{2}{5}$ (b) $a=\frac{1}{5}, b=-\frac{2}{5}$

(c) $a=-\frac{1}{5}, b=\frac{2}{5}$ (d) $a=-\frac{1}{5}, b=-\frac{2}{5}$

28. Let $I_n = \int \frac{dx}{(x^2+a^2)^n}$, where $n \in N$ and $n > 1$. If I_n and I_{n-1} are related by the relation

$PI_n = \frac{x}{(x^2+a^2)^{n-1}}+QI_{n-1}$. Then P and Q are respectively given by

(a) $(2n-1)a^2, 2n-3$ (b) $2a^2(n-1), 2n-3$

(c) $a^2(n+1), 2n+3$ (d) $a^2, a^2(n+1)$

29. If $U_n = \int x^n\sqrt{a^2-x^2}dx$, then $(n+2)u_n-(n-1)a^2u_{n-2} =$

(a) $x^n\sqrt{a^2-x^2}$ (b) $-x^{n-1}\sqrt{a^2-x^2}$

(c) $-x^{n-1}(a^2-x^2)^{3/2}$ (d) none of these

30. If $\int f(x)\sin x\cos x dx = \frac{1}{2(b^2-a^2)}\ln f(x)+c$, then $f(x)$ is

(a) $\frac{1}{a\sin x+b\cos x}$ (b) $\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$

(c) $\frac{1}{a^2\sin x+b^2\cos x}$ (d) $\frac{1}{a\sin^2 x+b\cos^2 x}$

31. $\int \frac{(ax^2-b)dx}{x\sqrt{c^2x^2-(ax^2+b)^2}} =$

(a) $\sin^{-1}\left(\frac{ax+bx^2}{c}\right)+k$ (b) $\tan^{-1}\left(\frac{ax+bx^2}{cx}\right)+k$

(c) $\sin^{-1}\left(\frac{ax^2+b}{cx}\right)+k$ (d) $\tan^{-1}(ax^2+bx+c)+k$

32. If $f(x)=\tan^{-1}x+\ln\sqrt{1+x}-\ln\sqrt{1-x}$, then the integral of $\frac{1}{2}f'(x)$ with respect to x^4 is

(a) $e^{-x^4}+c$ (b) $-\ln(1-x^4)+c$

(c) $e^{\sqrt{1-x^2}}+c$ (d) $\ln(1+x^4)+c$

33. Let $f:\left[0,\frac{\pi}{2}\right]\longrightarrow R$ be such that $f(0) = 3$ and $f'(x)=\frac{1}{1+\cos x}$. If $a < f\left(\frac{\pi}{2}\right) < b$, then a and b can be

(a) $\frac{\pi}{2}, \pi$ (b) 3, 4

(c) $3+\frac{\pi}{4}, 3+\frac{\pi}{2}$ (d) $3+\frac{\pi}{2}, 3+\frac{3\pi}{4}$

MARK YOUR RESPONSE					
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ
	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	

34. $\int x^3 d(\tan^{-1} x)$ equals

(a) $\frac{x^2}{2}-\frac{1}{2}\ln(1+x^2)+C$

(b) $x^2+\ln(1+x^2)+C$

(c) $x^2\tan^{-1}x+\ln(1+x^2)+C$

(d) $x\tan^{-1}x-\frac{1}{2}\ln(1+x^2)+C$

35. If $0<x<\frac{\pi}{2}$ then $\int\sqrt{1+2\tan x(\tan x+\sec x)}\,dx$ equals to

(a) $\frac{1}{2}\ln\sec x+C$

(b) $\ln\tan x(\sec x+\tan x)+C$

(c) $\ln\sec x(\sec x+\tan x)+C$

(d) $\frac{1}{2}\{\tan x(\sec x+\tan x)\}^{-1/2}+C$

36. The integral $\int\left(\frac{\sec 6\alpha}{\text{cosec}2\alpha}+\frac{\sec 18\alpha}{\text{cosec}6\alpha}+\frac{\sec 54\alpha}{\text{cosec}18\alpha}\right)d\alpha$ is equal to

(a) $\frac{\ln|\sec 54\alpha|}{108}-\frac{\ln|\sec 2\alpha|}{4}+c$

(b) $\frac{\ln|\sec 6\alpha|}{6}+\frac{\ln|\sec 18\alpha|}{18}+\frac{\ln|\sec 54\alpha|}{54}+c$

(c) $\frac{1}{6}\ln\left|\frac{\sec 6\alpha}{\text{cosec}2\alpha}\right|+\frac{1}{18}\ln\left|\frac{\sec 18\alpha}{\text{cosec}6\alpha}\right|+\frac{1}{54}\ln\left|\frac{\sec 54\alpha}{\text{cosec}18\alpha}\right|+\text{c}$

(d) none of these

37. $\int\left(\frac{\ell n x-1}{(\ell n x)^2+1}\right)^2 dx$ is equal to

(a) $\frac{x}{x^2+1}+c$ (b) $\frac{\ell n x}{(\ell n x)^2+1}+c$

(c) $\frac{\ell n x}{(\ell n x)^2+1}+c$ (d) $e^x\left(\frac{x}{x^2+1}\right)+c$

38. $\int\left(\ell n(1+\cos x)-x\tan\frac{x}{2}\right)dx$ is equal to

(a) $x\,\ell n(1+\cos x)+\text{c}$ (b) $x\,\ell n(1+\sec x)+\text{c}$

(c) $x^2\,\ell n(1+\cos x)+\text{c}$ (d) $x\,\ell n\tan x+\text{c}$

39. The integral $\int\left(3x^2\tan\frac{1}{x}-x\sec^2\frac{1}{x}\right)dx$ equals to

(a) $x\left(\tan\frac{1}{x}+\sec\frac{1}{x}\right)+c$ (b) $x^2\tan\frac{1}{x}-\sec\frac{1}{x}+c$

(c) $x^3\tan\frac{1}{x}+c$ (d) $(x^3-1)\tan\frac{1}{x}+c$

40. The value of $\int e^{\tan\theta}(\sec\theta-\sin\theta)d\theta$ is

(a) $e^{\tan\theta}\sec\theta+c$

(b) $e^{\tan\theta}\sin\theta+c$

(c) $e^{\tan\theta}(\sec\theta+\sin\theta)+c$

(d) $e^{\tan\theta}\cos\theta+c$

41. If $\int\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cos^2 x\sqrt{\tan^3 x+\tan^2 x+\tan x}}\text{dx}=-2\tan^{-1}u+c$

then u is equal to

(a) $\sqrt{1+\tan x+\cot x}$ (b) $\sqrt{1+\tan x+\tan^2 x}$

(c) $\sqrt{\tan x+\cot x}$ (d) $\tan^{-1}(\tan x+\cot x)$

42. The value of integral $\int\frac{1+(\sin x)^{2/3}}{1+(\sin x)^{4/3}}.d(\sqrt[3]{\sin x})$ is equal to

(a) $\sin^{-1}(1+\sqrt[3]{\sin x})+c$

(b) $\sin^{-1}\sqrt[3]{\sin x}+c$

(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt[3]{\sin x}-1}{\sqrt{2}.\sqrt[3]{\sin x}}\right)+c$

(d) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{\sqrt[3]{\sin x}}{\sqrt{2}}+c$

MARK YOUR RESPONSE					
	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ
	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	

43. The integral $\int \frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$ is equal to

(a) $-\frac{x^2}{x\tan x+1}+c$

(b) $2\log_e|x\sin x+\cos x|+c$

(c) $-\frac{x^2}{x\tan x+1}+2\log_e|x\sin x+\cos x|+c$

(d) $\frac{x^2}{x^2\tan x-1}-2\log_e|x\sin x+\cos x|+c$

44. If $\int \frac{dx}{(a+bx^2)\sqrt{b-ax^2}}$ =K tan^{-1} $(L\tan\theta)+M$, M being constant of integration then KL is equal to

(a) 1 (b) $\frac{1}{a}$

(c) $\frac{1}{\sqrt[a]{a}}$ (d) $\sqrt{a}$

45. If $xf(x)=3f^2(x)+2$ then

$\int \frac{2x^2-12xf(x)+f(x)}{(6f(x)-x)(x^2-f(x))^2}dx$ equals

(a) $\frac{1}{x^2-f(x)}+c$ (b) $\frac{1}{x^2+f(x)}+c$

(c) $\frac{1}{x-f(x)}+c$ (d) $\frac{1}{x+f(x)}+c$

MARK YOUR RESPONSE	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ		

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

In general if we have an integral of type $\int f(g(x))g'(x)dx$, we substitute $g(x)=t \Rightarrow g'(x)dx=dt$ and the integral becomes $\int f(t)dt$.

Some of the substitution can be guessed by keen observation of the nature of given integrand. For example, we have $\frac{d}{dx}\left(x+\frac{1}{x}\right)=1-\frac{1}{x^2}$. So if the integrand is of the type $f\left(x+\frac{1}{x}\right).\left(1-\frac{1}{x^2}\right)$, we can substitute $x+\frac{1}{x}=t$

Some more similar forms are given below

for integral $\int f\left(x-\frac{a}{x}\right).\left(1+\frac{a}{x^2}\right)dx$, put $x-\frac{a}{x}=t$

for integral $\int f\left(x+\frac{a}{x}\right).\left(1-\frac{a}{x^2}\right)dx$, put $x+\frac{a}{x}=t$

for integral $\int f\left(x^2-\frac{a}{x^2}\right).\left(x+\frac{a}{x^3}\right)dx$, put $x^2-\frac{a}{x^2}=t$

for integral $\int f\left(x^2+\frac{a}{x^2}\right).\left(x-\frac{a}{x^3}\right)dx$, put $x^2+\frac{a}{x^2}=t$

Many integrands can be brought into above forms by suitable reductions or transformations

1. $\int \frac{x^2+1}{x^4+1}dx=$

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}+C$ (b) $\sin^{-1}\frac{x^2+1}{\sqrt{2}x}+C$

(c) $\frac{1}{2}\log\frac{\sqrt{2}x+1}{\sqrt{2}x-1}+c$ (d) $x^2+\frac{1}{x^2}+C$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ				

2. $\int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}dx =$

(a) $\tan^{-1}\left(x+\frac{1}{x}\right)+C$

(b) $\left(x+\frac{1}{x}\right)\tan^{-1}\left(x+\frac{1}{x}\right)+C$

(c) $\ln\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right|+C$

(d) $\frac{1}{2}\ln\left|x+\frac{1}{x}\right|+C$

3. $\int \frac{x^4-2}{x^2\sqrt{x^4+x^2+2}}dx =$

(a) $\sqrt{x^2+1+\frac{1}{x^2}}+C$ (b) $\sqrt{x^2+1+\frac{2}{x^2}}+C$

(c) $\sqrt{x^2+\frac{1}{x^2}}+C$ (d) $\sqrt{x^2+\frac{2}{x^2}}+C$

4. The derivative of $x^{-4}+x^{-5}$ is $-(4x^{-5}+5x^{-6})$. So, $\int \frac{5x^4+4x^5}{(x^5+x+1)^2}dx =$

(a) $x^5+x+1+C$ (b) $\frac{1}{x^5+x+1}+C$

(c) $x^{-4}+x^{-5}+C$ (d) $\frac{x^5}{x^5+x+1}+C$

PASSAGE-2

Integrals of the form $\int f(x,\sqrt{ax^2+bx+c})dx$ can be evaluated with the help of the Euler's substitutions. There are normally three Euler's substitutions :

I. **First Euler's substitution**

If $a>0$, we put $\sqrt{ax^2+bx+c}=t\pm x\sqrt{a}$

or $ax^2+bx+c=t^2+ax^2\pm 2tx\sqrt{a}$

or $bx+c=t^2\pm 2tx\sqrt{a}$

II. **Second Euler's Substitutions**

If $c>0$, we put $\sqrt{ax^2+bx+c}=tx\pm\sqrt{c}$

or $ax+b=t^2x\pm 2t\sqrt{c}$

III. **Third Euler's substitution**

If the trinomial ax^2+bx+c has real roots α and β that is $ax^2+bx+c=a(x-\alpha)(x-\beta)$ then we put $\sqrt{ax^2+bx+c}=(x-\alpha)t$ or $(x-\beta)t$

5. $\int \frac{\left(x+\sqrt{1+x^2}\right)^{15}}{\sqrt{1+x^2}}dx =$

(a) $\frac{(x+\sqrt{1+x^2})^{16}}{16}+C$ (b) $\frac{(x+\sqrt{1+x^2})^{15}}{15}+C$

(c) $\frac{1}{15(\sqrt{1+x^2}+x)^{15}}+C$ (d) $\frac{15}{(\sqrt{1+x^2}-x)^{15}}+C$

6. $\int \frac{dx}{(x-1)\sqrt{-x^2+3x-2}}$ is equal to

(A) $-2\sqrt{\frac{x-2}{1-x}}+c$ (B) $-2\sqrt{\frac{x-2}{x-1}}+c$

(C) $2\sqrt{\frac{1-x}{x-2}}+c$ (D) $\sqrt{\frac{1-x}{x-2}}+c$

7. If $f(x)$ is the antiderivative obtained in Q. No.6 then the limit $\lim_{x\to 2}\frac{\sin f(x)}{\sqrt{2-x}}$ $(x<2)$ is

(a) 0 (b) 1

(c) –2 (d) not finite

PASSAGE-3

In some of the cases we can split the integrand into the sum of the two functions such that the integration of one of them by parts produces an integral which cancels the other integral.

Suppose we have an integral of the type $\int [f(x)\ h(x)+g(x)]\ dx$

Let $\int f(x)\ h(x)\,dx = I_1$ and $\int g(x)\ dx = I_2$

Integrating I_1 by parts we get

$I_1 = f(x)\int h(x)\ dx - \int\{f'(x)\int h(x)\ dx\}dx$

Suppose $\int\{f'(x)\int h(x)\ dx\}$ converts to I_2, then we get

MARK YOUR RESPONSE	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ
	7. ⓐⓑⓒⓓ				

$I_1 + I_2 = f(x)\int h(x)dx + C$, which is the desired integral.

In particular consider the integral of the kind

$I = \int e^x \{f(x) + f'(x)\}\, dx = \int e^x f(x)\, dx + \int e^x\ f'(x)\, dx$

Integrating first integral by parts, we get (e^x is second function)

$I = e^x f(x) - \int e^x f'(x)\, dx + \int e^x f'(x)\, dx = e^x f(x) + C$

8. The integral of $f(x) = \frac{1}{\ln x} - \frac{1}{(\ln x)^2}$ is

(a) $\ln(\ln x) + C$ (b) $x \ln x + C$

(c) $\frac{x}{\ln x} + C$ (d) $x + \ln x + C$

9. $\int \frac{x + \sin x}{1 + \cos x} dx =$

(a) $\tan\frac{x}{2} + C$ (b) $x\tan\frac{x}{2} + C$

(c) $x + \cos x + C$ (d) $e^x \tan\frac{x}{2} + C$

10. $\int \frac{xe^x}{(1+x)^2} dx =$

(a) $xe^x + C$ (b) $\frac{e^x}{(x+1)^2} + C$

(c) $e^x - \frac{1}{x+1} + C$ (d) $\frac{e^x}{x+1} + C$

11. Antiderivative of $f(x) = \log(\log x) + \frac{1}{(\log x)^2}$ is

(a) $\log(\log x)$ (b) $x\log(\log x) - \frac{x}{\log x}$

(c) $\frac{x}{\log x} - \log x$ (d) $\log(\log x) - \frac{x}{\log x}$

MARK YOUR RESPONSE	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options:

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.

(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is true but Statement-2 is false.

(d) Statement-1 is false but Statement-2 is true.

1. **Statement - 1 :** If $I_n = \int \tan^n x\, dx$ then $5(I_4 + I_6) = \tan^5 x$

Statement - 2 : If $I_n = \int \tan^n x\, dx$ then

$$I_n = \frac{\tan^{n+1} x}{n+1} - I_{n-2} \text{ where } n \in N$$

2. **Statement - 1 :** If $\int \frac{1}{f(x)} dx = \log_e (f(x))^2 + c$ then $f(x) = \frac{1}{2}x$.

Statement - 2 : $\int \frac{f'(x)}{f(x)} dx = \log_e |f(x)| + c$

3. **Statement - 1 :** $\int \frac{3 + 4\cos x}{(4 + 3\cos x)^2} dx = \left(\frac{\sin x}{4 + 3\cos x}\right) + C$

Statement - 2 : $\int \{f(x)\}^n f'(x) dx = \frac{\{f(x)\}^{n+1}}{n+1} + C,\ n \neq -1$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. If $f(x) = \lim\limits_{n\to\infty} e^{x\tan(1/n)ln(1/n)}$ and

$\int \frac{f(x)}{\sqrt[3]{\sin^{11}x\cos x}}dx = g(x)+C$ (C being the constant of integration), then

(a) $g\left(\frac{\pi}{4}\right) = \frac{3}{2}$

(b) $g(x)$ is continuous for all x

(c) $g\left(\frac{\pi}{4}\right) = -\frac{15}{8}$

(d) $g(x)$ is not differentiable at infinitely many points

2. $\int e^x\left\{\frac{2\tan x}{1+\tan x}+\cot^2\left(x+\frac{\pi}{4}\right)\right\}dx$ is equal to

(a) $e^x\tan\left(\frac{\pi}{4}-x\right)+C$ (b) $e^x\cot\left(\frac{3\pi}{4}-x\right)+C$

(c) $e^x\tan\left(x-\frac{\pi}{4}\right)+C$ (d) $e^x\cot\left(x+\frac{\pi}{4}\right)+C$

3. If $I = \int_0^1 \frac{dx}{1+x^{\pi/2}}$ then

(a) $I > \ln 2$ (b) $I < \ln 2$

(c) $I < \frac{\pi}{4}$ (d) $I > \frac{\pi}{4}$

4. If $\forall\ x \in [-1, 0)$, $\int\left(\cos^{-1}x+\cos^{-1}\sqrt{1-x^2}\right)dx$

$= Ax + f(x)\sin^{-1}x - 2\sqrt{1-x^2}+C$, then

(a) $A = \frac{\pi}{4}$ (b) $A = \frac{\pi}{2}$

(c) $f(x) = x$ (d) $f(x) = -2x$

5. If $\int x^{-1/2}(2+3x^{1/3})^{-2}dx =$

$A\tan^{-1}\left\{\sqrt{\frac{3}{2}}x^{1/6}\right\}+B\frac{x^{1/6}}{2+3x^{1/3}}+C$ then

(a) $A = \frac{1}{\sqrt{6}}$ (b) $A = -\frac{1}{\sqrt{6}}$

(c) $B = 1$ (d) $B = -1$

6. If $\int\frac{x^4+1}{x^6+1}dx = \tan^{-1}f(x)-\frac{2}{3}\tan^{-1}g(x)+C$ then

(a) $f(x) = x+\frac{1}{x}$ (b) $f(x) = x-\frac{1}{x}$

(c) $g(x) = x^{-3}$ (d) $g(x) = x^3$

7. The value of the integral $\int e^{\sin^2 x}(\cos x+\cos^3 x)\sin x\,dx$ is

(a) $\frac{1}{2}e^{\sin^2 x}(3-\sin^2 x)+c$

(b) $e^{\sin^2 x}\left(1+\frac{1}{2}\cos^2 x\right)+c$

(c) $e^{\sin^2 x}(3\cos^2 x+2\sin^2 x)+c$

(d) $e^{\sin^2 x}(2\cos^2 x+3\sin^2 x)+c$

8. If $l = \int\frac{(x^2+n)(n-1)x^{2n-1}}{(x\sin x+n\cos x)^2}dx = f(x)+g(x)+c$ then

(a) $f(x) = \frac{x^n}{x^n\sin x+n\cos x}$

(b) $f(x) = -\frac{x^n\sec x}{x^n\sin x+nx^{n-1}\cos x}$

(c) $g(x) = \tan x$

(d) $g(x) = \sec x$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Observe the following Columns :

Column-I	Column-II
(A) $\int \frac{e^x}{x+2}[(1+(x+2)\log(x+2)]\,dx$	p. $\sin^{-1}\left(\frac{2x+3}{\sqrt{17}}\right)+c$
(B) $\int \sin^2 x\cos^3 x\,dx$	q. $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2}\log(x^2+1)+c$
(C) $\int \frac{dx}{\sqrt{2-3x-x^2}}$	r. $e^x\log(x+2)+c$
(D) $\int \frac{x^5}{x^2+1}\,dx$	s. $\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+c$

2. Observe the following Columns :

Column-I	Column-II
(A) $\int \frac{dx}{\sqrt{x}(x+9)}=$	p. $\log\left\|1-\tan\left(\frac{x}{2}\right)\right\|+c$
(B) $\int e^x(1-\cot x+\cot^2 x)\,dx=$	q. $\log\left\|1-\cot\left(\frac{x}{2}\right)\right\|+c$
(C) $\int \frac{\sin^3 x+\cos^3 x}{\cos^2 x\sin^2 x}\,dx=$	r. $\sec x-\operatorname{cosec} x+c$
(D) $\int \frac{dx}{1-\cos x-\sin x}=$	s. $\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)+c$
	t. $-e^x\cdot\cot x+c$

3. Observe the following Columns :

Column-I	Column-II
(A) $\int (e^{a\log x}+e^{x\log a})\,dx$	p. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)+c$
(B) $\int \frac{e^{\log\left(1+\frac{1}{x^2}\right)}dx}{x^2+\frac{1}{x^2}}$	q. $\frac{1}{4}\tan^{-1}\left(\tan x+\frac{1}{2}\right)+c$
(C) $\int \frac{dx}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}$	r. $\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+c$
(D) $\int (\sqrt{\sin x}+\sqrt{\cos x})^{-4}\,dx$	s. $-\frac{1}{(1+\sqrt{\tan x})^2}+\frac{2}{3(1+\sqrt{\tan x})^3}+c$

MARK YOUR RESPONSE

1.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

2.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

3.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

4. If $\int \frac{\log_e(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}$ dx = fog(x) + C,Now match the entries from the following columns:

Column-I		**Column-II**
(A) $f(2)$ is equal to	p.	0
(B) g (0) is equal to	q.	1
(C) If $\int f(x)g(x)dx = ax^3 g(x) + b(1+x^2)^{3/2} + c(1+x^2)^{1/2} + d$, then a + c is equal to	r.	2
(D) If $\int e^{g(x)}dx = ax\left(x+\sqrt{1+x^2}+ag(x)+c\right)$ then a is equal to	s.	$\frac{1}{2}$
	t.	$\frac{1}{3}$

5. **Observe the following Column :**

Column-I		**Column-II**
(A) If $\int x^2 d(\tan^{-1}x) = x + f(x) + c$,then f(1)is equal to	p.	0
(B) If $\int \sqrt{1+2\tan x(\tan x+\sec x)}dx = a\log\left\|\cos\frac{x}{2}-\sin\frac{x}{2}\right\|+c$ then a is equal to $(0 < x < \frac{\pi}{2})$	q.	–2
(C) If $\int x^2 e^{2x}dx = e^{2x} f(x) + c$, then the minimum value of $f(x)$ is equal to	r.	$\frac{\pi}{4}$
(D) If $\int \frac{x^4+1}{x(x^2+1)^2}dx = a\ \log\|x\| + \frac{b}{x^2+1} + c$ then a – b is equal to	s.	1
	t.	$\frac{1}{8}$

6. **Observe the following columns :**

Column I		**Column II**
(A) If $\int \frac{x+(\cos^{-1}3x)^2}{\sqrt{1-9x^2}}dx$ $= A\sqrt{1-9x^2} + B(\cos^{-1}3x)^3 + C$, then	p.	$A = -\frac{1}{2}$
(B) If $\int \frac{(2x+1)dx}{x^4+2x^3+x^2-1}dx = A\ell n\left\|\frac{x^2+x+1}{x^2+x-1}\right\| + C$, then	q.	$A = -\frac{1}{9}$
(C) If $\int x^5(x^{10}+x^5+1)(2x^{10}+3x^5+6)^{1/5}dx$ $= -\frac{A}{4}(2x^{15}+3x^{10}+6x^5)^{6/5} + C$, then	r.	$B = -\frac{1}{9}$
(D) If $\int \frac{x^3-x}{\sqrt{1-x^2}}\frac{1}{(1+\sqrt{1-x^2})}dx$	s.	B = 1
$= -f(x) + Ax^2 + B\ln f(x) + C$, then	t.	A = B

MARK YOUR RESPONSE

4.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

5.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

6.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

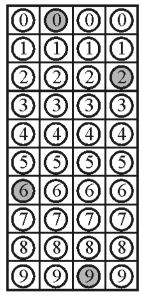
F — NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.



1. If $\int \dfrac{\sin^3 \frac{\theta}{2}\, d\theta}{\cos\frac{\theta}{2}\sqrt{\cos^3\theta + \cos^2\theta + \cos\theta}} = \tan^{-1}\sqrt{f(\theta)} + c$

then the least value of $f(\theta)$ for allowable values of θ is equal to

2. If $P = \int e^{ax}\cos bx\,dx$ and $Q = \int e^{ax}\sin bx\,dx$, without constant of integration then $e^{-2ax}(P^2+Q^2)(a^2+b^2)$ is equal to

3. If $\int \sin 4x\, e^{\tan^2 x} dx = -A\cos^4 x e^{\tan^2 x} + B$, then A is equal to

4. If $\int \operatorname{cosec}^2 x \, ln\left(\cos x + \sqrt{\cos 2x}\right) dx$

$= f(x)\, ln\left(\cos x + \sqrt{\cos 2x}\right) + g(x) + f(x) - x + c,$

then $f^2(x) - g^2(x)$ is equal to $\left(0 < x \le \dfrac{\pi}{2}\right)$.

5. If $\int \dfrac{\sqrt{\cot x} - \sqrt{\tan x}}{4 + 3\sin 2x} dx = \dfrac{1}{2\sqrt{2}} \ell n \left|\dfrac{f(x)+a}{f(x)-a}\right|$, where $0 < x < \dfrac{\pi}{2}$ then a is equal to

MARK YOUR RESPONSE

1. 2. 3. 4. 5.

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(c)	**11**	(b)	**21**	(a)	**31**	(c)	**41**	(a)
2	(a)	**12**	(b)	**22**	(a)	**32**	(b)	**42**	(c)
3	(b)	**13**	(a)	**23**	(d)	**33**	(c)	**43**	(c)
4	(c)	**14**	(d)	**24**	(b)	**34**	(a)	**44**	(c)
5	(d)	**15**	(a)	**25**	(b)	**35**	(c)	**45**	(a)
6	(b)	**16**	(a)	**26**	(a)	**36**	(a)		
7	(d)	**17**	(c)	**27**	(d)	**37**	(c)		
8	(d)	**18**	(a)	**28**	(b)	**38**	(a)		
9	(d)	**19**	(a)	**29**	(c)	**39**	(c)		
10	(c)	**20**	(b)	**30**	(b)	**40**	(d)		

B COMPREHENSION TYPE

1	(a)	**3**	(b)	**5**	(b)	**7**	(c)	**9**	(b)	**11**	(b)
2	(c)	**4**	(d)	**6**	(a)	**8**	(c)	**10**	(d)		

C REASONING TYPE

1	(c)	**2**	(b)	**3**	(a)

D MULTIPLE CORRECT CHOICE TYPE

1	(c, d)	**3**	(a,c)	**5**	(a,d)	**7**	(a, b)
2	(b,c)	**4**	(b,d)	**6**	(b,d)	**8**	(b,c)

E MATRIX-MATCH TYPE

1. **A-r; B-s; C-p; D-q**
2. **A-s; B-t; C-r; D-q**
3. **A-r; B-p; C-q ; D-s**
4. **A-r; B-p; C-t; D-s**
5. **A-r, B-q, C-t, D-p**
6. **A - p, q, t ; B - p ; C - q ; D - p, s**

F NUMERIC/INTEGER ANSWER TYPE

1	3	**2**	1	**3**	2	**4**	1	**5**	2

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(c)** $I=\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx$

$=\int \frac{x^2-1}{(x+1)^2\sqrt{x^3+x^2+x}}dx$

$=\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}+2\right)\sqrt{x+\frac{1}{x}+1}}dx$

Put $x+\frac{1}{x}+1=t^2 \Rightarrow \left(1-\frac{1}{x^2}\right)dx=2tdt$

$\therefore\ I=\int \frac{2tdt}{(t^2+1)t}=2\int \frac{dt}{t^2+1}=2\tan^{-1}t+C$

$=2\tan^{-1}\sqrt{x+\frac{1}{x}+1}+C$

2. **(a)** $I=\int e^{\sin x}.x\cos x\ dx-\int e^{\sin x}\sec x\tan x\ dx$

$=\left\{xe^{\sin x}-\int e^{\sin x}\ dx\right\}-\left\{e^{\sin x}.\sec x-\int e^{\sin x}\ dx\right\}$

$=xe^{\sin x}-e^{\sin x}\sec x+C$

3. **(b)** $I=\int \frac{\sin^3 xdx}{(\cos^4 x+3\cos^2 x+1)\tan^{-1}(\sec x+\cos x)}$

Let $\tan^{-1}(\sec x+\cos x)=t$

$\Rightarrow\ \frac{1}{1+(\sec x+\cos x)^2}(\sec x\tan x-\sin x)dx=dt$

or $\frac{\sin^3 x\,dx}{\cos^4 x+3\cos x^2+1}=dt$

$\therefore\ \mathrm{I}=\int \frac{\mathrm{dt}}{\mathrm{t}}=l\mathrm{n}\,|\,\mathrm{t}\,|+\mathrm{c}$

$=l\mathrm{n}\,|\tan^{-1}(\sec \mathrm{x}+\cos \mathrm{x})|+\mathrm{c}$

4. **(c)** We have $\int e^x\frac{1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}dx$

$=\int e^x\left[\sqrt{\frac{1+x^n}{1-x^n}}+\frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}\right]dx$

$=e^x\sqrt{\frac{1+x^n}{1-x^n}}+C$

Here, $f(x)=\sqrt{\frac{1+x^n}{1-x^n}}$, then

$f'(x)=\frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}$

and $\int e^x(f(x)+f'(x))dx=e^x f(x)+C$

5. **(d)** $\lim_{n\to\infty} n^2\left(x^{1/n}-x^{1/(n+1)}\right)$

$=\lim_{h\to 0}\frac{1}{h^2}\left(x^h-x^{\frac{h}{h+1}}\right)$

$=\lim_{h\to 0}\frac{1}{h^2}\left(x^{\left(h-\frac{h}{h+1}\right)}-1\right)x^{\frac{h}{h+1}}$

$=\lim_{h\to 0}\left(\frac{x^{\frac{h^2}{h+1}}-1}{\frac{h^2}{h+1}}\right).\left(\frac{1}{h+1}\right)x^{\frac{h}{h+1}}$

$=\ln x.1.1\qquad\left(\because \lim_{x\to 0}\frac{a^x-1}{x}=\ln a\right)$

$\therefore\ f(x)=\ln x$

So, $I=\int xf(x)dx=\int x\ln xdx$

$=\frac{x^2}{2}\ln x-\int\frac{x^2}{2}.\frac{1}{x}dx$

$=\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$

6. **(b)** $I=\int\frac{1+x^4}{(1-x^4)^{3/2}}dx$

$=\int\frac{x^3(x+1/x^3)dx}{(1-x^4)^{3/2}}=\int\frac{(x+1/x^3)dx}{\left(\frac{1}{x^2}-x^2\right)^{\frac{3}{2}}}$

Let $\frac{1}{x^2}-x^2=t \Rightarrow \left(\frac{-2}{x^3}-2x\right)dx=dt$

$\Rightarrow \left(x+\frac{1}{x^3}\right)dx=-\frac{1}{2}dt$

$\therefore I=-\frac{1}{2}\int\frac{dt}{t^{3/2}}=\frac{1}{\sqrt{t}}+c=\frac{1}{\sqrt{\frac{1}{x^2}-x^2}}+C$

7. (d) $I=\int\left(\frac{f(x)}{x^2}\right)^{\frac{1}{2}}dx=\int\left(\frac{x+2}{2x+3}\right)^{\frac{1}{2}}\frac{dx}{x}$

Put $\frac{x+2}{2x+3}=y^2 \Rightarrow x=\frac{3y^2-2}{1-2y^2}$

and $dx=\frac{-2ydy}{(1-2y^2)^2}$

$\therefore I=-\int y.\frac{2y}{(1-2y^2)^2}.\frac{1-2y^2}{3y^2-2}dy$

$=2\int\frac{y^2dy}{(2y^2-1)(3y^2-2)}$

$=2\int\left(\frac{2}{3y^2-2}-\frac{1}{2y^2-1}\right)dy$

$=\frac{4}{3}\int\frac{dy}{y^2-\frac{2}{3}}-\int\frac{dy}{y^2-\frac{1}{2}}$

$=\frac{4}{3}.\frac{1}{2\sqrt{\frac{2}{3}}}ln\left|\frac{y-\sqrt{\frac{2}{3}}}{y+\sqrt{\frac{2}{3}}}\right|-\frac{1}{2\times\frac{1}{\sqrt{2}}}ln\left|\frac{y-\frac{1}{\sqrt{2}}}{y+\frac{1}{\sqrt{2}}}\right|+\text{C}$

$=\frac{1}{\sqrt{2}}\log\left|\frac{1+\sqrt{2}y}{1-\sqrt{2}y}\right|-\sqrt{\frac{2}{3}}\log\left|\frac{\sqrt{3}y+\sqrt{2}}{\sqrt{3}y-\sqrt{2}}\right|+C$

$=\frac{1}{\sqrt{2}}\log\left|\frac{1+\sqrt{2f(x)}}{1-\sqrt{2f(x)}}\right|-\sqrt{\frac{2}{3}}\log\left|\frac{\sqrt{3f(x)}+\sqrt{2}}{\sqrt{3f(x)}-\sqrt{2}}\right|+C$

Thus $g(x)=\log|x|$ and $h(x)=\log|x|$

8. (d) $I=\int\sqrt{1+\text{cosec}\,x}dx=\int\frac{\sqrt{1+\sin x}}{\sqrt{\sin x}}$

$=\int\pm\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sqrt{2\sin\frac{x}{2}\cos\frac{x}{2}}}=\pm\int\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sqrt{1-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}dx$

Put $\sin\frac{x}{2}-\cos\frac{x}{2}=t \Rightarrow \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)dx=2dt$

$I=\pm\int\frac{2dt}{\sqrt{1-t^2}}=\pm2\sin^{-1}t=\pm2\sin^{-1}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)+c$

9. (d) Let $g_n(x)=1+x^2+x^4+....+x^{2n}=\frac{x^{2n+2}-1}{x^2-1}$

So, $2x+4x^3+.....+2nx^{2n-1}=$

$h_n(x)=g'_n(x)=\frac{2x(nx^{2n+2}-(n+1)x^{2n}+1)}{(x^2-1)^2}$

Now $f(x)=\lim_{n\to\infty}h_n(x)=\frac{2x}{(x^2-1)^2}$ as $0<x<1$

Thus $\int f(x)dx=\int\frac{2x}{(x^2-1)^2}dx$

$=-\frac{1}{x^2-1}=\frac{1}{1-x^2}+c$

10. (c) Here $-\frac{1}{2}-\frac{7}{2}=-4=$ negative even number

Put $\tan x=t \Rightarrow \sec^2x\,dx=dt$

$f(x)=\int\frac{dx}{\frac{\sin^{1/2}}{\cos^{1/2}}.\cos^4x}=\int\frac{(1+\tan^2x)\sec^2xdx}{\sqrt{\tan x}}$

$=\int\left(\frac{1+t^2}{\sqrt{t}}\right)dt=\int(t^{-1/2}+t^{3/2})dt$

$=2t^{1/2}+\frac{2}{5}t^{5/2}=2\sqrt{\tan x}+\frac{2}{5}(\tan x)^{5/2}$

$\therefore f\left(\frac{\pi}{4}\right)-f(0)=2+\frac{2}{5}=\frac{12}{5}$

11. **(b)** $f\left(\frac{3x-4}{3x+4}\right)=x+2$. Let, $\frac{3x-4}{3x+4}=t$

$3x-4=3xt+4t$

$x=\frac{4t+4}{3(1-t)}$ $\quad\therefore\ f(t)=\frac{4t+4}{3(1-t)}+2$

$\therefore\ f(x)=\frac{4x+4}{3(1-x)}+2=\frac{4(x-1)+8}{3(1-x)}+2$

$=2-\frac{4}{3}-\frac{8}{3(x-1)}$

$\int f(x)dx=\frac{2}{3}x-\frac{8}{3}ln|x-1|+c$

12. **(b)** The given integral is $I=\int\frac{(\sec\theta-1)\sqrt{\sec\theta}}{1+\sec^2\theta}\tan\theta d\theta$

$=\int\frac{(\sec\theta-1)\sec\theta\tan\theta}{(1+\sec^2\theta)\sqrt{\sec\theta}}d\theta$

Put $\sqrt{\sec\theta}=t\ \Rightarrow\ \frac{1}{2\sqrt{\sec\theta}}\sec\theta\tan\theta d\theta=dt$

$\therefore\ I=\int\frac{(t^2-1)}{1+t^4}.2dt$

$=2\int\frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt=2\int\frac{\left(1-\frac{1}{t^2}\right)dt}{\left(1+\frac{1}{t}\right)^2-2}$

Put $t+\frac{1}{t}=u\ \Rightarrow\ \left(1-\frac{1}{t^2}\right)dt=du$

$\therefore\ I=2\int\frac{du}{u^2-2}=\frac{2}{2\sqrt{2}}\log_e\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right|+C$

$=\frac{1}{\sqrt{2}}\log_e\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|+C$

$=\frac{1}{\sqrt{2}}\log_e\left|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right|+C$

$=\frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta-\sqrt{2\sec\theta}+1}{\sec\theta+\sqrt{2\sec\theta}+1}\right|+C$

13. **(a)** $f(x)=\lim\limits_{n\to\infty}\frac{x^{2n}-1}{x^{2n}+1}=-1\quad(\because 0<x<1)$, so

$\int\sin^{-1}x(f(x))dx=-\int\sin^{-1}x\,dx$

$=-[\mathrm{x}\sin^{-1}\mathrm{x}+\sqrt{1-\mathrm{x}^2}]+\mathrm{C}$

14. **(d)** Since, $x>1$ so $x^{-n}\longrightarrow 0$ as $n\longrightarrow\infty$.

Hence $f(x)=\lim\limits_{n\to\infty}\frac{1-x^{-2n}}{1+x^{-2n}}=1$

So, $\int\frac{xf(x)\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx$

$=\frac{1}{2}\int\frac{2x}{\sqrt{1+x^2}}\log(x+\sqrt{1+x^2})dx$

$=\sqrt{1+x^2}\log(x+\sqrt{1+x^2})$

$-\int\frac{\sqrt{1+x^2}}{x+\sqrt{1+x^2}}\times\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}$

$=\sqrt{1+x^2}\log(x+\sqrt{1+x^2})-x+C$

15. **(a)** $I_n=\int\cot^n x\,dx=\int\cot^{n-2}x\cot^2x\,dx$

$=\int\cot^{n-2}x(\mathrm{cosec}^2x-1)dx$

$=\int\cot^{n-2}x\mathrm{cosec}^2x\,dx-I_{n-2}$

Thus $I_n+I_{n-2}=\frac{\cot^{n-1}x}{n-1}$ ......(i)

$I_0+I_1+2(I_2+......+I_8)+I_9+I_{10}$

$=(I_2+I_0)+(I_3+I_1)+(I_4+I_2)+(I_5+I_3)$

$+(I_6+I_4)+(I_7+I_5).+(I_8+I_6)+(I_9+I_7)$

$+(I_{10}+I_8)$

$=-\left(\frac{\cot x}{1}+\frac{\cot^2x}{2}+.......+\frac{\cot^9x}{9}\right)$ [using (i)]

$=-\sum\limits_{k=1}^{9}\frac{\cot^k x}{k}$

16. **(a)** Let $I=\int\frac{(1-\cos\theta)^{2/7}}{(1+\cos\theta)^{9/7}}d\theta$

$=\int\frac{(2\sin^2\theta/2)^{2/7}}{(2\cos^2\theta/2)^{9/7}}d\theta$

$=\frac{1}{2}\int\frac{(\sin\theta/2)^{4/7}}{(\cos\theta/2)^{18/7}}d\theta$

Put $\frac{\theta}{2}=t$, $\therefore$ $\frac{d\theta}{2}=dt$

$$\therefore\ I=\int\frac{(\sin t)^{4/7}}{(\cos t)^{18/7}}dt \qquad \text{(Here } m+n=-2)$$

$$=\int(\tan t)^{4/7}\sec^2 t\,dt$$

Put $\tan t=u$ $\qquad\therefore\ \sec^2 t\,dt=du$

$$\therefore I=\int u^{4/7}du=\frac{u^{11/7}}{11/7}+c$$

$$=\frac{7}{11}(\tan t)^{11/7}+c=\frac{7}{11}\left(\tan\frac{\theta}{2}\right)^{11/7}+c$$

17. (c) $I_{4,3}=\int\cos^4 x\sin 3x\,dx$

Integrating by parts, we have

$$I_{4,3}=-\frac{\cos 3x\cos^4 x}{3}-\frac{4}{3}\int\cos^3 x\sin x\cos 3x\ dx$$

Now,

$\sin 2x=\sin(3x-x)=\sin 3x\cos x-\cos 3x\sin x$, so,

$$I_{4,3}=-\frac{\cos 3x\cos^4 x}{3}+\frac{4}{3}\int\cos^3 x\sin 2x$$

$$-\frac{4}{3}\int\cos^4 x\sin 3x\,dx+C$$

$$=-\frac{\cos 3x\cos^4 x}{3}+\frac{4}{3}I_{3,2}-\frac{4}{3}I_{4,3}+C$$

Therefore, $\frac{7}{3}I_{4,3}-\frac{4}{3}I_{3,2}=-\frac{\cos 3x\cos^4 x}{3}+C$

or $7I_{4,3}-4I_{3,2}=-\cos 3x\cos^4 x+$ const.

18. (a) Let $I=\int(x^{\sin x-1}.\sin x+x^{\sin x}.\cos x\ln x)dx$

Put $x^{\sin x}=t$

$\Rightarrow e^{\sin x\ln x}=t$

$\Rightarrow e^{\sin x\ln x}\left\{\sin x.\frac{1}{x}+\ln x.\cos x\right\}dx=dt$

$\Rightarrow x^{\sin x}\left\{\frac{\sin x}{x}+\cos x\ln x\right\}dx=dt$

$\Rightarrow (x^{\sin x-1}.\sin x+x^{\sin x}.\cos x\ln x)dx=dt$

$\therefore\ \mathrm{I}=\int\mathrm{dt}=\mathrm{t}+\mathrm{c}=\mathrm{x}^{\sin x}+\mathrm{c}$

19. (a) $f(xy)=f(x)f(y)$, Put $x=y=1$, we get $f(1)=f^2(1)$

$\Rightarrow f(1)=1$

Differentiation w.r.t x (partially),

we get $yf'(xy)=f'(x).f(y)$

Putting $x=1$, $yf'(y)=f'(1).f(y)$

$$\Rightarrow\frac{f(y)}{f'(y)}=\frac{y}{f'(1)}$$

$$\therefore\ \int\frac{f(x)}{f'(x)}dx=\int\frac{x}{f'(1)}dx=\frac{1}{f'(1)}\left(\frac{x^2}{2}+c\right)$$

$$\because\ f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{1+h+hg(h)-1}{h}=1$$

$$\therefore\ \int\frac{f(x)}{f'(x)}dx=\frac{x^2}{2}+c$$

20. (b) We have $\int\frac{dx}{x^2(x^n+1)^{(n-1)/n}}$

$$=\int\frac{dx}{x^2.x^{n-1}\left(1+\frac{1}{x^n}\right)^{(n-1)/n}}=\int\frac{dx}{x^{n+1}(1+x^{-n})^{(n-1)/n}}$$

Put $1+x^{-n}=t$

$\therefore\ -nx^{-n-1}dx=dt$ or $\frac{dx}{x^{n+1}}=-\frac{dt}{n}$ then

$$\int\frac{dx}{x^2(x^n+1)^{(n-1)/n}}=-\frac{1}{n}\int\frac{dt}{t^{(n-1)/n}}=-\frac{1}{n}\int t^{\frac{1}{n}-1}dt$$

$$=-\frac{1}{n}\frac{t^{1/n-1+1}}{1/n-1+1}+c=-t^{1/.n}+c=-(1+x^{-n})^{1/n}+c$$

21. (a) Let $I=\int\frac{dx}{(x-1)\sqrt{(-x^2+3x-2)}}$

The trinomial $-x^2+3x-2=-(x-2)(x-1)$

Put $\sqrt{(-x^2+3x-2)}=(x-2)t$

$\therefore\ t=\sqrt{\frac{-(x-1)}{(x-2)}}$ $\qquad(\because 1<x<2)$

We get $x=\left(\frac{2t^2+1}{t^2+1}\right)$ $\qquad\therefore\ dx=\frac{2tdt}{(t^2+1)^2}$

Also, $\sqrt{(-x^2+3x-2)}=\frac{t}{(t^2+1)}$

$$\therefore\ I=\int\frac{\frac{2tdt}{(t^2+1)^2}}{\left(\frac{t^2}{t^2+1}\right).\frac{t}{(t^2+1)}}=\int\frac{2}{t^2}dt=-\frac{2}{t}+c$$

$$=-2\sqrt{\left\{-\left(\frac{x-2}{x-1}\right)\right\}}+c$$

22. **(a)** Let $I=\int \frac{(x+\sqrt{1+x^2})^{15}}{\sqrt{1+x^2}}dx$

Put $\sqrt{(1+x^2)}=t-x$ (Euler's substitution)

$\Rightarrow x+\sqrt{(1+x^2)}=t \quad \therefore \left(1+\frac{x}{\sqrt{(1+x^2)}}\right)dx=dt$

$\Rightarrow \frac{tdx}{\sqrt{(1+x^2)}}=dt$ or $\frac{dx}{\sqrt{(1+x^2)}}=\frac{dt}{t}$

then $I=\int \frac{t^{15}dt}{t}$

$=\int t^{14}dt=\frac{t^{15}}{15}+c=\frac{(x+\sqrt{(1+x^2)})^{15}}{15}+c$

23. **(d)** Let $I=\int \frac{e^x(2-x^2)}{(1-x)\sqrt{(1-x^2)}}dx$

$I=\int \frac{e^x(1+1-x^2)}{(1-x)\sqrt{(1-x^2)}}dx$

$=\int e^x\left\{\frac{1}{(1-x)\sqrt{(1-x^2)}}+\sqrt{\left(\frac{1+x}{1-x}\right)}\right\}dx$

Let $f(x)=\sqrt{\left(\frac{1+x}{1-x}\right)}$; $f'(x)=\frac{1}{(x-1)\sqrt{(1-x^2)}}$

$\because \int e^x(f(x)+f'(x))dx=e^xf(x)+c$

$\therefore I=e^x\sqrt{\left(\frac{1+x}{1-x}\right)}+c=\frac{e^x(1+x)}{\sqrt{1-x^2}}+C$

24. **(b)** Let $I=\int \frac{\ln(1+\sin^2 x)}{\cos^2 x}dx$

$=\int \ln(1+\sin^2 x)\sec^2 x\,dx$

Intergrating by parts taking $\sec^2 x$ as the second fucntion, we have

$=ln(1+\sin^2 x).\tan x-2\int \frac{(\sin^2 x+1)-1}{(1+\sin^2 x)}dx$

$=\tan x\,ln(1+\sin^2 x)-2x+2\int \frac{dx}{(1+\sin^2 x)}$

$=\tan x\,ln(1+\sin^2 x)-2x+2\int \frac{\sec^2 x\,dx}{1+2\tan^2 x}$

Put $\sqrt{2}\tan x=t \qquad \therefore \sec^2 xdx=\frac{dt}{\sqrt{2}}$

$\therefore I=\tan x\,ln(1+\sin^2 x)-2x+\frac{2}{\sqrt{2}}\int \frac{dt}{1+t^2}$

$=\tan x\,ln(1+\sin^2 x)-2x+\sqrt{2}\tan^{-1}t+c$

$=\tan x\,ln(1+\sin^2 x)-2x+\sqrt{2}\tan^{-1}(\sqrt{2}\tan x)+c$

25. **(b)** Let $I=\frac{(\sin^{3/2}\theta+\cos^{3/2}\theta)d\theta}{\sqrt{\sin^3\theta\cos^3\theta\sin(\theta+\alpha)}}$

$=\int \frac{\sin^{3/2}\theta d\theta}{\sqrt{\sin^3\theta\cos^3\theta\sin(\theta+\alpha)}}+\int \frac{\cos^{3/2}\theta d\theta}{\sqrt{\sin^3\theta\cos^3\theta\sin(\theta+\alpha)}}$

$=\int \frac{d\theta}{\sqrt{\cos^3\theta(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}}$

$+\int \frac{d\theta}{\sqrt{\sin^3\theta(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}}$

$=\int \frac{\sec^2\theta d\theta}{\sqrt{(\cos\alpha\tan\theta+\sin\alpha)}}+\int \frac{\operatorname{cosec}^2\theta d\theta}{\sqrt{(\cos\alpha+\cot\theta\sin\alpha)}}$

Put $\cos\alpha\tan\theta+\sin\alpha=t^2$, in the first integral and $\cos\alpha+\cot\theta\sin\alpha=u^2$ in second integral

$\Rightarrow \sec^2 d\theta=\frac{2tdt}{\cos\alpha}$ and $\operatorname{cosec}^2\theta d\theta=-\frac{2u\,du}{\sin\alpha}$

$\therefore I=\int \frac{2tdt}{(\cos\alpha)t}-\int \frac{2udu}{\sin\alpha.u}=\frac{2}{\cos\alpha}t-\frac{2}{\sin\alpha}u+c$

$=\frac{2}{\cos\alpha}\sqrt{(\cos\alpha\tan\theta+\sin\alpha)}$

$-\frac{2}{\sin\alpha}\sqrt{(\cos\alpha+\cot\theta\sin\alpha)}+c$

26. **(a)** Let, $I=\int \frac{x^2dx}{(x\sin x+\cos x)^2}$

$\because \frac{d}{dx}(x\sin x+\cos x)=x\cos x$, we write

$I=\int\left(\frac{x}{\cos x}\right).\left(\frac{x\cos x}{(x\sin x+\cos x)^2}\right)dx$

Integrating by part taking $\frac{x\cos x}{(x\sin x+\cos x)^2}$ as second function, we get

$$I=\left(\frac{x}{\cos x}\right)\left(-\frac{1}{x\sin x+\cos x}\right)$$

$$+\int\frac{(x\sin x+\cos x)}{\cos^2 x}\cdot\frac{1}{(x\sin x+\cos x)}dx$$

$$=-\frac{x}{\cos x(x\sin x+\cos x)}+\int\sec^2 xdx$$

$$=-\frac{x}{\cos x(x\sin x+\cos x)}+\tan x+c$$

$$=-\frac{x}{\cos x(x\sin x+\cos x)}+\frac{\sin x}{\cos x}+c$$

$$=\frac{-x+\sin x(x\sin x+\cos x)}{\cos x(x\sin x+\cos x)}+c$$

$$=\frac{(\sin x-x\cos x)}{(x\sin x+\cos x)}+c$$

27. **(d)** Let $I=\int\frac{(\cos^2 x+\sin 2x)}{(2\cos x-\sin x)^2}dx$

$$=\int\frac{(\cos x+2\sin x)\cos x}{(2\cos x-\sin x)^2}dx$$

Integrating by part , taking cos x as the first and $\frac{(\cos x+2\sin x)}{(2\cos x-\sin x)^2}$ as the second function, we have

$$=\cos x\left\{\frac{1}{2\cos x-\sin x}\right\}-\int\frac{-\sin xdx}{(2\cos x-\sin x)}$$

$$=\cos x\left\{\frac{1}{2\cos x-\sin x}\right\}+\int\frac{-\sin x\ dx}{(2\cos x-\sin x)}$$

$$=\frac{\cos x}{(2\cos x-\sin x)}$$

$$+\int\frac{-\frac{1}{5}(2\cos x-\sin x)-\frac{2}{5}(-2\sin x-\cos x)}{(2\cos x-\sin x)}dx$$

$$\left[N^{\gamma}=\lambda Dr+\mu\frac{d}{dx}Dr\right]$$

$$=\frac{\cos x}{(2\cos x-\sin x)}-\frac{1}{5}\int dx-\frac{2}{5}\int\frac{(-2\sin x-\cos x)}{2\cos x-\sin x}dx$$

$$=\frac{\cos x}{(2\cos x-\sin x)}-\frac{1}{5}x-\frac{2}{5}ln|2\cos x-\sin x|+c$$

28. **(b)** $I_n=\int\frac{dx}{(x^2+a^2)^n}$

$$=\frac{1}{(x^2+a^2)^n}.x-\int\frac{(-n)2x}{(x^2+a^2)^{n+1}}.xdx$$

[Integrating by parts using 1 as second function]

$$\therefore\ I_n=\frac{x}{(x^2+a^2)^n}+2n\int\frac{x^2+a^2-a^2}{(x^2+a^2)^{n+1}}dx$$

$$=\frac{x}{(x^2+a^2)^n}+2n(I_n-a^2I_{n+1})$$

$$\Rightarrow\ 2na^2I_{n+1}=\frac{x}{(x^2+a^2)}+(2n-1)l_n$$

Replace n by $n-1$, then

$$2(n-1)a^2I_n=\frac{x}{(x^2+a^2)^{n-1}}+(2n-3)l_{n-1}$$

29. **(c)** $\because\ u_n=\int x^n\sqrt{(a^2-x^2)}dx=\int x^{n-1}\{x\sqrt{(a^2-x^2)}\}dx$

Integrating by parts taking x^{n-1} as first function, we have

$$=x^{n-1}\left\{\frac{-(a^2-x^2)^{3/2}}{3}\right\}$$

$$+\int(n-1)\frac{x^{n-2}(a^2-x^2)^{3/2}}{3}dx$$

$$\Rightarrow u_n=-\frac{x^{n-1}(a^2-x^2)^{3/2}}{3}$$

$$+\frac{(n-1)}{3}\int x^{n-2}(a^2-x^2)\sqrt{(a^2-x^2)}dx$$

$$\Rightarrow u_n=-\frac{x^{n-1}(a^2-x^2)^{3/2}}{3}+\frac{(n-1)a^2}{3}u_{n-2}-\frac{(n-1)}{3}u_n$$

$$\Rightarrow\frac{(n+2)u_n}{3}=-\frac{x^{n-1}(a^2-x^2)^{3/2}}{3}+\frac{(n-1)a^2u_{n-2}}{3}$$

$$\Rightarrow u_n=-\frac{x^{n-1}(a^2-x^2)^{3/2}}{(n+2)}+\frac{(n-1)a^2u_{n-2}}{(n+2)}$$

$$\Rightarrow\ (n+2)u_n-(n-1)a^2u_{n-2}=-x^{n-1}(a^2-x^2)^{3/2}$$

30. **(b)** Given $\int f(x)\sin x\cos x dx=\frac{1}{2(b^2-a^2)}\ ln\, f(x)+c$

Differentiating both sides w.r.t.x then

$$f(x)\sin x\cos x=\frac{1}{2(b^2-a^2)}.\frac{f'(x)}{f(x)}$$

$$\Rightarrow\ 2(b^2-a^2)\sin x\cos x=\frac{f'(x)}{\{f(x)\}^2}$$

$$\Rightarrow 2b^2\sin x\cos x-2a^2\sin x\cos x=\frac{f'(x)}{\{f(x)\}^2}$$

Integrating both side w.r.t. x we get

$$-b^2\cos^2 x-a^2\sin^2 x=-\frac{1}{f(x)}$$

or $f(x)=\frac{1}{(a^2\sin^2 x+\mathbf{b}^2\cos^2 x)}$

31. **(c)** Let $I=\int\frac{(ax^2-b)dx}{x\sqrt{c^2x^2-(ax^2+b)^2}}$

$$=\int\frac{(ax^2-b)dx}{x^2\sqrt{c^2-\left(ax+\frac{b}{x}\right)^2}}=\int\frac{\left(a-\frac{b}{x^2}\right)dx}{\sqrt{c^2-\left(ax+\frac{b}{x}\right)^2}}$$

Put $ax+\frac{b}{x}=c\sin\theta\quad\therefore\ \left(a-\frac{b}{x^2}\right)dx=c\cos\theta\, d\theta$

then $I=\int\frac{c\cos\theta d\theta}{\sqrt{c\cos\theta}}=\int d\theta=\theta+k$, k is constant of integration. So, $I=\sin^{-1}\left(\frac{ax^2+b}{cx}\right)+k$

32. **(b)** Since $f(x)=\tan^{-1}x+ln\sqrt{(1+x)}-ln\sqrt{(1-x)}$

$$\therefore\ f'(x)=\frac{1}{(1+x^2)}+\frac{1}{2(1+x)}+\frac{1}{2(1-x)}$$

$$=\frac{1}{(1+x^2)}+\frac{1}{(1-x^2)}=\frac{2}{(1-x^4)}$$

$$\Rightarrow\ \frac{1}{2}f'(x)=\frac{1}{(1-x^4)}$$

$$\therefore\ \int\frac{1}{2}f'(x)dx^4=\int\frac{dx^4}{(1-x^4)}$$

Put $1-x^4=t\ \therefore -dx^4=dt\ \ or\ \ dx^4=-dt$ then

$$\int\frac{1}{2}f'(x)dx^4=-\int\frac{dt}{t}=-\ln t+c=ln(1-x^4)+c$$

33. **(c)** $\because\ f'(x)=\frac{1}{1+\cos x}=\frac{1}{2\cos^2(x/2)}=\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

Integrating both sides with respect to x, we have

$$f(x)=\tan\left(\frac{x}{2}\right)+c$$

$\therefore\ f(0)=0+c=3$ then $f(x)=\tan\left(\frac{x}{2}\right)+3$

$$\therefore\ f\left(\frac{\pi}{2}\right)=\tan\left(\frac{\pi}{4}\right)+3=4$$

Now $3+\frac{\pi}{4}=3+\frac{22}{7\times 4}=3+\frac{11}{14}=\frac{53}{14}=3.78$

and $3+\frac{\pi}{2}=3+\frac{22}{14}=3+\frac{11}{14}=\frac{32}{7}=4.57$

$\because 3.78<4<4.57$

Hence, $3+\frac{\pi}{4}<f\left(\frac{\pi}{2}\right)<3+\frac{\pi}{2}$

It can be checked that other options do not satisfy the conditions.

34. **(a)** $\int x^2 d(\tan^{-1}x)=\int x^3.\frac{1}{1+x^2}dx$

$$=\int\left(x-\frac{x}{1+x^2}\right)dx=\frac{x^2}{2}-\frac{1}{2}\ln(1+x^2)+C$$

35. **(c)** $I=\int\sqrt{1+2\tan^2 x+2\tan x\sec x}\ dx$

$$=\int\sqrt{\sec^2 x+\tan^2 x+2\tan x\sec x}\ dx$$

$$=\int(\sec x+\tan x)\,dx=\ln(\sec x+\tan x)+\ln\sec x+C$$

36. **(a)** We have $\frac{\sec 6\alpha}{\text{cosec}2\alpha}+\frac{\sec 18\alpha}{\text{cosec}6\alpha}+\frac{\sec 54\alpha}{\text{cosec}18\alpha}$

$$=\frac{\sin 2\alpha}{\cos 6\alpha}+\frac{\sin 6\alpha}{\cos 18\alpha}+\frac{\sin 18\alpha}{\cos 54\alpha}$$

Now, $\frac{\sin 2\alpha}{\cos 6\alpha}=\frac{1}{2}\frac{\sin 4\alpha}{\cos 2\alpha\cos 6\alpha}$

$$=\frac{1}{2}\left[\frac{\sin 6\alpha\cos 2\alpha-\cos 6\alpha\sin 2\alpha}{\cos 2\alpha\cos 6\alpha}\right]$$

$$=\frac{1}{2}(\tan 6\alpha-\tan 2\alpha)$$

Similarly, $\frac{\sin 6\alpha}{\cos 18\alpha}=\frac{1}{2}(\tan 18\alpha-\tan 6\alpha)$

and $\frac{\sin 18\alpha}{\cos 54\alpha}=\frac{1}{2}(\tan 54\alpha-\tan 18\alpha)$

Thus integral $=\frac{1}{2}\int(\tan 54\alpha-\tan 2\alpha)\,dx$

$$=\frac{1}{2}\left[\frac{\ell n|\sec 54\alpha|}{54}-\frac{\ell n|\sec 2\alpha|}{2}\right]+c$$

37. (c) Put $\ell n\, x=t\Rightarrow x=e^t \Rightarrow dx=e^t dt$

$$I=\int e^t\left(\frac{t-1}{t^2+1}\right)^2 dt=\int e^t\left(\frac{1}{t^2+1}-\frac{2t}{(t^2+1)^2}\right)dt$$

$$=\frac{e^t}{t^2+1}+c=\frac{x}{(\ell n\, x)^2+1}+c.$$

38. (a) $I=\int\left(\ell n(1+\cos x)-x\tan\frac{x}{2}\right)dx$

$$=\int \ell n(1+\cos x).1dx-\int x\tan\frac{x}{2}dx$$

$$=\ell n(1+\cos x).x+\int\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}.x\,dx-\int x\tan\frac{x}{2}dx$$

$$=\ell n\,(1+\cos x).\,x+\int x\tan\frac{x}{2}dx-\int x\tan\frac{x}{2}dx+c$$

$=\ell n\,(1+\cos x).\,x+c.$

39. (c) $\int\left(3x^2\tan\frac{1}{x}-x\sec^2\frac{1}{x}\right)dx$

$$=\int 3x^2\tan\frac{1}{x}dx-\int x\sec^2\frac{1}{x}dx$$

$$=\tan\frac{1}{x}x^3-\int\left(\sec^2\frac{1}{x}\right)\left(-\frac{1}{x^2}\right).x^3dx-\int x\sec^2\frac{1}{x}dx$$

$$=x^3.\tan\frac{1}{x}+c$$

40. (d) Let $I=\int e^{\tan\theta}(\sec\theta-\sin\theta)d\theta$

Put $\tan\theta=t\Rightarrow\sec^2\theta\, d\theta=dt\Rightarrow d\theta=\frac{dt}{1+t^2}$

$$\Rightarrow I=\int e^t\left(\sqrt{1+t^2}-\frac{t}{\sqrt{1+t^2}}\right)\frac{dt}{1+t^2}$$

$$=\int e^t\left(\frac{1}{\sqrt{1+t^2}}-\frac{t}{(1+t^2)^{3/2}}\right)dt$$

Integrating first part by parts we have,

$$\frac{1}{\sqrt{1+t^2}}e^t+\int\frac{t}{(1+t^2)^{3/2}}.e^t dt-\int\frac{t}{(1+t^2)^{3/2}}e^t dt+c$$

$$=\frac{e^t}{\sqrt{1+t^2}}+c=e^{\tan\theta}\cos\theta+c$$

41. (a) $I=-\int\frac{(\tan x-1)\sec^2 x dx}{(\tan x+1)\sqrt{\tan^3 x+\tan^2 x+\tan x}}dx$

Put $\tan x=t$

$$I=-\int\frac{(t-1)}{(t+1)\sqrt{t^3+t^2+t}}dt$$

$$=-\int\frac{t^2-1}{(t^2+2t+1)\sqrt{t^3+t^2+t}}dt$$

$=-\int\frac{1-\frac{1}{t^2}}{\left(t+2+\frac{1}{t}\right)\sqrt{t+1+\frac{1}{t}}}dt$, put $1+t+\frac{1}{t}=u^2$

$$\therefore\ I=-\int\frac{2du}{1+u^2}=-2\tan^{-1}u+c,$$

where $u=\sqrt{1+\tan x+\frac{1}{\tan x}}$

42. (c) $\int\frac{1+(\sin x)^{2/3}}{1+(\sin x)^{4/3}}.d(\sqrt[3]{\sin x})$

$$=\int\frac{1+t^2}{1+t^4}dt=\int\frac{1+1/t^2}{(t-1/t)^2+2}dt=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-1/t}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt[3]{\sin x}-\frac{1}{\sqrt[3]{\sin x}}}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt[3]{\sin^2 x}-1}{\sqrt{2}\sqrt[3]{\sin x}}\right)+c$$

43. (c) We note that $\frac{d}{dx}(x\tan x+1) = x\sec^2 x + \tan x$

$\therefore$ integrating by parts with x^2 as first function, we get

$$I = \int x^2 \frac{x\sec^2 x + \tan x}{(x\tan x+1)^2}dx$$

$$= x^2\left(-\frac{1}{x\tan x+1}\right) - \int 2x\left(-\frac{1}{x\tan x+1}\right)dx$$

$$= -\frac{x^2}{x\tan x+1} + 2\int \frac{x\cos x}{x\sin x+\cos x}dx$$

$$= -\frac{x^2}{x\tan x+1} + 2\log_e|x\sin x+\cos x| + c$$

$(\because \frac{d}{dx}(x\sin x+\cos x) = x\cos x)$

44 (c) Let $ax^2 = b\sin^2\theta \Rightarrow x = \sqrt{\frac{b}{a}}\sin\theta$

$2ax\,dx = 2b\sin\theta\cos\theta d\theta$

$$\Rightarrow dx = \frac{2b\sin\theta\cos\theta}{2ax}d\theta = \sqrt{\frac{b}{a}}\cos\theta d\theta$$

$$I = \sqrt{\frac{b}{a}}\int \frac{\cos\theta d\theta}{\left(a + b.\frac{b}{a}\sin^2\theta\right).\sqrt{b}\cos\theta}$$

$$= \sqrt{a}\int \frac{d\theta}{(a^2+b^2\sin^2\theta)}$$

$$= \sqrt{a}\int \frac{\sec^2\theta d\theta}{(a^2\sec^2\theta + b^2\tan^2\theta)}$$

Putting $\tan\theta = z$

$\Rightarrow \sec^2\theta d\theta = dz$ then

$$I = \sqrt{a}\int \frac{dz}{a^2(1+z^2)+b^2z^2}$$

$$= \sqrt{a}\int \frac{dz}{(a^2+b^2)z^2+a^2}$$

$$= \frac{\sqrt{a}}{(a^2+b^2)}\int \frac{dz}{z^2 + \frac{a^2}{(a^2+b^2)}}$$

$$= \frac{\sqrt{a}}{a^2+b^2}.\frac{\sqrt{a^2+b^2}}{a}\tan^{-1}\left(\frac{z\sqrt{a^2+b^2}}{a}\right)+c$$

$$\therefore I = \frac{1}{\sqrt{a(a^2+b^2)}}\tan^{-1}\left(\frac{z\sqrt{a^2+b^2}}{a}\right)+c,$$ where

$z = \tan\theta$.

45. (a) Given $x f(x) = 3f^2(x)+2$

$\Rightarrow f(x) + xf'(x) = 6f(x)f'(x)$

$$\Rightarrow f'(x) = \frac{f(x)}{6f(x)-x}$$

Now $I = f\frac{2x(x-6f(x))+f(x)}{(6f(x)-x)(x^2-f(x))^2}dx$

$$\Rightarrow I = -\int \frac{2x-f'(x)}{(x^2-f(x))^2}dx = \frac{1}{x^2-f(x)}+c$$

B COMPREHENSION TYPE

1. (a) $$\int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}$$

Put $x - \frac{1}{x} = t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$$\therefore I = \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x} + C$$

2. (c) $$I = \int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}dx$$

$$= \int \frac{1-\frac{1}{x^2}}{\left(x^2+\frac{1}{x^2}+3\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx$$

Put $x + \frac{1}{x} = t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$ and $x^2 + \frac{1}{x^2} + 2 = t^2$

$$\therefore I = \int \frac{dt}{(t^2+1)\tan^{-1}t} = \ell n|\tan^{-1}t| + C$$

$$= \ell n\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right| + C$$

3.(b) $I=\int \frac{x^4-2}{x^2\sqrt{x^4+x^2+2}}dx=\int \frac{x^4-2}{x^3\sqrt{x^2+1+\frac{2}{x^2}}}dx$

$$=\int \frac{x-\frac{2}{x^3}}{\sqrt{x^2+1+\frac{2}{x^2}}}dx$$

Put $x^2+\frac{2}{x^2}+1=t \Rightarrow \left(x-\frac{2}{x^3}\right)dx=\frac{dt}{2}$,

we get $I=\int \frac{dt}{2\sqrt{t}}=\sqrt{t}+C=\sqrt{x^2+1+\frac{2}{x^2}}+C$

4.(d) Divide numerator and demoninator by x^{10} we get

$$I=\int \frac{5x^{-6}+4x^{-5}}{(1+x^{-4}+x^{-5})^2}dx$$

Put $1+x^{-4}+x^{-5}=t$ and evaluate

5.(b) Put $\sqrt{1+x^2}=(t-x)$

[Here $a=1>0$, we can put $\sqrt{1+x^2}=t+x$ also]

$\Rightarrow x+\sqrt{1+x^2}=t$

or $\left(1+\frac{x}{\sqrt{1+x^2}}dx=dt\right) \Rightarrow \frac{dx}{\sqrt{1+x^2}}=\frac{dt}{t}$

$$\therefore I=\int t^{15}.\frac{dt}{t}=\frac{t^{15}}{15}+C=\frac{(x+\sqrt{1+x^2})^{15}}{15}+C$$

$$=\frac{1}{15(\sqrt{1+x^2}-x)^{15}}+C$$

6. (a) Here $a<0$ and $c<0$, but $-x^2+3x-2=-(x-1)(x-2)$

So, we put $\sqrt{-x^2+3x-2}=(x-2)t$ or $(x-1)t$ or $t(1-x)$.

Put $-x^2+3x-2=t(x-2) \Rightarrow t=\sqrt{\frac{1-x}{x-2}}$ $\quad (\because 1<x<2)$

We get, $x=\frac{2t^2+1}{t^2+1}$ and $dx=\frac{2t\ dt}{(t^2+1)^2}$

So, the integral becomes

$$I=\int \frac{\frac{2t\ dt}{(t^2+1)^2}}{\left(\frac{t^2}{t^2+1}\right).\frac{t}{t^2+1}}=\int \frac{2}{t^2}\,dt$$

$$=-\frac{2}{t}+C=-2\sqrt{\frac{x-2}{1-x}}+C$$

7.(c) The antiderivative is $-2\sqrt{\frac{x-2}{1-x}}$. So, the limit is

$$\lim_{x\to 2}\frac{\sin\left(-2\sqrt{\frac{x-2}{1-x}}\right)}{\sqrt{2-x}}$$

$$=\lim_{x\to 2}\frac{\sin\left(-2\sqrt{\frac{x-2}{1-x}}\right)}{\left(-2\sqrt{\frac{x-2}{1-x}}\right)}\times -2\sqrt{\frac{x-2}{1-x}}\times\frac{1}{\sqrt{2-x}}$$

$=1\times -2=-2$

8.(c) $\int\left[\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\right]dx$

$$=\frac{1}{\ln x}.x-\int -\frac{1}{(\ln x)^2}.\frac{1}{x}.x\ dx-\int \frac{1}{(\ln x)^2}\ dx$$

$$=\frac{x}{\ln x}+C$$

9.(b) $\int \frac{x+\sin x}{1+\cos x}dx=\int \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$

$$=\int \frac{1}{2}x\sec^2\frac{x}{2}dx+\int \tan\frac{x}{2}dx$$

Integrating second integral by parts taking 1 as second function, we get

$$I=\int \frac{1}{2}x\sec^2\frac{x}{2}dx+x\tan\frac{x}{2}-\int x.\frac{1}{2}\sec^2\frac{x}{2}\ dx$$

$$=x\tan\frac{x}{2}+C$$

10.(d) $I=\int \frac{xe^x}{(x+1)^2}dx=\int \frac{(x+1-1)e^x}{(x+1)^2}\ dx$

$$=\int e^x\left\{\frac{1}{x+1}+\frac{-1}{(x+1)^2}\right\}dx=\frac{e^x}{x+1}+C$$

$\left[\because \frac{d}{dx}\left(\frac{1}{x+1}\right)=-\frac{1}{(x+1)^2}\right]$

11.(b) $I=\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx=\int\left(\log t+\frac{1}{t^2}\right)e^t\ dt$

[Putting $\log x=t$]

$$=\int e^t\left\{\left(\log t+\frac{1}{t}\right)-\left(\frac{1}{t}-\frac{1}{t^2}\right)\right\}dt$$

$$=e^t\left(\log t-\frac{1}{t}\right)=x\log(\log x)-\frac{x}{\log x}$$

C REASONING TYPE

1. **(c)** $I_n = \int \tan^n x\, dx$

$= \int \tan^{n-2} x \sec^2 x - \int \tan^{n-2} x\, dx$

$= \dfrac{\tan^{n-1} x}{n-1} - I_{n-2}$

Put $n = 6$, $5(I_6 + I_4) = \tan^5 x$

Statement -1 is true and statement -2 is false.

2. **(b)** $\int \dfrac{1}{f(x)} dx = \log(f(x))^2 + C \Rightarrow \dfrac{1}{f(x)} = \dfrac{2f'(x)}{f(x)}$

$\Rightarrow f'(x) = \dfrac{1}{2} \Rightarrow f(x) = \dfrac{x}{2}$

Statement -2 is clearly true.

3. **(a)** $I = \int \dfrac{3+4\cos x}{(4+3\cos x)^2} dx$

$= \int \dfrac{3\cos ec^2 x + 4\cot x \operatorname{cosec} x}{(4\operatorname{cosec} x + 3\cot x)^2} dx$

(Mulitplying N_r and D_r by $\operatorname{cosec}^2 x$)

$\therefore\ I = -\int \{f(x)\}^{-2} f'(x) dx,$

where $f(x) = 4\cos ec\, x + 3\cot x$

$= \dfrac{1}{f(x)} = \dfrac{1}{4\operatorname{cosec} x + 3\cot x} = \dfrac{\sin x}{4 + 3\cos x} + C$

Statement -2 is clearly true

D MULTIPLE CORRECT CHOICE TYPE

1. **(c,d)** $f(x) = \lim\limits_{n\to\infty} e^{x\tan(1/n)\log(1/n)}$

But $\lim\limits_{n\to\infty} \tan 1/n \log 1/n$

$= \lim\limits_{n\to\infty} \left(\dfrac{\log n}{n} \dfrac{\tan 1/n}{1/n} \right) = 0$

So, $f(x) = e^0 = 1$.

Hence, $\int \dfrac{f(x)}{\sqrt[3]{\sin^{11} x \cos x}} dx = \int \dfrac{\sec^4 x}{(\tan x)^{11/3}} dx$

$= \int \dfrac{1+t^2}{t^{11/3}} dt$ (Putting $t = \tan x$)

$= \int (t^{-11/3} + t^{-5/3}) dt = -\dfrac{3}{8} t^{-8/3} - \dfrac{3}{2} t^{-2/3} + C$

$= -\dfrac{3}{8} \dfrac{(1+4\tan^2 x)}{\tan^2 x \sqrt[3]{\tan^2 x}} + C$

Thus, $g(x) = -\dfrac{3}{8} \dfrac{(1+4\tan^2 x)}{\tan^2 x \sqrt[3]{\tan^2 x}}$

and $g\left(\dfrac{\pi}{4}\right) = -\dfrac{15}{8}$

Clearly g is not defined at $x = 0$ and odd multiples of $\dfrac{\pi}{2}$. So (b) is not correct.

2. **(b,c)** $I = \int e^x \left\{ \dfrac{2\tan x}{1+\tan x} + \cot^2\left(x + \dfrac{\pi}{4}\right) \right\} dx$

$= \int e^x \left\{ \dfrac{2}{1+\cot x} - 1 + \cos ec^2\left(x + \dfrac{\pi}{4}\right) \right\} dx$

$= \int e^x \left\{ -\cot\left(x + \dfrac{\pi}{4}\right) + \cos ec^2\left(x + \dfrac{\pi}{4}\right) \right\} dx$

$= -e^x \cot\left(x + \dfrac{\pi}{4}\right) + C = e^x \cot\left(\dfrac{3\pi}{4} - x\right) + C$

Again,

$I = e^x \cot\left(\dfrac{3\pi}{4} - x\right) + C = e^x \cot\left(\dfrac{\pi}{2} + \dfrac{\pi}{4} - x\right) + C$

$= e^x \tan\left(x - \dfrac{\pi}{4}\right) + C$

3. **(a,c)** As $0 < x < 1 \Rightarrow x^2 < x^{\frac{\pi}{2}} < x$

$\Rightarrow \dfrac{1}{1+x} < \dfrac{1}{1+x^{\pi/2}} < \dfrac{1}{1+x^2}$

or $\int_0^1 \dfrac{dx}{1+x} < \int_0^1 \dfrac{dx}{1+x^{\pi/2}} < \int_0^1 \dfrac{dx}{1+x^2}$

$\Rightarrow \ln 2 < I < \dfrac{\pi}{4}$

4. **(b,d)** $\cos^{-1}\sqrt{1-x^2} = -\sin^{-1}x, \quad \because\ x<0$

$$\therefore \int\left(\cos^{-1}x + \cos^{-1}\sqrt{1-x^2}\right)dx$$

$$= \int\left(\cos^{-1}x - \sin^{-1}x\right)dx$$

$$= \int\left(\frac{\pi}{2} - 2\sin^{-1}x\right)dx$$

$$= \frac{\pi}{2}x - 2x\sin^{-1}x + \int\frac{2x}{\sqrt{1-x^2}}dx$$

$$= \frac{\pi}{2}x - 2x\sin^{-1}x - 2\sqrt{1-x^2} + C$$

5. **(a, d)** Let $I = \int x^{-1/2}(2+3x^{1/3})^{-2}dx$

Put $x = t^6$ $\quad\therefore dx = 6t^5dt$

then $I = \int t^{-3}(2+3t^2)^{-2}.6t^5dt$

$$= 6\int\frac{t^2}{(2+3t^2)^2}dt = \frac{6}{9}\int\frac{t^2dt}{\left(\frac{2}{3}+t^2\right)^2}$$

Now put $t = \sqrt{\left(\frac{2}{3}\right)}\tan\theta$

$$\therefore\ dt = \sqrt{\left(\frac{2}{3}\right)}\sec^2\theta d\theta$$

$$\therefore I = \frac{6}{9}\int\frac{\frac{2}{3}\tan^2\theta.\sqrt{\frac{2}{3}}\sec^2\theta d\theta}{\frac{4}{9}\sec^4\theta} = \sqrt{\frac{2}{3}}\int\sin^2\theta d\theta$$

$$= \frac{1}{\sqrt{6}}\int(1-\cos 2\theta)d\theta = \frac{1}{\sqrt{6}}\left\{\theta - \frac{\sin 2\theta}{2}\right\} + C$$

$$= \frac{1}{\sqrt{6}}\left\{\theta - \frac{\tan\theta}{1+\tan^2\theta}\right\} + C$$

$$= \frac{1}{\sqrt{6}}\left\{\tan^{-1}\left\{\sqrt{\frac{3}{2}}t\right\} - \frac{t\sqrt{\frac{3}{2}}}{1+\frac{3}{2}t^2}\right\} + C$$

$$= \frac{1}{\sqrt{6}}\left\{\tan^{-1}\left\{\sqrt{\frac{3}{2}}x^{1/6}\right\} - \frac{\sqrt{6}x^{1/6}}{2+3x^{1/3}}\right\} + C$$

$$\left\{\because\ \tan\theta = \sqrt{\frac{3}{2}}t\right\}$$

6. **(b,d)** Let $I = \int\frac{(x^4+1)}{(x^6+1)}dx = \int\frac{(x^2+1)^2 - 2x^2}{(x^2+1)(x^4-x^2+1)}dx$

$$= \int\frac{(x^2+1)dx}{(x^4-x^2+1)} - 2\int\frac{x^2dx}{(x^6+1)}$$

$$= \int\frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x^2-1+\frac{1}{x^2}\right)} - 2\int\frac{x^2dx}{(x^3)^2+1}$$

$$= \int\frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+1} - 2\int\frac{x^2dx}{(x^3)^2+1}$$

In first integral put $x - \frac{1}{x} = t$

$\therefore \left(1+\frac{1}{x^2}\right)dx = dt$ and in second integral put $x^3 = u$

$\therefore\ x^2dx = \frac{du}{3}$ then $I = \int\frac{dt}{1+t^2} - \frac{2}{3}\int\frac{du}{1+u^2}$

$$= \tan^{-1}t - \frac{2}{3}\tan^{-1}u + c$$

$$= \tan^{-1}\left(x-\frac{1}{x}\right) - \frac{2}{3}\tan^{-1}(x^3) + c$$

7. **(a, b)** Put $t = \sin^2 x$

The integral reduces to

$$I = \frac{1}{2}\int e^t(2-t)dt = \frac{3}{2}e^t - \frac{te^t}{2} + c$$

$$= \frac{1}{2}e^{\sin^2 x}(3-\sin^2 x) + c$$

$$= e^{\sin^2 x}\left(1+\frac{1}{2}\cos^2 x\right) + c$$

8. **(b,c)** $I = \int\frac{x^2+n(n-1)}{(x\sin x + n\cos x)^2}dx$

Multiplying and dividing by x^{2n-2}

$$I = \int\frac{(x^2+n(n-1)).x^{2n-2}}{(x\sin x+n\cos x)^2.x^{2n-2}}dx$$

$$I = \int\frac{(x^2+n(n-1))x^{2n-2}}{(x^n\sin x + nx^{n-1}\cos x)^2}dx$$

Let $x^n\sin x + nx^{n-1}\cos x = t$

$\Rightarrow (nx^{n-1}\sin x + x^n\cos x + n(n-1))x^{n-2}$

$\cos x - nx^{n-1}\sin x)dx = dt$

$\Rightarrow\ x^{n-2}\cos x.(x^2 + n(n-1))dx = dt$

$$I = \int \frac{(x^2 + n(n-1)).x^{n-2}\cos x}{(x^n \sin x + nx^{n-1}\cos x)^2}.x^n.\sec x\, dx$$

Integrating by parts; we get

$$I = x^n \sec x.\left(-\frac{1}{x^n \sin x + nx^{n-1}\cos x}\right)$$

$$+\int \frac{x^n \sec x \tan x + nx^{n-1}.\sec x}{(x^n \sin x + nx^{n-1}\cos x)}dx$$

$$= -\frac{x^n \sec x}{x^n \sin x + nx^{n-1}\cos x} + \int \sec^2 x dx$$

$$\therefore\quad I = -\frac{x^n \sec x}{x^n \sin x + nx^{n-1}\cos x} + \tan x + c.$$

E MATRIX-MATCH TYPE

1. A-r; B-s; C-p; D-q

(A) $\int e^x\left[\frac{1}{x+2} + \log(x+2)\right]dx = e^x \log(x+2) + c$

(B) $\int \sin^2 x(1-\sin^2 x)\cos x dx$

$= \int \sin^2 x \cos x dx - \int \sin^4 x.\cos x\ dx$

$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(C) $\int \frac{dx}{\sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x+\frac{3}{2}\right)^2}} = \sin^{-1}\left(\frac{2x+3}{\sqrt{17}}\right) + c$

(D) $\int \frac{x^5}{x^2+1}dx = \int\left(x^3 - x + \frac{x}{x^2+1}\right)dx$

$= \frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2}\ell n(x^2+1) + C$

2. A-s; B-t; C-r; D-q

(A) Since $I = \int \frac{dx}{\sqrt{x}(x+9)}$

Put $\sqrt{x} = t \Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$

$\therefore\ I = \int \frac{2dt}{t^2+9} = \frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right) + C$

$= \frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right) + C$

(B) $\because \int e^x(1 - \cot x + \cot^2 x)dx$

$\Rightarrow \int e^x(\cos ec^2 x + (-\cot x))dx = e^x(-\cot x) + c$

(C) $\int(\tan x \sec x + \cot x \cos ec x)dx = \sec x - \cos ec x + c$

(D) $\int \frac{dx}{2\sin^2\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$

$= \int \frac{\cos ec^2\left(\frac{x}{2}\right)\frac{1}{2}}{1-\cot\left(\frac{x}{2}\right)}dx = \int \frac{f'(x)}{f(x)}dx = \log\left|1 - \cot\left(\frac{x}{2}\right)\right| + c$

3. A-r; B-p; C-q ; D-s

(A) $\int(x^a + a^x)dx = \frac{x^{a+1}}{a+1} + \frac{a^x}{(\log\ a)} + C$

(B) $\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx\ \ put\ \ x - \frac{1}{x} = t,\ \left(x^2 + \frac{1}{x^2}\right)dx = dt$

$\Rightarrow \int \frac{dt}{t^2 + (\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + C$

(C) $\frac{1}{4}\int \frac{\sec^2 x\ dx}{\tan^2 x + \tan x + \frac{5}{4}} = \frac{1}{4}\int \frac{\sec^2 x\ dx}{\left(\tan x + \frac{1}{2}\right)^2 + 1}$

$= \frac{1}{4}\tan^{-1}\left(\tan x + \frac{1}{2}\right) + C$

(D) $\int \frac{1}{(\sqrt{\sin x} + \sqrt{\cos x})^4}dx\ = \int \frac{\sec^2 x}{(\sqrt{\tan x} + 1)^4}dx$

(Using $\tan x = t^2 \Rightarrow \sec^2 x\,dx = 2t\,dt$)

$$= \int \frac{2t\,dt}{(t+1)^4} = 2\int \frac{1}{(t+1)^3} - \frac{1}{(t+1)^4}\,dt$$

$$= -\frac{1}{(t+1)^2} + \frac{1}{3(t+1)^3} + c$$

$$= \frac{-1}{(1+\sqrt{\tan x})^2} + \frac{2}{3(1+\sqrt{\tan x})^3} + c.$$

4. **A-r; B-p; C-t; D-s**

$$\int \frac{\ell n(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}\,dx = I$$

Put $\log(x+\sqrt{1+x^2}) = t \Rightarrow \frac{dx}{\sqrt{1+x^2}} = dt$

So, $I = \int t\,dt = \frac{t^2}{2} + C = \frac{1}{2}\left\{\log(x+\sqrt{1+x^2}\right\}^2 + C$.

Thus,

(A) $f(x) = \frac{x^2}{2}$

(B) $g(x) = \log(x+\sqrt{x^2+1})$

(C) Now, $\int \frac{x^2}{2}\log(x+\sqrt{x^2+1})dx = \frac{x^3}{6}\log(x+\sqrt{x^2+1})$

$$-\frac{1}{2}\int \frac{x^3}{3} \times \frac{1}{x+\sqrt{x^2+1}}\left\{1+\frac{2x}{2\sqrt{x^2+1}}\right\}dx$$

$$= \frac{x^3}{6}\log(x+\sqrt{x^2+1}) - \frac{1}{6}\int \frac{x^3 dx}{\sqrt{x^2+1}}$$

Putting $x^2 + 1 = t^2$

$$= \frac{x^3}{6}\log(x+\sqrt{x^2+1}) - \frac{1}{6}\int (t^2-1)\,dt$$

$$= \frac{x^3}{6}\log(x+\sqrt{x^2+1}) - \frac{1}{18}(1+x^2)^{3/2}$$

$$+\frac{1}{6}(1+x^2)^{1/2} + C$$

(D) $\int e^{g(x)}dx = \int (x+\sqrt{1+x^2})dx$

$$= \frac{x^2}{2} + \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\,(x+\sqrt{1+x^2}) + C$$

$$= \frac{1}{2}x(x+\sqrt{1+x^2}) + \frac{1}{2}g(x) + C$$

5. **A-r, B-q, C-t, D-p**

(A) $\int x^2 d(\tan^{-1}x) = \int \frac{x^2}{1+x^2}\,dx = \int\left(1-\frac{1}{1+x^2}\right)dx$

$$= x + \tan^{-1}x + C \Rightarrow f(x) = \tan^{-1}x$$

(B) $\int \sqrt{1+2\tan x(\tan x + \sec x)}\,dx$

$$= \int \sqrt{(\sec x + \tan x)^2}\,dx$$

$$= \ln|\sec x + \tan x| + \ln|\sec x| + C$$

$$= \ln\left|\frac{1+\sin x}{\cos^2 x}\right| + C = \ln\left|\frac{1}{1-\sin x}\right| + C$$

$$= \ln\left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|^{-2} + C = -2\ln\left|\cos\frac{x}{2} - \sin\frac{x}{2}\right| + C$$

(C) $\int x^2 e^{2x}dx = x^2\frac{e^{2x}}{2} - \int 2x\frac{e^{2x}}{2}\,dx$

$$= \frac{x^2}{2}e^{2x} - \left[x.\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}\,dx\right]$$

$$= \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right)e^{2x} + C = \frac{1}{4}(2x^2 - 2x + 1)e^{2x} + C$$

Now, $2x^2 - 2x + 1 \geq \frac{1}{2}$

$$\Rightarrow f(x) = \frac{1}{4}(2x^2 - 2x + 1) \geq \frac{1}{8}$$

(D) $\int \frac{x^4+1}{x(x^2+1)^2}\,dx = \int \frac{(x^2+1)^2 - 2x^2}{x(x^2+1)^2}dx$

$$= \int\left(\frac{1}{x} - \frac{2x}{(x^2+1)^2}\right)dx = \ln|x| + \frac{1}{x^2+1} + C$$

$\therefore a = b = 1$

6. **A→ p, q, t ; B→ p ; C → q ; D → p, s**

(A) Let $3x = \cos\theta \Rightarrow 3dx = -\sin\theta\,d\theta$, then integral is

$$-\frac{1}{3}\int \frac{\frac{\cos\theta}{3} + \theta^2}{\sin\theta}\sin\theta d\theta = -\frac{1}{3}\int\left(\frac{1}{3}\cos\theta + \theta^2\right)d\theta$$

$$= -\frac{1}{9}\sin\theta - \frac{1}{9}\theta^3 + c$$

$$= -\frac{1}{9}\sqrt{1-9x^2} - \frac{1}{9}(\cos^{-1}3x)^3 + c$$

Hence $A = -\frac{1}{9}$, $B = -\frac{1}{9}$.

(B) Given integral is

$$\int \frac{2x+1}{(x(x+1))^2-1}dx = \int \frac{dt}{t^2-1} = -\frac{1}{2}\ln\left|\frac{x^2+x+1}{x^2+x-1}\right| + c$$

(C) $\int x^5(x^{10}+x^5+1)(2x^{10}+3x^5+6)^{1/5}dx$

$$= \int (x^{14}+x^9+1)(2x^{15}+3x^{10}+6x^5)^{1/5}dx$$

Putting $2x^{15}+3x^{10}+6x^5 = t^5$

$$\Rightarrow (x^{14}+x^9+x^4)dx = \frac{5t^4}{30}dt$$

$$= \frac{1}{6}\int t^5 dt = \frac{1}{36}t^6$$

$$= \frac{1}{36}(2x^{15}+3x^{10}+6x^5)^{6/5} + c.$$

(D) $I = \int \frac{x^3-x}{\sqrt{1-x^2}} \cdot \frac{1}{(1+\sqrt{1-x^2})}dx$

let $1+\sqrt{1-x^2} = z$ $,\Rightarrow \frac{-2x}{2\sqrt{1-x^2}}dx = dz$

$$I = \int \frac{-x(1-x^2)}{\sqrt{1-x^2}} \cdot \frac{1}{(1+\sqrt{1-x^2})}dx$$

$$= \int (z-1)^2 \frac{dz}{z}$$

$$= \int \frac{z^2-2z+1}{z}dz = \int zdz - 2\int dz + \int \frac{1}{z}dz$$

$$= \frac{z^2}{2} - 2z + \ell n|z| + c$$

$$= \frac{(1+\sqrt{1-x})^2}{2} - 2(1+\sqrt{1-x})^2 + \ell n\left|1+\sqrt{1-x^2}\right| + c$$

$$= -\frac{(2+x^2+2\sqrt{1-x^2})}{2} + \ell n 1 + \sqrt{1-x^2} + c$$

$$= -(1+\sqrt{1-x^2}) - \frac{x^2}{2} + \ell n 1 + \sqrt{1-x^2} + c$$

F NUMERIC/INTEGER ANSWER TYPE

1. Ans. : 3

$$I = \int \frac{\sin^3\frac{\theta}{2}}{\cos\frac{\theta}{2}\sqrt{\cos^3\theta+\cos^2\theta+\cos\theta}}d\theta$$

$$= \int \frac{\left(2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right).\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}\sqrt{\cos^3\theta+\cos^2\theta+\cos\theta}}d\theta$$

$$= \int \frac{2\sin^2\frac{\theta}{2}\sin\theta\, d\theta}{2\cos^2\frac{\theta}{2}\sqrt{\cos^3\theta+\cos^2\theta+\cos\theta}}$$

Put $\cos\theta = t \Rightarrow -\sin\theta d\theta = dt$

Also $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2} = t$

$$\therefore \quad I = \int \frac{\frac{1-t}{2}(-dt)}{(1+t)\sqrt{t^3+t^2+t}} = \frac{1}{2}\int \frac{(t^2-1)dt}{(t+1)^2\sqrt{t^3+t^2+t}}$$

$$= \frac{1}{2}\int \frac{\left(1-\frac{1}{t^2}\right)(dt)}{\left(t+\frac{1}{t}+2\right)\sqrt{t+\frac{1}{t}+1}}$$

Put $t+\frac{1}{t}+1 = u^2 \Rightarrow \left(1-\frac{1}{t^2}\right)dt = 2u\,du$

$$I = \frac{1}{2}\int \frac{2udu}{\left(1+u^2\right)u} = \tan^{-1}u = \tan^{-1}\sqrt{1+\frac{1}{t}+1} + c$$

$$= \tan^{-1}(\cos\theta+\sec\theta+1)^{1/2} + c$$

So, $f(\theta) = \cos\theta + \sec\theta + 1 \geq 2 + 1 = 3$

2. Ans. : 1

Let $P = \int e^{ax}\cos bxdx$, $Q = \int e^{ax}\sin bxdx$

$$P + iQ = \int e^{ax}(\cos bx + i\sin bx)dx$$

[We may apply integration by parts twice also]

$$\therefore \quad \int e^{ax}.e^{ibx}dx = \int e^{(a+ib)x}dx$$

$$= \frac{e^{(a+ib)x}}{(a+ib)} + c = \frac{e^{ax}(\cos bx + i\sin bx)(a-ib)}{a^2+b^2} + c$$

$$= \frac{e^{ax}\{(a\cos bx + b\sin bx)\} + ie^{ax}\{(a\sin bx - b\cos bx)\}}{(a^2+b^2)}$$

$$= \frac{e^{ax}(a\cos bx + b\sin bx)}{(a^2+b^2)} + i\frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}$$

Equating real and imaginary parts on both sides, we get

$$P = \int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{(a^2+b^2)} + c$$

$$= \frac{1}{r}e^{ax}\cos(bx-\phi) + c \text{ and}$$

$$Q = \int e^{ax}\sin bx dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{(a^2+b^2)} + c$$

$$= \frac{1}{r}e^{ax}\sin(bx+\phi) + c$$

where $r = \sqrt{a^2+b^2}$ and $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

$$\therefore\ (P^2+Q^2)r^2 = e^{2ax}$$

(neglecting constant of integration)

$$\therefore\ (P^2+Q^2)(a^2+b^2) = e^{2ax}$$

3. Ans. : 2

$$I = \int \sin 4x\, e^{\tan^2 x}dx = \int 2\sin 2x\cos 2x e^{\tan^2 x}dx$$

$$= 4\int \sin x\cos x\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)e^{\tan^2 x}dx$$

$$= 4\int \tan x.\sec^2 x.\cos^6 x(1-\tan^2 x)e^{\tan^2 x}dx$$

Put $\tan^2 x = t \Rightarrow 2\tan x\sec^2 x\, dx = dt$

$$\therefore\ I = 2\int \frac{(1-t)e^t}{(1+t)^3}dt = -2\int\left[\frac{t+1-2}{(t+1)^3}\right]e^t dt$$

$$= -2\int\left[\frac{1}{(t+1)^2} + \frac{-2}{(t+1)^3}\right]e^t dt$$

$$= -2\frac{e^t}{(t+1)^2} + c = -2\cos^4 x\,.\,e^{\tan^2 x} + c$$

4. Ans. : 1

$$I = \int \operatorname{cosec}^2 x\, ln\left(\cos x + \sqrt{\cos 2x}\right)dx$$

$$= -\cot x.\log_e\left(\cos x + \sqrt{\cos 2x}\right)$$

$$-\int(-\cot x), \frac{1}{\cos x + \sqrt{\cos 2x}}$$

$$\left\{-\sin x + \frac{1}{2}(\cos 2x)^{-1/2}(-\sin 2x).2\right\}dx$$

$$= -\cot x\, ln\left(\cos x + \sqrt{\cos 2x}\right)$$

$$-\int \cot x\frac{\sin x\sqrt{\cos 2x} + \sin 2x}{\sqrt{\cos 2x}\left(\cos x + \sqrt{\cos 2x}\right)}dx$$

$$= -\cot x\, ln\left(\cos x + \sqrt{\cos 2x}\right)$$

$$-\int \frac{\cos x\sqrt{\cos 2x} - \cos^2 x\cos 2x}{\cos 2x\sin^2 x}dx$$

$$= -\cot x\, ln\left(\cos x + \sqrt{\cos 2x}\right)$$

$$-\int \frac{\cos x}{\sqrt{\cos 2x}\sin^2 x}dx + \int \cot^2 x dx$$

Now, $I_1 = \int \frac{\cos x dx}{\sqrt{\cos 2x}\sin^2 x}$

$$= \int \frac{\cos x\, dx}{\sin^2 x\sqrt{1-2\sin^2 x}} = \int \frac{dt}{t^2\sqrt{1-2t^2}}$$

Put $t = \frac{1}{u} \Rightarrow dt = -\frac{1}{u^2}du$

$$\therefore\ I_1 = -\int \frac{udu}{\sqrt{u^2-2}} = -\sqrt{u^2-2} = -\sqrt{\cos ec^2 x - 2}$$

Thus $I = -\cot x\, ln\left(\cos x + \sqrt{\cos 2x}\right)$

$$+\sqrt{\cos ec^2 x - 2} - \cot x - x + c$$

$$\therefore\ f(x) = -\cot x \text{ and } g(x) = \sqrt{\cos ec^2 x - 2}$$

5. Ans: 2

$$I = \sqrt{2}\int \frac{(\cos x - \sin x)}{\sqrt{\sin 2x}(4+3\sin 2x)}dx$$

Put $\cos x + \sin x = z$

$(\cos x - \sin x)\, dx = dz$

$$\therefore\ I = \sqrt{2}\int \frac{dz}{\sqrt{(z^2-1)}(4+3)(z^2-1))}$$

Put $z = \sec\theta$ $\qquad dz = \sec\theta\tan\theta d\theta$

$$I = \sqrt{2}\int \frac{\sec\theta\tan\theta d\theta}{\tan\theta(3\sec^2\theta+1)}$$

$$\therefore I = \sqrt{2}\int \frac{\frac{\sin\theta}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta}\left(\frac{3+\cos^2\theta}{\cos^2\theta}\right)}d\theta$$

$$= \sqrt{2}\int \frac{\cos\theta}{4-\sin^2\theta}d\theta$$

Let $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$$\therefore I = \sqrt{2}\int \frac{dt}{4-t^2} \Rightarrow \frac{1}{2\sqrt{2}}\ell n\left|\frac{t+2}{t-2}\right| + c$$

$$= \frac{1}{2\sqrt{2}}\ell n\left|\frac{\sin\theta+2}{\sin\theta-2}\right| + c,$$

where $\sin\theta = \frac{\sqrt{z^2-1}}{z} = \frac{\sqrt{\sin 2x}}{\sqrt{1+\sin 2x}}$

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