CHAPTER TWENTY SEVEN

Heights and Distances

ANGLES OF ELEVATION AND DEPRESSION

Let *OP* be a horizontal line in the vertical plane in which an object *R* is given and let *OR* be joined.

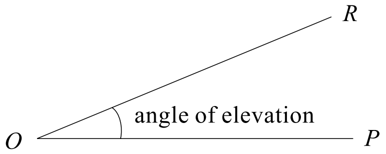

Fig. 27(a)

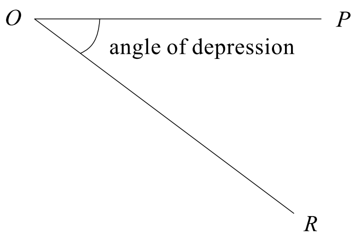

Fig. 27(b)

In Fig. 27(a), where the object *R* is above the horizontal line *OP*, the angle *POR* is called the *angle of elevation* of the object *R* as seen from the point *O*. In Fig. 27(b), where the object *R* is below the horizontal line *OP*, the angle *POR* is called the *angle of depression* of the object *R* as seen from the point *O*.

Remark

Unless stated to the contrary, it is assumed that the height of the observer is neglected, and that the angles of elevation are measured from the ground.

The following results regarding the sides and angles of a triangle are useful in solving the problems on heights and distances.

Illustration 1

Solution:

$$\frac{OP}{OA} = \tan 30° = 1/\sqrt{3}$$

$$\Rightarrow OP = (1/\sqrt{3})\, OA = 10/\sqrt{3}$$

So height of the tower is $10\sqrt{3}$

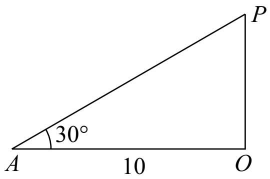

Fig. 27.1

Illustration 2

A car is moving towards a pole of height *a*. The angle of depression of the car from the top of the pole at a time is observed to be α. After 10 minutes it is observed to be *B*. Find the distance travelled by the car in 10 minutes.

Solution: Let *OP* be the pole of height *a*. *A* and *B* be the two positions of the car then

$\angle OAP = \alpha, \angle OBP = \beta$

$OA = a \cot \alpha,\ OB = a \cot \beta$

$AB = OA - OB = a(\cot \alpha - \cot \beta)$

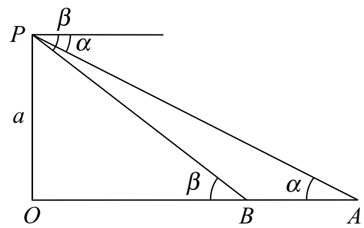

Fig. 27.2

If the angle of elevation of the top *P* of a tower *OP* at a point A on the ground is 30° and *OA* = 10 m. Find the height of the tower.

In a triangle *ABC*, the angles are denoted by the capital letters *A*, *B*, *C* and the lengths of the sides opposite these angles are denoted by *a*, *b*, *c*, respectively (Fig. 27.3).

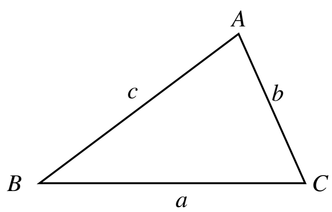

Fig. 27.3

1. *The law of sines* In any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

where R is the radius of the circumcircle of the triangle ABC. R is also known as the *circumradius* of the triangle.

2. *The law of cosines* In any triangle ABC

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

or $a^2 = b^2 + c^2 - 2bc \cos A$

Similar formulae for $\cos B$, $\cos C$ exist.

3. *Projection rule* In any triangle ABC
$a = b \cos C + c \cos B$, $b = c \cos A + a \cos C$ and $C = a \cos B + b \cos A$.

4. *Area of the triangle* If Δ represents the area of the triangle ABC, then

$$\Delta = (1/2)bc \sin A = (1/2)ca \sin B = (1/2)ab \sin C.$$

$$= \frac{abc}{4R} = rs = \sqrt{s(s-a)(s-b)(s-c)}$$

where $2s = a + b + c$, R is the radius of the circum circle of triangle ABC and r is the radius of the circle inscribed in the triangle ABC.

SOLVED EXAMPLES

Concept-based Straight Objective Type Questions

Example 1: The angle of elevation of the top of two poles at a point on the line joining the foot of the towers on the ground is 45°. If the distance between the towers is 1 m., the difference between the heights of the tower is

(a) 2m (b) 1m
(c) 1/2 m (d) 3/2 m

Ans. (b)

Solution: Let PQ and LM be the two towers

$$\angle PAQ = \angle LAM = 45°$$

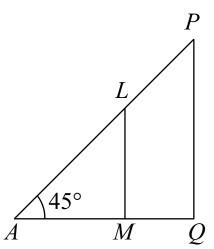

Fig. 27.4

$\Rightarrow$ $AQ = PQ$ and $AM = LM$.

$\Rightarrow$ $PQ - LM = AQ - AM = MQ = 1$m.

Hence the required difference is 1m.

Example 2: Apoorv is standing in the centre of a rectangular park and observes that the angle of elevation of the top of a lamp post at a corner of the park in 60°. He then moves diagonally towards the opposite corner of the park and observes that the angle of elevation is now β, then the value of β is

(a) 45° (b) 30°
(c) $\tan^{-1}(\sqrt{3}/2)$ (d) $\tan^{-1}(2/\sqrt{3})$

Ans. (c)

Solution: Let P and Q be the opposite corners of the park with centre O. If h is the height of the lampost at P

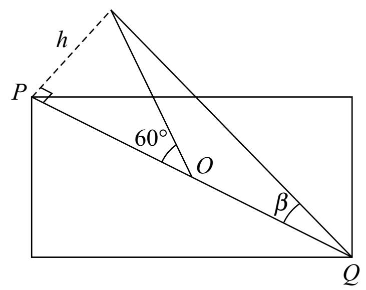

Fig. 27.5

then $OP = h \cot 60° = h/\sqrt{3}$.

$PQ = h \cot \beta = 2\, OP = 2h/\sqrt{3}$.

$\Rightarrow \cot \beta = (2/\sqrt{3}) \Rightarrow \beta = \tan^{-1}(\sqrt{3}/2)$.

Example 3: A bird is sitting on the top of a vertical pole 20m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (m/s) of the bird is

(a) $40(\sqrt{2}-1)$ (b) $40(\sqrt{3}-\sqrt{2})$
(c) $20\sqrt{2}$ (d) $20(\sqrt{3}-1)$

Ans. (d)

Solution: Let PQ be the pole of height 20 m $\angle POQ = 45°$

Let P' be the position of the bird after one second

$\angle P'OQ' = 30°$.

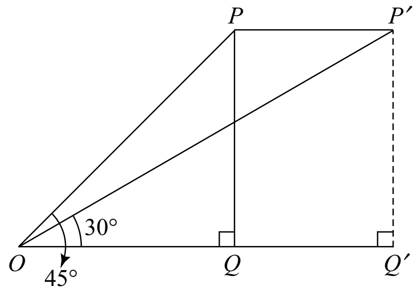

Fig. 27.6

$$PP' = QQ' = PQ' - PQ = 20 \cot 30° - 20 \cot 45°$$
$$= 20\left(\sqrt{3} - 1\right)$$

Example 4: Two ships A and B are sailing straight away from the foot of a tower OP along routes such that $\angle AOB$ is always 120°. At a certain instance, the angles of depression of the ships A and B from the top P of the towers are 60° and 30° respectively. The distance between the ships when the height of the tower is 15m is

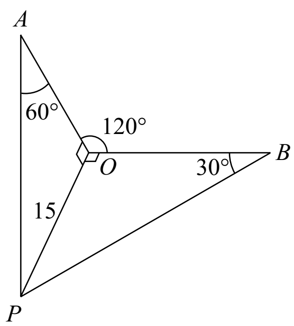

Fig. 27.7

(a) $5\sqrt{39}$ m (b) $5\sqrt{30}$ m

(c) $5\sqrt{21}$ m (d) $5\sqrt{3}$ m

Ans. (a)

Solution: $OP = 15$m

$\angle PAO = 60°$

$\angle PBO = 30°$

$\angle AOB = 120°$

$$OA = OP \cot 60° = 15/\sqrt{3}$$
$$OB = OP \cot 30° = 15\sqrt{3}$$

From ΔAOB

$$(AB)^2 = (OA)^2 + (OB)^2 - 2OA \times OB \times \cos 120°$$
$$= \left(15^2\right)\left[\frac{1}{3} + 3 - 2\left(-\frac{1}{2}\right)\right]$$
$$= 15^2 \times \frac{13}{3} = \frac{15 \times 15 \times 13}{3} = 5\sqrt{39}$$

Example 5: A vertical pole stands at a point A on the boundary of a circular park of radius 2 km. and subtends an angle 60° at another point B on the boundary. If the chord AB subtends the same angle 60° at the centre of the park, the height of the pole is

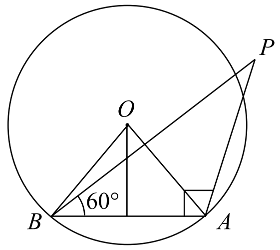

Fig. 27.8

(a) $2\sqrt{3}$ km (b) $\sqrt{3}$ km

(c) $2/\sqrt{3}$ km (d) 1 km.

Ans. (a)

Solution: Let AP be the pole, $\angle PBA = 60°$

$\Rightarrow AB = AP \cot 60° = \dfrac{h}{\sqrt{3}}$, h being the height of the pole.

Let O be the centre of the park.

Then $\angle AOB = 60° \Rightarrow \Delta OAB$ is equilateral and hence $AB = OB = 2$km.

$\Rightarrow \dfrac{h}{\sqrt{3}} = 2 \Rightarrow h = 2\sqrt{3}$ km .

LEVEL 1

Straight Objective Type Questions

Example 6: A flagstaff stands in the centre of a rectangular field whose diagonal is 1200 m, and subtends angles 15° and 45° at the mid points of the sides of the field. The height of the flagstaff is

(a) 200 m (b) $300\sqrt{2+\sqrt{3}}$ m

(c) $300\sqrt{2-\sqrt{3}}$ m (d) 400 m

Ans. (c)

◎ **Solution:** Let OP be the flagstaff of height h standing at the centre O of the rectangular field $ABCD$ subtending angles 15° and 45° at E and F the mid points of the sides AD and DC of the field.(Fig. 27.9)

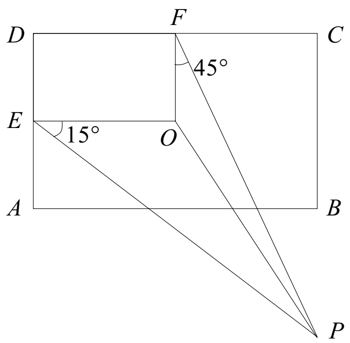

Fig. 27.9

then $OE = h \cot 15°$

$= h(2 + \sqrt{3})$

and $OF = h.\cot 45° = h.$

$\Rightarrow \quad EF = h\sqrt{1+\left(2+\sqrt{3}\right)^2}$

$= 2h\sqrt{2+\sqrt{3}}$

$\Rightarrow \quad 1200 = AC = 2EF = 4h\sqrt{2+\sqrt{3}}$

$\Rightarrow \quad h = \dfrac{300}{\sqrt{2+\sqrt{3}}} = 300\sqrt{2-\sqrt{3}}.$

◉ **Example 7:** Two flagstaffs stand on a horizontal plane. A and B are two points on the line joining their feet and between them. The angles of elevation of the tops of the flagstaffs as seen from A are 30° and 60° and as seen from B are 60° and 45°. If AB is 30 m, the distance between the flagstaffs in metres is

(a) $30 + 15\sqrt{3}$ (b) $45 + 15\sqrt{3}$

(c) $60 - 15\sqrt{3}$ (d) $60 + 15\sqrt{3}$

Ans. (d)

◎ **Solution:** Let x and y be the heights of the flagstaffs at P and Q respectively.

Then $AP = x \cot 60° = x/\sqrt{3}$,

$AQ = y \cot 30° = y\sqrt{3}$

$BP = x \cot 45° = x,$

$BQ = y \cot 60° = y/\sqrt{3}$ (Fig. 27.10)

$\Rightarrow \quad BP - AP = x - x/\sqrt{3} = AB$

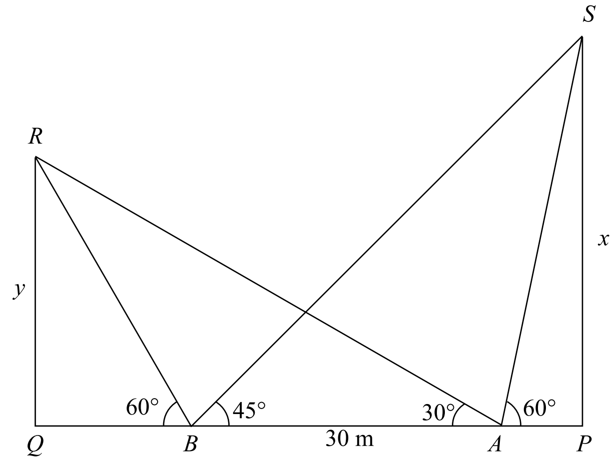

Fig. 27.10

$\Rightarrow \quad 30\sqrt{3} = \left(\sqrt{3}-1\right)x$

$\Rightarrow \quad x = 15\left(3+\sqrt{3}\right)$

Similarly $30 = y\left(\sqrt{3} - 1/\sqrt{3}\right)$

$\Rightarrow \quad y = 15\sqrt{3}.$

so that $PQ = BP + BQ = x + y/\sqrt{3}$

$= 15\left(3+\sqrt{3}\right) + 15$

$= (60 + 15\sqrt{3})$ m.

◉ **Example 8:** In a cubical hall $A\ B\ C\ D\ P\ Q\ R\ S$ with each side 10 m, G is the centre of the wall $B\ C\ R\ Q$ and T is the mid point of the side AB. The angle of elevation of G at the point T is

(a) $\sin^{-1}(1/\sqrt{3})$ (b) $\cos^{-1}(1/\sqrt{3})$

(c) $\tan^{-1}(1/\sqrt{3})$ (d) $\cot^{-1}(1/\sqrt{3})$

Ans. (a)

◎ **Solution:** Let H be the mid point of BC since $\angle TBH = 90°$, $TH^2 = BT^2 + BH^2 = 5^2 + 5^2 = 50$

Also since $\angle THG = 90°$, $TG^2 = TH^2 + GH^2 = 50 + 25 = 75$

Let θ be the required angle of elevation of G at T. (Fig. 27.11)

then $\sin\theta = \dfrac{GH}{TG} = \dfrac{5}{5\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = \sin^{-1}(1/\sqrt{3})$

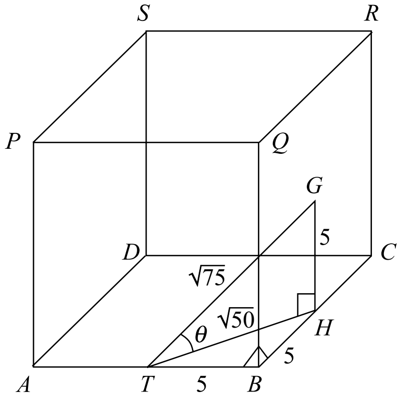

Fig. 27.11

Example 9: Two vertical poles 20 m and 80 m high stand apart on a horizontal plane. The height of the point of intersection of the lines joining the top of each pole to the foot of the other is

(a) 15 m (b) 16 m
(c) 18 m (d) 50 m

Ans. (b)

Solution: Let PQ and RS be the poles of height 20 m and 80 m subtending angles α and β at R and P respectively. Let h be the height of the point T, the intersection of QR and PS. (Fig. 27.12)

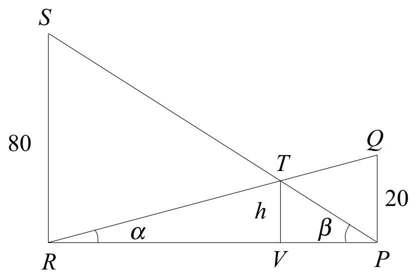

Fig. 27.12

Then $PR = h \cot \alpha + h \cot \beta = 20 \cot \alpha = 80 \cot \beta$

$\Rightarrow \quad \cot \alpha = 4 \cot \beta$ or $\dfrac{\cot \alpha}{\cot \beta} = 4$

Again $h \cot \alpha + h \cot \beta = 20 \cot \alpha$

$\Rightarrow \quad (h - 20) \cot \alpha = -h \cot \beta$

$\Rightarrow \quad \dfrac{\cot \alpha}{\cot \beta} = \dfrac{h}{20 - h} = 4$

$\Rightarrow \quad h = 80 - 4h \quad \Rightarrow \quad h = 16$ m.

Example 10: A man from the top of a 100 metres high tower observes a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in metres) travelled by the car during this time is

(a) $100\sqrt{3}$ (b) $200/\sqrt{3}$
(c) $100/\sqrt{3}$ (d) $200\sqrt{3}$

Ans. (b)

Solution: Let OP be the tower and C_1, C_2 be the positions of the car (Fig. 27.13)

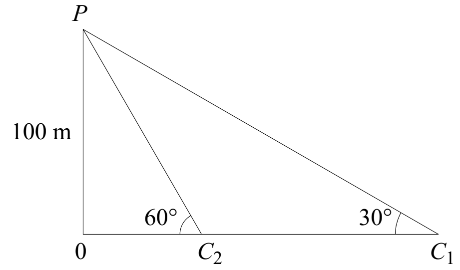

Fig. 27.13

We have $\quad \dfrac{OC_1}{OP} = \cot 30° = \sqrt{3}$

$\Rightarrow \quad OC_1 = 100\sqrt{3}$

Similarly $\quad OC_2 = 100/\sqrt{3}$

Now $\quad C_1 C_2 = OC_1 - OC_2$

$= 100(\sqrt{3} - 1/\sqrt{3})$

$= 200/\sqrt{3}$.

Example 11: A pole stands vertically, inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in ΔABC, the foot of the pole is at the

(a) centroid (b) circumcentre
(c) incentre (d) orthocentre

Ans. (b)

Solution: Let OP be the pole with O as the foot, and the angle of elevation of the top P of the pole at each of the corner A, B, C be α. (Fig. 27.14)

then $\quad \dfrac{OP}{OA} = \tan \alpha$

$\Rightarrow \quad OA = OP \cot \alpha$

Similarly $\quad OB = OC = OP \cot \alpha$

Since $OA = OB = OC$, O is the circumcentre of the triangle ABC.

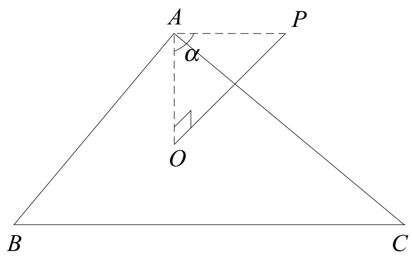

Fig. 27.14

Example 12: A man observes that the angle of elevation of the top of a tower from a point P on the ground is θ.

He moves a certain distance towards the foot of the tower and finds that the angle of elevation of the top has doubled. He further moves a distance 3/4 of the previous and finds that the angle of elevation is three times that at P. The angle θ is given by

(a) $\sin\theta = \sqrt{5/12}$ (b) $\cos\theta = \sqrt{5/12}$
(c) $\sin\theta = 3/4$ (d) $\cos\theta = 3/8$

Ans. (a)

◎ **Solution:** Let AB be the tower,

$$\angle APB = \theta,\ \angle AQB = 2\theta \text{ and } \angle ARB = 3\theta$$

Then $QR = (3/4)\ PQ$ and $\angle PBQ = \angle QBR = \theta$

$\Rightarrow$ BQ is the bisector of $\angle PBR$ (Fig. 27.15)

$$\Rightarrow \frac{PB}{BR} = \frac{PQ}{QR} \Rightarrow \frac{AB\operatorname{cosec}\theta}{AB\operatorname{cosec}3\theta} = \frac{4}{3} \Rightarrow \frac{\sin 3\theta}{\sin\theta} = \frac{4}{3}$$

$$\Rightarrow 3 - 4\sin^2\theta = 4/3 \Rightarrow 12\sin^2\theta = 5 \Rightarrow \sin\theta = \sqrt{5/12}\,.$$

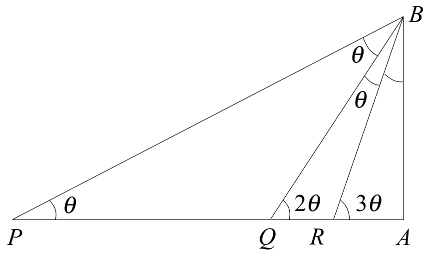

Fig. 27.15

⦿ **Example 13:** A vertical pole subtends an angle $\tan^{-1}(1/2)$ at a point P on the ground. The angle subtended by the upper half of the pole at the point P is

(a) $\tan^{-1}(1/4)$ (b) $\tan^{-1}(2/9)$
(c) $\tan^{-1}(1/8)$ (d) $\tan^{-1}(2/3)$

Ans. (b)

◎ **Solution:** Let the pole AB subtend angle θ at P and the upper half BC of the pole subtend angle α at P (Fig. 27.16)

then $\tan\theta = \dfrac{1}{2} = \dfrac{AB}{AP}$

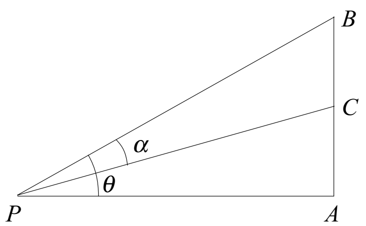

Fig. 27.16

$$\tan(\theta - \alpha) = \frac{AC}{AP} = \frac{(1/2)AB}{AP} = \frac{1}{4}.$$

Now $\tan\alpha = \tan(\theta - (\theta - \alpha))$

$$= \frac{\tan\theta - \tan(\theta-\alpha)}{1 + \tan\theta\tan(\theta-\alpha)} = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2}\times\frac{1}{4}} = \frac{2}{9}$$

$\Rightarrow$ $\alpha = \tan^{-1}(2/9)$.

⦿ **Example 14:** An aeroplane flying at a height of 3000 m above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. The height of the lower plane from the ground is

(a) $1000\sqrt{3}$ m (b) $1000/\sqrt{3}$ m
(c) 500 m (d) $1500(\sqrt{3}+1)$ m

Ans. (a)

◎ **Solution:** Let Q and P be the position of the two planes and A be the point on the ground such that $\angle QAO = 60°$, $\angle PAO = 45°$ (Fig. 27.17) and OP, the height of lower plane be h

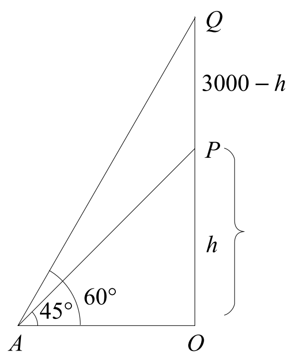

Fig. 27.17

Also $OQ = 3000$ m
Then $OA = OP = h$
and $OQ = h\tan 60° = 3000$

$$\Rightarrow h = 3000\cot 60° = \frac{3000}{\sqrt{3}} = 1000\sqrt{3}\ \text{m}$$

⦿ **Example 15:** A pole of height h stands at one corner of a park in the shape of an equilateral triangle. If α is the angle which the pole subtends at the midpoint of the opposite side, the length of each side of the park is

(a) $(\sqrt{3}/2)\ h\cot\alpha$ (b) $(2/\sqrt{3})\ h\cot\alpha$
(c) $(\sqrt{3}/2)\ h\tan\alpha$ (d) $(2/\sqrt{3})\ h\tan\alpha$

Ans. (b)

◎ **Solution:** Let ABC be the triangular park, AP the pole at A, and D the midpoint of BC. Let each side of the equilateral triangle ABC be a. Then (Fig. 27.18)

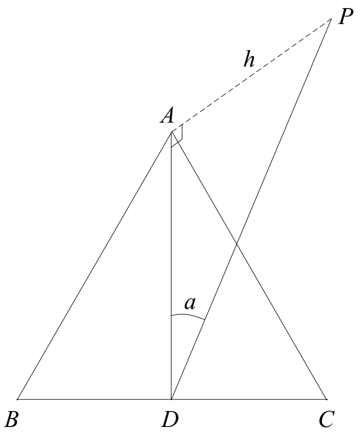

Fig. 27.18

$$AD^2 = AB^2 - BD^2 = a^2 - a^2/4 = 3a^2/4$$

$\Rightarrow \quad AD = a\sqrt{3}/2$

and, since $\quad AP = h$ and $\angle ADP = \alpha,$

we have $\quad AD = h \cot \alpha.$

Therefore, $\quad a\sqrt{3}/2 = h \cot \alpha$

$\Rightarrow \quad a = \left(2/\sqrt{3}\right) h \cot \alpha$

Example 16: If each side of length a of an equilateral triangle subtends an angle of 60° at the top of a tower h metre high situated at the centre of the triangle, then

(a) $3a^2 = 2h^2$ (b) $2a^2 = 3h^2$
(c) $a^2 = 3h^2$ (d) $3a^2 = h^2$

Ans. (b)

Solution: Let O be the centre of the equilateral triangle ABC and OP be the tower of height h. (Fig. 27.19)
Then each of the triangles PAB, PBC and PCA are equilateral and thus

$$PA = PB = AB = a.$$

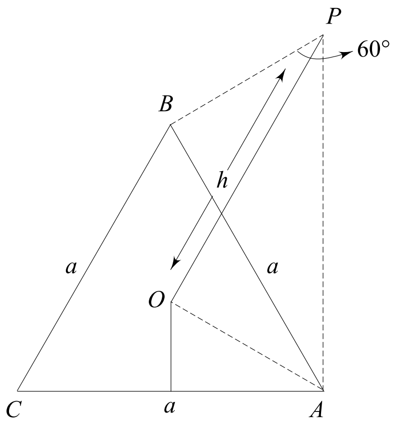

Fig. 27.19

In triangle ABC, OA is the bisector of the angle A,

So $\quad \dfrac{OA}{a/2} = \sec 30°$

$\Rightarrow \quad OA = \dfrac{a}{2} \cdot \dfrac{2}{\sqrt{3}} = \dfrac{a}{\sqrt{3}}.$

Now from right angled triangle POA

$$PA^2 = OP^2 + OA^2$$

$\Rightarrow \quad a^2 = h^2 + \dfrac{a^2}{3} \Rightarrow \dfrac{2a^2}{3} = h^2$

$\Rightarrow \quad 2a^2 = 3h^2.$

Example 17: A pole 50 m high stands on a building 250 m high. To an observer at a height of 300 m, the building and the pole subtend equal angles. The horizontal distance of the observer from the pole is

(a) 25 m (b) 50 m
(c) $25\sqrt{6}$ m (d) $25\sqrt{3}$ m

Ans. (c)

Solution: Let PQ be the pole on the building QR and O be the observer.

Then $\quad PQ = 50$, $QR = 250$ (Fig. 27.20)

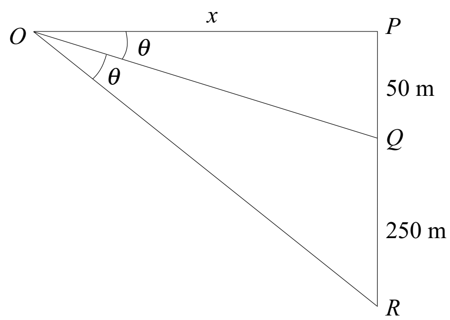

Fig. 27.20

$\Rightarrow \quad PR = 300$ so the observer is at the same height as the top P of the pole. Let $OP = x$. Then from right angled triangles OPQ and OPR,

$\tan \theta = \dfrac{50}{x}$ and $\tan 2\theta = \dfrac{300}{x}.$

so that $\quad \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{300}{x}$

$$\Rightarrow \frac{2\times(50/x)}{1-(50/x)^2} = \frac{300}{x} \Rightarrow 3\left\{1-\left(\frac{50}{x}\right)^2\right\} = 1$$

$$\Rightarrow \left(\frac{50}{x}\right)^2 = \frac{2}{3} \Rightarrow x = \frac{50\sqrt{3}}{\sqrt{2}} = 25\sqrt{6}.$$

Example 18: Two vertical poles of height a and b subtend the same angle 45° at a point on the line joining their feet, the square of the distance between their tops is

(a) $(1/2)(a^2 + b^2)$ (b) $a^2 + b^2$

(c) $2(a^2 + b^2)$ (d) $(a + b)^2$

Ans. (c)

Solution: Let OP and QR be the poles of height a and b respectively subtending angle 45° at S, a point between O and Q. (Fig. 27.21)

then $OS = a$, $QS = b$

$PS^2 = 2a^2$ and $RS^2 = 2b^2$

Since $\angle PSR = 180° - (45° + 45°) = 90°$

$PR^2 = PS^2 + RS^2 = 2(a^2 + b^2)$

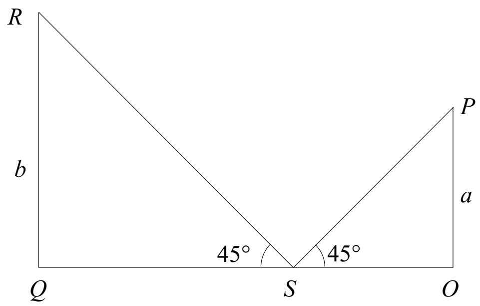

Fig. 27.21

Example 19: A monument $ABCD$ stands at A on a level ground. At a point P on the ground the portions AB, AC, AD subtend angles α, β, γ respectively. If $AB = a$, $AC = b$, $AD = c$, $AP = x$ and $\alpha + \beta + \gamma = 180°$ then x^2 is equal to

(a) $\dfrac{a}{a+b+c}$ (b) $\dfrac{b}{a+b+c}$

(c) $\dfrac{c}{a+b+c}$ (d) $\dfrac{abc}{a+b+c}$

Ans. (d)

Solution: We have

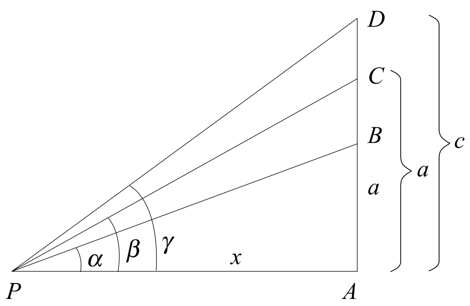

Fig. 27.22

$a = AB = AP \tan \alpha = x \tan \alpha$ (Fig. 27.22)

$b = AC = x \tan \beta$

and $c = AD = x \tan \gamma$

so that $a + b + c = x(\tan \alpha + \tan \beta + \tan \gamma)$

$= x \tan \alpha \tan \beta \tan \gamma$ [$\because \alpha + \beta + \gamma = 180°$]

and $abc = x^3 \tan \alpha \tan \beta \tan \gamma$

$$\Rightarrow \quad x^2 = \frac{abc}{a+b+c}$$

Example 20: A vertical tower CP subtends the same angle θ, at point B on the horizontal plane through C, the foot of the tower, and at point A in the vertical plane. If the triangle ABC is equilateral with length of each side equal to 4 m, the height of the tower is

(a) $8\sqrt{3}$ m (b) $4\sqrt{3}/3$ m

(c) $4\sqrt{3}$ m (d) $8/\sqrt{3}$ m

Ans. (b)

Solution: Since $\angle CBP = \angle CAP = \theta$ (angles in the same segment are equal) $PCBA$ is a cyclic quadrilateral. (Fig. 27.23)

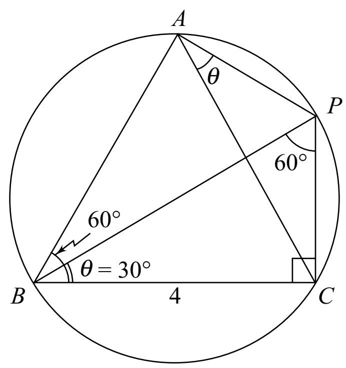

Fig. 27.23

$\Rightarrow \quad \angle BAP + \angle BCP = 180°$

$\Rightarrow \quad \angle BAP = 90°$ $[\because \angle BCP = 90°]$

$\Rightarrow \quad \angle PBC = \theta = \angle BAP - \angle BAC$

$= 90° - 60° = 30°$

If h is the height of the tower CP, then $\dfrac{CP}{CB} = \tan \theta = \tan A$

$$\Rightarrow \quad \frac{h}{4} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad h = \frac{4}{\sqrt{3}} \text{ m}$$

Example 21: A man on the ground observes that the angle of elevation of the top of a tower is 68° 11′, and a flagstaff 24 m high on the summit of the tower subtends an angle of 2° 10′ at the observer's eye. If tan 70° 21′ = 2.8 and cot 68° 11′ = 0.4, the height of the tower is

(a) 120 m (b) 168 m

(c) 200 m (d) 300 m

Ans. (c)

◎ **Solution:** Let PQ be the tower of height h whose elevation at A is $68° 11'$ and QR be the flagstaff of height 24 m subtending an angle of $2° 10'$ at A. (Fig. 27.24)

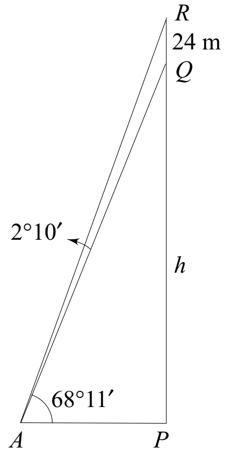

Fig. 27.24

Then $h \cot 68° 11' = AP$

$= (h + 24) \cot (68° 11' + 2° 10')$

$\Rightarrow \quad h \cot 68°11' = \dfrac{(h+24)}{\tan 70° 21'}$

$\Rightarrow \quad h\ (0{\cdot}4)\ (2{\cdot}8) = h + 24$

$\Rightarrow \quad h = \dfrac{24}{1{\cdot}12 - 1} = \dfrac{24}{{\cdot}12}$

$= 200$ m

◉ **Example 22:** A statue, standing on the top of a pillar 25 m high, subtends an angle whose tangent is 0.125 at a point 60 m from the foot of the pillar. The best approximation for the height of the statue is

(a) 9.28 m (b) 9.29 m
(c) 9.30 m (d) 10 m

Ans. (b)

◎ **Solution:** Let PQ be the statue of height x on the top of the pillar OP, 25 m high, subtending an angle α at the point A, 60 m from O, the foot of the tower and $\angle OAP = \theta$. (Fig. 27.25)

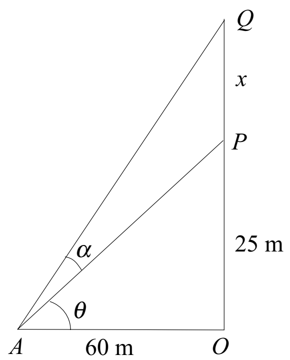

Fig. 27.25

Then $\tan \alpha = {\cdot}125$ (given)

and $\tan \theta = \dfrac{OP}{OA} = \dfrac{25}{60} = \dfrac{5}{12}$

then $(x + 25) \cot (\alpha + \theta) = 60$

$\Rightarrow \quad x + 25 = 60 \tan (\alpha + \theta)$

$= 60\left[\dfrac{\tan\alpha + \tan\theta}{1 - \tan\alpha \tan\theta}\right]$

$\Rightarrow \quad x = 60\left[\dfrac{(1/8)+(5/12)}{1-(1/8)\times(5/12)}\right] - 25$

$= 60 \times (52/91) - 25 = 60 \times (4/7) - 25$

$= 65/7$

$= 9.2857 = 9.29$ (approximately).

◉ **Example 23:** A tower BCD surmounted by a spire DE stands on a horizontal plane. At the extremity A of a horizontal line BA it is found that BC and DE subtend equal angles. If $BC = 3$ m, $CD = 28$ m and $DE = 5$ m, then BA is equal to

(a) $\sqrt{18 \times 93}$ (b) $\sqrt{36 \times 93}$
(c) $\sqrt{34 \times 93}$ (d) $\sqrt{34 \times 36}$

Ans. (a)

◎ **Solution:** Let $\angle BAC = \angle DAE = \theta$

$\angle DAB = \alpha$, $AB = x$. (Fig. 27.26)

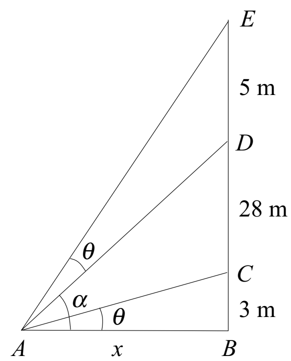

Fig. 27.26

Now $BC = 3$ m, $CD = 28$ m, $DE = 5$ m.

$\therefore \quad \tan(\alpha + \theta) = 36/x$

$\tan(\alpha) = 31/x, \ \tan\theta = 3/x$

so, $\tan(\alpha + \theta) = \dfrac{\tan\alpha + \tan\theta}{1 - \tan\alpha \tan\theta}$

$$\Rightarrow \quad \frac{36}{x} = \frac{\frac{31}{x} + \frac{3}{x}}{1 - \frac{31}{x} \times \frac{3}{x}}$$

$\Rightarrow \quad 36(x^2 - 93) = 34x^2 \Rightarrow 2x^2 = 36 \times 93$

$\Rightarrow \quad x^2 = 18 \times 93$

$\Rightarrow \quad x = \sqrt{18 \times 93}$.

◉ **Example 24:** A lamp post standing at a point A on a circular path of radius r subtends an angle α at some point B on the path, and AB subtends an angle of 45° at any other point on the path, then height of the lampost is

(a) $\sqrt{2}\ r \cot \alpha$ (b) $(r/\sqrt{2}) \tan \alpha$

(c) $\sqrt{2}\ r \tan \alpha$ (d) $(r/\sqrt{2}) \cot \alpha$

Ans. (c)

Solution: Let AP be the lampost of height h at a point A on a circular path of radius r and centre C. Let B be the point on this path such that $\angle PBA = \alpha$. $\Rightarrow$ $AB = h \cot \alpha$ (Fig. 27.27)

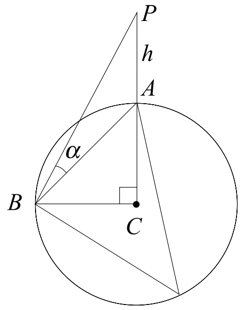

Fig. 27.27

Since AB subtends an angle 45° at another point of the path, it subtends an angle of 90° at the centre C so that $\angle BCA = 90°$

Also $CA = CB = r \Rightarrow AB = \sqrt{2}\ r$

and then $h \cot \alpha = \sqrt{2}\ r$

$\Rightarrow \quad h = \sqrt{2}\ r \tan \alpha.$

Example 25: A tree is broken by wind, its upper part touches the ground at a point 10 m from the foot of the tree and makes an angle 45° with the ground. The entire length of the tree was

(a) 15m (b) 20m

(c) $10(\sqrt{2} + 1)$m (d) $5(\sqrt{3} + 2)$ m

Ans. (c)

Solution: Let OPQ be the tree whose upper part PQ makes an angle 45° with the ground.

Then the length of the tree (Fig. 27.28)

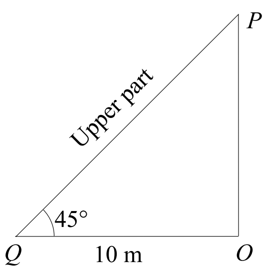

Fig. 27.28

$= OP + PQ = 10 \tan 45° + 10 \sec 45°$

$= 10 + 10\sqrt{2} = 10(\sqrt{2} + 1)$ m

Example 26: From the top of a cliff x m high, the angle of depression of the foot of a tower is found to be double the angle of elevation of the tower. If the height of the tower is h, the angle of elevation is

(a) $\sin^{-1}\sqrt{x/(2-h)}$ (b) $\tan^{-1}\sqrt{3-2h/x}$

(c) $\sin^{-1}\sqrt{2h/x}$ (d) $\cos^{-1}\sqrt{2h/x}$

Ans. (b)

Solution: Let OQ be the tower of height h with O as its foot. AB be the cliff of height x. (Fig. 27.29)

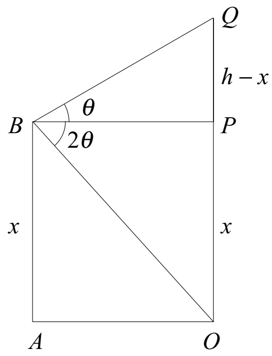

Fig. 27.29

$PQ = h - x, \quad \angle QBP = \theta$ and $\angle PBO = 2\theta$

Then $(h - x) \cot \theta = x \cot 2\theta$

$\Rightarrow \quad (h - x) \tan 2\theta = x \tan \theta$

$$\Rightarrow \quad \frac{h-x}{x} = \frac{\tan \theta}{\tan 2\theta}$$

$$\Rightarrow \quad \frac{h-x}{x} + 1 = \frac{1-\tan^2 \theta}{2} + 1$$

$$\Rightarrow \quad \frac{2h}{x} = 3 - \tan^2 \theta$$

$\Rightarrow \theta = \tan^{-1}\sqrt{3 - 2h/x}$.

Example 27: A river flows due North, and a tower stands on its left bank. From a point A upstream and on the same bank as the tower, the elevation of the tower is 60°, and from a point B just opposite A on the other bank the elevation is 45°. If the tower is 360 m high, the breadth of the river is

(a) $120\sqrt{6}$ m (b) $240/\sqrt{3}$ m

(c) $240\sqrt{3}$ m (d) $240\sqrt{6}$ m

Ans. (a)

Solution: Let PQ be the tower of height 360 m standing on the left bank of the river subtending an angle 60° at A on the same bank as the tower. (Fig. 27.30) Then PA = 360 cot 60°

Let B be the point opposite to A on the other bank where the elevation of the tower is 45°. So that $PB = 360 \cot 45° = 360$.

If x be the width of the river, then $AB = x$ and from the right angled triangle PAB,

$$(PA)^2 + (AB)^2 = (PB)^2$$

$$\Rightarrow \quad \left(360 \times \frac{1}{\sqrt{3}}\right)^2 + x^2 = (360)^2$$

$$\Rightarrow \quad (360)^2 \left(1 - \frac{1}{3}\right) = x^2$$

$$\Rightarrow \quad x = 360 \times \frac{\sqrt{2}}{\sqrt{3}} = 120\sqrt{6} \text{ m.}$$

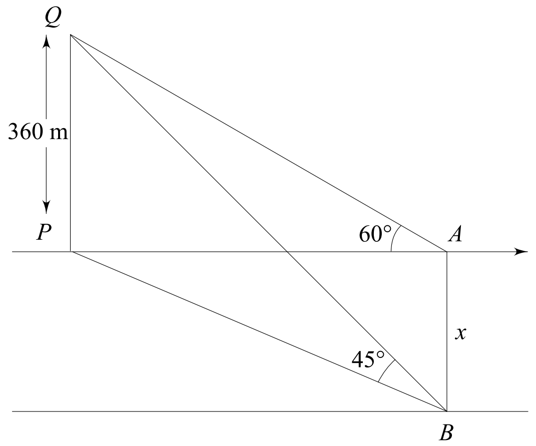

Fig. 27.30

Example 28: The angle of elevation of a cloud at a height h above the level of water in a lake is α, and the angle of depression of its image in the lake is β. The height of the cloud above the surface of the lake is

(a) $\dfrac{h(\cot\alpha + \cot\beta)}{\cot\beta - \cot\alpha}$ (b) $\dfrac{h(\tan\alpha - \tan\beta)}{\tan\alpha + \tan\beta}$

(c) $\dfrac{h\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ (d) $\dfrac{h\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$

Ans. (c)

Solution: Let the height of the cloud above the water level be x. Let A be the point at a height h above the water level from which the cloud's angles of depression and elevation are measured. Let L and L' be the locations of the cloud and its image, respectively. Further, as shown in Fig. 27.31, let P be the point directly below the cloud, midway between L and L', and Q the point directly above P at a height h. Then $\angle LAQ = \alpha$, $\angle QAL' = \beta$, $PQ = h$, $QL = x - h$, $PL' = x$, and we have $(x - h)\cot\alpha = (x + h)\cot\beta = AQ$, i.e.,

$$\frac{x+h}{x-h} = \frac{\tan\beta}{\tan\alpha} \Rightarrow \frac{2x}{2h} = \frac{\tan\beta + \tan\alpha}{\tan\beta - \tan\alpha}$$

$$\Rightarrow \quad x = h\left(\frac{\tan\beta + \tan\alpha}{\tan\beta - \tan\alpha}\right) = h\left(\frac{\cot\alpha + \cot\beta}{\cot\alpha - \cot\beta}\right)$$

$$= h\left[\frac{\sin(\alpha+\beta)}{\sin(\beta-\alpha)}\right],$$

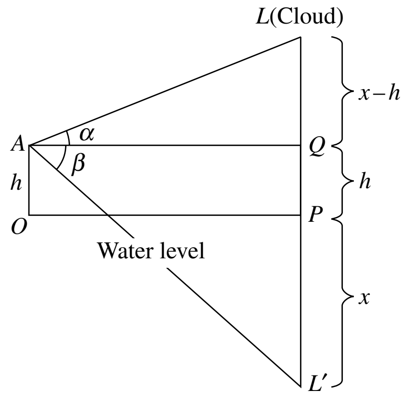

Fig. 27.31

Example 29: A balloon of radius r subtends an angle α at the eye of an observer and the elevation of the centre of the balloon from the eye is β, the height h of the centre of the balloon is given by

(a) $\dfrac{r\sin\beta}{\sin\alpha}$ (b) $r \sin b \sin a$

(c) $\dfrac{r\sin\beta}{\sin(\alpha/2)}$ (d) $\dfrac{r\sin\alpha}{\sin(\beta/2)}$

Ans. (c)

Solution: Let O be the centre of the balloon of radius r which subtends an angle α at E, the eye of the observer. If EA and EB are the tangents to the balloon, then $\angle OEA = \angle OEB = \alpha/2$ (Fig. 27.32)

Let $OL = h$ be the height of the centre of the balloon, then from ΔOLE

$$h = OE \sin\beta$$
$$= r \operatorname{cosec}(\alpha/2) \sin\beta$$
$$= r \sin\beta / \sin(\alpha/2).$$

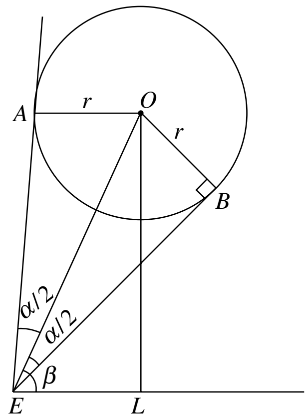

Fig. 27.32

Example 30: Two poles of height a and b stand at the centres of two circular plots which touch each other externally at a point and the two poles subtend angles of 30° and 60° respectively at this point, then distance between the centres of these poles is

(a) $a + b$ (b) $(3a + b)/\sqrt{3}$

(c) $(a + 3b)/\sqrt{3}$ (d) $a\sqrt{3} + b$

Ans. (b)

Solution: Let A and B be the centres of the two circles where the poles of height a and b respectively stand making angles 30° and 60° respectively at the point O where these circle touch each other externally. (Fig. 27.33)

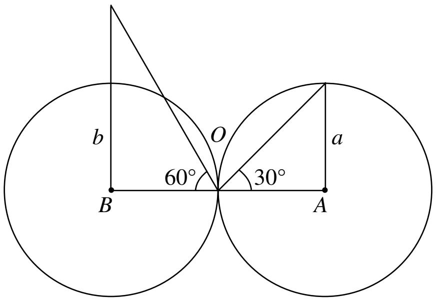

Fig. 27.33

sum of the radii of the two circles

$= a \cot 30° + b \cot 60°$

$= a\sqrt{3} + \dfrac{b}{\sqrt{3}} = \dfrac{3a+b}{\sqrt{3}}$

= distance between the centres of the two circles.

Example 31: A tower is standing in the centre of an elliptic field. If Adya observes that the angle of elevation of the top of the tower at an extremity of the major axis of the field is α, at its focus is β and an extremity of the minor axis is γ, then

(a) $\cot^2\alpha = \cot^2\beta - \cot^2\gamma$
(b) $\cot^2\beta = \cot^2\gamma - \cot^2\alpha$
(c) $\cot^2\gamma = \cot^2\alpha - \cot^2\beta$
(d) none of these

Ans. (c)

Solution: Let the equation of the elliptic field be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

OP be the tower of height h at O, the centre of the field, Let A & B be extremities of major and minor axis respectively and S be a focus, then

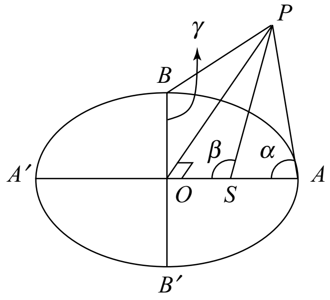

Fig. 27.34

$OA = h\cot\alpha = a$

$OS = h\cot\beta = ae$

$OB = h\cot\gamma = b$

Since $b^2 = a^2(1 - e^2)$

$\Rightarrow \cot^2\gamma = \cot^2\alpha - \cot^2\beta$

Example 32: A tower of height h stands at a point O on the ground. Two poles of height a and b stand at the points A and B respectively such that O lies on the line joining A and B. If the angle of elevation of the top of the tower at the foot of one pole is same as at the top of the other pole, then h is equal to

(a) $\dfrac{a+b}{ab}$ (b) $\dfrac{ab}{a+b}$

(c) $a + b$ (d) $\dfrac{a+b}{|a-b|}$

Ans. (b)

Solution: Let L and M be the tops of the poles at A and B respectively. Angle of elevation of P at A and M be α, and at B and L be β.

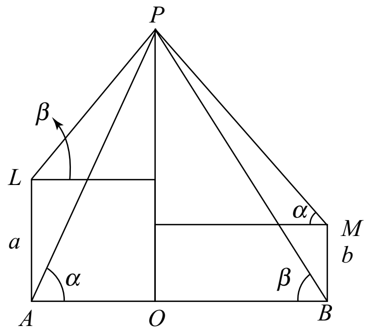

Fig. 27.35

then $OA = h\cot\alpha = (h - a)\cot\beta$

and $OB = h\cot\beta = (h - b)\cot\alpha$

$\Rightarrow \dfrac{h-a}{h} = \dfrac{h}{h-b} \Rightarrow h = \dfrac{ab}{a+b}$

Example 33: Rajat observes that the angle of elevation of the first floor of a building at a point A on the ground is $30°$. He moves $\sqrt{3}$ units towards the building to the point B and finds that the angle of elevation of the second floor of the building is $60°$. If each floor has the same height, height of the 7th floor from the ground in units is

(a) 5 (b) 7
(c) 21 (d) 35

Ans. (c)

Solution: Let the height of each floor be x units

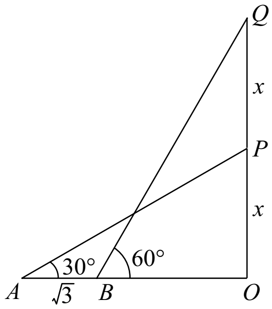

Fig. 27.36

$$\begin{aligned}\sqrt{3} &= AB = OA - OB\\ &= x\cot 30° - 2x\cot 60°\\ &= x\sqrt{3} - \frac{2x}{\sqrt{3}}\end{aligned}$$

$\Rightarrow$ $x = 3$ and hence the required height is 21 units.

Example 34: A pole of height h stands in the centre of a circular platform in the centre of a circular field. Another pole of equal height is at a point on the boundary of the field. The angles of elevation of the top of the first pole from the bottom and top of the second pole are respectively α and β. Height of the platform from the ground is

(a) $\dfrac{h\cot\alpha}{\cot\beta - \cot\alpha}$ (b) $\dfrac{h\cot\beta}{\cot\alpha - \cot\beta}$
(c) $\cot\alpha - \cot\beta$ (d) none of these

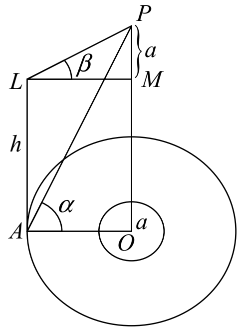

Fig. 27.37

Ans. (a)

Solution: Let a be the height of the platform then height of the first pole from the ground is $h + a$ and from the top of the second pole is a, so

$$(h + a)\cot\alpha = a\cot\beta$$

$$\Rightarrow \quad a = \frac{h\cot\alpha}{\cot\beta - \cot\alpha}$$

Example 35: The tangents of the angles subtended by a tower at four points A, B, C and D on the ground are in H.P. If O be the foot of the tower on the ground, then

(a) $OA + OC = OB + OD$
(b) $OA + OB = OC + OD$
(c) $OA + OD = OB + OC$
(d) $AB + CD = BC + CD$

Ans. (c)

Solution: If α, β, γ, δ be the four angles such that $\tan\alpha$, $\tan\beta$, $\tan\gamma$, $\tan\delta$ are in H.P, then $\cot\alpha$, $\cot\beta$, $\cot\gamma$, $\cot\delta$ are in A.P

$\Rightarrow$ $h\cot\alpha$, $h\cot\beta$, $h\cot\gamma$, $h\cot\delta$ are in A.P.

$\Rightarrow$ OA, OB, OC, OD are in A.P

$\Rightarrow$ $OA + OD = OB + OC$

Assertion-Reason Type Questions

Example 36: Statement-1: A pole standing in the centre of a rectangular field of area 2500 sq. units subtends angle α and β respectively at the mid-points of two adjacent sides of the field such that $\alpha + \beta = \pi/2$, the height of the pole is 25 sq. units.

Statement-2: Area of a rectangle is equal to the product of the length of the adjacent sides.

Ans. (a)

Solution: Statement-2 is True. In statement-1 if the height of the pole is h, then the length of the adjacent sides of the field are $2h\cot\alpha$ and $2h\cot\beta$ and the area is $4h^2\cot\alpha\cot\beta = 4h^2$ as $\alpha + \beta = \pi/2 \Rightarrow \cot\alpha\cot\beta = 1$. So $4h^2 = 2500 \Rightarrow h = 25$ sq. units and the statement-1 is True using statement-2

Example 37: Statement-1: Apoorv, standing on the ground wants to observe the angle α of elevation of the top of a tower in front of him. He walks half the distance towards the foot of the tower and finds the angle of elevation is $\pi/4$. He observes $\alpha = \tan^{-1}(1/2)$

Statement-2: If the angles of elevation of the top of a tower at three distinct points on the ground is α, the points lie on a circle with centre at the foot of the tower.

Ans. (b)

Solution: Statement-2 is True, because if h is the height of the tower, the points are at a distance of $h \cot \alpha$ from the foot of the tower and hence lie on a circle. In statement-1, if h is the height of the tower then $h \cot \alpha = 2h \cot \pi/4 = 2h \Rightarrow \alpha = \tan^{-1} 1/2$

$\Rightarrow$ Statement-1 is true but does not follow from statement-2

Example 38: A and B are two points in a line on the horizontal plane through the foot O of a tower lying on opposite sides of the tower.

Statement-1: If the angles of elevation of the top of the tower at A and B are α and 2α respectively and the distance between the points is twice the height of the tower, then $\tan^2 \alpha + 4 \tan \alpha = 3$.

Statement-2: If $OB = 2(OA)$, α, β are respectively the angles of elevation of the top of the tower at A and B, then $\beta = 2\alpha$.

Ans. (c)

Solution: If h is the height of the tower, in statement-1 $OA = h \cot \alpha$, $OB = h \cot 2\alpha$ so $h (\cot \alpha + \cot 2\alpha) = 2h$.

$$\Rightarrow \quad \frac{1}{\tan\alpha} + \frac{1}{\tan 2\alpha} = 2$$

$$\Rightarrow \quad \frac{1}{\tan\alpha} + \frac{1-\tan^2\alpha}{2\tan\alpha} = 2$$

$\Rightarrow$ $\tan^2 \alpha + 4 \tan \alpha = 3$

$\Rightarrow$ statement-1 is True.

In statement 2, $h \cot \beta = 2h \cot \alpha$

$\Rightarrow$ $\cot \beta = 2\cot \alpha \Rightarrow \beta = 2\alpha$ and the statement is false.

Example 39: Mansi observes that the angle of elevation of a vertical pole of height h at two points A and B on the horizontal plane through the foot O of the pole is $\pi/3$. $AB = a$.

Statement-1: If AB subtends an angle $\pi/2$ at the foot of the tower, then $2h = 3a$.

Statement-2: If AB subtends an angle $\pi/3$ at the foot of the tower then $h = a\sqrt{3}$

Ans. (d)

Solution: $OA = OB = h \cot \pi/3 = h/\sqrt{3}$.

In statement-1 AOB is a right angled triangle.

so $(AB)^2 = (OA)^2 + (OB)^2 \Rightarrow a^2 = 2h^2/3 \Rightarrow 3a^2 = 2h^2$ and the statement is false.

In statement-2 AOB is an equilateral triangle.

so $AB = OA \Rightarrow a = h/\sqrt{3}$

$\Rightarrow$ $h = a\sqrt{3}$ and the statement is True.

Example 40: ABC is an equilateral triangle on the horizontal ground with length of each side equal to a.

Statement-1: If a tower standing at the centre O of the triangle makes an angle α at each corner such that

$\alpha = \tan^{-1} 9$, then height of the tower is $3\sqrt{3}$.

Statement-2: If a tower of height $2a$ standing at one corner of the triangle makes an α at any other corner, then $a = \tan^{-1} 2$.

Ans. (b)

Solution: Since ABC is an equilateral triangle.

$OA = OB = OC = a/\sqrt{3}$. In statement-1 if the height of the tower is h, $h \cot \alpha = a/\sqrt{3}$

$$\Rightarrow \quad h = \frac{a}{\sqrt{3}} \times 9 = 3\sqrt{3}\,a.$$

$\Rightarrow$ statement-1 is True.

In statement-2, $2a \cot \alpha = a$

$\Rightarrow$ $\tan \alpha = 2 \Rightarrow \alpha = \tan^{-1} 2$

so the statement-2 is True but does not lead to statement-1.

LEVEL 2

Straight Objective Type Questions

Example 41: A and B are two points 30 m apart in a line on the horizontal plane through the foot of a tower lying on opposite sides of the tower. If the distances of the top of the tower from A and B are 20 m and 15 m respectively, the angle of elevation of the top of the tower at A is

(a) $\cos^{-1}$ (43/48)
(b) $\sin^{-1}$ (43/48)
(c) $\cos^{-1}$ (29/36)
(d) $\sin^{-1}$ (29/36)

Ans. (a)

◎ **Solution:** Let OP be the tower, then $AB = 30$, $AP = 20$ and $BP = 15$. (Fig. 27.38)

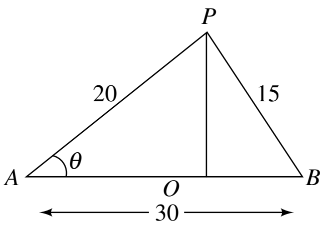

Fig. 27.38

If θ be the elevation at A of the top P of the tower, then from triangle ABP

$$\cos\theta = \frac{20^2 + 30^2 - 15^2}{2 \times 20 \times 30} = \frac{1075}{1200} = \frac{43}{48}$$

$\Rightarrow \quad \theta = \cos^{-1}(43/48)$.

Example 42: The angle of elevation of the top C of a vertical tower CD of height h from a point A in the horizontal plane is 45° and from a point B at a distance a from A on the line making an angle 30° with AD, it is 60°, then

(a) $a = h(\sqrt{3} + 1)$ (b) $h = a(\sqrt{3} + 1)$

(c) $a = h(\sqrt{3} - 1)$ (d) $h = a(\sqrt{3} - 1)$

Ans. (c)

◎ **Solution:** We have

$\angle CAD = 45°$, $\angle BAD = 30°$, $\angle CBH = 60°$ (Fig. 27.39)

$\Rightarrow \angle ACD = 45°$, $\angle BCH = 30°$ so that $\angle ACB = 15°$

and $\angle CAB = 45° - 30° = 15°$

$\Rightarrow \angle ABC = 150°$

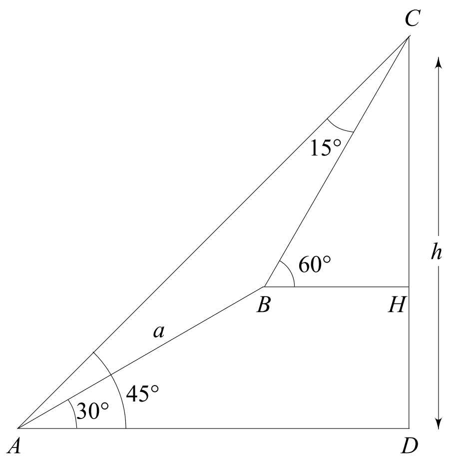

Fig. 27.39

From $\Delta\, ADC$, $AC^2 = h^2 + h^2 = 2h^2$ and from $\Delta\, ABC$

$$\frac{AB}{\sin 15^\circ} = \frac{AC}{\sin 150^\circ} \Rightarrow \frac{a}{\sin 15^\circ} = \frac{\sqrt{2}\,h}{\sin 150^\circ}$$

$$\Rightarrow \quad a = \frac{\sqrt{2}\,h \cdot \sin 15^\circ}{\sin 30^\circ} \quad \Rightarrow a = h(\sqrt{3} - 1).$$

Example 43: The angles of elevation of a vertical tower standing inside a triangular field at the vertices of the field are each equal to θ. If the length of the sides of the field are 30 m, 50 m and 70 m, the height of the tower is

(a) $(70\sqrt{3})\tan\theta$ m (b) $(70/\sqrt{3})\tan\theta$ m

(c) $(50/\sqrt{3})\tan\theta$ m (d) $(75\sqrt{3})\tan\theta$ m

Ans. (b)

◎ **Solution:** Let OP be the tower of height h at the point O in the triangular field ABC with sides $BC = 30$, $CA = 50$ and $AB = 70$.

Then $OA = OB = OC = h\cot\theta$

$\Rightarrow$ O is the circumcentre of the triangle ABC

$\Rightarrow$ $h\cot\theta = R$ (the radius of the circumcircle)

$\Rightarrow$ $h = R\tan\theta$

We know that $R = abc/4\Delta$

where $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{75 \times 45 \times 25 \times 5} = 25 \times 5 \times 3\sqrt{3}$

Thus, $h = \dfrac{30 \times 50 \times 70}{4 \times 25 \times 5 \times 3\sqrt{3}}\tan\theta$

$= \dfrac{70}{\sqrt{3}}\tan\theta$ m.

Example 44: In a triangular plot ABC with $BC = 7$ m, $CA = 8$ m and $AB = 9$ m. A lamp post is situated at the middle point E of the side AC and subtends an angle $\tan^{-1} 3$ at the point B, the height of the lamp post is

(a) 21 m (b) 24 m

(c) 27 m (d) cannot be determined

Ans. (a)

◎ **Solution:** In triangle ABC

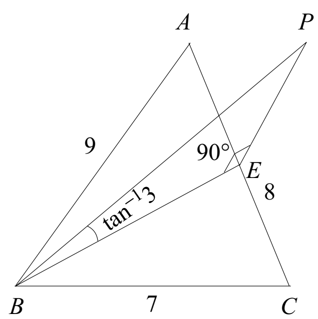

Fig. 27.40

$$a = BC = 7,\ b = CA = 8 \text{ and } c = AB = 9$$

so that $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{49 + 64 - 81}{2 \times 7 \times 8} = \dfrac{2}{7}$

In triangle BEC, $BC = 7$, $CE = 4$ and $\cos C = \frac{2}{7}$

so that $BE^2 = 4^2 + 7^2 - 2 \times 4 \times 7 \times \frac{2}{7} = 49$

$\Rightarrow \quad BE = 7.$

If h is the height of the lamp post EP at E

then $\quad \frac{PE}{BE} = \frac{h}{7} = \tan(\tan^{-1} 3) = 3 \quad \Rightarrow h = 21$ m.

Example 45: Two objects at the points P and Q subtend an angle of 30° at a point A. Lengths $AR = 20$ m and $AS = 10$ m are measured from A at right angles to AP and AQ respectively. If PQ subtends equal angles of 30°, at R and S, then length of PQ is

(a) $\sqrt{300 - 200\sqrt{3}}$ (b) $\sqrt{500 - 200\sqrt{3}}$

(c) $\sqrt{500\sqrt{3} - 200}$ (d) $\sqrt{300}$

Ans. (b)

Solution: Since PQ subtends the same angle of 30° at each of the points A, R and S, the points A, P, Q, R and S lie on a circle. (Fig. 27.41)

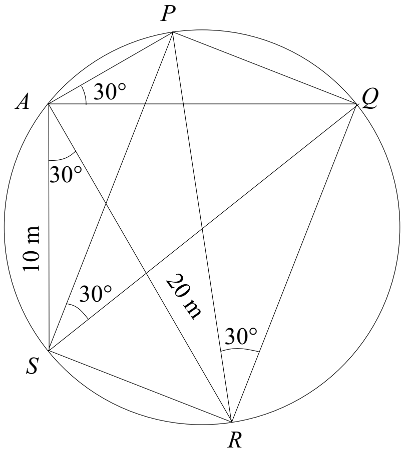

Fig. 27.41

Also $AR \perp AP$ and $AS \perp AQ$

$\Rightarrow \quad \angle RAS = 90° - \angle RAQ = \angle PAQ = 30°$

As it is given

$$AR = 20 \text{ m}, AS = 10 \text{ m}.$$

Then PQ and RS are chords of the same circle making an angle of 30° at a point A on the circumference of the circle and hence are equal in length.

Now from $\Delta\, SAR$

$$RS^2 = 20^2 + 10^2 - 2 \times 20 \times 10 \cos 30°$$

$$\Rightarrow \quad PQ^2 = 500 - 200\sqrt{3}$$

$$\Rightarrow \quad PQ = \sqrt{500 - 200\sqrt{3}}.$$

Example 46: From a ship at sea it is observed that the angle subtended by feet A and B of two light houses, at the ship is 30°. The ship sails 4 km towards A and this angle is then 48°, the distance of B from the ship at the second observation is

(a) 6.460 km (b) 6.472 km

(c) 6.476 km (d) 6.478 km

Ans. (b)

Solution: If S_1 and S_2 are the two positions of the ship then $S_1S_2 = 4$.

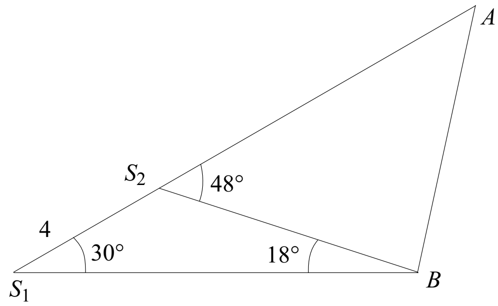

Fig. 27.42

$$\angle AS_1B = 30° \text{ and } \angle AS_2B = 48°$$

$\Rightarrow \quad \angle S_1BS_2 = 18°$ (Fig. 27.42)

Now from ΔS_1BS_2

$$\frac{S_1S_2}{\sin 18°} = \frac{S_2B}{\sin 30°}$$

$$\Rightarrow S_2B = \frac{4 \times (1/2)}{\sin 18°} = 2(\sqrt{5} + 1) = 6.472.$$

Example 47: From a point on the horizontal plane, the elevation of the top of a hill is 45°. After walking 500 m towards its summit up a slope inclined at an angle of 15° to the horizon the elevation is 75°, the height of the hill is

(a) $500\sqrt{6}$ m (b) $500\sqrt{3}$ m

(c) $250\sqrt{6}$ m (d) $250\sqrt{3}$ m

Ans. (c)

Solution: Let P be the top of the hill CP, A and B be the points where the angles of elevation of the top P of the hill are 45° and 75° respectively. Let BE be the line parallel to AC through B. (Fig. 27.43)

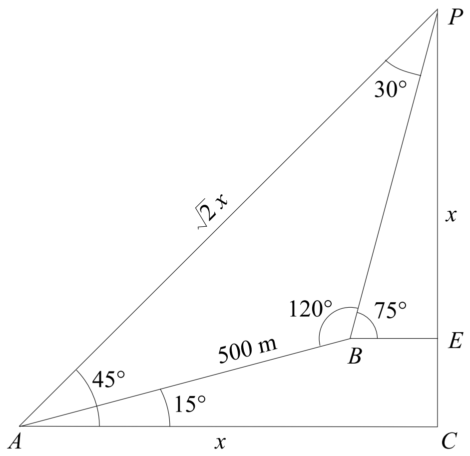

Fig. 27.43

Then $\angle PAC = 45°$ and $\angle BAC = 15°$

$\Rightarrow \quad CA = CP = x$ (say)

$\Rightarrow \quad PA = \sqrt{2}\, x$

$\angle PBE = 75° \quad \Rightarrow \quad \angle BPE = 15°$

$\Rightarrow \quad \angle APB = 30° = \angle BAP$

and $\quad \angle ABP = 120° \quad \therefore$ From $\Delta\, ABP$

$$\frac{\sqrt{2}x}{\sin 120^\circ} = \frac{500}{\sin 30} \quad \Rightarrow \quad x = 250\sqrt{6} \text{ m.}$$

Example 48: The elevation of a steeple at a place due south of it is 45° and at a place B due west of A the elevation is 15°. If $AB = 2a$, the height of the steeple is

(a) $a\dfrac{(\sqrt{3}-1)}{\sqrt{2}}$ (b) $a\dfrac{(\sqrt{3}+1)}{\sqrt{2}}$

(c) $a\left[3^{\frac{1}{4}} - 3^{-\frac{1}{4}}\right]$ (d) $a\left[3^{\frac{1}{4}} + 3^{-\frac{1}{4}}\right]$

Ans. (c)

Solution: Let OP be the steeple of height h. A be due south of OP and $\angle OAP = 45°$ and B be due west of A and $\angle OBP = 15°$ then $OA = h \cot 45° = h$ and $OB = h \cot 15°$ and $AB = 2a$ (given) so that from right angled ΔOAB, $OB^2 = OA^2 + AB^2$

$\Rightarrow \quad h^2 \cot^2 15 = h^2 + 4a^2$ (Fig. 27.44)

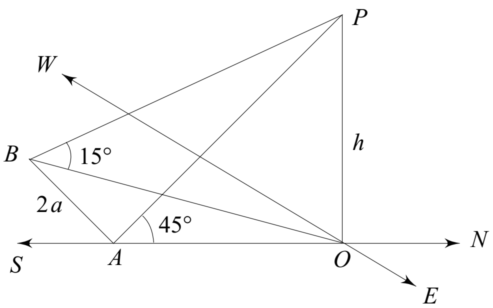

Fig. 27.44

$$\Rightarrow \quad h^2 \left[\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 - 1\right] = 4a^2$$

$$\Rightarrow \quad h^2 \times 4\sqrt{3} = 4a^2 \left(\sqrt{3}-1\right)^2$$

$$\Rightarrow \quad h = \frac{\sqrt{3}-1}{\left(\sqrt{3}\right)^{1/2}} a = [3^{\frac{1}{4}} - 3^{-\frac{1}{4}}]\, a.$$

Example 49: The top of a pole, placed against a wall at an angle α with the horizon, just touches the coping, and when its foot is moved a m, away from the wall and its angle of inclination is β, it rests on the sill of a window; the vertical distance of the sill from the coping is

(a) $a \sin ((\alpha + \beta)/2)$ (b) $a \cos ((\alpha + \beta)/2)$

(c) $a \cot ((\alpha + \beta)/2)$ (d) $a \tan ((\alpha + \beta)/2)$

Ans. (c)

Solution: Let L be the coping and M the sill of the window and A and B be the two positions of the pole such that $\angle LAX = \alpha$ and $\angle MBX = \beta$. $LM = x$, X being the foot of the vertical wall. (Fig. 27.45)

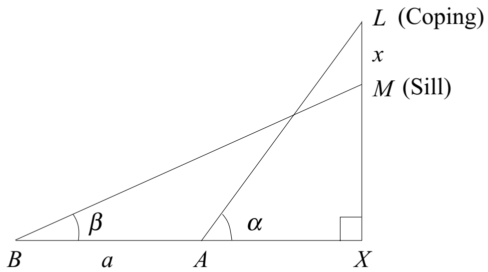

Fig. 27.45

Let $\quad AL = BM = l$ and $AB = a$ (given)

then $\quad x = LX - MX = l(\sin \alpha - \sin \beta)$

$\quad a = BA = BX - AX = l(\cos \beta - \cos \alpha)$.

$$\Rightarrow \quad \frac{x}{a} = \frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}}{2\sin\cdot\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}}$$

$$\Rightarrow \quad x = a \cot \frac{\alpha+\beta}{2}.$$

Example 50: OAB is a triangle in the horizontal plane through the foot P of the tower at the middle point of the side OB of the triangle. If $OA = 2$ m, $OB = 6$ m, $AB = 5$ m and $\angle AOB$ is equal to the angle subtended by the tower at A then the height of the tower is

(a) $\sqrt{\dfrac{11\times 39}{25\times 3}}$ (b) $\sqrt{\dfrac{11\times 39}{25\times 2}}$

(c) $\sqrt{\dfrac{11\times 25}{39\times 2}}$ (d) none of these

Ans. (b)

◎ **Solution:** Let PQ be the tower of height h at the middle point P of the side OB of the triangle OAB, where (Fig. 27.46)

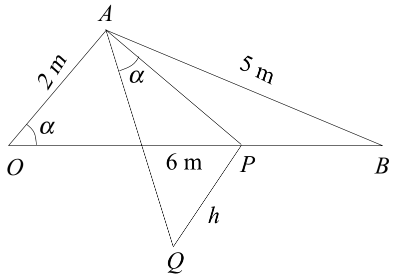

Fig. 27.46

$OA = 2$, $OB = 6$, $AB = 5$ and $\angle AOB = \angle PAQ = \alpha$,

then $AP = h \cot \alpha$, $OP = 3$

From triangle OAB, $\cos \alpha = \dfrac{2^2 + 6^2 - 5^2}{2 \times 2 \times 6} = \dfrac{5}{8}$

and from Δ OAP

$$\cos \alpha = \frac{2^2 + 3^2 - h^2 \cot^2 \alpha}{2 \times 2 \times 3}$$

$$\Rightarrow \quad \frac{5}{8} = \frac{13 - h^2 \times \frac{25}{39}}{12}$$

$$\Rightarrow \quad h^2 = \frac{11 \times 39}{25 \times 2}$$

Example 51: If two vertical towers PQ and RS of lengths a and b ($a > b$) respectively subtend the same angle α at a point A on the line joining their feet P and R in the horizontal plane and angles β and γ at another point B on this line nearer the towers on the same side of the towers as A, then $\dfrac{\sin(\beta - \gamma)}{\sin(\beta - \alpha)}$ is equal to

(a) $\dfrac{b \sin \alpha}{(a-b)\sin \gamma}$ (b) $\dfrac{(b-a)\sin \gamma}{b \sin \alpha}$

(c) $\dfrac{\sin \gamma}{\sin \alpha}$ (d) $\dfrac{(b-a)\sin \alpha}{b \sin \gamma}$

Ans. (b)

◎ **Solution:** We have

$$\angle PAQ = \alpha = \angle RAS$$

$$\angle PBQ = \beta, \angle RBS = \gamma$$

$$\Rightarrow \quad \angle QBS = \gamma - \beta$$

and $\angle BQS = 90° - \alpha - (90° - \beta) = \beta - \alpha$.

$\therefore$ From Δ SBQ, we have

$$\frac{\sin(\gamma - \beta)}{\sin(\beta - \alpha)} = \frac{SQ}{SB} = \frac{a-b}{\sin \alpha} \times \frac{\sin \gamma}{b}$$

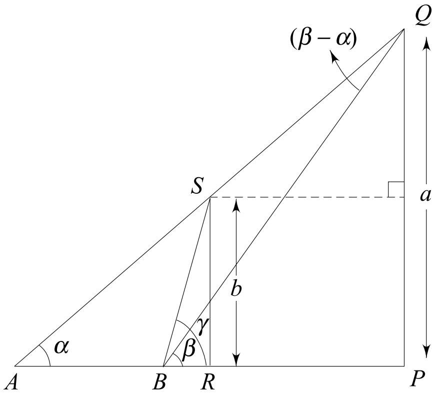

Fig. 27.47

Example 52: A person standing on the ground observes the angle of elevation of the top of a tower to be 30°. On walking a distance a in a certain direction, he finds the elevation of the top to be the same as before. He then walks a distance $5a/3$ at right angles to his former direction, and finds that the elevation of the top has doubled. The height of the tower is

(a) a (b) $\sqrt{85/48}\,a$

(c) $\sqrt{6/5}\,a$ (d) $\sqrt{48/85}\,a$

Ans. (b)

◎ **Solution:** Let PQ be the tower of height h and A, B, C be the three positions of the person from initial to final then (Fig. 27.48)

$$\angle PAQ = \angle PBQ = 30° \text{ and } \angle PCQ = 60°$$

$$\Rightarrow \quad AQ = BQ = h \cot 30° = h\sqrt{3}$$

and $\quad CQ = h \cot 60° = h/\sqrt{3}$.

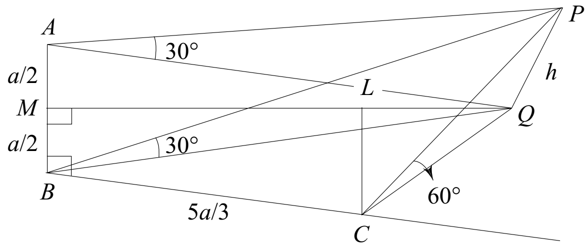

Fig. 27.48

Let QM be perpendicular to AB and CL be perpendicular to QM

Now $AB = a$ and $BC = 5a/3$ and $\angle ABC = 90°$

$\Rightarrow \quad ML = BC = 5a/3$ and $CL = BM = a/2$

From right angled triangle BMQ,

$$QM^2 = BQ^2 - BM^2 = 3h^2 - (a/2)^2 = 3h^2 - (a^2/4)$$

and from right angled triangle CLQ.

$$QL^2 = QC^2 - CL^2 = \frac{h^2}{3} - \frac{a^2}{4}$$

Since $QM = QL + LM$

$$\Rightarrow \quad 3h^2 - \frac{a^2}{4} = \frac{h^2}{3} - \frac{a^2}{4} + \frac{25a^2}{9} + \frac{10a}{3}\sqrt{\frac{h^2}{3} - \frac{a^2}{4}}$$

$$\Rightarrow \quad \frac{8h^2}{3} - \frac{25a^2}{9} = \frac{10a}{3}\sqrt{\frac{h^2}{3} - \frac{a^2}{4}}$$

$$\Rightarrow \quad 64h^4 - \frac{400a^2h^2}{3} + \frac{625a^4}{9} = \frac{100a^2h^2}{3} - 25a^4$$

$\Rightarrow \quad 576h^4 - 1500a^2h^2 + 850a^4 = 0$

$\Rightarrow \quad 288h^4 - 750a^2h^2 + 425a^4 = 0$

$\Rightarrow \quad (48h^2 - 85a^2)(6h^2 - 5a^2) = 0$

$\Rightarrow \quad h = \sqrt{85/48}\, a$ or $\sqrt{5/6} \cdot a$

Example 53: A tower PQ subtends an angle α at a point A on the same level as the foot Q of the tower. It also subtends the same angle α at a point B where AB subtends the angle α with AP then

(a) $AB = BQ$ (b) $BQ = 2AQ$

(c) $\dfrac{AB}{BQ} = (1/2)\sin\alpha$ (d) $\dfrac{AB}{BQ} = (1/2)\operatorname{cosec}\alpha$

Ans. (d)

Solution: Since $\angle PAQ = \angle PBQ = \alpha$ (angles in the same segment are equal) $PQAB$ is a cyclic quadrilateral (Fig. 27.49)

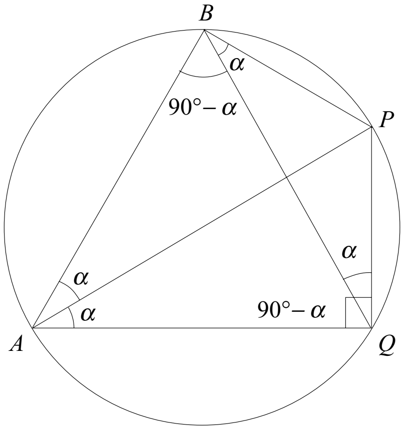

Fig. 27.49

Also since $\angle PAB = \alpha$ given, $\angle PQB = \alpha$ (angles in the same segment.)

$\Rightarrow \quad$ Now $\angle PBQ = \alpha = \angle PQB$

$\Rightarrow \quad \angle ABQ = \angle AQB = 90° - \alpha$

$\Rightarrow \quad ABQ$ is an isosceles triangle.

$\Rightarrow \quad AB = AQ$, and

$$\frac{AB}{\sin(90° - \alpha)} = \frac{BQ}{\sin 2\alpha} \Rightarrow 2AB \sin\alpha = BQ.$$

$$\Rightarrow \quad \frac{AB}{BQ} = \frac{1}{2\sin\alpha} = \frac{1}{2}\operatorname{cosec}\alpha.$$

Example 54: The angle of elevation of the top of a tree at a point B due south of it is 60° and at a point C due north of it is 30°. D is a point due north of C where the angle of elevation is 15°, then given $\sqrt{3} = 1\frac{8}{11}$ and $BC \times CD = 2^3 \times 3^2 \times 19 \times 11$, the height of the tree is

(a) 33 (b) 38

(c) 57 (d) 88

Ans. (c)

Solution: Let A be the top of the tree $OA = h$ (Fig. 27.50) B, C, D be the three point of observation such then $\angle ABO = 60°$, $\angle ACO = 30°$ and $\angle ADO = 15°$

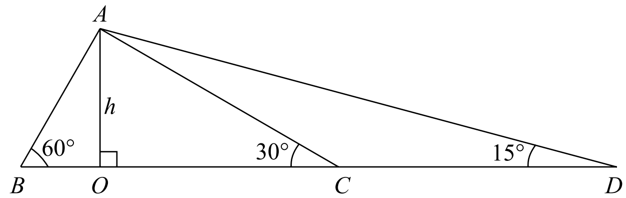

Fig. 27.50

Then $BC = BO + OC = h(\cot 60° + \cot 30)$

$$= h\left(\sqrt{3} + 1/\sqrt{3}\right) = \left(4/\sqrt{3}\right) h$$

$$CD = h\cot 15° - h\cot 30°$$

$$= h\left(2 + \sqrt{3} - \sqrt{3}\right) = 2h$$

So that $\quad 2^3 \times 3^2 \times 19 \times 11$

$$= \frac{4}{\sqrt{3}} \times 2h^2 = \frac{4 \times 2 \times 11}{19} h^2 \quad \left(\because \sqrt{3} = \frac{19}{11}\right)$$

$\Rightarrow \quad h^2 = 3^2 \times 19^2 \Rightarrow h = 57$

Example 55: n poles standing at equal distances on a straight road subtend the same angle α at a point O on the road. If the height of the largest pole is h and the distance of the foot of the smallest pole from O is a, the distance between two consecutive poles is

(a) $\dfrac{h\sin\alpha - a\cos\alpha}{(n-1)\sin\alpha}$ (b) $\dfrac{h\cos\alpha - a\cos\alpha}{(n-1)\cos\alpha}$

(c) $\dfrac{h\cos\alpha - a\sin\alpha}{(n-1)\sin\alpha}$ (d) $\dfrac{h\sin\alpha - a\cos\alpha}{(n-1)\cos\alpha}$

Ans. (c)

Solution: Let $A_1, A_2 \ldots A_n$ be the feet of the n poles subtending angle α at O, such that $OA_1 = a$, if d be the distance between two consecutive poles

$$OA_2 = a + d$$
$$OA_3 = a + 2d$$
$$\vdots$$
$$OA_n = a + (n-1)d$$

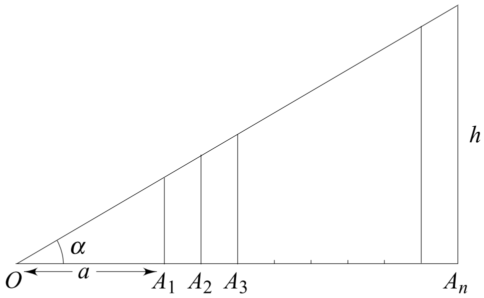

Fig. 27.51

Since the height of the pole at A_n is h;

$$OA_n = h\cot\alpha.$$

$$\Rightarrow \quad a + (n-1)d = h\cot\alpha$$

$$\Rightarrow \quad d = \frac{h\cot\alpha - a}{n-1} = \frac{h\cos\alpha - a\sin\alpha}{(n-1)\sin\alpha}.$$

EXERCISE

Concept-based Straight Objective Type Questions

1. A pole is standing at a point O between two milestones at A and B such that the angles of elevation of the top of the pole at A and B are respectively α and β. If the distance between the milestones is half the height of the pole then
   (a) $2\sin(\alpha+\beta) = \sin\alpha\sin\beta$
   (b) $2\sin(\alpha+\beta) = \cos\alpha\cos\beta$
   (c) $\sin(\alpha+\beta) = 2\sin\alpha\sin\beta$
   (d) $\sin(\alpha+\beta) = 2\cos\alpha\cos\beta$

2. OP is a tower of height 20 m and AB is a pole of height 5 m. The angle of elevation of the top P of the tower from the top B of the pole is 45°. Both pole and tower stand on the same ground. The angle of elevation of the top P of the tower from the base A of the pole is
   (a) $\cos^{-1}\dfrac{3}{5}$ (b) $\sin^{-1}\dfrac{3}{5}$
   (c) $\tan^{-1}\dfrac{3}{4}$ (d) $\cot^{-1}\dfrac{4}{3}$

3. A circular path is 50 m. wide. The angle of elevation of the top of a pole at the centre of the circular park at a point on the inner circle is α and at a point on the outer circle is 45°. The height of the pole is
   (a) $\dfrac{50\cos\alpha}{\cos\alpha - \sin\alpha}$ (b) $\dfrac{50\sin\alpha}{\sin\alpha - \cos\alpha}$
   (c) $\dfrac{50}{\cos\alpha - \sin\alpha}$ (d) $\dfrac{50}{\cos\alpha + \sin\alpha}$

4. A pole 10 m high stands on a tower 30 m. high. The angle of elevation of the top of the pole at a point A on the ground in 45° and the pole subtends an angle α at the same point A then α is equal to
   (a) $\cot^{-1}(1/7)$ (b) $\cot^{-1} 7$
   (c) $\cos^{-1}(1/7)$ (d) $\sin^{-1}(1/7)$

   with each side equal to 100 m.

5. $ABCD$ is a square field with each side equal to 100 m. Two poles of equal heights stand at E, the mid point of DC and at the corner B of the field, subtending respectively angles α and 30° at the corner A of the field. The value of α satisfies
   (a) $\cos 2\alpha = \dfrac{11}{19}$ (b) $\sin 2\alpha = \dfrac{15}{19}$
   (c) $\tan 2\alpha = \dfrac{4}{19}$ (d) $\tan 2\alpha = \dfrac{19}{15}$

LEVEL 1

Straight Objective Type Questions

6. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45°. If the angle of elevation of the top of the pillar after completion at the same point is 60°, then the height to be increased for the completion of the pillar in metres is
   (a) $50\sqrt{3}$ (b) $100\sqrt{2}$
   (c) $100\sqrt{3}$ (d) $100(\sqrt{3}-1)$

7. The angles of elevation of the top of a tower at the top and the foot of a pole 10 m high are 30° and 60° respectively. The height of the tower is
   (a) 15 m (b) 20 m
   (c) $10\sqrt{3}$ m (d) $25\sqrt{3}$ m

8. A tower subtends an angle α at a point A on the ground, and the angle of depression of its foot from a point B just above A and at distance b from A, is β. The height of the tower is
   (a) $b \tan\alpha \tan\beta$ (b) $b \tan\alpha \cot\beta$
   (c) $b \cot\alpha \cot\beta$ (d) $b \cot\alpha \tan\beta$

9. A person walking along a straight road observes that at two points 1 km apart, the angles of elevation of a pole in front of him are 30° and 75°. The height of the pole is
   (a) $250(\sqrt{3}+1)$ m (b) $250(\sqrt{3}-1)$ m
   (c) $500(\sqrt{2}+1)$ m (d) $500(\sqrt{2}-1)$ m

10. If a flagstaff subtends the same angle at the points A, B, C and D on the horizontal plane through its foot, then $ABCD$ is a
    (a) square (b) cyclic quadrilateral
    (c) rectangle (d) none of these

11. From a point on the ground 100 m away from the base of a building, the angle of elevation of the top of the building is 60°. Which of the following is the best approximation for the height of the building?
    (a) 172 m (b) 173 m
    (c) 174 m (d) 175 m

12. From the top of a tower 100 m height, the angles of depression of two objects 200 m apart on the horizontal plane and in a line passing through the foot of the tower and on the same side of the tower are $45° - A$ and $45° + A$, then angle A is equal to
    (a) 15° (b) 22.5°
    (c) 30° (d) 35°

13. An observer finds that the angular elevation of a tower is A, on advancing 3 m towards the tower the elevation is 45° and on advancing 2 m nearer, the elevation is $90° - A$, the height of the tower is
    (a) 1m (b) 5m
    (c) 6m (d) 8m

14. ABC is a triangular park with all sides equal. If a pillar at A subtends an angle of 45° at C, the angle of elevation of the pillar at D, the middle point of BC is
    (a) $\tan^{-1}(\sqrt{3}/2)$ (b) $\tan^{-1}(2/\sqrt{3})$
    (c) $\cot^{-1}\sqrt{3}$ (d) $\tan^{-1}\sqrt{3}$

15. A kite is flying with the string inclined at 75° to the horizon. If the length of the string is 25 m, the height of the kite is
    (a) $(25/2)(\sqrt{3}-1)^2$ (b) $(25/4)(\sqrt{3}+1)\sqrt{2}$
    (c) $(25/2)(\sqrt{3}+1)^2$ (d) $(25/2)(\sqrt{6}+\sqrt{2})$

16. AB is a vertical pole. The end A is on the level ground, C is the middle point of AB. P is a point on the level ground. The portion BC subtends an angle β at P. If $AP = n\,AB$, then $\tan\beta =$
    (a) $\frac{n}{2n^2+1}$ (b) $\frac{n}{n^2-1}$
    (c) $\frac{n}{n^2+1}$ (d) none of these

17. A man in a boat rowing away from a cliff 150 metres high observes that it takes 2 minutes to change the angle of elevation of the top of the cliff from 60° to 45°. The speed of the boat is
    (a) $(1/2)(9-3\sqrt{3})$ km/h
    (b) $(1/2)(9+3\sqrt{3})$ km/h
    (c) $(1/2)(9\sqrt{3})$ km/h
    (d) none of these

18. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°, when he retires 40 metres from the banks he finds the angle to 30°. The breadth of the river is
    (a) 40 m (b) 60 m
    (c) 20 m (d) 30 m

19. The elevation of the top of a mountain at each of the three angular points A, B and C of a plane horizontal triangle is α, if $BC = a$ the height of the mountain is
(a) $(a/2)$ cosec $A \tan \alpha$ (b) $(a/2) \sec A \tan \alpha$
(c) $(a/2)$ cosec $\alpha \cot A$ (d) $(a/2) \sec \alpha \tan A$

20. The angles of elevation of the top of a tower standing on a horizontal plane, from two points on a line passing through its foot at distances a and b, respectively, are complementary angles. If the line joining the two points subtends an angle θ at the top of the tower, then if $a > b$, $\sin \theta =$
(a) $\dfrac{a-b}{a+b}$ (b) $\dfrac{a+b}{a-b}$
(c) $\dfrac{2\sqrt{ab}}{a+b}$ (d) $\dfrac{2\sqrt{ab}}{a-b}$

21. The upper three-quarters of a vertical pole subtends an angle $\tan^{-1}(3/5)$ at a point in the horizontal plane through its foot and distant 40 m from it. The height of the pole is
(a) 80 m (b) 100 m
(c) 160 m (d) 200 m

22. PQ is a vertical tower and A, B, C are three points on a horizontal line through Q, the foot of the tower and on the same side of the tower. If the angles of elevation of the top of the tower from A, B and C are α, β, γ respectively, then $AB/BC =$
(a) $\dfrac{\cot \alpha - \cot \gamma}{\cot \beta - \cot \gamma}$ (b) $\dfrac{\cot \alpha - \cot \beta}{\cot \beta - \cot \gamma}$
(c) $\dfrac{\cot \alpha - \cot \beta}{\cot \alpha - \cot \gamma}$ (d) $\dfrac{\cot \alpha - \cot \gamma}{\cot \alpha - \cot \beta}$

23. $ABCD$ is a rectangular park with $AB = a$. A tower standing at C makes angles α and β at A and B respectively, the height of the tower is
(a) $\dfrac{a}{\sqrt{\cot^2 \alpha + \cot^2 \beta}}$ (b) $\dfrac{a}{\sqrt{\cot^2 \alpha - \cot^2 \beta}}$
(c) $\dfrac{a \tan \alpha \tan \beta}{\sqrt{\tan^2 \beta + \tan^2 \alpha}}$ (d) $\dfrac{a \cot \alpha \cot \beta}{\sqrt{\cot^2 \alpha - \cot^2 \beta}}$

24. Two circular path of radii a and b intersect at a point O and AB is a line through O meeting the circles at A and B respectively. Chords OA and OB subtend equal angles of 60° at their respective centres. A vertical pole at O subtends angles α and β respectively at A and B then height of the pole is
(a) $a \cot \alpha$ (b) $b \cot \beta$
(c) $\dfrac{a+b}{\cot \alpha + \cot \beta}$ (d) none of these

25. Three poles of height a, b, c stand on the same side of a road and subtend an angle of 45° at a point on the line joining their feet. The pole of height a subtends an angle α at the foot of the pole of height b which subtends an angle β at the foot of the pole with height c, if $a > b > c$, then $\cot \alpha - \cot \beta =$
(a) $\dfrac{ac - b^2}{ab}$ (b) $\dfrac{bc - a^2}{ab}$
(c) $\dfrac{ab - c^2}{bc}$ (d) $\dfrac{ac - b^2}{bc}$

26. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. If after 10 seconds, the elevation is observed to be 30°, then the uniform speed of the aeroplane per hour is
(a) 120 km (b) 240 km
(c) $240\sqrt{3}$ km (d) $240/\sqrt{3}$ km

27. If a flagstaff 6 metres high placed on the top of a tower throws a shadow of $2\sqrt{3}$ metres along the ground then the angle (in degrees) that the sun makes with the ground is
(a) 15° (b) 30°
(c) 60° (d) $\tan^{-1} 2\sqrt{3}$

28. Three poles whose feet lie on a circle subtend angle α, β, γ respectively at the centre of the circle. If the height of the poles are in A.P. then $\cot \alpha$, $\cot \beta$, $\cot \gamma$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

29. A, B, C are three points on a vertical pole whose distances from the foot of the pole are in A.P. and whose angles of elevation at a point on the ground are α, β and γ respectively. If $\alpha + \beta + \gamma = \pi$, then $\tan \alpha \tan \gamma$ is equal to
(a) 3 (b) 2
(c) 1 (d) – 1

30. A ladder rests against a wall at an angle of 35°. Its foot is pulled away through a distance a, so that it slides a distance b down the wall, finally making an angle of 25° with the horizontal, then $a/b =$
(a) 1 (b) $1/\sqrt{3}$
(c) $\sqrt{3}$ (d) $\sqrt{3}/2$

Assertion-Reason Type Questions

31. **Statement-1:** Three poles of height a, b, c stand at the points A, B, C respectively and subtend the same angle α at a point O on the horizontal line through the feet of the poles. If a, b, c are in A.P., then $AB = BC$.

    Statement-2: O is the centre of a circular field and A is any point on its boundary. Two poles standing at A and O subtend the same angle α at a point B on the other end of the diameter through A. Height of the pole at A is twice the height of the plot at O.

32. **Statement-1:** Rajat observes that the angle of elevation of the top P of a tower OP at a point A on the ground is α. He travels a distance a in the direction AP and reaches the point B. He then travels a horizontal distance a towards the tower and reaches the point C, where the angle of elevation of the top of the tower is $\pi/4$, the height of the tower is $\dfrac{a(\cos\alpha + 1 - \sin\alpha)}{\cot\alpha - 1}$

    Statement-2: On the top of building a pole of height equal to 1/3 of the height of the building is placed so that the angles of elevations of the top of the pole and the top of the building at a point on the ground are α and β respectively then $\alpha = (3/4)\beta$.

33. **Statement-1:** ABC is a triangular field with $AC = b$ and $AB = c$. A pole standing at a point D on BC subtends angles α at B and β at C.

    If $\angle BAD = \angle DAC$ then $b \cot \alpha = c \cot \beta$.

    Statement-2: Bisector of an angle of a triangle divides the opposite side in the ratio of the side containing the angle.

34. **Statement-1:** The angle of elevation of the top P of a tower OP at a point A on the ground is α, the angle of elevation of the mid-point Q of the tower at the mid-point B of OA is also α.

    Statement-2: The line joining the mid-points of two sides of a triangle is parallel to the third side.

35. **Statement-1:** A tower standing at the centre of a square field subtends an angle α at each corner. If the height of the tower is twice the length of a side of the square, then $\alpha = \tan^{-1} 2$.

    Statement-2: A, B, C are three points on the horizontal line through the foot of a tower and the angles of elevation of the top of the tower at these points are 30°, 45° and 60° respectively, $\dfrac{AB}{BC} = \sqrt{3}$.

LEVEL 2

Straight Objective Type Questions

36. The angle of elevation of the top Q of a tower PQ at a point A on the horizontal plane through P the foot of the tower is α. At a point B on AQ at a vertical height of a, the angle of elevation of the middle point R of the tower PQ is β, then the height of the tower is

    (a) $\dfrac{2a(\tan\alpha - \tan\beta)}{\tan\alpha - 2\tan\beta}$ (b) $\dfrac{2a(\tan\alpha - 2\tan\beta)}{\tan\alpha - \tan\beta}$

    (c) $\dfrac{2a(\tan\alpha\tan\beta - 1)}{2\tan\alpha\cot\beta - 1}$ (d) $\dfrac{2a(\tan\alpha\cot\beta - 1)}{2\tan\alpha\cot\beta - 1}$

37. A lamppost stands in the centre of a circular garden and makes angle α at points A and B on the boundary where AB subtends an angle 2β at the foot of the lamppost. If γ is the angle which the lamppost subtends at C, the middle point of the line joining A and B, then $\tan\gamma =$

    (a) $\tan\alpha\tan\beta$ (b) $\sec\alpha\tan\beta$
    (c) $\tan\alpha\sec\beta$ (d) none of these

38. From a point on the ground, if the angles of elevation of a bird flying at constant speed in a horizontal direction, measured at equal intervals of time are α, β, γ and δ, then

    (a) $\cot^2\beta - \cot^2\gamma = 3(\cot^2\alpha - \cot^2\delta)$
    (b) $\cot^2\beta - \cot^2\delta = 3(\cot^2\alpha - \cot^2\gamma)$
    (c) $\cot^2\gamma - \cot^2\delta = 3(\cot^2\alpha - \cot^2\beta)$
    (d) $\cot^2\alpha - \cot^2\delta = 3(\cot^2\beta - \cot^2\gamma)$

39. A vertical tower standing at O has marks P, Q, R, S at heights of 1m, 2m, 3m and 4m from the foot O and A is a point on the horizontal plane through O. If PQ and RS subtend angles α and β respectively at A where $OA = 2$m then $\cos(\alpha + \beta) =$

(a) $5/\sqrt{26}$ (b) $24/\sqrt{650}$
(c) $23/\sqrt{650}$ (d) $1/\sqrt{26}$

40. $ABCD$ is a rectangular field with $AB = a$ and $BC = b$. A lamp post of height h at A subtends an angle α at P, the middle point of CD and another lamp post of equal height at D subtends an angle β at Q, the middle point of BC. If PQ subtends an angle θ at A, then $\cot^2 \alpha \cot^2 \beta \cos^2 \theta = k^2$, where $k =$
(a) $(a^2 + b^2)/2h^2$ (b) $(a^2 - b^2)/2h^2$
(c) $2h^2/(a^2 + b^2)$ (d) $2(a^2 + b^2)h^2$

41. A vertical tower OP of height h subtends angle α, β, γ respectively at the points A, B, C on the horizontal plane through the foot O of the tower. A is due west of the tower. B is due east of A and on the same side of the tower as A. C is due south of B, then $AC =$
(a) $h(\cot \alpha - \cot \beta)$
(b) $h\sqrt{\cot^2 \gamma - \cot^2 \beta}$
(c) $h\sqrt{\cot^2 \alpha + \cot^2 \gamma - 2\cot \alpha \cot \beta}$
(d) $h\sqrt{\cot^2 \gamma + \cot^2 \beta - 2\cot \alpha \cot \beta}$

42. PQ and RS are two vertical towers of the same height where S is on the ground Q is above the ground. The line joining the top P and the foot S of the two towers meets the horizontal line through Q at a point A where the angles of elevation of the tops P and R of the two towers are α and β respectively. If $AS = a$, the height of the towers is
(a) $\dfrac{a\sin(\beta+\alpha)}{\cos\beta}$ (b) $\dfrac{a\cos(\beta+\alpha)}{\cos\beta}$
(c) $\dfrac{a\sin(\beta+\alpha)}{\sin\beta}$ (d) $\dfrac{a\cos(\beta+\alpha)}{\sin\beta}$

43. From the top of a building of height h, a tower standing on the ground is observed to make an angle θ. If the horizontal distance between the building and the tower is h, the height of the tower is
(a) $\dfrac{2h\cos\theta}{\sin\theta+\cos\theta}$ (b) $\dfrac{2h}{1+\cot\theta}$
(c) $\dfrac{2h}{1+\tan\theta}$ (d) $\dfrac{2h}{\sin\theta+\cos\theta}$

44. A tower stands at the foot of a hill whose inclination to the horizon is 9°; at a point 40 m up the hill the tower subtends an angle of 54°. The height of the tower is
(a) 17.56 m (b) 45.76 m
(c) 54.76 m (d) none of these

45. The angle of elevation of a stationary cloud from a point 2500 metres above a lake is 15° and the angle of depression of its reflection in the lake is 45°. The height of the cloud above the lake level is
(a) $2500/\sqrt{3}$ m (b) 2500 m
(c) $2500\sqrt{3}$ m (d) $5000\sqrt{3}$ m

46. A tower PQ stands at a point P with in the triangular park ABC such that the sides a, b, c of the triangle subtend equal angles at P, the foot of the tower and the tower subtends angles α, β, γ at A, B, C respectively, then $a^2(\cot \beta - \cot \gamma) + b^2(\cot \gamma - \cot \alpha) + c^2(\cot \alpha - \cot \beta)$ is equal to
(a) – 1 (b) 0
(c) 1 (d) $a + b + c$

47. A spherical balloon subtends an angle 2α at a man's eye and the elevation of its centre is β. If θ is the elevation of the highest point of the balloon at A then $\tan \theta$ is equal to
(a) $\dfrac{\sin\alpha+\cos\beta}{\sin\beta}$ (b) $\dfrac{\sin\alpha+\sin\beta}{\cos\beta}$
(c) $\dfrac{\sin\alpha+\cos\beta}{\sin\alpha}$ (d) $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha}$

48. A person stands at a point A due south of a tower and observes that its elevation is 60°. He then walks westwards towards B, where the elevation is 45°. At a point C on AB produced, he finds it to be 30°. Then AB/BC is equal to
(a) 1/2 (b) 1
(c) 2 (d) 5/2

49. A pole stands at a point A on the boundary of a circular park of radius a and subtends an angle α at another point B on the boundary. If the chord AB subtends an angle α at the centre of the path, the height of the pole is
(a) $2a \cos(\alpha/2) \tan \alpha$ (b) $2a \sin(\alpha/2) \cot \alpha$
(c) $2a \sin(\alpha/2) \tan \alpha$ (d) $2a \cos(\alpha/2) \cot \alpha$

50. A, B, C are three points on a horizontal line through the base O of a pillar OP, such that OA, OB, OC are in A.P. If α, β, γ the angles of elevation of the top of the pillar at A, B, C respectively are also in A.P. then $\sin \alpha$, $\sin \beta$, $\sin \gamma$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

Previous Years' AIEEE/JEE Main Questions

1. The upper 3/4th portion of a vertical pole subtends an angle $\tan^{-1}$ (3/5) at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is
   (a) 40 m (b) 60 m
   (c) 80 m (d) 20 m **[2003]**
2. A person standing on the bank of a river observe that the angle of elevation of the top of a tree on the opposite bank of the river is 60º and when he retires 40 m away from the tree the angle of elevation becomes 30º. The breadth of the river is
   (a) 40 m (b) 30 m
   (c) 20 m (d) 60 m **[2004]**
3. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60º at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30º. The height of the tower is
   (a) $2a/\sqrt{3}$ (b) $2a\sqrt{3}$
   (c) $a/\sqrt{3}$ (d) $a\sqrt{3}$ **[2007]**
4. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7m. From D the angle of elevation of the point A is 45°. Then the height of the pole is
   (a) $\frac{7\sqrt{3}}{2}.\frac{1}{\sqrt{3}+1}m.$ (b) $\frac{7\sqrt{3}}{2}.\frac{1}{\sqrt{3}-1}m.$
   (c) $\frac{7\sqrt{3}}{2}.\left(\sqrt{3}+1\right)m.$ (d) $\frac{7\sqrt{3}}{2}.\left(\sqrt{3}-1\right)m.$
   [2008]
5. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is
   (a) $40\left(\sqrt{2}-1\right)$ (b) $40\left(\sqrt{3}-\sqrt{2}\right)$
   (c) $20\sqrt{2}$ (d) $20\left(\sqrt{3}-1\right)$ **[2014]**
6. The angle of elevation of the top of a vertical tower from a point P on the horizontal ground was observed to be α. After moving a distance 2 meters from P towards the foot of the tower, the angle of elevation changes to β. Then the height (in meters) of the tower is:
   (a) $\frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ (b) $\frac{\sin\alpha\sin\beta}{\cos(\beta-\alpha)}$
   (c) $\frac{2\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ (d) $\frac{\cos(\beta-\alpha)}{\sin\alpha\sin\beta}$
   [2014, online]
7. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, art 30°, 45° and 60° respectively, then the ration AB: BC, is:
   (a) $\sqrt{3}:1$ (b) $\sqrt{3}:\sqrt{2}$
   (c) $1:\sqrt{3}$ (d) 2 : 3 **[2015]**
8. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation α at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is 'h' and the distance of the foot of the smallest pole from O is 'a' then the distance between two consecutive poles, is:
   (a) $\frac{h\sin\alpha+a\cos\alpha}{9\sin\alpha}$ (b) $\frac{h\cos\alpha-a\sin\alpha}{9\cos\alpha}$
   (c) $\frac{h\cos\alpha-a\sin\alpha}{9\sin\alpha}$ (d) $\frac{h\sin\alpha+a\cos\alpha}{9\cos\alpha}$
   [2014, online]
9. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar is
   (a) 6 (b) 10
   (c) 20 (d) 5 **[2016]**
10. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in meters) is;
    (a) 108 (b) $36\sqrt{3}$
    (c) $54\sqrt{3}$ (d) 54 **[2016, online]**

Previous Years' B-Architecture Entrance Examination Questions

1. A vertical pole stands at a point A on the boundary of a circular park of radius a and subtends an angle α at another point β on the boundary. If the chord AB subtends an angle α at the centre of the park, the height of the pole is

   (a) $2a \sin \frac{\alpha}{2} \tan \alpha$ (b) $2a \cos \frac{\alpha}{2} \tan \alpha$

   (c) $2a \sin \frac{\alpha}{2} \cot \alpha$ (d) $2a \cos \frac{\alpha}{2} \cot \alpha$. **[2014]**

2. Two vehicles C_1 and C_2 start from a point P and travel east of P at the speeds 20 km/hr and 60 km/hr respectively. If an observer, one kilometre north of P, is able to see both the vehicles at the same time, then the maximum angle of sight between the observer's view of C_1 and C_2, is:

   (a) $\frac{\pi}{3}$ (b) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

   (c) $\frac{\pi}{6}$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **[2015]**

Answers

Concept-based

1. (a)	**2.** (a)	**3.** (b)	**4.** (b)
5. (a)			

Level 1

6. (d)	**7.** (a)	**8.** (b)	**9.** (a)
10. (b)	**11.** (b)	**12.** (b)	**13.** (c)
14. (b)	**15.** (b)	**16.** (a)	**17.** (a)
18. (c)	**19.** (a)	**20.** (a)	**21.** (c)
22. (b)	**23.** (b)	**24.** (c)	**25.** (a)
26. (c)	**27.** (c)	**28.** (c)	**29.** (a)
30. (b)	**31.** (b)	**32.** (c)	**33.** (a)
34. (a)	**35.** (d)		

Level 2

36. (a)	**37.** (c)	**38.** (d)	**39.** (c)
40. (a)	**41.** (c)	**42.** (a)	**43.** (b)
44. (b)	**45.** (c)	**46.** (b)	**47.** (b)
48. (b)	**49.** (c)	**50.** (b)	

Previous Years' AIEEE/JEE Main Questions

1. (a)	**2.** (c)	**3.** (c)	**4.** (c)
5. (d)	**6.** (a)	**7.** (a)	**8.** (c)
9. (d)	**10.** (d)		

Previous Years' B-Architecture Entrance Examination Questions

1. (a) **2.** (c)

Hints and Solutions

Concept-based

1. $h \cot \alpha + h \cot \beta = \frac{h}{2}$

   $\Rightarrow 2 \sin (\alpha + \beta) = \sin \alpha \sin \beta$

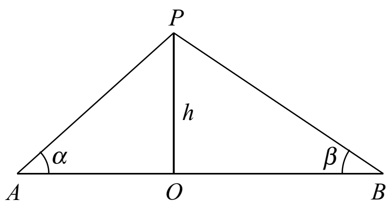

Fig. 27.52

2. $OA = BL = 15 \cot 45° = 15$

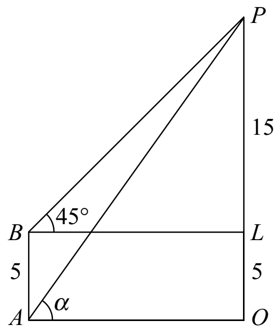

Fig. 27.53

$$\tan \alpha = \frac{OP}{OA} = \frac{20}{15} = \frac{4}{3}$$

$$\Rightarrow \cos \alpha = \frac{3}{5}.$$

3. $OA = h \cot \alpha$

$OB = h \cot 45° = h$

$50 = OB - OA$

$= h\,[1 - \cot \alpha]$

$\Rightarrow h = \dfrac{50}{1 - \cot\alpha} = \dfrac{50 \sin\alpha}{\sin\alpha - \cos\alpha}$

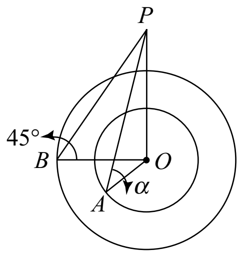

Fig. 27.54

4. $OA = 40 \cot - 45°\ \alpha = 40$

$= 30 \cot (45° - \alpha)$

$\Rightarrow 4 \tan (45° - \alpha) = 3$

$\Rightarrow 4\,(1 - \tan \alpha) = 3\,(1 + \tan \alpha)$

$\Rightarrow \tan \alpha = 1/7 \Rightarrow \cot \alpha = 7$

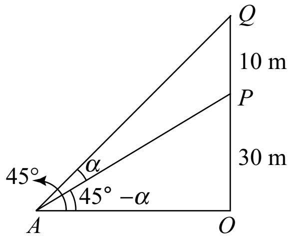

Fig. 27.55

5. Let the height of the poles be h.

then $h \cot \alpha = AE = \dfrac{100\sqrt{5}}{2}$

$h \cot 30° = 100$

$h = \dfrac{100}{\sqrt{3}}.$

$\cot \alpha = \dfrac{\sqrt{15}}{2}$

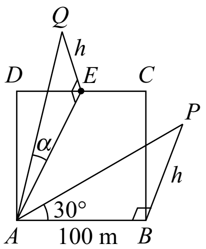

Fig. 27.56

$\cos^2\alpha = \dfrac{15}{19}, \ \sin^2 \alpha = \dfrac{4}{19}$

$\cos 2\alpha = \dfrac{11}{19}.$

Level 1

6. $PQ = OQ - OP = 100 \tan 60° - 100 \tan 45°$
$= 100(\sqrt{3} - 1)$

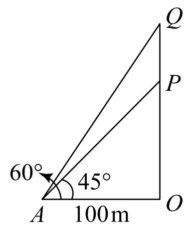

Fig. 27.57

7. $(h - 10) \cot 30° = h \cot 60°$

$\Rightarrow h = \dfrac{10 \cot 30°}{\cot 30° - \cot 60°}$

$= \dfrac{10\sqrt{3}}{\sqrt{3} - (1/\sqrt{3})} = 15$

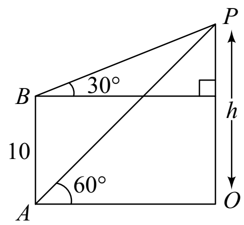

Fig. 27.58

8. $h \cot \alpha = b \cot \beta$
$\Rightarrow \quad h = b \tan \alpha \cot \beta$

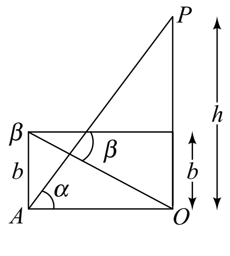

Fig. 27.59

9. $h \cot 30° - h \cot 75° = 1000$ m.

$\Rightarrow \quad h = \dfrac{1000}{\cot 30° - \cot 75°}$

$= \dfrac{1000}{\sqrt{3} - \dfrac{\sqrt{3}-1}{\sqrt{3}+1}} = 250(\sqrt{3}+1)$m

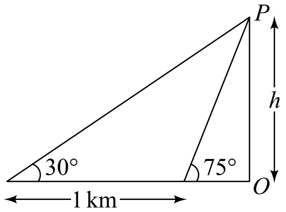

Fig. 27.60

10. A, B, C, D lie on a circle with centre at O, the foot of the flagstaff as $OA = OB = OC = OD = h \cot \alpha$, h being the height of the flagstaff and α the angle given.

11. Height = 100 tan 60º = $100 \times \sqrt{3} = 100 \times 1.73$ = 173 m.

12. $100(\cot (45° - A) - \cot(45° + A)) = 200$

$$\Rightarrow \frac{1+\tan A}{1-\tan A} - \frac{1-\tan A}{1+\tan A} = 2$$

$$\Rightarrow 4 \tan A = 2(1 - \tan^2 A)$$

$$\Rightarrow \frac{2 \tan A}{1-\tan^2 A} = 1 = \tan 45°$$

$$\Rightarrow \tan 2A = 45° \Rightarrow A = 22\frac{1}{2}°.$$

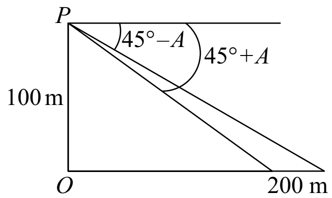

Fig. 27.61

13. $h \cot A = OM + ML = h + 3$

$h \cot (90° - A) = h - 2$

$h^2 = (h + 3)(h - 2)$

$\Rightarrow \quad h = 6$ m

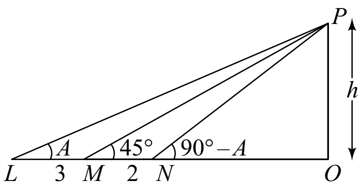

Fig. 27.62

14. $AC = h \cot 45° = h$

$$AD = \frac{\sqrt{3}\,h}{2} = h \cot \theta$$

$$\Rightarrow \theta = \tan^{-1}(2/\sqrt{3})$$

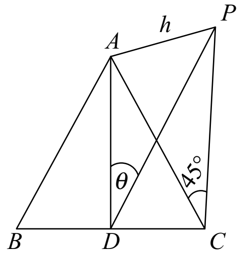

Fig. 27.63

15. $\sin 75° = h/25$

$$\Rightarrow \quad h = 25 \times \frac{\sqrt{3}+1}{2\sqrt{2}} = (25/4)\,(\sqrt{3}+1)\sqrt{2}.$$

16. $AC = AP \tan \alpha$

$\Rightarrow (1/2)\, AB = n\, AB \tan \alpha$

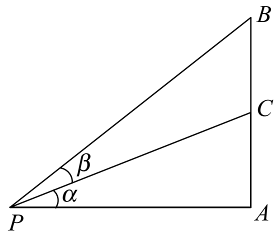

Fig. 27.64

$$\Rightarrow \quad \tan \alpha = \frac{1}{2n}$$

$$\tan(\alpha + \beta) = \frac{AB}{AP} = \frac{1}{n}.$$

$$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{1}{n}.$$

$$\Rightarrow n\left(\frac{1}{2n} + \tan \beta\right) = 1 - \frac{1}{2n} \tan \beta$$

$$\Rightarrow \quad \tan \beta = \frac{n}{2n^2+1}.$$

17. AB = (cot 45° – cot 60°) 150

$$= 150\left(1 - \frac{1}{\sqrt{3}}\right) = 50(3-\sqrt{3})$$

$$\text{speed} = \frac{50(3-\sqrt{3})}{2} \text{ m/min.}$$

$$= 25(3-\sqrt{3}) \times \frac{60}{1000} \text{ km/hr}$$

$$= \frac{9-3\sqrt{3}}{2} \text{ km/h.}$$

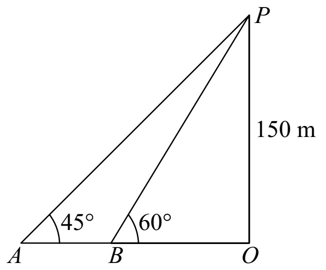

Fig. 27.65

18. $OP = x \tan 60° = (40 + x) \tan 30°$

$$\Rightarrow \quad x = \frac{40 \tan 30°}{\tan 60° - \tan 30°}$$

$$= \frac{40}{\sqrt{3}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = 20 \text{ m}$$

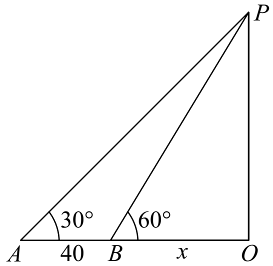

Fig. 27.66

19. The base of the mountain is at the circumcentre of the circle passing through A, B and C if O is the centre and R the radius of the circle, then $R = h \cot \alpha$.

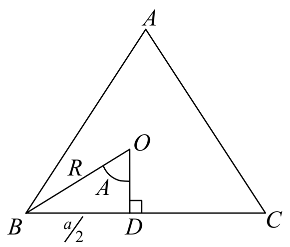

Fig. 27.67

$$\sin A = \frac{a}{2R} = \frac{a}{2h \cot \alpha}$$

$$\Rightarrow h = \frac{a}{2} \tan \alpha \cdot \operatorname{cosec} A$$

20. $\tan \alpha = h/a$, $\cot \alpha = h/b$.

$$\Rightarrow \quad h = \sqrt{ab}.$$

$\tan (\alpha + \theta) = a/h$.

$$\Rightarrow \frac{\tan \alpha + \tan \theta}{1 - \tan \alpha \tan \theta} = \frac{a}{h}.$$

$$\Rightarrow h \tan \alpha + h \tan \theta = a - a \tan \alpha \tan \theta$$

$$\Rightarrow \tan \theta = \frac{a - b}{h + a \tan \alpha} = \frac{a - b}{2\sqrt{ab}}.$$

$$\Rightarrow \sin \theta = \frac{a - b}{a + b}$$

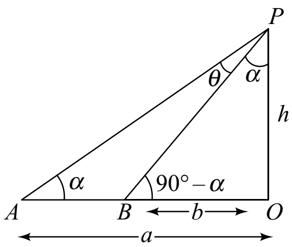

Fig. 27.68

21. $\tan\theta = \dfrac{h}{160}$, $\tan (\alpha + \theta) = \dfrac{h}{40}$.

$$\Rightarrow \frac{\tan \alpha + \frac{h}{160}}{1 - \frac{h}{160} \cdot \tan \alpha} = \frac{h}{40}$$

$$\Rightarrow \frac{\frac{3}{5} + \frac{h}{160}}{1 - \frac{h}{160} \times \frac{3}{5}} = \frac{h}{40}$$

$$\Rightarrow 3h^2 - 600h + 19200 = 0$$

$$\Rightarrow h = 40 \text{ or } 160.$$

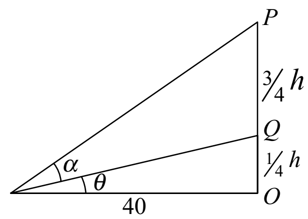

Fig. 27.69

22. $AB = h[\cot \alpha - \cot \beta]$

$BC = h[\cot \beta - \cot \gamma]$

$$\Rightarrow \frac{AB}{BC} = \frac{\cot \alpha - \cot \beta}{\cot \beta - \cot \gamma}.$$

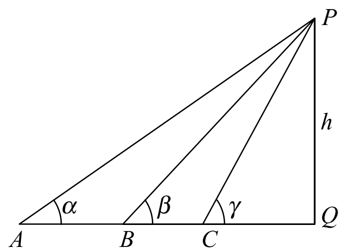

Fig. 27.70

23. $BC = h \cot \beta$

$AC = h \cot \alpha.$

$(AC)^2 = a^2 + (BC)^2$

$\Rightarrow h^2 \cot^2 \alpha = a^2 + h^2 \cot^2 \beta$

$$\Rightarrow \quad h = \frac{a}{\sqrt{\cot^2 \alpha - \cot^2 \beta}}.$$

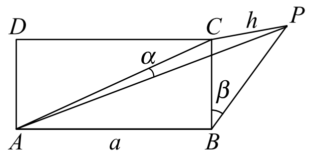

Fig. 27.71

24. $OA = a$, $OB = b$.

$h \cot \alpha = a$, $h \cot \beta = b$

$$\Rightarrow \quad h = \frac{a+b}{\cot \alpha + \cot \beta}.$$

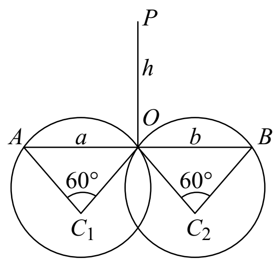

Fig. 27.72

25. $PQ = a - b \;\Rightarrow\; QL = a - b$

$\Rightarrow \quad OA = a - b$

$\Rightarrow \quad a \cot \alpha = a - b$

$$\Rightarrow \quad \cot \alpha = \frac{a-b}{a}$$

Similarly $\cot \beta = \dfrac{b-c}{b}$.

So that $\cot \alpha - \cot \beta = \dfrac{c}{b} - \dfrac{b}{a} = \dfrac{ac - b^2}{ab}$

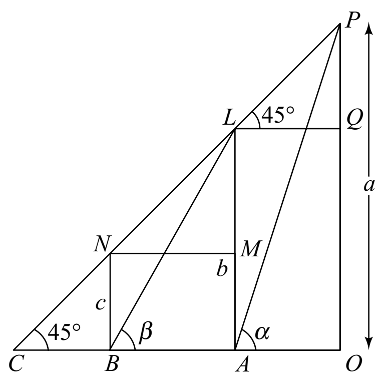

Fig. 27.73

26. Distance travelled in

10 sec $= AB = PQ$.

$= OQ - OP$

$= (\cot 30° - \cot 60°)$ km.

in 1 hour $= \dfrac{\sqrt{3} - 1/\sqrt{3}}{10} \times 60 \times 60$ km.

$= \dfrac{2 \times 60 \times 60 \sqrt{3}}{30} = 240\sqrt{3}$ km.

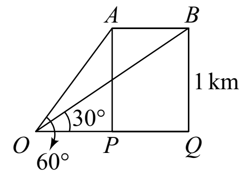

Fig. 27.74

27. AB is the shadow of the tower PQ (= 6 m) on the ground.

So $2\sqrt{3} = (x + 6) \cot \theta - x \cot \theta$

$\Rightarrow \cot \theta = \dfrac{2\sqrt{3}}{6} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = 60°.$

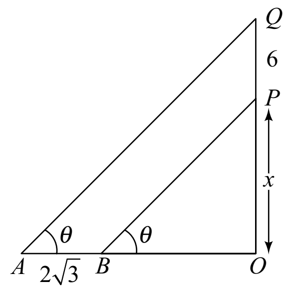

Fig. 27.75

28. If r is the radius of the circle, then the heights of the poles are $r \tan \alpha$, $r \tan \beta$, $r \tan \gamma$. Since they are in A.P. $\cot \alpha$, $\cot \beta$, $\cot \gamma$ are in H.P.

29. If a is the distance of the point from the foot O, of the poles, then the distances of the points A, B, C from O are $a \tan \alpha$, $a \tan \beta$, $a \tan \gamma$, since they are in A.P; $\tan \alpha$, $\tan \beta$, $\tan \gamma$ are in A.P.

So $2 \tan \beta = \tan \alpha + \tan \gamma$.

Also $\alpha + \beta + \gamma = \pi \;\Rightarrow\; \tan \alpha + \tan \beta + \tan \gamma = \tan \alpha \tan \beta \tan \gamma$

$\Rightarrow \tan \alpha \tan \gamma = 3.$

30. AP and BQ be the two positions of the ladder of length l.

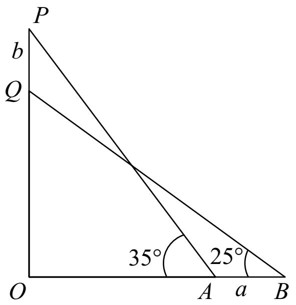

Fig. 27.76

$$\frac{a}{b} = \frac{l\,(\cos 25° - \cos 35°)}{l\,(\sin 35° - \sin 25°)}$$

$$= \frac{2 \sin 30° \sin 5°}{2 \cos 30° \sin 5°}$$

$= \tan 30° = 1/\sqrt{3}$.

31. $AB = OA - OB = (a - b) \cot \alpha$

$BC = OB - OC = (b - c) \cot \alpha$

$\Rightarrow AB = BC$

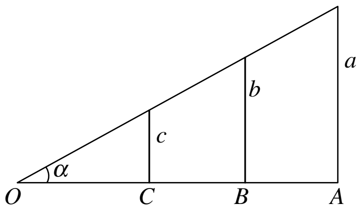

Fig. 27.77

[$\because$ a, b, c are in A.P. $\Rightarrow$ $b - a = c - b$]

$\Rightarrow$ Statement-1 is True.

In statement-2 if r is the radius of the circular field, then $OB = r$, $AB = 2r$

Height of the pole at $A = 2r \tan \alpha = 2 \times$ height of the pole at O.

$\Rightarrow$ Statement-2 is also correct but does not lead to statement-1.

32. Let h be the height of the tower, $BD \perp OA$; $BE \perp OP$ then (Fig. 27.78)

$OA = AD + BC + CE$

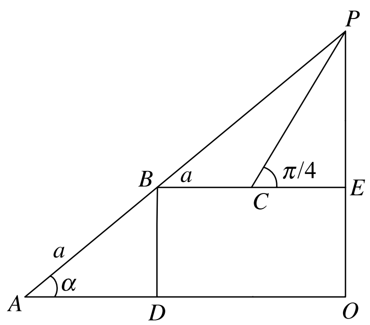

Fig. 27.78

$$h \cot \alpha = a \cos \alpha + a + h - a \sin \alpha.$$

$$h\,(\cot \alpha - 1) = a(\cos \alpha + 1 - \sin \alpha)$$

$\Rightarrow$ statement-1 is True.

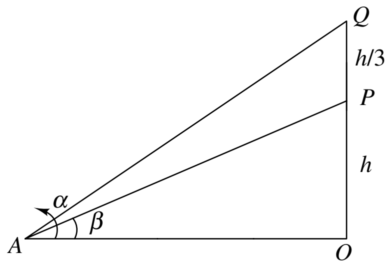

Fig. 27.79

In statement-2 (Fig. 27.79)

$$OA = h \cot \beta = (h + h/3) \cot \alpha.$$

$\Rightarrow$ $\cot \alpha = (3/4) \cot \beta$

$\not\Rightarrow$ $\alpha = (3/4)\, \beta$

$\Rightarrow$ statement-2 is False.

33. $BD = h \cot \alpha$, $CD = h \cot \beta$.

Statement-2 is True, so using it $\dfrac{BD}{CD} = \dfrac{c}{b} = \dfrac{h \cot \alpha}{h \cot \beta}$.

$\Rightarrow$ $b \cot \alpha = c \cot \beta$ $\Rightarrow$ statement-1 is True.

34. Statement-2 is True. Using it $BQ \parallel AP$

$\Rightarrow$ $\underline{|QBO} = \underline{|PAO} = \alpha$ and the statement-1 is also True.

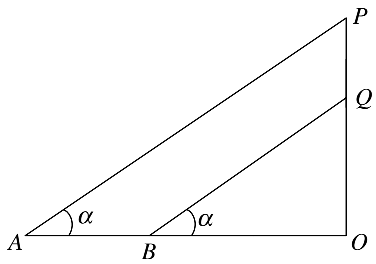

Fig. 27.80

35. Let h be the height of the tower in statement-1 and x be the side of the square, then diagonal of the square is $2h \cot \alpha$

$\Rightarrow$ $(2h \cot \alpha)^2 = 2x^2$

$\Rightarrow$ $4h^2 \cot^2 \alpha = 2x^2$

$\Rightarrow$ $16 \cot^2 \alpha = 2$ [$\because$ $h = 2x$]

$\Rightarrow$ $\tan \alpha = 2\sqrt{2}$

$\Rightarrow$ statement-1 is false.

In statement-2

$$\frac{AB}{BC} = \frac{\cot 30° - \cot 45°}{\cot 45° - \cot 60°} = \frac{\sqrt{3} - 1}{1 - \left(1/\sqrt{3}\right)} = \sqrt{3}$$

$\Rightarrow$ statement-2 is True.

Level 2

36. Let the height of the tower be h

$\angle PAQ = \alpha$, $\angle RBN = \beta$, $BM = a$

$\Rightarrow RN = (h/2) - a$

$AP = h \cot \alpha = AM + MP = a \cot\alpha + (h/2 - a) \cot\beta$

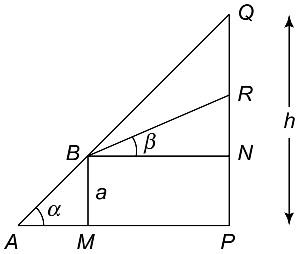

Fig. 27.81

$\Rightarrow h (\cot\alpha - (h/2) \cot\beta) = a (\cot\alpha - \cot\beta)$

$$\Rightarrow h = 2a \frac{(\tan\beta - \tan\alpha)}{2 \tan\beta - \tan\alpha}$$

37. If h is the height of the tower OP at the centre O of the circular garden.

Then $\angle AOB = 2\beta \Rightarrow \angle AOC = \beta$

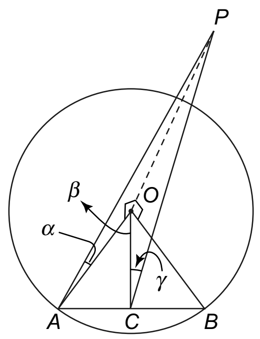

Fig. 27.82

$OA = OB = h \cot \alpha$

$OC = h \cot \gamma$.

$$\Rightarrow \frac{OA}{OC} = \sec\beta \Rightarrow \tan\gamma = \tan\alpha \sec\beta.$$

38. Let P, Q, R, S be the positions of the bird when it makes angles α, β, γ, δ respectively at a point, O, on the ground.

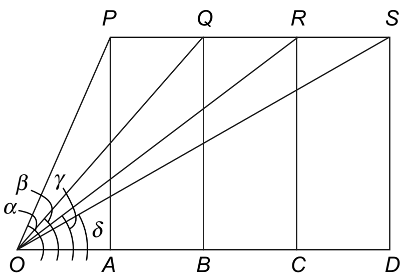

Fig. 27.83

$\angle POA = \alpha$, $\angle QOB = \beta$, $\angle ROC = \gamma$, $\angle SOD = \delta$

$\Rightarrow OA = h \cot\alpha$, $OB = h \cot\beta$, $OC = h \cot \gamma$

and $OD = h \cot\delta$, $AB = BC = CD$.

$AD = h (\cot \delta - \cot\alpha) = 3\ BC = 3h (\cot\gamma - \cot \beta)$

$\Rightarrow \cot\alpha - \cot\delta = 3 (\cot\beta - \cot\gamma)$

Also $\cot\alpha + \cot\delta = \cot\beta + \cot\gamma$

So $\cot^2\alpha - \cot^2\delta = 3 (\cot^2\beta - \cot^2\gamma)$

39. $(AP)^2 = 4 + 1 = 5$

$(AQ)^2 = 4 + 4 = 8$

$(AR)^2 = 4 + 9 = 13$

$(AS)^2 = 4 + 4 = 16$

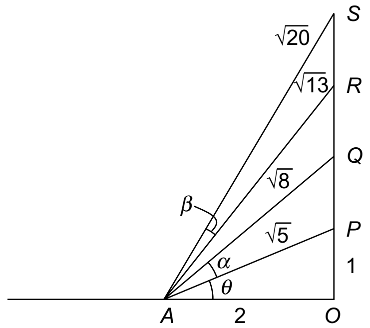

Fig. 27.84

$$\cos\alpha = \frac{8+5-1}{2\sqrt{8\times 5}} = \frac{3}{\sqrt{10}} \Rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$$

$$\cos\beta = \frac{20+13-1}{2\sqrt{20\times 13}} = \frac{8}{\sqrt{65}} \Rightarrow \sin\beta = \frac{1}{\sqrt{65}}$$

$$\cos(\alpha+\beta) = \frac{3\times 8}{\sqrt{650}} - \frac{1\times 1}{\sqrt{650}} = \frac{23}{\sqrt{650}}$$

40. Let the height of the lamppost be h.

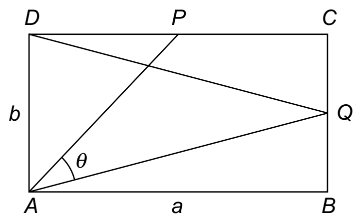

Fig. 27.85

$$AP = h \cot\alpha = \sqrt{b^2 + a^2/4}$$

$$AQ = DQ = \text{h}\cot\beta = \sqrt{a^2 + b^2/4}$$

$$\cos\theta = \frac{(AP)^2 + (AQ)^2 - (a^2/4 + b^2/4)}{2\,AP \cdot AQ \cdot}$$

$$= \frac{a^2 + b^2}{2\,h\cot\alpha \times h\cot\beta}$$

$$\Rightarrow k = \cot\alpha\,\cot\beta\,\cos\theta = \frac{a^2 + b^2}{2h^2}$$

41. $AB = h\,(\cot\alpha - \cot\beta)$

$OB = h\cot\beta,\ OC = h\cot\gamma$

From right angled triangle OBC

$(BC)^2 = h^2\,[\cot^2\gamma - \cot^2\beta]$

From right angled triangle ABC

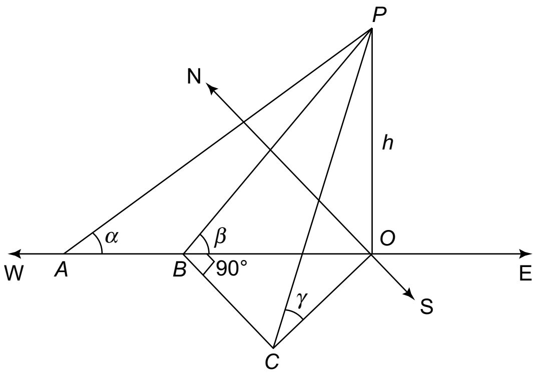

Fig. 27.86

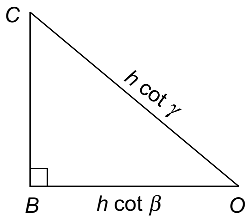

Fig. 27.87

$(AC)^2 = h^2\,[(\cot\alpha - \cot\beta)^2 + \cot^2\gamma - \cot^2\beta]$

$= h^2\,[\cot^2\alpha + \cot^2\gamma - 2\cot\alpha\cot\beta]$

42. $h = PQ = RS$

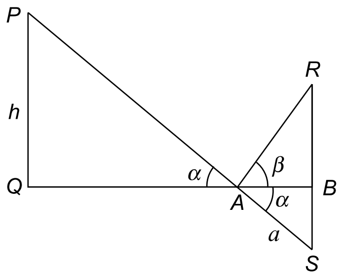

Fig. 27.88

$= RB + BS$

$= AB\,(\tan\alpha + \tan\beta)$

$= a\cos\alpha\,(\tan\alpha + \tan\beta)$

$= a\,\dfrac{\sin(\alpha + \beta)}{\cos\beta}$

43. $\angle PBO = \theta,\ \angle QBO = 45°$

$\Rightarrow \angle PBQ = \theta - 45°$

$PQ = h\tan(\theta - 45°)$

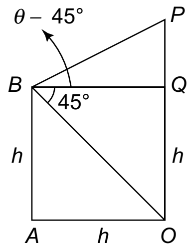

Fig. 27.89

Height of the tower is $PQ + OQ$.

$$= h\left[\frac{\tan\theta - 1}{\tan\theta + 1} + 1\right] = \frac{2h}{1 + \cot\theta}$$

44. $\angle ROX = 9° = \angle ORQ$.

$\angle ORP = 54° \Rightarrow \angle QRP = 45°$

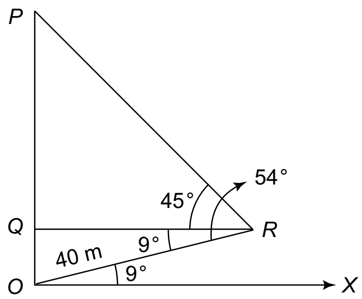

Fig. 27.90

$\Rightarrow \angle QPR = 45° \Rightarrow PQ = QR$

$OR = 40$ m

$\Rightarrow QR = 40\cos 9°,\ OQ = 40\sin 9°$

Height of the tower $OP = OQ + QP$

$= 40\,(\sin 9° + \cos 9°)$

$$= 40\sqrt{1 + \sin 18°} = 40\sqrt{1 + \frac{\sqrt{5} - 1}{4}}$$

$$= 20\sqrt{3 + \sqrt{5}} = 45.76 \text{ m.}$$

45. $\angle QPC = 15°,\ \angle QPR = 45°,\ CQ = x$

$PQ = QR = 2500 + 2500 + x$

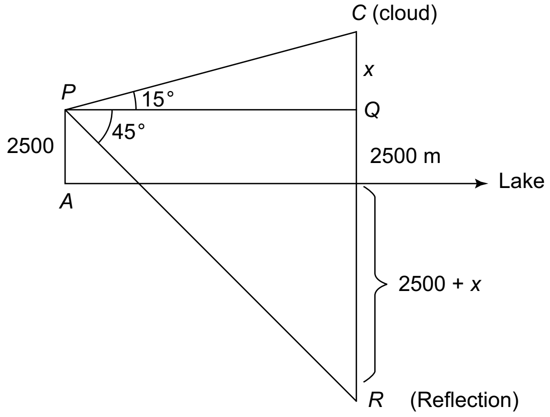

Fig. 27.91

Also $PQ = x \cot 15°$

So $x = \dfrac{5000}{\cot 15° - 1} = 2500\left(\sqrt{3} - 1\right)$

Height of the cloud above the lake

$= x + 2500 = 2500\sqrt{3}$ m.

46. Let h be the height of the tower at P, then

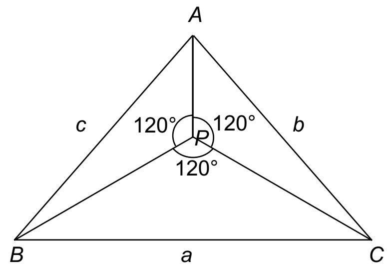

Fig. 27.92

$PA = h \cot\alpha$, $PB = h \cot\beta$, $PC = h \cot \gamma$.

From triangle PBC

$$\cos 120° = \frac{h^2 \cot^2 \gamma + h^2 \cot^2 \beta - a^2}{2\, h^2 \cot\beta \cot\gamma}$$

$\Rightarrow a^2 = h^2 [\cot^2 \beta + \cot^2 \gamma + \cot \beta \cot \gamma]$

$\Rightarrow a^2 (\cot \beta - \cot \gamma) = h^2 [\cot^3 \beta - \cot^3 \gamma]$

Similarly for $b^2 (\cot \gamma - \cot \alpha)$ and $c^2(\cot \alpha - \cot \beta)$

47. Let r be the radius of the sphere with centre O

$\underline{|OPA} = \alpha$, $\underline{|OPQ} = \beta$, $\underline{|RPQ} = \theta$

$OP = r \operatorname{cosec} \alpha$ (From $\Delta\, OAP$)

$OQ = OP \sin \beta = r \operatorname{cosec} \alpha \sin \beta$

$PQ = OP \cos \beta = r \operatorname{cosec} \alpha \cos \beta$

$$\tan \theta = \frac{QR}{PQ} = \frac{r + r \operatorname{cosec} \alpha \sin\beta}{r \operatorname{cosec} \alpha \cos\beta} = \frac{\sin\alpha + \sin\beta}{\cos\beta}$$

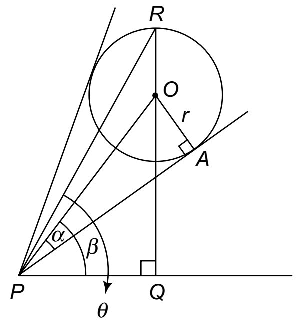

Fig. 27.93

48. Let OP be the tower of height h.

$\underline{|OAP} = 60°$, $\underline{|OBP} = 45°$, $\underline{|OCP} = 30°$,

$\underline{|OAC} = 90°$

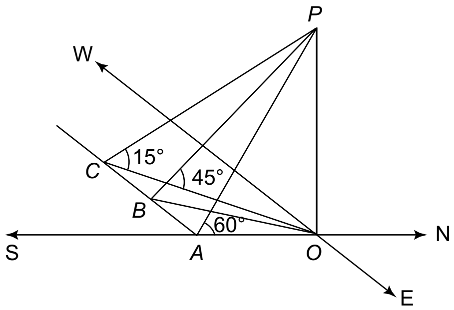

Fig. 27.94

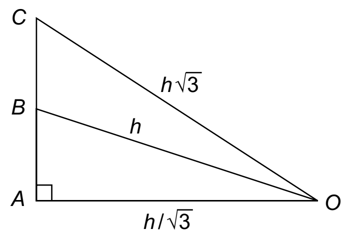

Fig. 27.95

$OA = h \cot 60° = h/\sqrt{3}$

$OB = h$, $OC = h \cot 30° = h/\sqrt{3}$

$(AB)^2 = h^2 (1 - 1/3) = (2/3)\, h^2$

$(AC)^2 = h^2 (3 - 1/3) = (8/3)\, h^2$

$BC = AC - AB = \left(2\sqrt{2/3} - \sqrt{2/3}\right) h = \sqrt{2/3}\, h = AB$

Hence $AB/BC = 1$

49. Let AP be the pole of height h.

$\underline{|PBA} = \alpha \Rightarrow AB = h \cot a$

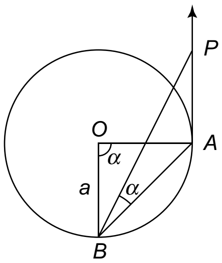

Fig. 27.96

$OA = OB = a$

From triangle OAB

$$\cos \alpha = \frac{a^2 + a^2 - h^2 \cot^2 \alpha}{2a^2}$$

$\Rightarrow h^2 \cot^2 \alpha = 2a^2 (1 - \cos \alpha) = 4a^2 \sin^2 (\alpha/2)$

$\Rightarrow h = 2a \sin (\alpha/2) \tan\alpha$

50. Let OP be the pillar of height h

$\underline{|OAP} = \alpha$, $\underline{|OBP} = \beta$, $\underline{|OCP} = \gamma$

$2\ h \cot\beta = h (\cot\alpha + \cot \gamma)$

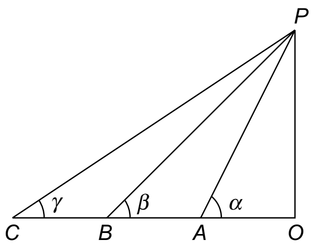

Fig. 27.97

$$\Rightarrow 2\frac{\cos\beta}{\sin\beta} = \frac{\sin(\alpha+\gamma)}{\sin\alpha\sin\gamma}$$

$$= \frac{\sin 2\beta}{\sin\alpha\sin\gamma}$$

$$= \frac{2\sin\beta\cos\beta}{\sin\alpha\sin\gamma}$$

$\Rightarrow \sin \alpha \sin \gamma = \sin^2 \beta \Rightarrow \sin\alpha, \sin \beta, \sin \gamma$ are in G.P.

Previous Year's AIEEE/JEE Main Questions

1. $\tan \theta = \dfrac{h}{160}$, $\tan(\alpha + \theta) = \dfrac{h}{40}$.

$$\Rightarrow \frac{\tan\alpha + \dfrac{h}{160}}{1 - \dfrac{h}{160}\cdot\tan\alpha} = \frac{h}{40}$$

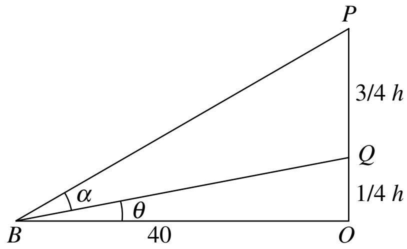

Fig. 27.98

$$\Rightarrow \frac{\dfrac{3}{5} + \dfrac{h}{160}}{1 - \dfrac{h}{160}\times\dfrac{3}{5}} = \frac{h}{40}$$

$\Rightarrow 3h^2 - 600h + 19200 = 0$

$\Rightarrow h = 40$ or 160.

2. We have

$\dfrac{h}{b} = \tan 60°$

$\Rightarrow h = \sqrt{3}\, b$

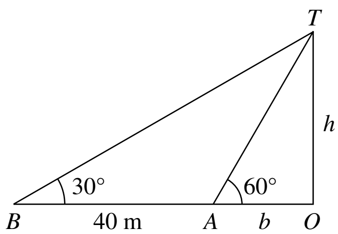

Fig. 27.99

and $\dfrac{h}{40+b} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \left(\sqrt{3}b\right)\sqrt{3} = 40 + b \Rightarrow 2b = 40 \Rightarrow b = 20$

3. $OA = OB = AB = a$

$\Rightarrow \dfrac{h}{a} = \tan 30°$

$\Rightarrow h = \dfrac{a}{\sqrt{3}}$

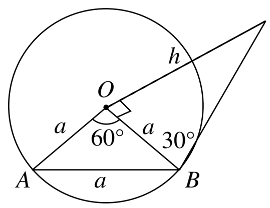

Fig. 27.100

4. $h - h \cot 60° = 7$

$$\Rightarrow h = \frac{7}{1 - \dfrac{1}{\sqrt{3}}}$$

$$= \frac{7\sqrt{3}}{2}\left(\sqrt{3}+1\right)$$

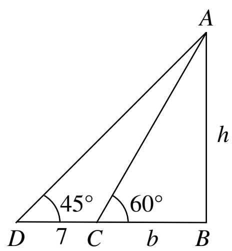

Fig. 27.101

5. $RS = PQ = 20$

$\Rightarrow OS = 20$

Let $SP = d = 20 \cot 30° - 20$

$= 20\left(\sqrt{3}-1\right)$

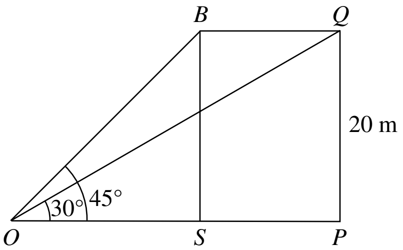

Fig. 27.102

Required speed is

$20\left(\sqrt{3}-1\right)$ m/s

6. $h \cot \alpha - h \cot \beta = 2$

$\Rightarrow h = \dfrac{2}{\cot\alpha - \cot\beta}$

$= \dfrac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$

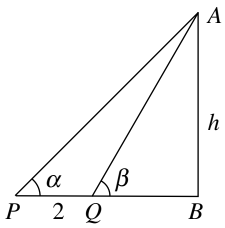

Fig. 27.103

7. $\dfrac{PQ}{CQ} = \tan 60°$

$\Rightarrow$ $CQ = h \cot 60° = hl\sqrt{3}$

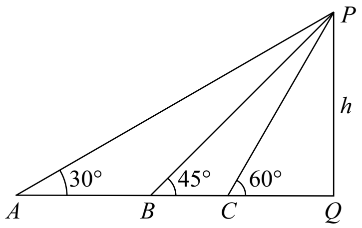

Fig. 27.104

Similarly

$BQ = h$ and $AQ = \sqrt{3}\,h$

Now,

$\dfrac{AB}{BC} = \dfrac{AQ - BQ}{BQ - CQ} = \dfrac{\sqrt{3}h - h}{h - h/\sqrt{3}} = \dfrac{\sqrt{3}}{1}$

8. Let $OA_1 = a$ and $A_1A_2 = A_2A_3 = \ldots d.$

$A_1B_1 = C$

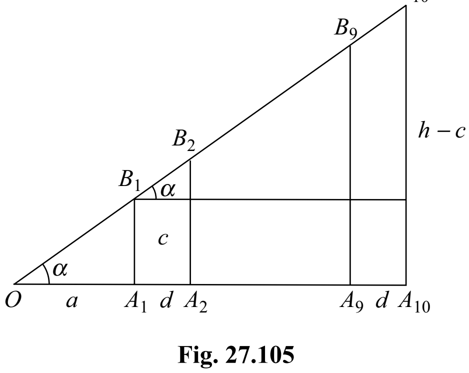

Fig. 27.105

Then $OA_{10} = a + 9d$

Now,

$\tan\alpha = \dfrac{c}{a}$

and $\tan\alpha = \dfrac{h-c}{9d} = \dfrac{h - a\tan\alpha}{9d}$

$9d\tan\alpha = h - a\tan\alpha$

$\Rightarrow \quad d = \dfrac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$

9. Refer figure

$\dfrac{h}{a+b} = \tan 30°$

and $\dfrac{h}{b} = \tan 60°$

$\Rightarrow \quad a + b = \sqrt{3}h,\ b = \dfrac{h}{\sqrt{3}}$

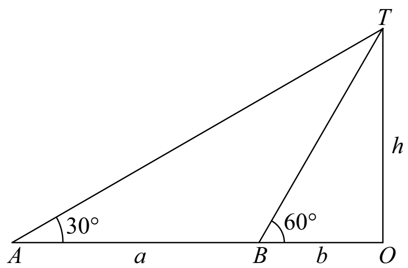

Fig. 27.106

$\Rightarrow \quad a = \left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right)h = \dfrac{2}{\sqrt{3}}h$

Time taken to cover $a = \dfrac{2}{\sqrt{3}}h$ units is 10 minutes

$\Rightarrow$ time taken to cover $b = \dfrac{1}{\sqrt{3}}h$ units is 5 minutes.

10. Let $OT = h$

In right triangle

ΔAOT,

$\dfrac{OT}{OB} = \tan 45° = 1$

$\Rightarrow \quad OA = OT = h$

In right triangle, ΔBOT

$\dfrac{OT}{OB} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad OB = \sqrt{3}OT = \sqrt{3}h$

Also, $OB^2 = OA^2 + AB^2$

$\Rightarrow \quad 3h^2 = h^2 + (54\sqrt{2})^2$

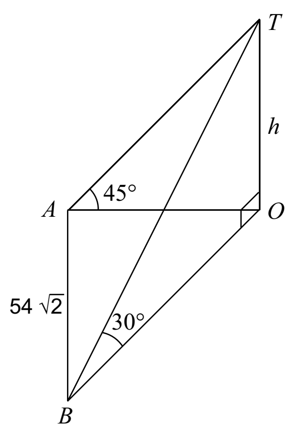

Fig. 27.107

$\Rightarrow \quad 2h^2 = (54\sqrt{2})^2$

$\Rightarrow \quad h = 54$

Previous Years' B-Architecture Entrance Examination Questions

1. $AP = h$

$\underline{|ABP} = \alpha, \underline{|AOB} = \alpha$

$\Rightarrow \quad \underline{|BOQ} = \dfrac{\alpha}{2}.\ OB = a$

$AB = h \cot \alpha$

Also $AB = 2BQ$

$\Rightarrow \quad h\cot\alpha = 2a\sin\dfrac{\alpha}{2}$

$\Rightarrow \quad h = 2a\sin\dfrac{\alpha}{2}\tan\alpha$

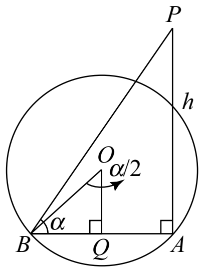

Fig. 27.108

2. At time t, $PC_1 = 20t$

and $PC_2 = 60t$

As $OP = 1$,

$$\tan\theta_1 = \frac{PC_1}{OP} = 20t,\ \tan\theta_2 = \frac{PC_2}{OP} = 60t$$

Now

$$\tan\theta = \tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$$

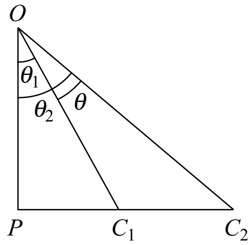

Fig. 27.109

$$= \frac{40t}{1+1200t^2}$$

$$\sec^2\theta \frac{d\theta}{dt} = 40\frac{\left(1+1200t^2\right)(1) - t(2400t)}{\left(1+1200t^2\right)^2}$$

$$= 40\frac{1-1200t^2}{\left(1+1200t^2\right)^2}$$

We have $\dfrac{d\theta}{dt} = 0 \Rightarrow t = \dfrac{1}{20\sqrt{3}}$

and $\dfrac{d\theta}{dt} > 0$ for $0 < t < \dfrac{1}{20\sqrt{3}}$

and $\dfrac{d\theta}{dt} < 0$ for $t < \dfrac{1}{20\sqrt{3}}$

$\therefore\ \theta$ is maximum when

$$t = \frac{1}{20\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$$