Introduction toAlternating Current

1 Mark Questions

1.Why is the use of AC voltage preferred over DC voltage? Give two reasons. [All India 2014]

Ans. The use of AC voltage is preferred over DC voltage because of

(i) the loss of energy in transmitting the AC voltage over long distances with the help of step-up transformers is negligible as compared to DC voltage.

(ii) AC voltage can be stepped up and stepped down as per the requirement by using a transformer.

2.The current flowing through a pure inductance 2mH is, I =(15 cos 300t) A. What is the (i) rms and average value of current for a complete cycle? [Foreign 2011]

To calculate different values for AC with the help of given equation, compare the given equation with standard equation of AC.

Current flowing through the inductor,

$$I = 15 \cos 300t$$

Comparing with $I = I_0 \sin \omega t$

Here, peak value of current,

$$I_0 = 15 \text{ A}$$

(*i*) For complete cycle, rms value of current

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{15}{\sqrt{2}} \text{ A}$$

(*ii*) For complete cycle, average value of current is zero

i.e. $I_{av} = 0$ **(1/2 × 2 = 1)**

3. Define the term wattless current. [Delhi 2011]

Wattless Current The current in an AC circuit when average power consumption in AC circuit is zero, is referred as wattless current.

If ϕ is the phase difference between voltage and current then power associated with $I \sin \phi$ component of current is termed as wattless current. **(1)**

4. Define the term rms value of the current. How is it related to the peak value? [All India 2010c]

Ans. It is defined as the value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle .It is also called virtual value or effective value of AC.

Let the peak value of the current be I_0

$$\therefore \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

where, I_0 = peak value of AC.

5. How much average power over a complete cycle, does an AC source supply to a capacitor? [Delhi 2009c]

Average power, $P_{av} = V_{rms} \times I_{rms} \times \cos\phi$

But for pure capacitive circuit, $\phi = 90° = \frac{\pi}{2}$

$$P_{av} = V_{rms} \times I_{rms} \times \cos 90° = 0$$

$$\therefore \quad P_{av} = 0$$

6. An AC current, $I = I_0 \sin \omega t$ produces certain heat H in a resistor R over a time $T = 2\pi/\omega$. Write the value of the DC current that would produce the same heat in the same resistor in the same time.

[All India 2009C]

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7. The instantaneous current and voltage of an AC circuit are given by I =(10 sin 300t) A and V= (200 sin 300t) V. What is the power dissipation in the circuit? [All India 2008]

Given, $I = 10 \sin 300t$ and $V = 200 \sin 300t$.

Comparing with $I = I_0 \sin \omega t$ and $V = V_0 \sin \omega t$, we get

$$I_0 = 10 \text{ A}, \; V_0 = 200 \text{ V}$$

$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}} \text{ A}, \; V_{rms} = \frac{200}{\sqrt{2}} \text{ V}$$

Phase difference, $\phi = 0°$

$\therefore$ Power dissipated,

$$P_{av} = V_{rms} \times I_{rms} \times \cos\phi$$

$$= \left(\frac{10}{\sqrt{2}}\right)\left(\frac{200}{\sqrt{2}}\right) \cos 0°$$

$$P_{av} = 1000 \text{ W}$$ **(1)**

8. The instantaneous current and voltage of an AC circuit are given by I = (10 sin 314t) A and V= (50 sin 314t) V. What is the power dissipation in the circuit? [All India 2008]

Given, $I = 10 \sin 300t$ and $V = 200 \sin 300t$.

Comparing with $I = I_0 \sin \omega t$ and $V = V_0 \sin \omega t$, we get

$$I_0 = 10 \text{ A}, \; V_0 = 200 \text{ V}$$

$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}} \text{ A}, \; V_{rms} = \frac{200}{\sqrt{2}} \text{ V}$$

Phase difference, $\phi = 0°$

$\therefore$ Power dissipated,

$$P_{av} = V_{rms} \times I_{rms} \times \cos \phi$$

$$= \left(\frac{10}{\sqrt{2}}\right)\left(\frac{200}{\sqrt{2}}\right) \cos 0°$$

$$P_{av} = 1000 \text{ W}$$ **(1)**

9. The instantaneous current and voltage of an AC circuit are given by $I = (10 \sin 314t)$ A and $V = 50 \sin\left(314t + \frac{\pi}{2}\right)$ V. What is the power dissipation in the circuit?

[All India 2008]

Given, $I = 10 \sin 314t$

$$V = 50 \sin\left(314t + \frac{\pi}{2}\right)$$

Comparing with $I = I_0 \sin \omega t$

and $V = V_0 \sin(\omega t + \phi)$

$$I_0 = 10 \text{ A}, V_0 = 50 \text{ V}$$

$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ A}$$

Phase difference, $\phi = \frac{\pi}{2}$

$$V_{rms} = \frac{50}{\sqrt{2}} \text{ V}$$

$$\therefore \quad P_{av} = V_{rms} I_{rms} \cos \phi$$

$$= \frac{10}{\sqrt{2}} \times \frac{50}{\sqrt{2}} \times \cos \frac{\pi}{2} = 0$$

$$\because \quad \cos \frac{\pi}{2} = 0$$

2 Marks Questions

10.A light bulb is rated 150 W for 220 V AC supply of 60 Hz. Calculate

- the resistance of the bulb
- the rms current through the bulb. [All India 2012]

(i) $P = 150$ W, $V = 220$ V

Resistance of the bulb,

$$R = \frac{V^2}{P}$$ **(1/2)**

$$R = \frac{220 \times 220}{150} = 322.7\ \Omega$$ **(1/2)**

(ii) As, $I_{rms} = \frac{V_{rms}}{R} = \frac{220}{322.7}$ $(V_{rms} = V = 220\text{ V})$ **(1/2)**

$\Rightarrow$ $I_{rms} = 0.68\text{A}$ **(1/2)**

11. Prove that an ideal capacitor in an AC circuit does not dissipate power. [Delhi 2011]

Since, average power consumption in an AC circuit is given by

$$P_{av} = V_{rms} \times I_{rms} \times \cos\phi$$ **(1)**

But in pure capacitive circuit, phase difference between voltage and current is given by

$$\phi = \frac{\pi}{2}$$

$\therefore$ $P_{av} = V_{rms} \times I_{rms} \times \cos\frac{\pi}{2}$

$\Rightarrow$ $P_{av} = 0$ $\left(\because \cos\frac{\pi}{2} = 0\right)$

Thus, no power is consumed in pure capacitive AC circuit. **(1)**

3 Marks Questions

12. (i) For a given AC, $i = i_m \sin\omega t$, show that the average power dissipated in a resistor R over a complete cycle is $\frac{1}{2} i_m^2 R$.

(ii) A light bulb is rated at 100 W for a 220 V AC supply. Calculate the resistance of the bulb. **[All India 2013]**

(i) The average power dissipated,

$$\overline{P} = \langle i^2 R\rangle = \langle i_m^2 R \sin^2\omega t\rangle = i_m^2 R\langle \sin^2\omega t\rangle$$

$\because$ $\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t)$

$\therefore$ $\langle \sin^2\omega t\rangle = \frac{1}{2}\left(1 - \langle \cos 2\omega t\rangle\right) = \frac{1}{2}$

$(\because \langle \cos 2\omega t\rangle = 0)$

$\therefore$ $\overline{P} = \frac{1}{2} i_m^2 R$ **(2)**

(*ii*) Power of the bulb, $P = 100$ W and voltage, $V = 220$ V

The resistance of the bulb is given as $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$ **(1)**

13. (i) When an AC source is connected to an ideal capacitor, then show that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with a capacitor. Predict your observations when the system is connected first across a DC and then AC source. What happens in each case if the capacitance of the capacitor is reduced? [Delhi 2013C]

(*i*) When a source of AC is connected to a capacitor of capacitance C_1 the charge on it glows from zero to maximum steady value Q_0.

The energy stored in a capacitor is

$$E = \frac{1}{2}CV_0^2$$

where, V_0 is maximum potential difference across the plates of the capacitor.

The alternating voltage applied is

$$E = E_0 \sin \omega t$$

and the current leads the emf by a phase angle of $\pi/2$.

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= I_0 \cos \omega t$$

$\therefore$ Work done over a complete cycle is

$$W = \int_0^T EIdt = \int_0^T (E_0 \sin \omega t)(I_0 \cos \omega t)\, dt$$

$$= \frac{E_0 I_0}{2} \int_0^T 2 \sin \omega t \cos \omega t\, dt$$

$$W = \frac{E_0 I_0}{2} \int_0^T \sin 2\omega t\, dt$$

$$= \frac{E_0 I_0}{2} \left[1 - \frac{\cos 2\omega t}{2\omega}\right]_0^T = 0$$ $\left(1\frac{1}{2}\right)$

(*ii*) When DC source is connected, the condenser is charged but no current flows in the circuit. Therefore, the lamp does not glow. No change occurs even when capacitance of capacitor is reduced.

When AC source is connected, the capacitor offers capacitative reactance $X_C = \frac{1}{\omega C}$. The current flows in the circuit and the lamp glows. On reducing C_1, X_C increases. Therefore, the glowing of the bulb reduces. (1\1)