CHAPTER – 11

FACTORISATION

Exercise 11.1

Factorise the following (1 to 8) polynomials:

1.

(i) $8xy^3 + 12x^2y^2$

(ii) $15ax^3 - 9ax^2$

Solution:

(i) $8xy^3 + 12x^2y^2 = 4xy^2 (2y + 3x)$

(ii) $15ax^3 - 9ax^2 = 3ax^2 (5x - 3)$

2.

(i) $21 py^2 - 56py$

(ii) $4x^3 - 6x^2$

Solution:

(i) $21 py^2 - 56py = 7py (3y - 8)$

(ii) $4x^3 - 6x^2 = 2x^2 (2x - 3)$

3.

(i) $25abc^2 - 15a^2b^2c$

(ii) $x^2yz + xy^2z + xyz^2$

Solution:

(i) $25abc^2 – 15a^2b^2c = 5abc (5c – 3ab)$

(ii) $x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

4.

(i) $8x^3 – 6x^2 + 10x$
(ii) $14mn + 22m – 62p$
Solution:

(i) $8x^3 – 6x^2 + 10x = 2x (4x^2 – 3x + 5)$

(ii) $14mn + 22m – 62p = 2 (7mn + 11m – 31p)$

5.

(i) $18p^2q^2 – 24pq^2 + 30p^2q$
(ii) $27a^3b^3 – 18a^2b^3 + 75a^3b^2$
Solution:
(i) $18p^2q^2 – 24 pq^2 + 30p^2q$

$= 6pq (3pq -4q + 5p)$

(ii) $27a^3b^3 – 18a^2b^3 + 75a^3b^2$

$= 3a^2b^2 (9ab – 6b + 25a)$

6.

(i) $15a (2p – 3p) – 106 (2p – 3q)$
(ii) $3a (x^2 + y^2) + 6b (x^2 + y^2)$
Solution:

(i) $15a (2p – 3q) – 10b (2p – 3q)$

$= (2p – 3q)(15a – 10b)$

$= (2p – 3q) (5) (3a – 2b)$

$= 5 (2p- 3q) (3a – 2b)$

(ii) $3a (x^2 + y^2) + 66 (x^2 + y^2)$

$= (x^2 + y^2) (3a + 6b)$

$= (x^2 + y^2) (3) (a + 2b)$

$= 3 (x^2 + y^2) (a + 2b)$

7.

(i) $6(x + 2y)^3 + 8(x + 2y)^2$

(ii) $14(a – 3b)^3 – 21p(a – 3b)$

Solution:

(i) $6(x + 2y)^3 + 8(x + 2y)^2$

$(x + 2y)^2 [6 (x + 2y) + 8]$

$= (x + 2y)^2 [6x + 12y + 8]$

$= (x + 2y)^2 (2) (3x + 6y + 4)$

$= 2 (x + 2y)^2 (3x + 6y + 4)$

(ii) $14(a – 3b)^3 – 21 p(a – 3b)$

$= 7 [2 (a – 3b)^3 – 3p(a- 3b)]$

$= 7 [(a - 3b) \{2 (a - 3b)^2 - 3p\}]$

$= 7 (a - 3b) [2 (a - 3b)^2 - 3p]$

8. $10a (2p + q)^3 - 15b (2p + q)^2 + 35(2p + q)$

Solution:

$10a (2p + q)^3 - 15b (2p + q)^2 + 35(2p + q)$

$= 5 [2a (2p + q)]^3 - 3b (2p + q)^2 + 7 (2p + q)$

$= 5(2p + q) [2a (2p + q)^2 - 3b(2p + q) + 7]$

Exercise 11.2

Factorise the following (1 to 11) polynomials:

1.

(i) $x^2 + xy - x - y$
(ii) $y^2 - yz - 5y + 5z$

Solution:

(i) $x^2 + xy - x - y$

$= x (x + y) -1 (x + y)$

$= (x + y) (x - 1)$

(ii) $y^2 - yz - 5y + 5z$

$= y (y - z) -5 (y - z)$

$= (y - z) (y - 5)$

2.

(i) $5xy + 7y - 5y^2 - 7x$
(ii) $5p^2 - 8pq - 10p + 16q$

Solution:

(i) $5xy + 7y - 5y^2 - 7x$

$= 5xy - 5y^2 + 7y - 7x$

$= 5y (x - y) -7 (x - y)$

$= (x - y)(5y - 1)$

(ii) $5p^2 - 8pq - 10p + 16q$

$= 5p^2 - 10p - 8pq + 16q$

$= 5p (p - 2) - 8q (p - 2)$

$= (p - 2) (5p - 5q)$

$= (5p - 8q) (p - 2)$

3.

(i) $a^2b - ab^2 + 3a - 3b$
(ii) $x^3 - 3x^2 + x - 3$
Solution:
(i) $a^2b - ab^2 + 3a - 3b$

$= ab (a - b) + 3 (a - b) = (a - b) (ab + 3)$

(ii) $x^3 - 3x^2 + x - 3$

$= x^2 (x - 3) + 1 (x - 3)$

$= (x - 3) (x^2 + 1)$

4.

(i) $6xy^2 - 3xy - 10y + 5$
(ii) $3ax - 6ay - 8by + 4bx$
Solution:

(i) $6xy^2 - 3xy - 10y + 5$

$3xy (2y - 1) -5(2y - 1)$

$= (2y - 1) (3xy - 5)$

(ii) $3ax - 6ay - 8by + 4bx$

$= 3ax - 6ay + 4bx - 8by$

$= 3a (x - 2y) + 4b (x - 2y)$

$= (x - 2y) (3a + 4b)$

5.

(i) $x^2 + xy (1 + y) + y^3$

(ii) $y^2 - xy (1 - x) - x^3$

Solution:

(i) $x^2 + xy (1 + y) + y^3$

$= x^2 + xy + xy^2 + y^3$

$= x(x + y) + y^2(x + y)$

$= (x + y) (x + y^2)$

(ii) $y^2 - xy (1 - x) - x^3$

$= y^2 - xy + x^2y - x^3$

$= y (y - x) + x^2 (y - x)$

$= (y - x) (y + x^2)$

6.

(i) $ab^2 + (a – 1) b – 1$
(ii) $2a – 4b – xa + 2bx$
Solution:

(i) $ab^2 + (a – 1) b – 1$

$= ab^2 + ab – b – 1$

$= ab (b + 1) -1 (b + 1)$

$= (b + 1) (ab – 1)$

(ii) $2a – 4b – xa + 2bx$

$= 2 (a – 2b) -x (a – 2b)$

$= (a – 2b) (2 – x)$

7.

(i) $5ph – 10qk + 2rph – 4qrk$
(ii) $x^2 – x(a + 2b) + 2a^2$
Solution:

(i) $5ph – 10qk + 2rph – 4qrk$

$= 5 (ph – 2qk) + 2r (ph – 2qk)$

$= (ph – 2qk) (5 + 2r)$

(ii) $x^2 – x(a + 2b) + 2ab$

$= x^2 – xa – 2bx + 2ab$

$= x(x – a) – 2b(x – a)$

$= (x – a) (x – 2b)$

8.

(i) ab $(x^2 + y^2)$ – xy $(a^2 + b^2)$
(ii) $(ax + by)^2 + (bx – ay)^2$
Solution:
(i) ab $(x^2 + y^2)$ – xy $(a^2 + b^2)$

$= abx^2 + aby^2 – a^2xy – b^2xy$

$= (abx^2 – b^2xy) + (aby^2 – a^2xy)$

$= bx (ax – by) – ay (ax – by)$

$= (ax – by) (bx – ay)$
(ii) $(ax + by)^2 + (bx – ay)^2$

$= (a^2x^2 + b^2y^2 + 2abxy) + (b^2x^2 + a^2y^2 – 2abxy)$

$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 – 2abxy$

$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$

$= a^2x^2 + a^2y^2 + b^2x^2 + a^2y$

$= a^2 (x^2 + y^2) + b^2 (x^2 + y^2)$

$= (a^2 + b^2) (x^2 + y^2)$

9.

(i) $a^3 + ab(1 - 2a) - 2b^2$
(ii) $3x^2y - 3xy + 12x - 12$
Solution:
(i) $a^3 + ab - 2a^2b - 2b^2$

$= a^3 + ab - 2a^2b - 2b^2$

$= a(a^2 + b) - 2b(a^2 + b)$

$= (a^2 + b)(a - 2b)$

(ii) $3x^2y - 3xy + 12x- 12$

$= 3(x^2y - xy + 4x - 4)$

$= 3[xy(x - 1) + 4(x - 1)]$
$= 3(x - 1)(xy + 4)$

10.

(i) $a^2b + ab^2 - abc - b^2c + axy + bxy$
(ii) $ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$
Solution:

(i) $a^2b + ab^2 - abc - b^2c + axy + bxy$

$= ab(a + b) - bc(a + b) + xy(a + b)$

$= (a + b)(ab - bc + xy)$

(ii) $ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$

$= x^2 (a – b) + y^2 (a – b) + z^2 (a – b)$

$= (a – b)(x^2 + y^2 + z^2)$

11.

(i) $x – 1 – (x – 1)^2 + ax – a$
(ii) $ax + a^2x + aby + by – (ax + by)^2$
Solution:
(i) $x – 1 – (x – 1)^2 + ax – a$

$= (x – 1) – (x – 1)^2 + a (x – 1)$

$= (x – 1) [1 – (x – 1) + a]$
$= (x – 1) (1 – x + 1 + a)$

$= (x- 1) (2 – x + a)$
(ii) $ax + a^2x + aby + by – (ax + by)^2$

$= (ax + by) + (a^2x + aby) – (ax + by)^2$

$= (ax + by) + a (ax + by) – (ax + by)^2$

$= (ax + by) [1 + a – (ax + by)]$
$= (ax + by) (1 + a – ax – by)$

Exercise 11.3

1. Factorise the following expressions using algebraic identities:

(i) $x^2 – 12x + 36$

(ii) $36p^2 – 60pq + 25q^2$

(iii) $9y^2 + 66xy + 121y^2$

(iv) $a^4 + 6a^2b^2 + 9b^4$

(v) $x^2 + \frac{1}{x^2} + 2$

(vi) $x^2 + x + \frac{1}{4}$

Solution:

Using $(a + b)^2 = a^2 + 2ab +b^2$ and $(a – b)^2 = a^2 – 2ab + b^2$

(i) $y^2 – 12x + 36$

$= (x)^2 – 2 \times x \times 6 + (6)2^2$

$= (x – 6)^2$

(ii) $36p^2 – 60pq + 25q^2$

$= (6p)^2 – 2 \times 6p \times 5q + (5q)^2$

$= (6p – 5q)^2$

(iii) $9x^2 + 66xy + 121 y^2$

$= (3x)^2 + 2 \times 3x \times 11y + (11y)^2$

$= (3x + 11 y)^2$

(iv) $a^4 + 6a^2b^2 + 9b^4$

$= (a^2)^2 + 2 \times 2a^2 \times 3b^2 + (3b^2)^2$

$= (a^2 + 3b^2)^2$

(v) $x^2 + \frac{1}{x^2} + 2$

$= (x)^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$

$= \left(x + \frac{1}{x}\right)^2$

(vi) $x^2 + x + \frac{1}{4}$

$= (x)^2 + 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

$= \left(x + \frac{1}{2}\right)^2$

Factorise the following (2 to 13) expressions:

2.

(i) $4p^2 – 9$
(ii) $4x^2 – 169y^2$
Solution:

(i) $4p^2 – 9$

$= (2p)^2 – (3)^2$

$= (2p + 3)(2p – 3)$

(ii) $4x^2 - 169y^2$

$= (2x)^2 - (13y)^2$

$= (2x + 13y)(2x - 13y)$

3.

(i) $9x^2y^2 - 25$

(ii) $16x^2 - \frac{1}{144}$

Solution:

(i) $9x^2y^2 - 25$

$= (3xy)^2 - (5)^2$

$= (3xy + 5)(3xy - 5)$

(ii) $16x^2 - \frac{1}{144}$

$= (4x)^2 - \left(\frac{1}{12}\right)^2$

$= \left(4x + \frac{1}{12}\right)\left(4x - \frac{1}{12}\right)$

4.

(i) $20x^2 - 45y^2$

(ii) $\frac{9}{16} - 25a^2b^2$

Solution:

(i) $20x^2 - 45y^2$

$= 5(4x^2 - 9y^2)$

$= 5[(2x)^2 - (3y)^2]$

$= 5(2x + 3y)(2x - 3y)$

(ii) $\frac{9}{16} - 25a^2b^2$

$= \left(\frac{3}{4}\right)^2 - (5ab)^2$

$= \left(\frac{3}{4} + 5ab\right)\left(\frac{3}{4} - 5ab\right)$

5.

(i) $(2a + 3b)^2 - 16c^2$
(ii) $1 - (b - c)^2$
Solution:
(i) $(2a + 3b)^2 - 16c^2$

$= (2a + 3b)^2 - (4c)^2$

$= (2a + 3b + 4c)(2a + 3b - 4c)$

(ii) $1 - (b - c)^2$

$= (1)^2 - (b - c)^2$

$= [1 + b - c)] [1 - (b - c)]$
$= (1 + b - c)(1 - b + c)$

6.

(i) $9(x + y)^2 - x^2$
(ii) $(2m + 3n)^2 - (3m + 2n)^2$
Solution:

(i) $9(x + x)^2 - x^2$

$= [3(x + y)]^2 - [x]^2$

$= [3 (x + y) + x] [3 (x + y) – x]$
$= (3x + 3y + x) (3x + 3y – x)$

$= (4x + 3y) (2x + 3x)$

(ii) $(2m + 3n)^2 – (3m + 2n)^2$

$= (4m^2 + 9n^2 + 12mn) – (9m^2 + 4n^2 + 12mn)$

$= 4m^2 + 9n^2 + 12mn – 9m^2 – 4m^2 – 12mn$

$= 4m^2 + 9n^2 – 9m^2 – 4n^2$

$= – 5m^2 + 5n^2$

$= 5 (n^2 – m^2)$

$= 5 (m + n) (n – m)$

7.

(i) $25 (a + b)^2 – 16 (a – b)^2$
(ii) $9 (3x + 2)^2 – 4 (2x – 1)^2$
Solution:

(i) $25 (a + b)^2 – 16 (a – b)^2$

$= [5 (a + b)]^2 – [4 (a – b)]^2$

$= (5a + 5b)^2 – (4a – 4b)^2$

$= [(5a + 5b)^2 + (4a – 4b)] [(5a + 5b) – (4a – 4b)]$

$= (5a + 5b + 4a – 4b) (5a + 5b – 4a + 4b)$

$= (9a + b) (a + 9b)$

(ii) $9 (3x + 2)^2 – 4 (2x – 1)^2$

$= [3 (3x + 2)]^2 – [2 (2x – 1)]^2$

$= (9x + 6)^2 – (4x – 2)^2$

$= [(9x + 6) + (4x – 2)] [(9x + 6) – (4x – 2)]$
$= (9x + 6 + 4x – 2) (9x + 6 – 4x + 2)$

$= (13x + 4) (5x + 8)$

8. (i) $x^3 – 25x$
(ii) $63p^2q^2 – 7$
Solution:
(i) $x^3 – 25x$

$= x (x^2 – 25) = x [(x)^2 – (5)^2]$
$= x (x + 5) (x – 5)$

(ii) $63p^2q^2 – 7$

$= 7 (9p^2q^2 – 1)$

$= 7 [(3pq)^2 – (1)^2]$
$= 7 (3pq + 1) (3pq – 1)$

9. (i) $32a^2b - 72b^3$
(ii) $9(a + b)^3 - 25(a + b)$
Solution:

(i) $32a^2b - 72b^3$

$= 8b(4a^2 - 9b^2) \Rightarrow 8b[(2a)^2 - (3b)^2]$
$= 8b(2a + 3b)(2a - 3b)$

(ii) $9(a + b)^3 - 25(a + b)$
$= (a + b)[9(a + b)^2 - 25]$
$= (a + b)[\{3(a + b)\}^2 - (5)^2]$
$= (a + 6)[(3a + 3b)^2 - (5)^2]$
$= (a + b)[(3a + 3b + 5)(3a + 36 - 5)]$
$= (a + b)(3a + 3b + 5)(3a + 3b - 5)$

10.

(i) $x^2 - y^2 - 2y - 1$
(ii) $p^2 - 4pq + 4q^2 - r^2$
Solution:
(i) $x^2 - y^2 - 2y - 1$

$= x^2 - (y^2 + 2y + 1)$

$= (x)^2 - (y + 1)^2$

$= [x + (y + 1)][x - (y + 1)]$
$= (x + y + 1)(x - y - 1)$

(ii) $p^2 - 4pq + 4q^2 - r^2$

$= (p)^2 - 2 \times p \times 2q + (2q)^2 - r^2$ [$\because (a - b)^2 = a2 - 2ab + b^2$]
$= (p - 2q)^2 - (r)^2$

$= (p - 2q + r)(p - 2q - r)$ [$\because a^2 - b^2 = (a + b)(a - b)$]

11. (i) $9x^2 - y^2 + 4y - 4$
(ii) $4a^2 - 4b^2 + 4a + 1$
Solution:
(i) $9x^2 - y^2 + 4y - 4$

$= 9x^2 - (y^2 - 4y + 4)$

$= 9x^2 - (y - 2)^2$

$= (3x)^2 (y - 2)^2$

$= [3x + (y - 2)] [3x - (y - 2)]$
$= (3x + y - 2) (3x - y + 2)$

(ii) $4a^2 - 4b^2 + 4a + 1$

$= (4a^2 + 4a + 1) - 4b^2$

$= (2a + 1)^2 - (2b)^2$

$= (2a + 2b + 1) (2a - 2b + 1)$

12.

(i) $625 - p^4$
(ii) $5y^5 - 405y$

Solution:

(i) $625 - p^4$

$= (25)^2 - (p^2)^2$

$= (25 + p^2)(25 - p^2)$

$= (25 + p^2)[(5)^2 - (p)^2]$

$= (25 + p^2)(5 + p)(5 - p)$

(ii) $5y^5 - 405y$

$= 5y(y^4 - 81)$

$= 5y[(y^2)^2 - (9)^2]$

$= 5y(y^2 + 9)(y^2 - 9)$

$= 5y(y^2 + 9)[(y)^2 - (3)^2$

$= 5y(y^2 + 9)(y + 3)(y - 3)$

13.

(i) $x^4 - y^4 + x^2 - y^2$

(ii) $64a^2 - 9b^2 + 42bc - 49c^2$

Solution:

(i) $x^4 - y^4 + x^2 - y^2$

$= [(x^2)^2 - (y^2)^2] + (x^2 - y^2)$ [Using, $a^2 - b^2 = (a + b)(a - b)$]

$= (x^2 + y^2)(x^2 - y^2) + 1(x^2 - y^2)$

$= (x^2 - y^2)(x^2 + y^2 + 1)$

$= (x + y(x - y)(x^2 + y^2 + 1)$

(ii) $64a^2 - 9b^2 + 42bc - 49c^2$

$= 64a^2 - [9b^2 - 42bc + 49c^2]$

$= (8a)^2 - [(3b)^2 - 2 \times 3b \times 7c + (7c)^2]$

$[\because a^2 + b^2 - 2ab = (a - b)^2$ and $a^2 - b^2 = (a + b)(a - b)]$

$= (8a)^2 - (3b - 7c)^2$

$= (8a + 3b - 7c)(8a - 3b + 7c)$

Exercise 11.4

1.

(i) $x^2 + 3x + 2$,
(ii) $z^2 + 10z + 24$

Solution:

(i) $x^2 + 3x + 2$

$= x^2 + 2x + x + 2$

$= x(x + 2) + 1 (x + 2)$

$= (x + 2) (x + 1)$

(ii) $z^2 + 10z + 24$

$= z^2 + 6z + 4z + 24$

$= z(z + 6) + 4 (z + 6)$

$= (z + 6) (z + 4)$

2.

(i) $y^2 – 7y + 12$
(ii) $m^2 – 23m + 42$
Solution:

(i) $y^2 – 7y + 12$

$= y^2 – 3y – 4y + 12$ [Since, $12 = -3 \times (-4)$ and $– 7 = -3 – 4$]

$= y(y – 3) -4(y – 3)$

$= (y – 3) (y – 4)$

(ii) $m^2 – 23m + 42$

$= m^2 – 2m – 21m + 42$ [Since, $42 = -2 \times (-21)$ and $– 23 = -21 – 2$]
$= m(m – 2) – 21(m – 2)$

$= (m – 2) (m – 21)$

3.

(i) $y^2 – 5y – 24$,
(ii) $t^2 + 23t – 108$
Solution:

(i) $y^2– 5y – 24$

$= y^2 – 8y + 3y – 24$

$= y(y – 8) + 3(y – 8)$

$= (y – 8) (y + 3)$

(ii) $t^2 + 23t – 108$

$= t^2 + 27t – 4t – 108$

$= t (t + 27) – 4 (t + 27)$

$= (t + 27) (t – 4)$

4.

(i) $3x^2 + 14x + 8$,
(ii) $3y^2 + 10y + 8$
Solution:

(i) $3x^2 + 14x + 8$

$= 3x^2 + 12x + 2x + 8$

$= 3x (x + 4) + 2 (x + 4)$

$= (x + 4) (3x + 2)$

(ii) $3y^2 + 10y + 8$

$= 3y^2 + 6y + 4y + 8$

$= 3y (y + 2) + 4 (y + 2)$

$= (y + 2) (3y + 4)$

5.

(i) $14x^2 – 23x + 8$,
(ii) $12x^2 – x – 35$
Solution:
(i) $14x^2 – 23x + 8$

$= 14x^2 – 16x – 7x + 8$

$= 2x (7x – 8) – 1 (7x – 8)$

$= (7x – 8) (2x – 1)$

(ii) $12x^2 - x - 35$

$= 12x^2 - 21x + 20x - 35$

$= 3x (4x - 7) + 5 (4x - 7)$

$= (4x - 7) (3x + 5)$

6.

(i) $6x^2 + 11x - 10$
(ii) $5 - 4x - 12x^2$
Solution:
(i) $6x^2 + 11x - 10$

$= 6x^2 + 15x - 4x - 10$

$= 3x (2x + 5) - 2 (2x + 5)$

$= (2x + 5) (3x - 2)$

(ii) $5 - 4x- 12x^2$

$= 5 - 10x + 6x - 12x^2$

$= 5 (1 - 2x) + 6x (1 - 2x)$

$= (1 - 2x) (5 + 6x)$

7.

(i) $1 - 18y - 63y^2$,
(ii) $3x^2 - 5xy - 12y^2$

Solution:

(i) $1 - 18y - 63y^2$

$= 1 - 21y + 3y - 63y^2$

$= 1(1- 21y) + 3y (1 - 21y)$

$= (1 - 21y) (1 + 3y)$

(ii) $3x^2 - 5xy - 12y^2$

$= 3x^2 - 9xy + 4xy- 12y^2$

$= 3x (x - 3y) + 4y (x - 3y)$

$= (x - 3y) (3x + 4y)$

8.

(i) $x^2 - 3xy - 40y^2$

(ii) $10p^2q^2 - 21pq + 9$

Solution:

(i) $x^2 - 3xy - 40y^2$

$= x^2 - 8xy + 5xy - 40y^2$

$= x (x - 8y) + 5y (x - 8y)$

$= (x - 8y) (x + 5y)$

(ii) $10p^2q^2 - 21pq + 9$

$= 10p^2q^2 – 15pq – 6pq + 9$

$= 5pq (2pq – 3) – 3 (2pq – 3)$

$= (2pq – 3) (5pq – 3)$

9.

(i) $2a^2b^2 + ab – 45$
(ii) $x (12x + 7) – 10$
Solution:

(i) $2a^2b^2 + ab – 45$

$= 2a^2b^2 + 10ab – 9ab – 45$

$= 2ab (ab + 5) – 9 (ab + 5)$

$= (ab + 5) (2ab – 9)$

(ii) $x (12x + 7) – 10$

$= 12x^2 + 7x – 10$

$= 12x^2 + 15x – 8x – 10$

$= 3x (4x + 5) – 2 (4x + 5)$

$= (4x + 5) (3x – 2)$

10.

(i) $(a + b)^2 – 11(a + b) – 42$
(ii) $8 + 6(p + q) – 5(p + q)$

Solution:

(i) $(a + b)^2 – 11(a + b) – 42$

Let $(a + b) = x$, then we have

$= x^2 – 11x – 42$

$= x^2 – 14x + 3x – 42$ [$\because -42 = -14 \times 3$ and $-11 = -14 + 3$]
$= x(x – 14) + 3 (x – 14)$

$= (x – 14) (x + 3)$

Substituting the value of x we get,

$= (a + b – 14) (a + b + 3)$

(ii) $8 + 6(p + q) – 5(p + q)^2$

Let $p + q = x$, then we have

$= 8 + 6x – 5x^2$

$= -5x^2 + 6x + 8$

$= -(5x^2 – 6x – 8)$

$= 5x^2 – 10x + 4x – 8$

[$\because 5 \times (-8) = 40 \Rightarrow -40 = -10 \times 4$ and $-6 = -10 + 4$]

$= (x – 2) (5x + 4)$

Substituting the value of x, then

$= -(p + q - 2)(5p + 5q + 4)$

$= (4 + 5p + 5q)(-p - q + 2)$

$= (4 + 5p + 5q)(2 - p - q)$

11.

(i) $(x - 2y)^2 - 6(x - 2y) + 5$
(ii) $7 + 10(2x - 3y) - 8(2x - 3y)^2$
Solution:

(i) Let $x - 2y = z$

Then, $(x - 2y)^2 - 6(x - 2y) + 5$ becomes

$= z^2 - 6z + 5$

$= z^2 - 5z - z + 5$

$= z(z - 5) - 1(z - 5)$

$= (z - 5)(z - 1)$

Now, on substituting $z = x - 2y$, we get

$= [(x - 2y) - 5][(x - 2y) - 1]$
$= (x - 2y - 5)(x - 2y - 1)$

(ii) $7 + 10(2x - 3y) - 8(2x - 3y)^2$

Let $2x - 3y = z$

Then, $7 + 10 (2x - 3y) - 8 (2x - 3y)^2$ becomes

$= 7 + 10z - 8z^2$

$= 7 + 14z - 4z - 8z^2$

$= 7 (1 + 2z) - 4z (1 + 2z)$

$= (1 + 2z) (7 - 4z)$

Now, on substituting $z = 2x - 3y$, we get

$= [(1 + 2 (2x - 3y)] [7 - 4 (2x - 3y)]$
$= (1 + 4x - 6y) (7 - 8x + 12y)$

Exercise 11.5

Work out the following divisions:
(i) (35x + 28) ÷ (5x + 4)
(ii) $7p^2q^2(9r - 27) \div 63pq(r - 3)$
Solution:

(i) (35x + 28) ÷ (5x + 4)

$\frac{7(5x+4)}{(5x+4)} = 7$

(ii) $7p^2q^2(9r - 27) \div 63pq(r - 3)$

$= \frac{7p^2q^2 \times 9(r-3)}{63pq(r-3)}$

$= p^{2-1}\, q^{2-1} \times 9 = 9pq$

2. Divide as directed:
(i) 6(2x + 7) (5x – 3) ÷ 3(5x – 3)
(ii) 33pq (p + 3) (2q – 5) ÷ 11p (2q – 5)
Solution:

(i) 6(2x + 7) (5x – 3) ÷ 3(5x – 3)

$= \frac{6(2x+7)(5x-3)}{3(5x-3)}$

= 2(2x + 7)

(ii) 33pq (p + 3) (2q – 5) ÷ 11p (2q – 5)

$= \frac{33pq(p+3)(2q-5)}{11p(2q-5)}$

$= 3q(p + 3)$

3. Factorise the expression and divide them as directed:
(i) $(7x^2 - 63x) \div 7(x - 3)$
(ii) $(3p^2 + 17p + 10) \div (p + 5)$
(iii) $10xy(14y^2 + 43y - 21) \div 5x(7y - 3)$
(iv) $12pqr(6p^2 - 13pq + 6q^2) \div 6pq(2p - 3q)$
Solution:

(i) $(7x^2 - 63x) \div 7(x - 3)$

$= \frac{7x(x^2-9)}{7(x-3)}$

$= \frac{7x[(x)^2-(3)^2]}{7(x-3)}$

$= \frac{7x(x+3)(x-3)}{7(x-3)}$

$= x(x+3)$

(ii) $(3p^2 + 17p + 10) \div (p + 5)$

$= \frac{3p^2 + 17p + 10}{p + 5}$ $\left\{\begin{matrix} \because 3 \times 10 = 30 \\ \therefore 30 = 2 \times 15 \\ 17 = 2 + 15 \end{matrix}\right\}$

$= \frac{3p^2 + 2p + 15p + 10}{p + 5}$

$= \frac{p(3p+2)+5(3p+2)}{p+5}$

$= \frac{(3p+2)(p+5)}{(p+5)}$

$= 3p + 2$

(iii) $10xy(14y^2 + 43y - 21) \div 5x(7y - 3)$

$= \frac{10xy[14y^2+49y-6y-21]}{5x(7y-3)}$ $\left\{\begin{matrix} \because -21 \times 14 = -294 \\ \therefore -294 = 49 \times (-6) \\ 43 = 49 - 6 \end{matrix}\right\}$

$= \frac{10xy[7y(2y+7)-3(2y+7)]}{5x(7y-3)}$

$= \frac{10xy(2y+7)(7y-3)}{5x(7y-3)}$

$= 2x(2y + 7)$

(iv) $12pqr\ (6p^2 - 13pq + 6q^2) \div 6pq\ (2p - 3q)$

$= \frac{12pqr[6p^2-9pq-4pq+6q^2]}{6pq(2p-3q)}$ $\left\{\begin{matrix} \because 6 \times 6 = 36 \\ \therefore 36 = -9 \times (-4) \\ -13 = -9 - 4 \end{matrix}\right\}$

$= \frac{12pqr[3p(2p-3q)-2q(2p-3q)]}{6pq(2p-3q)}$

$= \frac{12pqr(2p-3q)(3p-2q)}{6pq(2p-3q)}$

$= 2r(3p - 2q)$

Mental Maths

Question 1: Fill in the blanks:
(i) When an algebraic expression can be written as the product of two or more expressions then each of these expressions is called ……….. of the given expression.
(ii) The process of finding two or more expressions whose product is the given expression is called ………..
(iii) HCF of two or more monomials = (HCF of their ……….. coefficients) × (HCF of their literal coefficients)
(iv) HCB of literal coefficients = product of each common literal raised to the ……….. power.
(v) To factorise the trinomial of the form $x^2 + px + q$, we need to find two integers a and b such that a + b = ……….. and ab = ………..
(vi) To factorise the trinomial of the form $ax^2 + bx + c$, where a, b and c are integers, we split b into two parts such that ……….. of these parts is b and their is ……….. ac.
Solution:
(i) When an algebraic expression can be written as the product of two or more expressions then each of these expressions is called factor of the given expression.
(ii) The process of finding two or more expressions whose product is the given expression is called factorization.
(iii) HCF of two or more monomials
= (HCF of their numerical coefficients) × (HCF of their literal coefficients)
(iv) HCF of literal coefficients
= product of each common literal raised to the lowest power.
(v) To factorise the trinomial of form $x^2 + px + q$,
we need to find two integers a and b such that a + b= p and ab = q.
(vi) To factorise the trinomial of the form $ax^2 + bx + c$,
where a, b and c are integers, we split b into two parts such that algebraic sum of these parts is b and their product is ac.

Question 2: State whether the following statements are true (T) or false (F):
(i) Factorisation is the reverse process of multiplication.
(ii) HCF of two or more polynomials (with integral coefficients) is the smallest common factor of the given polynomials.
(iii) HCF of $6x^2y^2$ and $8xy^3$ is $2xy^2$.
(iv) Factorisation by grouping is possible only if the given polynomial contains an even number of terms.
(v) To factorise the trinomial of the form $ax^2 + bx + c$ where, a, b, c are integers we want to find two integers A and B such that
$A + B = ac$ and $AB = b$
(vi) Factors of $4x^2 – 12x + 9$ are $(2x – 3) (2x – 3)$.
Solution:
(i) Factorisation is the reverse process of multiplication. True
(ii) HCF of two or more polynomials (with integral coefficients) is the smallest common factor of the given polynomials. False
(iii) HCF of $6x^2y^2$ and $8xy^2$ is $2xy^2$. True
(iv) Factorisation by grouping is possible only
if the given polynomial contains an even number of terms. True
(v) To factorise the trinomial of the form $ax^2 + bx + c$
where, a, b, c are integers we want to find two integers A and B such that
$A + B = ac$ and $AB = b$ False
Correct :
A + B should be equal to ft. and $AB = ac$
(vi) Factors of
$4x^2 – 12x + 9$ are $(2x – 3) (2x – 3)$. True

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 14):

Question 3: H.C.F. of 6abc, $24ab^2$, $12a^2b$ is
(a) 6ab
(b) $6ab^2$
(c) $6a^2b$
(d) 6abc
Solution:
H.C.F. of babe, $24ab^2$, $12a^2b$
= H.C.F. of 6, 24, 12 × H.C.F. of abc, ab^2, a^2b
= 6 × a × b = 6ab (a)

Question 4: Factors of $12a^2b + 15ab^2$ are
(a) 3a(4ab + 5b2)
(b) 3ab(4a + 5b)
(c) $3b(4a^2 + 5ab)$
(d) none of these
Solution:
$12a^2b + 15\ ab^2 = 3ab(4a + 5b)$ (b)

Question 5: Factors of 6xy – 4y + 6 – 9x are
(a) (3y – 2) (2x – 3)
(b) (3x – 2) (2y – 3)
(c) (2y – 3) (2 – 3x)
(d) none of these
Solution:
6xy – 4y + 6 – 9x
= 6xy – 9x – 4y + 6
= 3x(2y – 3) -2(2y – 3)
= (2y – 3) (3x – 2)

Question 6: Factors of $49p^3q - 36pq$ are
(a) p(7p + 6q) (7p – 6q)
(b) q(7p – 6) (7p + 6)
(c) pq(7p + 6) (7p – 6)
(d) none of these
Solution:
$49p^2q - 36pq$
$= pq(49p^2 - 36)$
$=pq[(7p)^2 - (6)^2]$
= pq(7p + 6) (7p – 6)

Question 7: Factors of y(y – z) + 9(z – y) are
(a) (y – z) (y + 9)
(b) (z – y) (y + 9)
(c) (y – z) (y – 9)
(d) none of these
Solution:
y(y – z) + 9(z – y)
= y(y – z) – 9(y – z)
= (y – z) (y – 9) (c)

Question 8: Factors of (lm + l) + m + 1 are
(a) (lm + l)(m + l)
(b) (lm + m)(l + 1)
(c) l(m + 1)
(d) (l + 1)(m + 1)
Solution:
Factors of lm + l + m + 1 are
l(m + 1) + l (m + 1) = (m + 1)(l + 1) (d)

Question 9: Factors of $z^2 - 4z - 12$ are
(a) $(z + 6)(z - 2)$
(b) $(z - 6)(z + 2)$
(c) $(z - 6)(z - 2)$
(d) $(z + 6)(z + 2)$
Solution:
Factors of $z^2 - 4z - 12$
$\Rightarrow z^2 - 6z + 2z - 12$
$= z(z - 6) + 2(z - b)$
$= (z - 6)(z + 2)$ (b)

Question 10: Factors of $63a^2 - 112b^2$ are
(a) $63 (a - 2b)(a + 2b)$
(b) $7(3a + 2b)(3a - 2b)$
(c) $7(3a + 4b)(3a - 4b)$
(d) none of these
Solution:
Factors of $63a^2 - 112b^2$ are
$= 7(9a^2 - 16b^2)$
$= 7[(3a)^2 - (4b)^2]$
$= 7(3a + 4b)(3a - 4b)$ (c)

Question 11: Factors of $p^4 - 81$ are
(a) $(p^2 - 9)(p^2 + 9)$
(b) $(p + 3)^2 (p - 3)^2$
(c) $(p + 3) (p - 3) (p^2 + 9)$
(d) none of these
Solution:
$p^4 - 81 = (p^2)^2 - (9)^2$
$= (p^2 + 9)(p^2 - 9)$

$= (p^2 + 9)\{(p)^2 - (3)^2\}$
$= (p^2 + 9)(p + 3)(p - 3)$ (c)

Question 12: Factors of 3x + 7x – 6 are
(a) (3x – 2)(x + 3)
(b) (3x + 2) (x – 3)
(c) (3x – 2)(x – 3)
(d) (3x + 2) (x + 3)
Solution:
$3x^2 + 7x - 6$
$= 3x^2 + 9x - 2x - 6$
$= 3x(x + 3) - 2(x + 3)$
$= (3x - 2)(x + 3)$ (a)

Question 13: Factors of $16x^2 + 40x + 25$ are
(a) (4x + 5)(4x + 5)
(b) (4x + 5)(4x – 5)
(c) (4x + 5)(4x + 8)
(d) none of these
Solution:
$16x^2 + 40x + 25$
$= (4x)^2 + 2 \times 4x \times 5 + (5)^2$
$= (4x + 5)^2$
$= (4x + 5)(4x + 5)$ (a)

Question 14: Factors of $x^2 - 4xy + 4y^2$ are
(a) (x – 2y)(x + 2y)
(b) (x-2y)(x-2y)
(c) (x + 2y)(x + 2y)
(d) none of these

Solution:

$x^2 – 4xy + 4y^2$

$= (x)^2 – 2 \times x \times 2y + (2y)^2 = (x – 2y)^2$

$= (x- 2y)(x – 2y)$ (b)

Higher Order Thinking Skills (Hots)
Factorise the following

Question 1: $x^2 + \left(a + \frac{1}{a}\right) x + 1$

Solution:

$x^2 + \left(a + \frac{1}{a}\right)x + 1$

$= x^2 + ax + \frac{x}{a} + 1$

$= x(x + a) + \frac{1}{a}(x + a)$

$= (x + a) \left(x + \frac{1}{a}\right)$

Question 2: $36a^4 – 97a^2b^2 + 36b^4$

Solution:

$= 36a^4 – 97a^2b^2 + 36b^4$

$= 36a^4 – 72a^2b^2 + 36b^4 – 25a^2b^2$

$= (6a^2)^2 – 2 \times 6a^2 \times 6b^2 + (6b^2)^2 – (5ab)^2$

$= (6a^2 – 6b^2)^2 – (5ab)^2$

$= (6a^2 – 6b^2 + 5ab)(6a^2 – 6b^2 – 5ab)$

$= (6a^2 + 5ab – 6b^2)(6a^2 – 5ab – 6b^2)$

$= [6a^2 + 9ab – 4ab – 6b^2] [6a^2 – 9ab + 4ab – 6b^2]$

$= [3a(2a + 3b) – 2b(2a + 3b)] [3a(2a – 3b) + 2b(2a – 3b)]$

$= (2a + 3b)(3a – 2b)(2a – 3b)(3a + 2b)$

Question 3: $2x^2 - \sqrt{3}x - 3$
Solution:
$2x^2 - \sqrt{3}x - 3$
$= 2x^2 - 2\sqrt{3}x + \sqrt{3}x - 3$
$\{\because 2 \times (-3) = -6 \therefore -6 = -2\sqrt{3} \times \sqrt{3} - \sqrt{3} = -2\sqrt{3} + \sqrt{3}\}$
$= 2x(x - \sqrt{3}) + \sqrt{3}\,(x - \sqrt{3})$
$= (x - \sqrt{3}) + (2x + \sqrt{3})$

Question 4: $y(y^2 - 2y) + 2(2y - y^2) - 2 + y$
Solution:
$y(y^2 - 2y) + 2(2y - y^2) - 2 + y$
$= y^3 - 2y^2 + 4y - 2y^2 - 2 + y$
$= y^3 - 4y^2 + 5y - 2$
$= y^3 - 2y^2 + y - 2y^2 + 4y - 2$
$= y(y^2 - 2y + 1) - 2(y^2 - 2y + 1)$
$= (y^2 - 2y + 1)(y - 2)$
$= [(y)^2 - 2 \times y \times 1 + (1)^2]\,(y - 2)$
$= (y - 1)^2(y - 2)$

Check Your Progress

1. Find the HCF of the given polynomials:
(i) $14pq, 28p^2q^2$
(ii) $8abc, 24ab^2, 12a^2b$
Solution:

(i) $14pq, 28p^2q^2$

HCF of 14, 28 = 14

HCF of $14pq, 28p^2q^2 = 14pq$

(ii) $8abc, 24ab^2, 12a^2b$

HCF of 8, 24, 12 = 4

HCF of $8abc, 24ab^2, 12a^2b = 4ab$

2. Factorise the following:
(i) $10x^2 – 18x^3 + 14x^4$
(ii) $5x^2y + 10xyz + 15xy^2$
(iii) $p^2x^2 + c^2x^2 – ac^2 – ap^2$
(iv) $15(x + y)^2 – 5x – 5y$
(v) $(ax + by)^2 + (ay – bx)^2$
(vi) $ax + by + cx + bx + cy + ay$
(vii) $49x^2 – 70xy + 25y^2$
(viii) $4a^2 + 12ab + 9b^2$
(ix) $49p^2 – 36q^2$
(x) $100x^3 – 25xy^2$
(xi) $x^2 – 2xy + y^2 – z^2$

(xii) $x^8 - y^8$
(xiii) $12x^3 - 14x^2 - 10x$
(xiv) $p^2 - 10p + 21$
(xv) $2x^2 - x - 6$
(xvi) $6x^2 - 5xy - 6y^2$
(xvii) $x^2 + 2xy - 99y^2$
Solution:

(i) $10x^2 - 18x^3 + 14x^4$

HCF of 10, 18, 14 = 2

So, $10x^2 - 18x^3 + 14x^4$

$= 2x^2 (5 - 9x + 7x^2)$

(ii) $5x^2y + 10xyz + 15xy^2$

HCF of 5, 10, 15 = 5

So, $5x^2y + 10xyz + 15xy^2$

$= 5xy (x + 2z + 3y)$

(iii) $p^2x^2 + c^2x^2 - ac^2 - ap$

$= p^2x^2 - ap^2 + c^2x^2 - ac^2$

$= p^2(x^2 - a) + c^2(x^2 - a)$

$= (x^2 - a) (p^2 + c^2)$

(iv) $15(x + y)^2 - 5x - 5y$

$= 15(x + y)^2 - 5(x + y)$

$= 5(x + y)\,[3(x + y) – 1]$
$= 5(x + y)\,(3x + 3y – 1)$

(v) $(ax + by)^2 + (ay – bx)^2$

On expanding, we have

$= a^2x^2 + b^2y^2 + 2abxy + a^2y^2 + b^2x^2 – 2abxy$

$= a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$

$= a^2(x^2 + y^2) + b^2(x^2 + y^2)$

$= (x^2 + y^2)\,(a^2 + b^2)$

(vi) $ax + by + cx + bx + cy + ay$

$= ax + bx + cx + ay + by + cy$ [On grouping the like variables]
$= x(a + b + c) + y(a + b + c)$

$= (a + b + c)\,(x + y)$

(vii) $49x^2 – 70xy + 25y^2$

$= (7x)^2 – 2 \times 7x \times 5y + (5y)^2$ [$\because (a – b)^2 = a^2 – 2ab + b^2$]
$= (7x – 5y)^2$

(viii) $4a^2 + 12ab + 9b^2$

$= (2a)2^2 + 2 \times 2a \times 3b + (3b)^2$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]
$= (2a + 3b)^2$

(ix) $49p^2 – 36q^2$

$= (7p)^2 – (6q)^2$
$= (7p + 6q) (7p – 6q)$ [∵ $a^2 – b^2 = (a + b) (a – b)$]

(x) $100x^3 – 25xy^2$

$= 25x(x^2 – y^2) = 25x\{(x)^2 – (y)^2\}$

$= 25x(x + y) (x – y)$

(xi) $x^2 – 2xy + y^2 – z^2$

$= (x – y)^2 – (z)^2$ [∵ $a^2 -2ab + b^2 = (a – b)^2$ and $a^2 – b^2 = (a + b) (a – b)$]
$= (x – y + z)(x – y – z)$

(xii) $x^8 – y^8$

$= (x^4)^2 – (y^4)^2$ [∵ $a^2 – b^2 = (a + b)(a- b)$]
$= (x^4 + y^4) (x^4 – y^4)$

$= (x^4 + y^4) [(x^2)^2 – (y^2)^2]$
$= (x^4 + y^4) (x^2 + y^2) (x^2 – y^2)$

$= (x^4 + y^4 (x^2 + y^2) (x + y) (x – y)$

(xiii) $12x^3 – 14x^2 – 10x$

$= 2x(6x^2 – 7x – 5)$ [Now, as $6 \times (-5) = -30 \Rightarrow -30 = -10 \times 3$ and $-7 = -10 + 3$]
$= 2x(6x^2 + 3x – 10x – 5)$

$= 2x\{3x(2x + 1) – 5(2x + 1)\}$

$= 2x(2x + 1) (3x – 5)$

(xiv) $p^2 - 10p + 21$

$= p^2 - 3p - 7p + 21$ [Now, as 21 =-3 × (-7) and -10 = -3 – 7]
$= p(p - 3) - 7(p - 3)$

$= (p - 3)(p - 7)$

(xv) $2x^2 - x - 6$

$= 2x^2 - 4x + 3x - 6$ [Now, as -6 × 2 = -12 ⇒ -12 = -4 × 3 and -1 = -4 + 3]
$= 2x(x - 2) + 3(x - 2)$

$= (x - 2)(2x + 3)$

(xvi) $6x^2 - 5xy - 6y^2$

$= 6x^2 - 9xy + 4xy - 6y^2$ [Now, as 6 × (-6) = -36 ⇒ – 36 = -9 × 4 and – 5 = -9 + 4]
$= 3x(2x - 3y) + 2y(2x - 3y)$

$= (2x - 3y)(3x + 2y)$

(xvii) $x^2 + 2xy - 99y^2$

$= x^2 + 11xy - 9xy - 99y^2$ [Now, as -99 = -11 × 9 and -2 = -11 + 9 }

$= x(x + 11y) - 9y(x + 11y)$

$= (x + 11y)(x - 9y)$

3. Divide as directed:
(i) $15(y + 3)(y^2 - 16) \div 5(y^2 - y - 12)$
(ii) $(3x^3 - 6x^2 - 24x) \div (x - 4)(x + 2)$
(iii) $(x^4 - 81) \div (x^3 + 3x^2 + 9x + 27)$
Solution:

(i) $15(y + 3)(y^2 - 16) \div 5(y^2 - y - 12)$

$y^2 - 16 = (y)^2 - (4)^2$

$= (y + 4)(y - 4)$

$y^2 - y - 12 = y^2 - 4y + 3y - 12$

$= y(y - 4) + 3(y - 4)$

$= (y - 4)(y + 3)$

Now,

$$\frac{15(y+3)(y^2-16)}{5(y^2-y-12)}$$

$$= \frac{15\times(y+3)(y+4)(y-4)}{5(y-4)(y+3)}$$

$= 3(y + 4)$

(ii) $(3x^3 - 6x^2 - 24x) \div (x - 4)(x + 2)$

$3x^3 - 6x^2 - 24x = 3x(x^2 - 2x - 8)$

$= 3x\{x^2 - 4x + 2x - 8\}$

$= 3x\{x(x - 4) + 2(x - 4)\}$

$= 3x(x - 4)(x + 2)$

Now,

$\frac{3x^3-6x^2-24x}{(x-4)(x+2)}$

$= \frac{3x(x-4)(x+2)}{(x-4)(x+2)}$

$= 3x$

(iii) $(x^4 - 81) \div (x^3 + 3x^2 + 9x + 27)$

$x^4 - 81 = (x^2)^2 - (9)^2 = (x^2 + 9)(x^2 - 9)$

$= (x^2 + 9)[(x)^2 - (3)^2]$
$= (x^2 + 9)(x + 3)(x - 3)$

And,

$x^3 + 3x^2 + 9x + 27 = (x)^2 + (x + 3) + 9(x + 3)$

$= (x^2 + 9)(x + 3)$

Now,

$\frac{x^4-81}{x^3+3x^2+9x+27}$

$= \frac{(x^2+9)(x+3)(x-3)}{(x^2+9)(x+3)}$

$= (x - 3)$