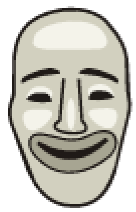
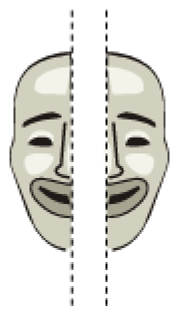
# Symmetry

## Symmetrical Figures and Lines of Symmetry

Let us consider the following mask.



If we cut this mask exactly from middle, we obtain two halves of the mask as shown below.



We can observe that both left and right half faces are exact copies of each other.

Similarly, if we fold the following pictures from the middle, then the left half of the picture will exactly overlap over the right half of the picture.

India gate

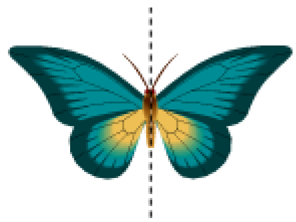
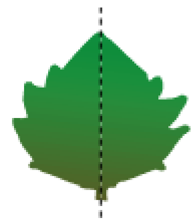
Taj Mahal

Gateway of India

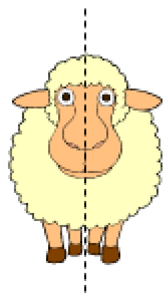
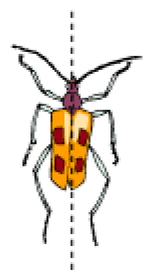
Such figures are known as **symmetrical figures** or **symmetric.**

If we place a mirror on the line where we folded these pictures, then we will find that the image of one half of the figure is exactly the same as the other half.

Symmetry is something that we observe in many places in our daily lives without even noticing it. It is easily noticeable in various arts, buildings, and monuments. Nature uses symmetry to make things beautiful. For example, consider the pictures of the butterfly and the leaf drawn below.

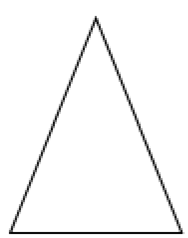
Let us also consider the following pictures.

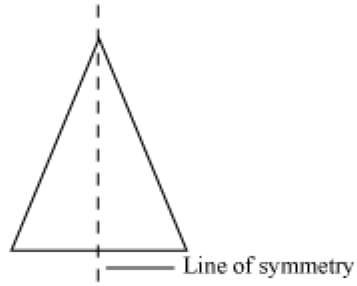
In each of these pictures, the dotted line divides the picture into two parts such that the left half of the picture is exactly same as the right half. This dotted line is known as the **line of symmetry** or **mirror line**. It can be defined as:

**The line through which the figure can be folded to form two identical figures is called line of symmetry or axis of symmetry or mirror line.**

Let us consider the following isosceles triangle.



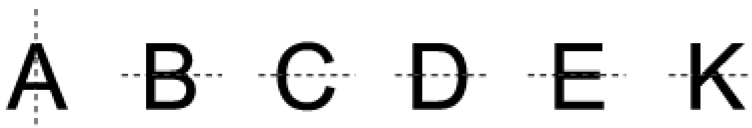
If we draw a dotted line at the middle of the triangle, then we get two parts such that the left part is exactly same as right part.




Here, the dotted line is the line of symmetry. We cannot draw more lines of symmetry for this triangle. Therefore, we can say that an isosceles triangle has only **one line of symmetry**.

Some letters of the English alphabet have only one line of symmetry.

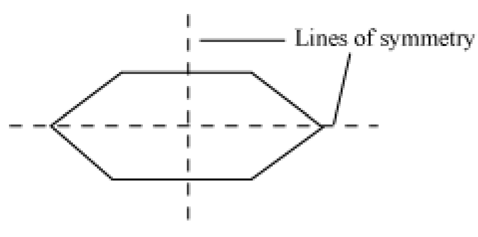
For example, A, B, C, D, E, K, etc.




Here, letter A has a vertical line of symmetry, while each of the letters B, C, D, E, and K has a horizontal line of symmetry.

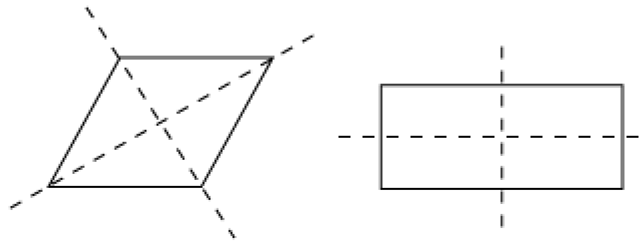
However, it is not necessary that a figure has only one line of symmetry. A figure can have more than one line of symmetry. Let us consider the following figure.

We can draw the lines of symmetry as shown below.



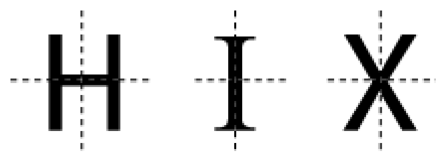

For this figure, we cannot draw more lines of symmetry except these two. Therefore, we can say that this figure has **two lines of symmetry.**

Some more figures with two lines of symmetry are shown below.



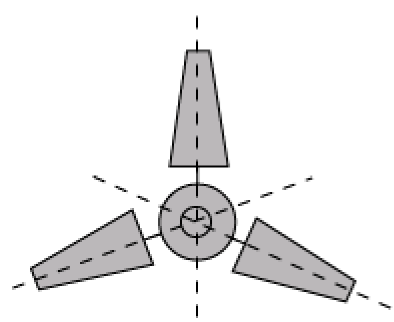
Some letters of the English alphabet contains two lines of symmetry.

For example, H, I, X, etc.

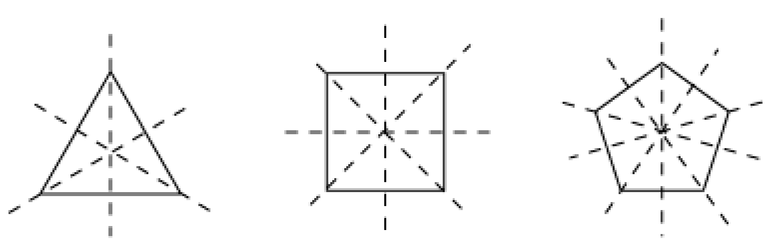



Note that each of the letters H, I, and X has both vertical and horizontal lines of symmetry.

Let us look at the fan drawn in the following figure. It has **three lines of symmetry**. When a figure has more than two lines of symmetry, we say that it has **multiple lines of symmetry**. Thus, we can also say that the fan drawn below has multiple lines of symmetry.



Some geometric figures with multiple lines of symmetry are shown below.

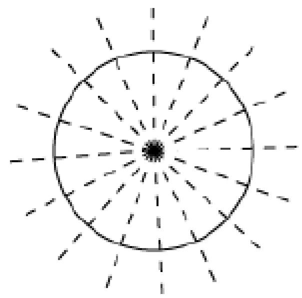


The figures drawn above represent

- an equilateral triangle with three lines of symmetry
- a square with four lines of symmetry
- a regular pentagon with five lines of symmetry

Using the concept of line of symmetry, can we tell how many lines of symmetry are there in a circle?

A circle has infinite number of lines of symmetry as shown in the figure below.

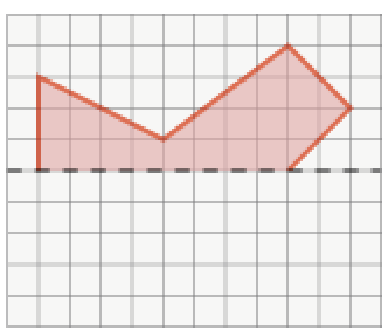


**Now, if suppose we are given a part of a figure and its line(s) of symmetry, then can we draw the complete figure?**

Yes, we can draw the complete figure by tracing the given part of the figure on the other side of the given line of symmetry.

Let us see this with the help of an example.

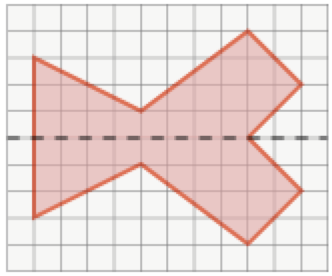
Consider the following figure:



Here, the dotted line is the line of symmetry of the figure.

The complete figure can be obtained by tracing the given figure below its line of symmetry.

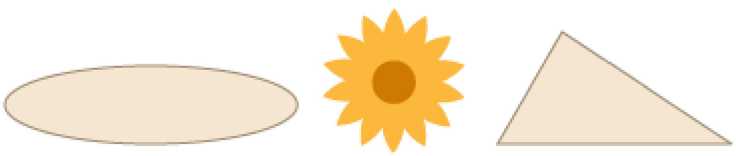
Thus, the complete figure will be represented as:



Now, we know how to identify symmetrical figures and their lines of symmetry. Let us discuss some examples based on these concepts.

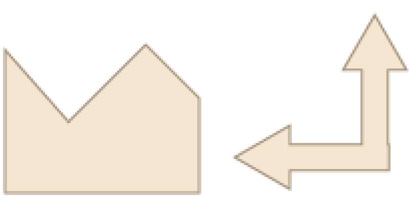
**Example 1:**

**Identify the symmetrical figures out of the following figures. Also draw their lines of symmetry.**



**(i)** **(ii)** **(iii)**

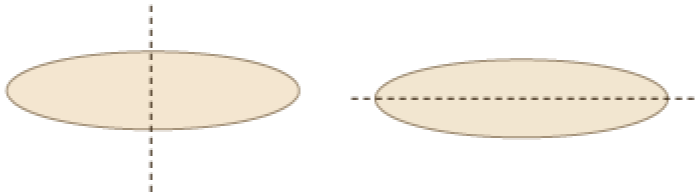
**(iv)** **(v)** **(vi)**



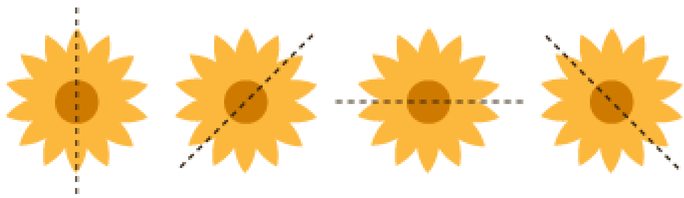
**(vii)** **(viii)**

**Solution:**

**(i)** The given figure is oval (egg-shaped). It has two lines of symmetry. That means the figure is symmetrical.

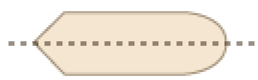


**(ii)** We can fold the given figure from the centre in any way as it has infinite lines of symmetry. Therefore, the figure is symmetrical. Some of its lines of symmetry are as follows.



**(iii)** We cannot fold the given triangle to form two identical halves. Therefore, this figure is not symmetrical and does not have any line of symmetry.

**(iv)** The given figure has a horizontal line of symmetry. Therefore, the figure is symmetrical.

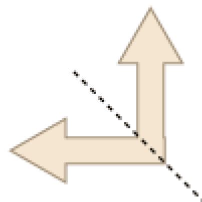


**(v)** The bottle shown in the given figure is symmetrical, as it has a vertical line of symmetry that divides it into two identical halves.

**(vi)** The given figure is not symmetrical, as we cannot divide it into two identical halves. Thus, it does not have any line of symmetry.

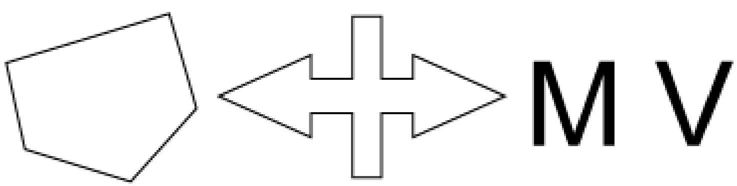
**(vii)** We cannot fold the figure in any way in order to divide it into two identical halves. Thus, the figure is not symmetrical and does not have any line of symmetry.

**(viii)** When we fold the given figure along the dotted line, we obtain two identical halves. Thus, the figure is symmetrical and has one line of symmetry.

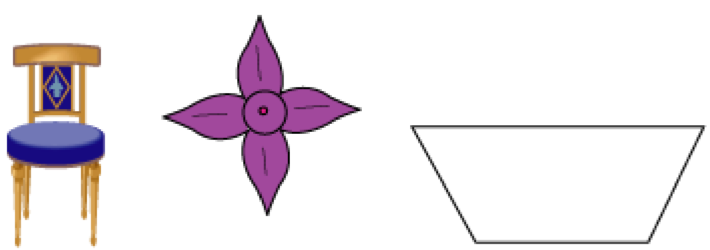


**Example 2:**

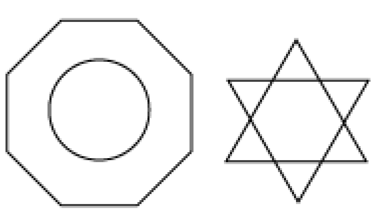
**State whether the following figures have single, double, or multiple lines of symmetry. Also, draw their line or lines of symmetry.**




**(i)** **(ii)** **(iii)** **(iv)**



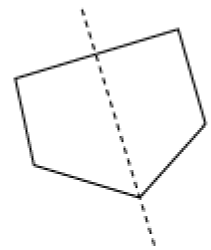
**(v)** **(vi)** **(vii)**



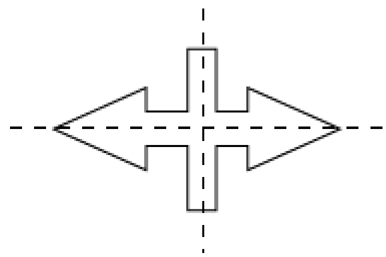
**(viii)** **(ix)**

**Solution:**

**(i)** The given figure has one line of symmetry.

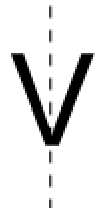


**(ii)** The given figure has two lines of symmetry.

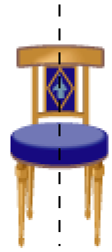


**(iii)** The given figure has one line of symmetry.

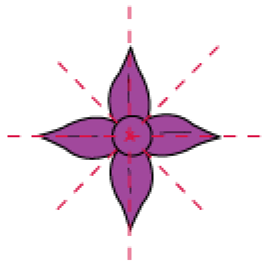

**(iv)** The given figure has one line of symmetry.



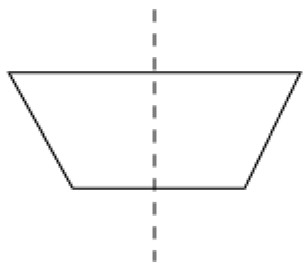

**(v)** The given figure has one line of symmetry.



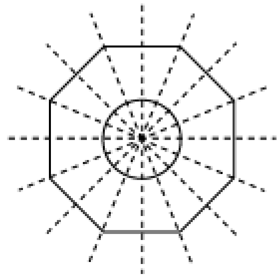
**(vi)** The given figure has four lines of symmetry, i.e., it has multiple lines of symmetry.



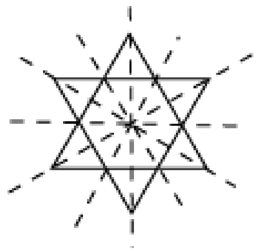
**(vii)** The given figure has one line of symmetry.



**(viii)** The given figure has eight lines of symmetry, i.e., it has multiple lines of symmetry.

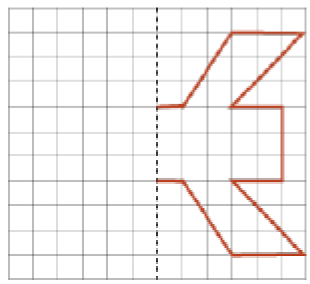


**(ix)** The given figure has six lines of symmetry, i.e., it has multiple lines of symmetry.

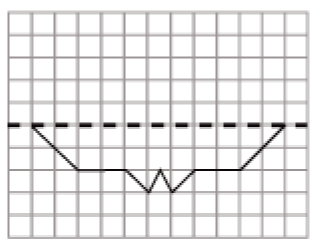


**Example 3:**
**Complete the following figures in which the dotted line shows the line of symmetry.**

1.



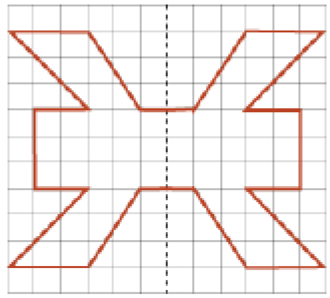
2.



**Solution:**

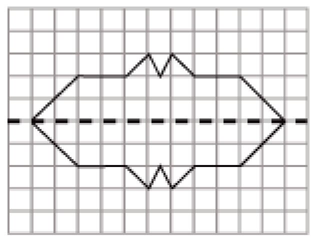
1. The complete figure can be obtained by tracing the given figure to the left of the line of symmetry.

   Thus, the complete figure is represented as:



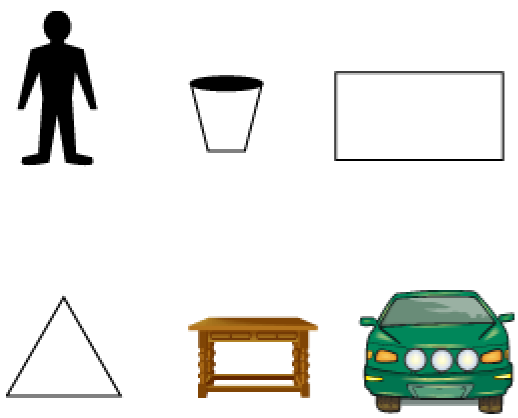
2. The complete figure can be obtained by tracing the given figure above the line of symmetry.

   Thus, the complete figure is represented as:

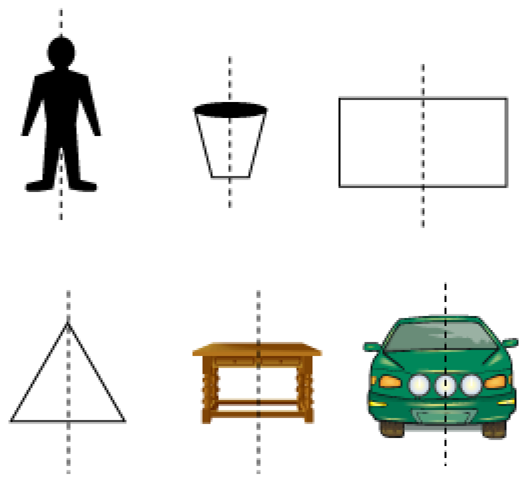


## Reflection of Figures

Let us consider the following pictures.



For each picture, let us draw vertical lines exactly at the middle as shown below.



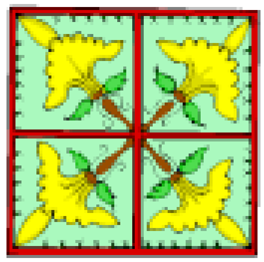
After drawing lines, we can observe that the left half of the pictures is exactly the same as the right half of the pictures. These pictures are known as symmetrical pictures. The line through which the figure is divided is called **line of symmetry.**

Here, the dotted lines of these pictures are lines of symmetry. If we consider only one-half of these images and place a mirror instead of the dotted line, then we will get a mirror reflection of the image, which will be the missing half of the original image.

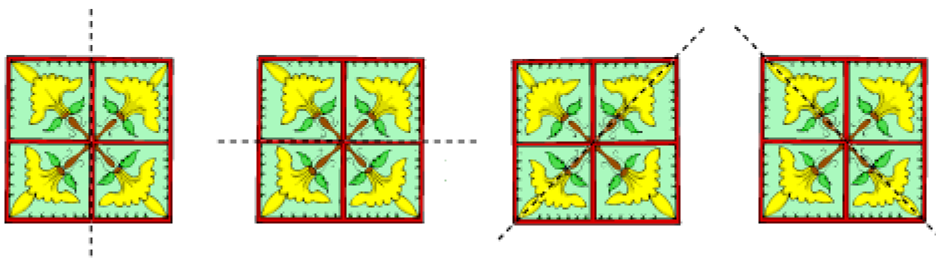
Thus, we can also say that the two halves obtained by dividing the figure through the line of symmetry are mirror images of each other. For example, the left portion of each image is the mirror image of the right portion of the image and vice-versa.

We come across different objects in our day to day life where symmetry is shown by mirror reflection. For example, if we look at the following picture, we can see the reflected image of the buildings in the water. The water surface acts as a mirror or as a line of symmetry. We can observe the symmetry of the objects by their reflection, though the image is not very clear.

We can also take the example of Rangoli patterns.

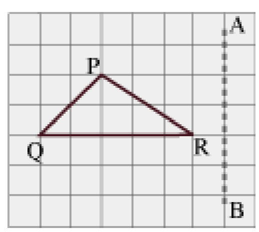


There are many lines of symmetry for these types of patterns as shown below. We can observe reflection pattern in the patterns along their lines of symmetry acting as mirror lines.

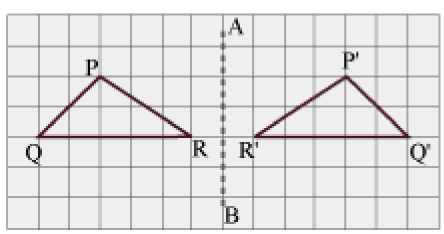


We can see that in each figure, one half of the pattern is the reflection of the other half.

Let us consider the given figure of ΔPQR on a grid paper, where AB is a mirror line.




Let us draw the image of ΔPQR with reference to the mirror line AB.




Here, ΔPQR and ΔP′Q′R′ are symmetrical with reference to the mirror line AB.

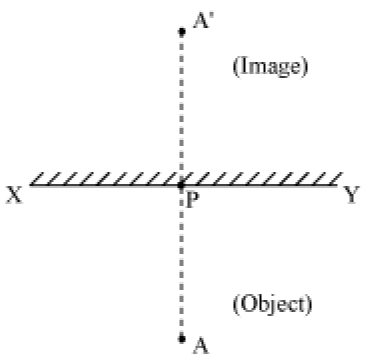
Now, we can say that ΔP′Q′R′ is the mirror image of ΔPQR with reference to the mirror line AB. If we fold the grid paper along the mirror line, then we will observe that the two triangles overlap. It can also be observed that:

- The lengths of the sides of ΔP′Q′R′ are equal to the corresponding sides of ΔPQR.
- The angles of ΔP′Q′R′ are equal to the corresponding angles of ΔPQR.
- Every portion of ΔP′Q′R′ is at the same distance from the mirror line as that of the corresponding portions of ΔPQR.

Now, let us extend the concept of mirror further to study about image of a point.

If we place a point in front of a mirror, then what is the nature of the image formed?

Let XY be a mirror. Let A be a point (object) placed in front of it. We obtain its image A′ as shown below:




**Can we notice anything in the above figure?**

We can notice that:

1. The distance of the image (A′) behind the mirror is same as the distance of the object (A) from it i.e., PA = PA′
2. The mirror line XY is perpendicular to the line joining the object and the image i.e., XY ⊥ AA′

Here, XY (the mirror line) is called the **axis of reflection** or **mediator**.

What would happen if the point A lies on XY?

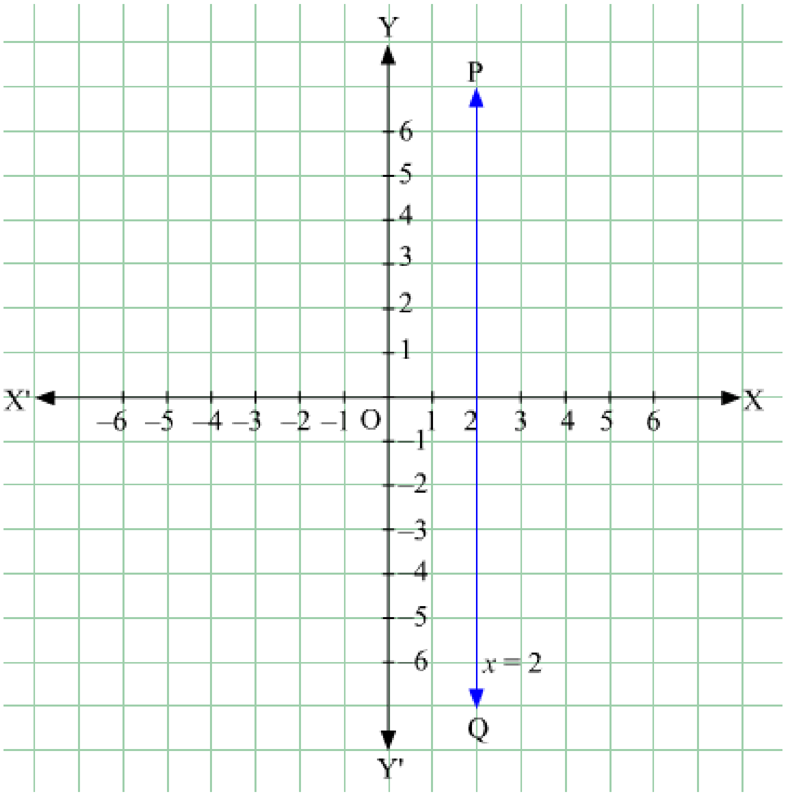
If the point A lies on XY, then its image will be this point itself. In such case, A is called an **invariant point** with respect to mirror line XY.

**Reflection of a point in the lines *x* = *a* and *y* = *a*.**

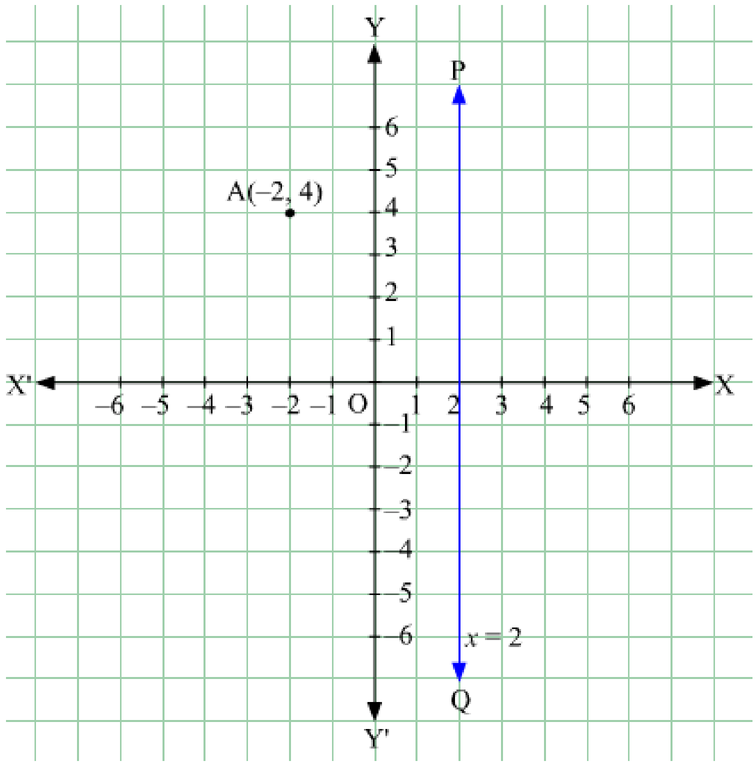
***x* = *a*** is a line parallel to the *y*-axis and at a distance of *a* units from it.

If we have to find the reflection of point A(−2, 4) from the line *x* = 2 we follow the below given steps:

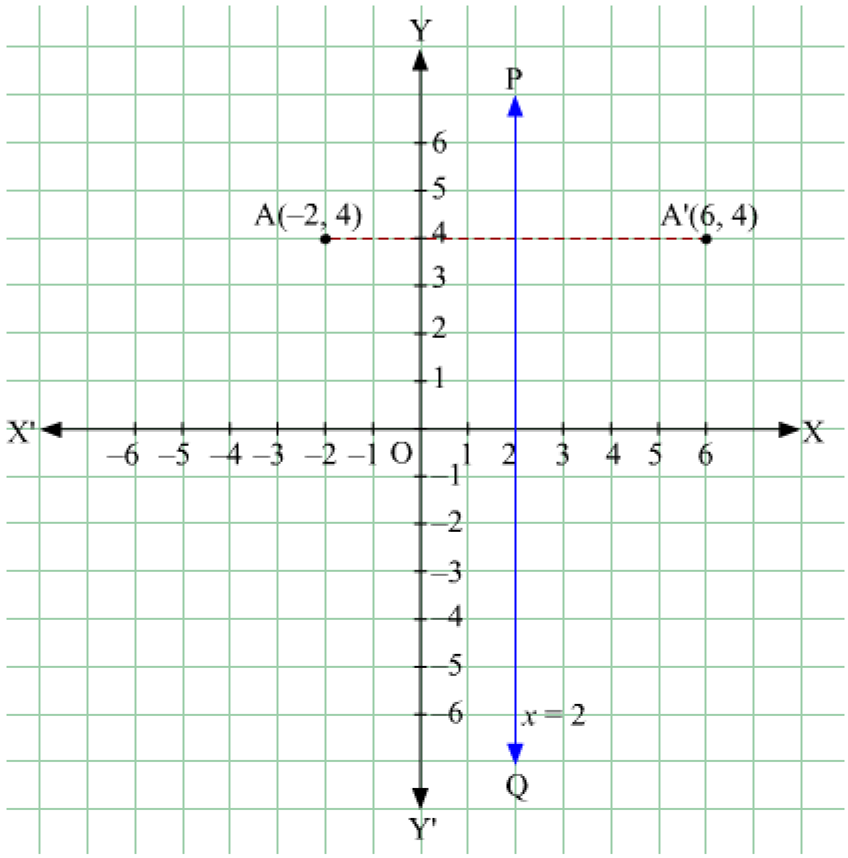
Step 1: Line PQ represents *x* = 2 which is a straight line parallel to the *y*-axis and at a distance of 2 units from it.




Step 2: Mark a point A(−2, 4) on the same graph.

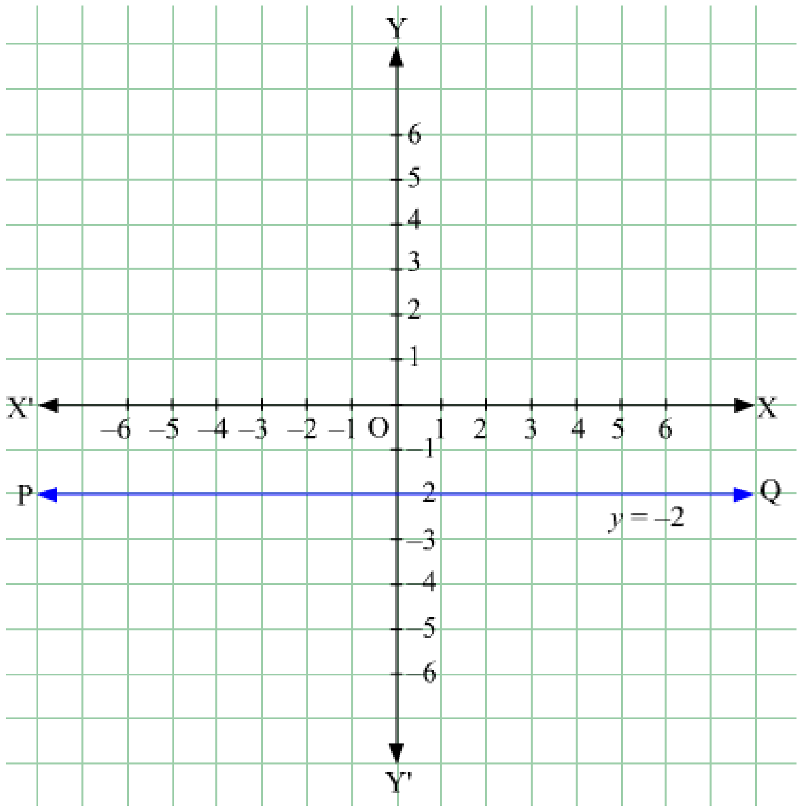



Step 3: From the point A, draw a straight line perpendicular to PQ. Mark a point A' behind the straight line PQ at the same distance as A(−2, 4) is before it. A'(6, 4) is the required reflection of the point A(−2, 4) in the line $x = 2$.

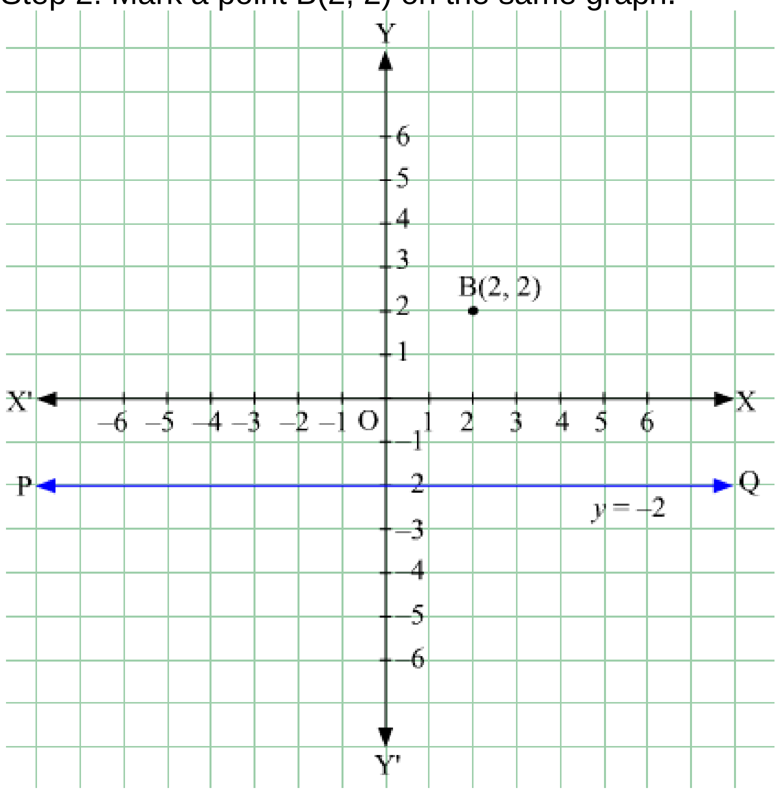



Similarly, we have ***y* = *a*** which is parallel to *x*-axis and is at a distance of *a* units from it. Suppose we have to find the reflection of point B(2, 2) from the line *y* + 2 = 0 we follow the below given steps:

Step 1: Line PQ represents *y* = − 2 which is a straight line parallel to the *x*-axis and at a distance of 2 units from it.

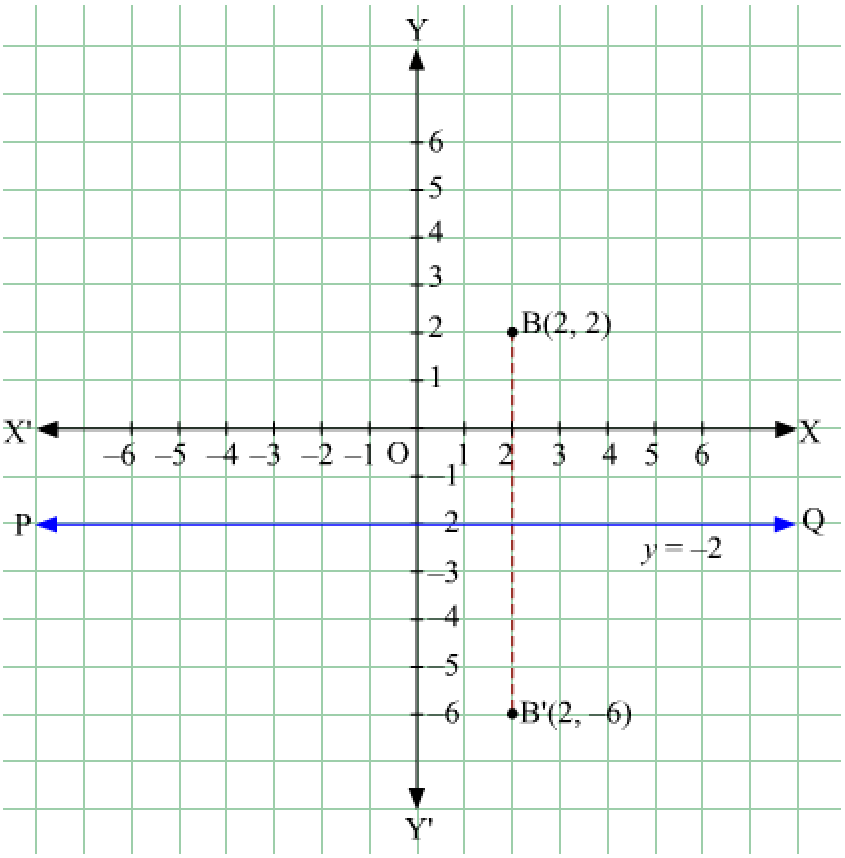



Step 2: Mark a point B(2, 2) on the same graph.




Step 3: From the point B, draw a straight line perpendicular to PQ. Mark a point B'

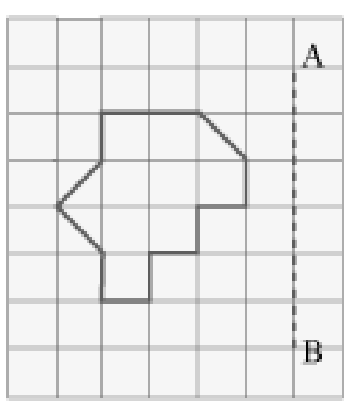
below this straight line PQ at the same distance as B(2, 2) is above it. B'(2, −6) is the required reflection of the point B(2, 2) in the line $y + 2 = 0$.




In order to understand these concepts better, let us look at some examples.

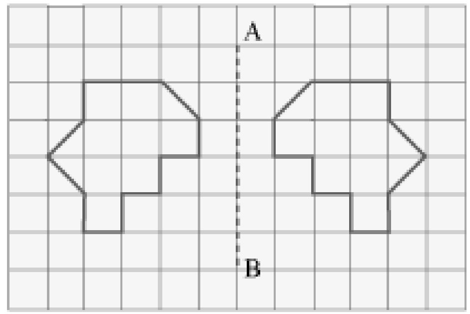
**Example 1:**

**Draw the mirror reflection of the following figure where AB is the mirror line.**



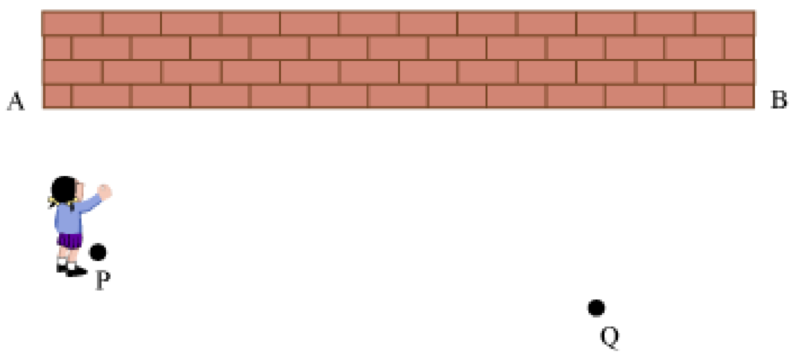

**Solution:**

The mirror reflection of the given figure with respect to mirror line AB can be drawn as



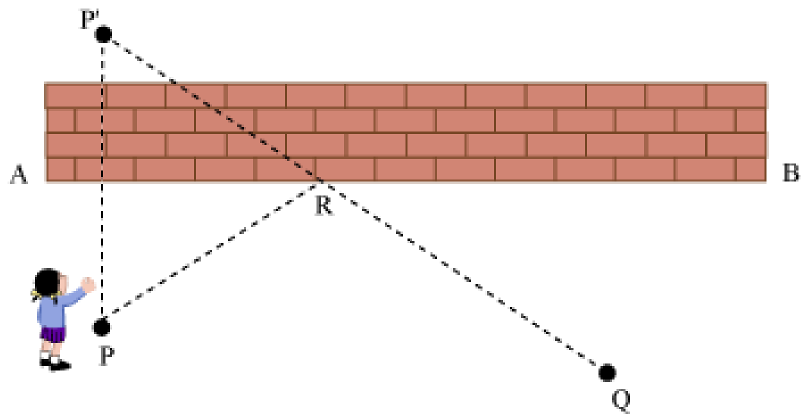

**Example 2:**

**The given figure shows a wall with end points A and B. Sanjana is standing at position P. She has to come to position Q after touching the wall. Find the shortest path for Sanjana to come from P to Q.**




**Solution:**

Let us imagine that wall AB acts as a mirror. Then P′ is the position of the image of point P. The object and its mirror image are at the same distance from the mirror. Therefore, points P and P′ are at the same distance from the wall.




The shortest distance between two points is the straight line joining the two points.

Therefore, the shortest distance between P′ and Q is P′Q. Let us join the points P′ and Q by a straight line which passes through the wall at point R.

Now, P′Q = P′R + RQ

But PR is the mirror reflection of P`R.

Or we can say that PR = P′R

Therefore, P′Q = PR + RQ

The path from P to R and then from R to Q is the shortest path which should be followed by Sanjana.

**Example 3:**

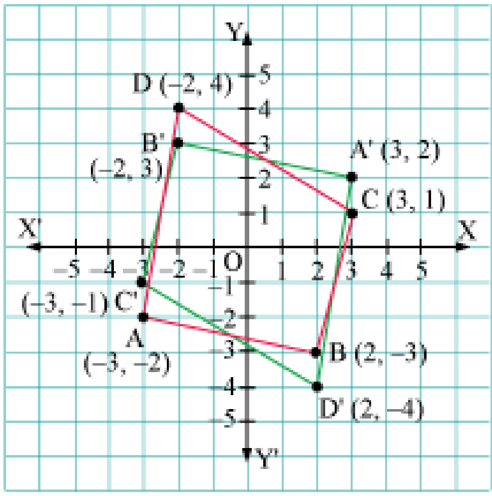
**The quadrilateral ABCD whose vertices are A (−3, −2), B (2, −3), C (3, 1), and D (−2, 4) is on a co-ordinate plane. Draw its reflection $A'B'C'D'$ in origin.**

**Solution:**

(1) The reflection of the points A (−3, −2), B (2, −3), C (3, 1), and (D (−2, 4) in the origin are:

$A'(3,2), B'(-2,3), C'(-3,-1)$, and $D'(2,-4)$

By joining $A'B', B'C', C'D'$, and $A'D'$, we obtain the quadrilateral $A'B'C'D'$, which is the reflection of the given quadrilateral ABCD in the origin as shown below.




**Example 4:**

**The line AB joining the points A (−3, −4) and B (2, 1) is on co-ordinate plane.**

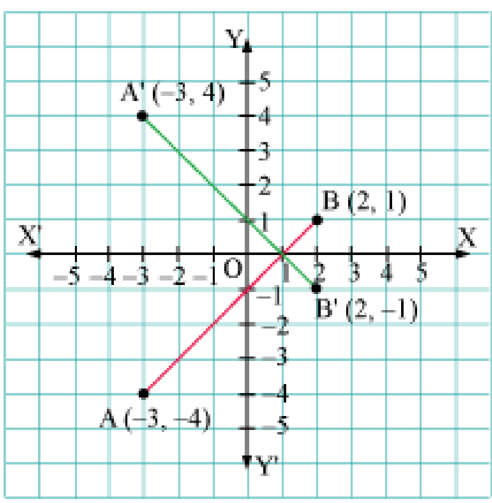
**Draw its reflection**

**(i) $A'B'$ about *x*-axis**

**(ii) $A''B''$ about *y*-axis**

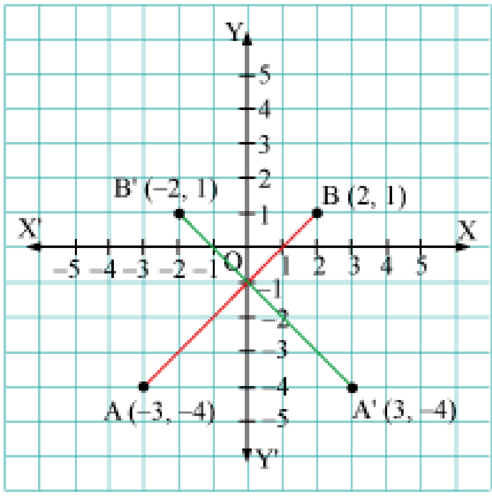
**Solution:**

(i) The refection of points A (−3, −4) and B (2, 1) about *x*-axis are $A'$ (−3, 4) and $B'$ (2, −1). By joining $A'B'$, we obtain the reflection of the given line AB as shown below.



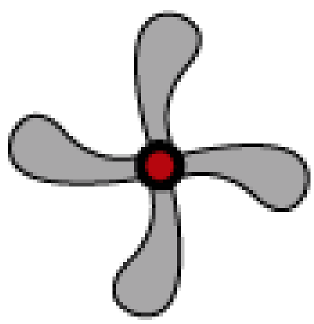
(ii) The reflection of points A (−3, −4) and B (2, 1) about *y*-axis are $A''$ (3, −4) and $B''$ (−2, 1).

By joining $A''B''$ , we obtain the reflection of the given line AB as shown below.




## Concept Of Rotational Symmetry

Look at the following figure of a fan with four blades.

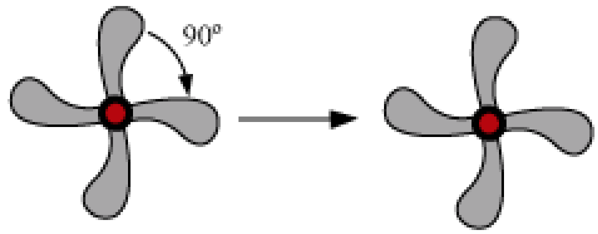


Does the fan have any line of symmetry?

A line which divides an object into two equal halves such that one half is the mirror image of the other is called the line of symmetry.

Therefore, we can see that the fan does not have any line of symmetry.

Now, rotate the fan through 90° about its centre.




What have you observed? Have you found any difference?

It looks exactly the same as the original figure after rotating it through 90°. Therefore, we can say that the fan has a **rotational symmetry**.

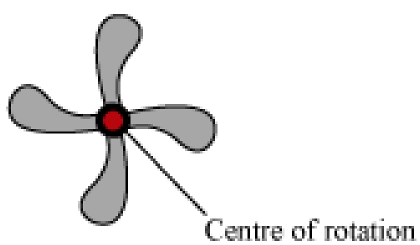
Thus, we can say that

> “**If an object remains identical after a rotation through certain degrees, then the object is said to possess rotational symmetry”.**

However, if we do not rotate the fan through the centre, then the identical structures would not be observed.

What is the centre of the fan called?

> “**The fixed point about which an object is rotated to attain the structure identical to the original one is called the centre of rotation.”**

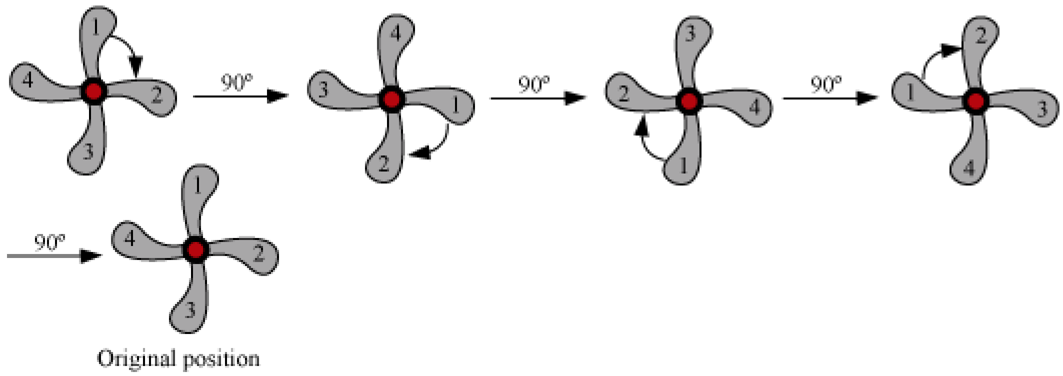



> “**The minimum angle through which an object is rotated to attain a structure identical to the original one is called angle of rotation.”**

In the above case, the fan is rotated through an angle of 90°. Therefore, the angle of rotation is 90°.

An object can be rotated in both the directions - clockwise and anti-clockwise.

Let us rotate the fan in clockwise direction.




After a rotation of 90°, the fan looks exactly the same as the original figure but it is not the same as the original one as the numbering on the blades is different.

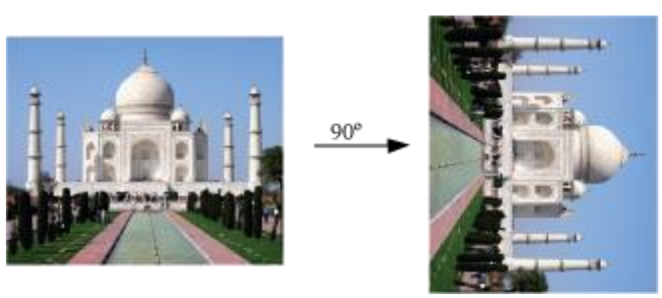
Only the last fan has the original position. It requires four rotations each of 90° to attain the original structure.

**“The number of rotations (when rotated through the angle of rotation) required by an object to rotate about the centre of rotation to attain its original structure is known as the order of rotational symmetry.”**

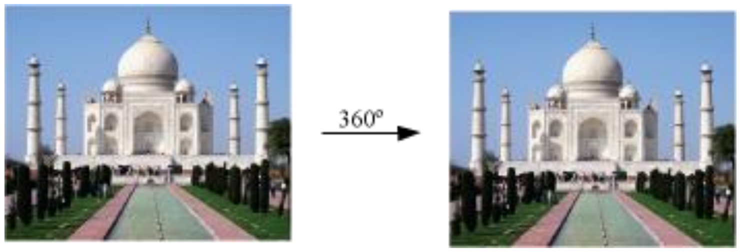
Therefore, the fan is said to have a rotational symmetry of order 4 about the centre of rotation.

Consider the following figure.

If we rotate this figure through 90°, would it coincide with the original figure? No, they will not coincide.



However, if we rotate this figure through an angle of 360°, then we will obtain the original figure.




Thus, we can say that this figure has a rotational symmetry of order one.

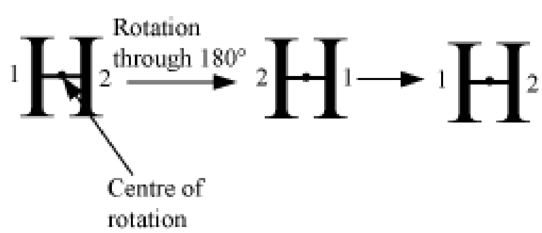
We can also say that **every object has a rotational symmetry of order at least one.**

Let us discuss this with the help of some examples. Consider the figures of a butterfly and a guitar.

If we rotate these figures by 360°, then they will retain their original figure.

This is true for every object in this world i.e., every object retains its original figure after rotating through 360°.

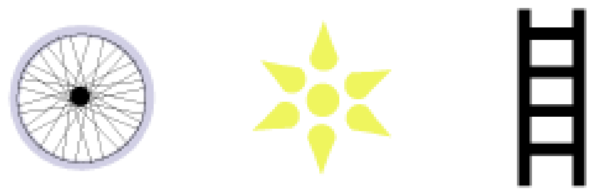
Some letters in the English alphabet also show rotational symmetries. Let us take the example of letter H.




Rotational symmetry

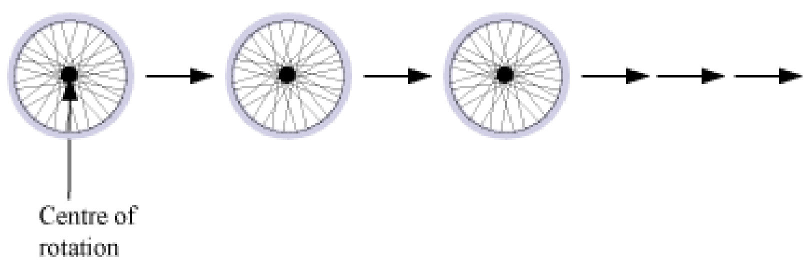
We can see that the letter H has rotational symmetry of order 2.

Let us consider the following objects one by one.



**Wheel**

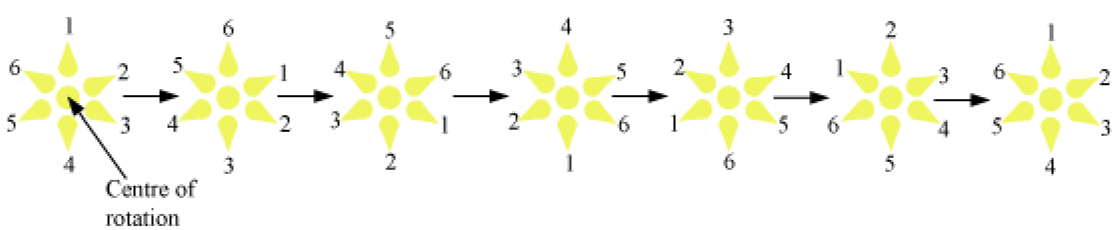
We can rotate the wheel through its centre at any angle to obtain the structure identical to the original wheel.




Therefore, we can say that the wheel has a rotational symmetry of infinite order.

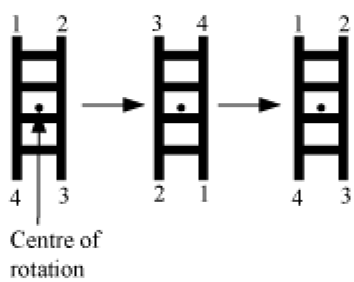
**Flower**

The flower shows a rotational symmetry of order 6.




**Ladder**

The ladder has a rotational symmetry of order 2.

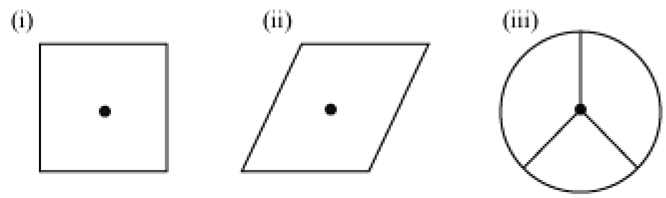


Now, in order to understand this concept in greater detail, let us take a look at the following video.

Let us go through some more examples to understand the concept better.

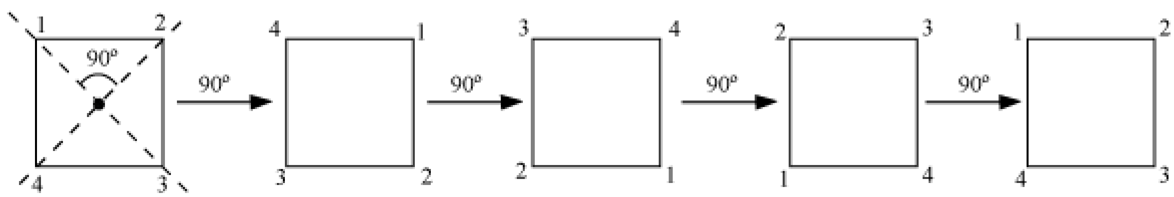
**Example 1:**

**What is the order of rotational symmetry in the following figures about the centre of rotation marked as dot ( • )?**

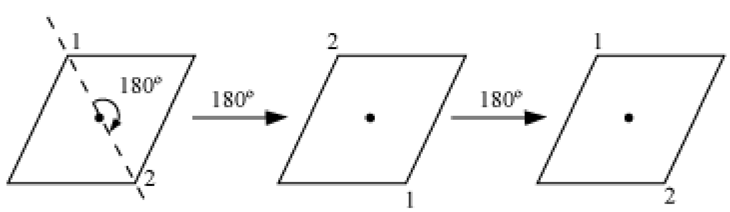



**Solution:**

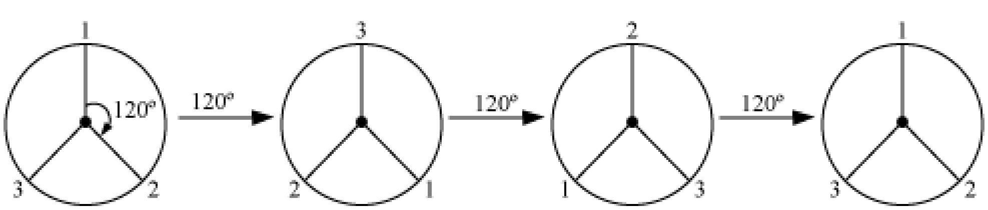
**(i)** The given figure shows a square. It has to be rotated through 90° four times to attain its original position. Therefore, it has a rotational symmetry of order 4.



**(ii)** For the given figure of parallelogram, the order of rotational symmetry is 2.

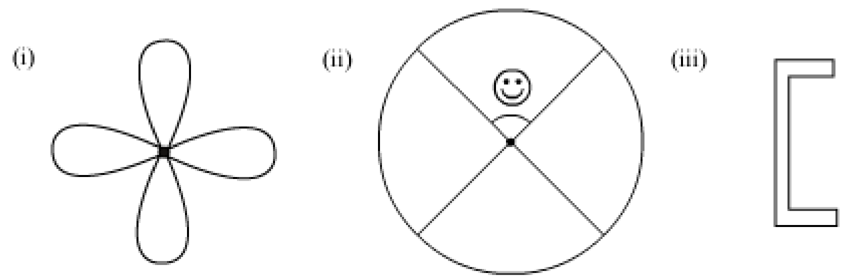


**(iii)** Here, the order of rotational symmetry is 3, as after rotating the figure through 120° three times, the figure comes back to its original position.



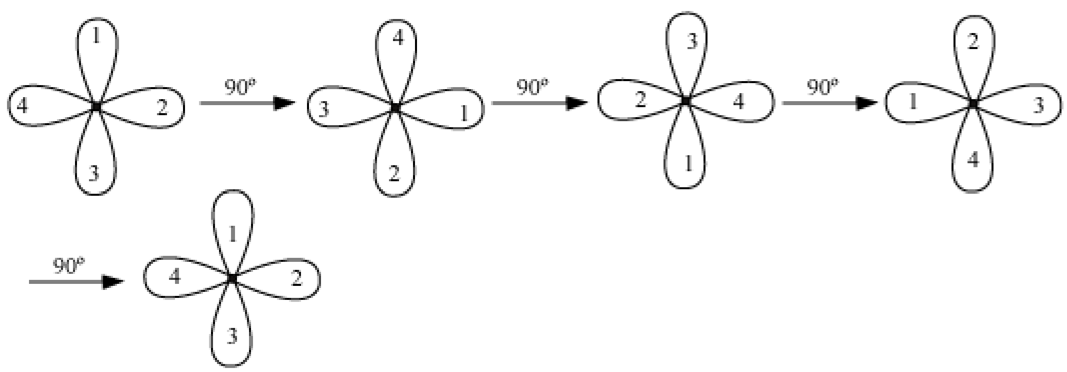
**Example 2:**

**Which of the following figures have rotational symmetry of order more than 1?**

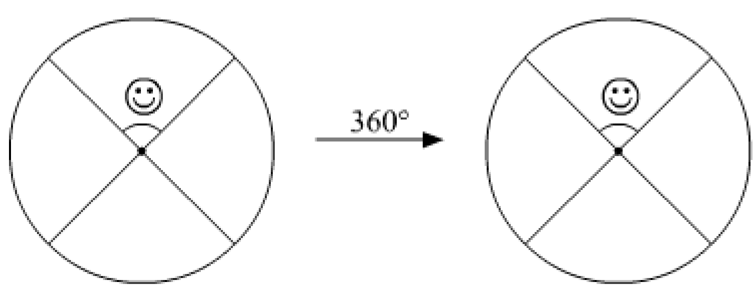



**Solution:**

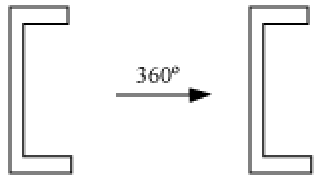
**(i)** The given figure has a rotational symmetry of order 4.




**(ii)** It has a rotational symmetry of order 1. It can attain the original and identical structure only by rotation through 360°.




**(iii)** It has a rotational symmetry of order 1.




Thus, figure **(i)** has rotational symmetry of order more than 1.

**Example 3:**

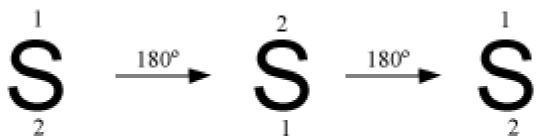
**Which of the following letters of English alphabet have rotational symmetry? What are their orders?**

**(i) M (ii) S**

**Solution:**

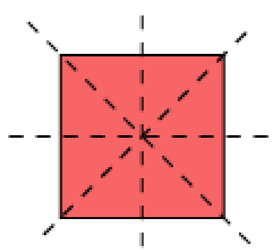
1. The letter M does not have rotational symmetry. It can attain the original and identical structure only by rotation through 360°.

2. The letter S has a rotational symmetry of order 2. It has been represented in the following figure.

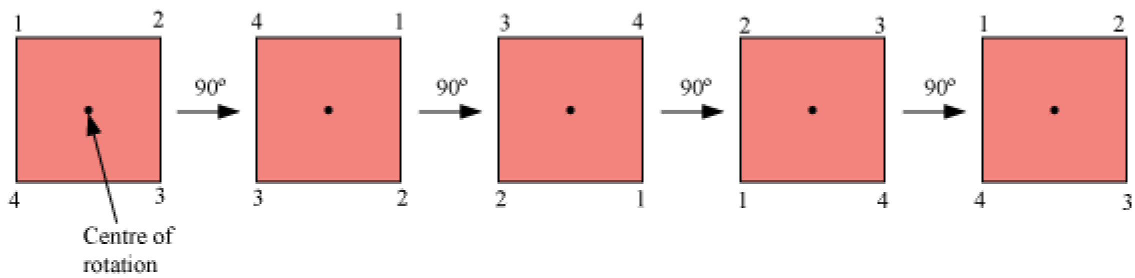



## Lines Of Symmetry And Rotational Symmetry

We know that a square has four lines of symmetry as shown in the following figure.


4 line symmetry

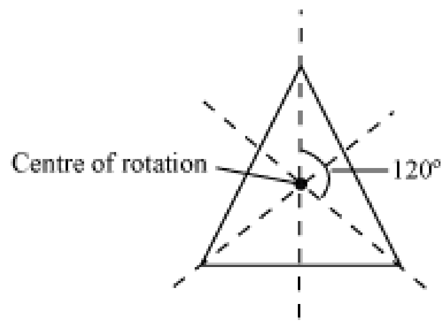
If we rotate the square about the point of intersection of its diagonals through an angle of 90° four times then an original figure will be obtained as shown in the following figure.



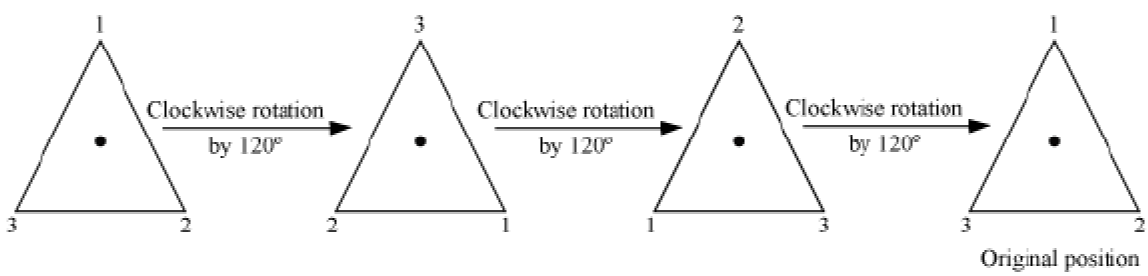

Rotational symmetry

Thus, we can say that a square has a rotational symmetry of order 4.

Now, let us look at the following figure of an equilateral triangle.




An equilateral triangle has three lines of symmetry. If we rotate the triangle about the centre of rotation through an angle of 120° then a similar shape will be obtained. Here, we have to rotate the triangle through 120° three times to take the triangle to its original position.



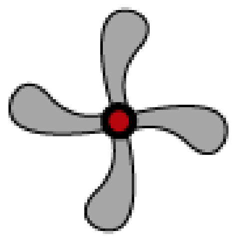

Therefore, the equilateral triangle is said to have a **rotational symmetry of order 3,** about the centre of rotation.

From the above examples, we can observe that

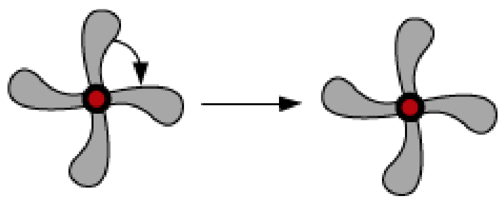
"***If a figure has more than one line of symmetry then it has rotational symmetry of order equal to the number of its lines of symmetry".***

**What happens when the lines of symmetry do not exist? Can we say anything about the figure then?**

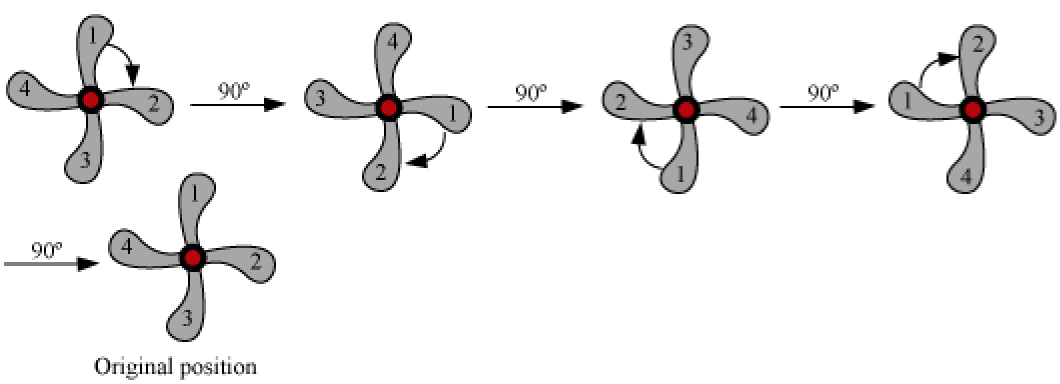
Consider the figure of a fan with four blades.



It does not have any line of symmetry but still it has rotational symmetry of order 4. If we rotate the fan about its centre through an angle of 90°, then a similar shape will be obtained.



We have to rotate the fan through 90° fourtimes to take the fan to its original position.




Therefore, the fan is said to have a **rotational symmetry of order 4,** about the centre of rotation.

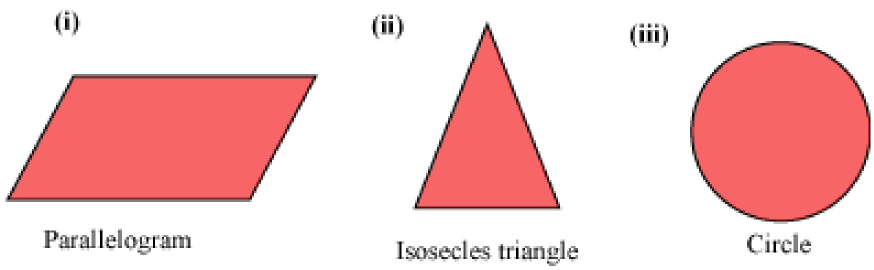
Thus, we can say that

"***When the lines of symmetry do not exist for a figure, the figure can still have rotational symmetry".***

Let us solve some more examples to understand the concept better.

**Example 1:**

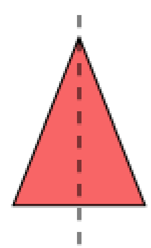
**Which of the following figures has only line symmetry, only rotational symmetry, and both line and rotational symmetries?**




**Solution:**

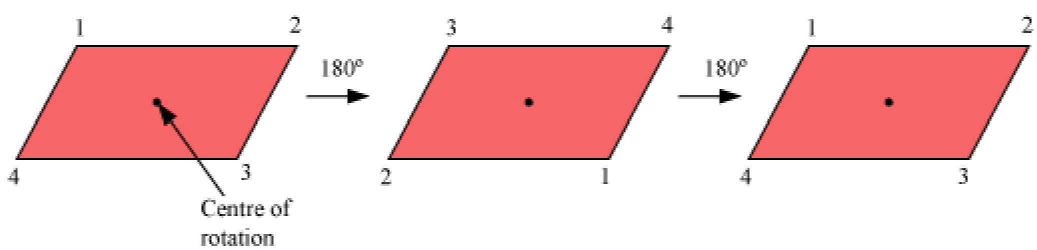
Here, we do not consider the rotational symmetry of order 1 as every object has rotational symmetry of order 1.

The figure (**ii**), an isosceles triangle, has only one of line symmetry. It does not have any rotational symmetry.

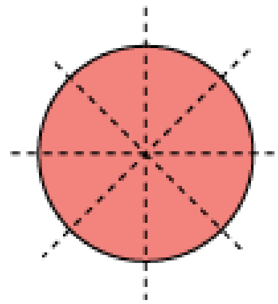


One line of symmetry

The figure (**i**), a parallelogram, has rotational symmetry of order 2 only, and does not possess any line of symmetry.




The figure **(iii)**, a circle, has infinite lines of symmetry and rotational symmetry of order infinite.



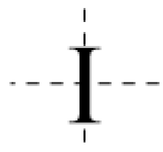
**Example 2:**

**Which of the following letters have both line and rotational symmetry?**

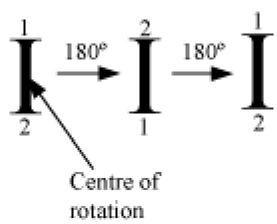
**(i) C (ii) W (iii) I (iv) X**

**Solution:**

The letters I and X have both line symmetry and rotational symmetry.



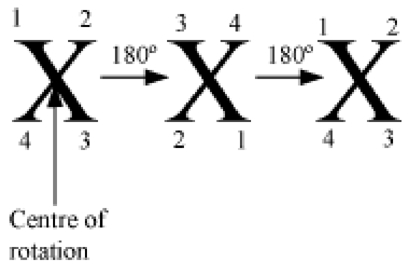
2 line symmetry




Rotational symmetry

The letter I has two lines of symmetry and rotational symmetry of order 2 about its centre.

2 line symmetry



Rotational symmetry

The letter X has two lines of symmetry and rotational symmetry of order 2 about its centre.

**Example 3:**

**Which of the letters in English alphabets have rotational symmetry but no line of symmetry?**

**Solution:**

The letters N, S, and Z have rotational symmetry of order 2 but no line symmetry.