Binomial Theorem

Exercise 8.1 : Solutions of Questions on Page Number : **166**

Q1 :

Expand the expression (1- 2*x*)5

Answer :

By using Binomial Theorem, the expression (1â€“ 2*x*)5 can be expanded as

$$(1-2x)^5$$
$$= {}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)^1(2x)^4 - {}^5C_5(2x)^5$$
$$= 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - (32x^5)$$
$$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

Q2 :

Expand the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Answer :

By using Binomial Theorem, the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ can be expanded as

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2$$
$$- {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5$$
$$= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32}$$
$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$$

Q3 :

Expand the expression (2*x* - 3)6

Answer :

By using Binomial Theorem, the expression (2xâ€“ 3)6 can be expanded as

$$\begin{aligned}(2x-3)^6 &= {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 \\ &\quad + {}^6C_4(2x)^2(3)^4 - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6 \\ &= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) \\ &\quad + 15(4x^2)(81) - 6(2x)(243) + 729 \\ &= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729\end{aligned}$$

Q4 :

Expand the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Answer :

By using Binomial Theorem, the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$ can be expanded as

$$\begin{aligned}\left(\frac{x}{3}+\frac{1}{x}\right)^5 &= {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 \\ &\quad + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5 \\ &= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5} \\ &= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}\end{aligned}$$

Q5 :

Expand $\left(x+\frac{1}{x}\right)^6$

Answer :

By using Binomial Theorem, the expression $\left(x+\frac{1}{x}\right)^6$ can be expanded as

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2$$
$$+ {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$
$$= x^6 + 6(x)^5\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x^2}\right) + 20(x)^3\left(\frac{1}{x^3}\right) + 15(x)^2\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6}$$
$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

Q6 :

Using Binomial Theorem, evaluate (96)^3

Answer :

96 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that, 96 = 100 â€“ 4

$$\therefore (96)^3 = (100-4)^3$$
$$= {}^3C_0(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3$$
$$= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3$$
$$= 1000000 - 120000 + 4800 - 64$$
$$= 884736$$

Q7 :

Using Binomial Theorem, evaluate (102)^5

Answer :

102can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that, 102 = 100 + 2

$$\therefore (102)^5 = (100+2)^5$$
$$= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5$$
$$= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5$$
$$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$$
$$= 11040808032$$

Q8 :

Using Binomial Theorem, evaluate (101)^4

Answer :

101 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that, 101 = 100 + 1

$$\therefore (101)^4 = (100+1)^4$$
$$= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4$$
$$= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + (1)^4$$
$$= 100000000 + 4000000 + 60000 + 400 + 1$$
$$= 104060401$$

Q9 :

Using Binomial Theorem, evaluate (99)^5

Answer :

99 can be written as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that, 99 = 100 â€“ 1

$$\therefore (99)^5 = (100-1)^5$$
$$= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3 + {}^5C_4(100)(1)^4 - {}^5C_5(1)^5$$
$$= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1$$
$$= 10000000000 - 500000000 + 10000000 - 100000 + 500 - 1$$
$$= 10010000500 - 500100001$$
$$= 9509900499$$

Q10 :

Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$or 1000.

Answer :

By splitting 1.1 and then applying Binomial Theorem, the first few terms of $(1.1)^{10000}$can be obtained as

$$(1.1)^{10000} = (1+0.1)^{10000}$$
$$= {}^{10000}C_0 + {}^{10000}C_1(1.1) + \text{Other positive terms}$$
$$= 1 + 10000 \times 1.1 + \text{Other positive terms}$$
$$= 1 + 11000 + \text{Other positive terms}$$
$$> 1000$$

Hence, $(1.1)^{10000} > 1000$

Q11 :

Find $(a+b)^4$â€“ $(a$â€“ $b)^4$. Hence, evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$ **.**

Answer :

Using Binomial Theorem, the expressions, $(a+b)^4$and $(a$ â€“ $b)^4$, can be expanded as

$$(a+b)^4 = {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$$
$$(a-b)^4 = {}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4$$
$$\therefore (a+b)^4 - (a-b)^4 = {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$$
$$-\left[{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4\right]$$
$$= 2\left({}^4C_1a^3b + {}^4C_3ab^3\right) = 2\left(4a^3b + 4ab^3\right)$$
$$= 8ab\left(a^2 + b^2\right)$$

By putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\left(\sqrt{3}\right)\left(\sqrt{2}\right)\left\{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right\}$$
$$= 8\left(\sqrt{6}\right)\{3+2\} = 40\sqrt{6}$$

Q12 :

Find (x+ 1)^6+ (x â€“ 1)^6. Hence or otherwise evaluate $\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$.

Answer :

Using Binomial Theorem, the expressions, (x+ 1)^{6}and (x â€“ 1)^6, can be expanded as

$$(x+1)^6 = {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6$$
$$(x-1)^6 = {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6$$
$$\therefore (x+1)^6 + (x-1)^6 = 2\left[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6\right]$$
$$= 2\left[x^6 + 15x^4 + 15x^2 + 1\right]$$

By putting $x = \sqrt{2}$, we obtain

$$\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6 = 2\left[\left(\sqrt{2}\right)^6 + 15\left(\sqrt{2}\right)^4 + 15\left(\sqrt{2}\right)^2 + 1\right]$$
$$= 2\left(8 + 15\times4 + 15\times2 + 1\right)$$
$$= 2\left(8 + 60 + 30 + 1\right)$$
$$= 2(99) = 198$$

Q13 :

Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever *n* is a positive integer.

Answer :

In order to show that $9^{n+1} - 8n - 9$ is divisible by 64, it has to be proved that,

$9^{n+1} - 8n - 9 = 64k$, where *k* is some natural number

By Binomial Theorem,

$$(1+a)^m = {}^mC_0 + {}^mC_1 a + {}^mC_2 a^2 + \ldots + {}^mC_m a^m$$

For *a* = 8 and *m* = *n*+ 1, we obtain

$$(1+8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$$
$$\Rightarrow 9^{n+1} = 1 + (n+1)(8) + 8^2\left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \ldots + {}^{n+1}C_{n+1}(8)^{n-1}\right]$$
$$\Rightarrow 9^{n+1} = 9 + 8n + 64\left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \ldots + {}^{n+1}C_{n+1}(8)^{n-1}\right]$$
$$\Rightarrow 9^{n+1} - 8n - 9 = 64k,\ \text{where } k = {}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \ldots + {}^{n+1}C_{n+1}(8)^{n-1} \text{ is a natural number}$$

Thus, $9^{n+1} - 8n - 9$ is divisible by 64, whenever *n* is a positive integer.

Q14 :

Prove that $\sum_{r=0}^{n} 3^r\, {}^nC_r = 4^n$ **.**

Answer :

By Binomial Theorem,

$$\sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r = (a+b)^n$$

By putting *b*= 3 and *a*= 1 in the above equation, we obtain

$$\sum_{r=0}^{n} {}^nC_r (1)^{n-r} (3)^r = (1+3)^n$$
$$\Rightarrow \sum_{r=0}^{n} 3^r\, {}^nC_r = 4^n$$

Hence, proved.

Exercise 8.2 : Solutions of Questions on Page Number : **171**

Q1 :

Find the coefficient of x^5 in $(x + 3)^8$

Answer :

It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that x^5 occurs in the $(r+ 1)^{th}$ term of the expansion $(x+ 3)^8$, we obtain

$$T_{r+1} = {}^8C_r (x)^{8-r} (3)^r$$

Comparing the indices of x in x^5 and in T_{r+1}, we obtain r=

3

Thus, the coefficient of x^5 is $${}^8C_3 (3)^3 = \frac{8!}{3!5!} \times 3^3 = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2.5!} \cdot 3^3 = 1512$$

Q2 :

Find the coefficient of a^5b^7 in $(a - 2b)^{12}$

Answer :

It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that a^5b^7 occurs in the $(r+ 1)^{th}$ term of the expansion (aâ€“ $2b)^{12}$, we obtain

$$T_{r+1} = {}^{12}C_r (a)^{12-r} (-2b)^r = {}^{12}C_r (-2)^r (a)^{12-r} (b)^r$$

Comparing the indices of a and b in a^5b^7 and in T_{r+1}, we obtain

r= 7

Thus, the coefficient of a^5b^7 is $${}^{12}C_7 (-2)^7 = -\frac{12!}{7!5!} \cdot 2^7 = -\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8.7!}{5 \cdot 4 \cdot 3 \cdot 2.7!} \cdot 2^7 = -(792)(128) = -101376$$

Q3 :

Write the general term in the expansion of $(x^2 - y)^6$

Answer :

It is known that the general term T_{r+1} {which is the $(r + 1)^{th}$ term} in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the general term in the expansion of (x^2â€“ y^6) is

$$T_{r+1} = {}^6C_r (x^2)^{6-r} (-y)^r = (-1)^r \, {}^6C_r . x^{12-2r} . y^r$$

Q4 :

Write the general term in the expansion of $(x^2 - yx)^{12}$, $x \neq 0$

Answer :

It is known that the general term T_{r+1} {which is the $(r + 1)^{th}$ term} in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the general term in the expansion of(x^2â€“ $yx)^{12}$is

$$T_{r+1} = {}^{12}C_r \left(x^2\right)^{12-r} \left(-yx\right)^r = \left(-1\right)^r\, {}^{12}C_r . x^{24-2r} . y^r . x^r = \left(-1\right)^r\, {}^{12}C_r . x^{24-r} . y^r$$

Q5 :

Find the 4th term in the expansion of $(x - 2y)^{12}$.

Answer :

It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the 4thterm in the expansion of (xâ€“ $2y)^{12}$is

$$T_4 = T_{3+1} = {}^{12}C_3 \left(x\right)^{12-3} \left(-2y\right)^3 = \left(-1\right)^3 \cdot \frac{12!}{3!9!} \cdot x^9 \cdot \left(2\right)^3 \cdot y^3 = -\frac{12 \cdot 11 \cdot 10}{3 \cdot 2} \cdot \left(2\right)^3 x^9 y^3 = -1760 x^9 y^3$$

Q6 :

Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, \ x \neq 0$ **.**

Answer :

It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$ is

$$T_{13} = T_{12+1} = {}^{18}C_{12} \left(9x\right)^{18-12} \left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= \left(-1\right)^{12} \frac{18!}{12!6!} \left(9\right)^6 \left(x\right)^6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{\sqrt{x}}\right)^{12}$$

$$= \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 . 12!}{12! . 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \cdot x^6 . \left(\frac{1}{x^6}\right) . 3^{12} \left(\frac{1}{3^{12}}\right) \qquad \left[9^6 = \left(3^2\right)^6 = 3^{12}\right]$$

$$= 18564$$

Q7 :

Find the middle terms in the expansions of $\left(3-\frac{x^3}{6}\right)^7$

Answer :

It is known that in the expansion of $(a+b)^n$, if n is odd, then there are two middle terms, namely, $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{th}$ term.

Therefore, the middle terms in the expansion of $\left(3-\frac{x^3}{6}\right)^7$ are $\left(\frac{7+1}{2}\right)^{th}=4^{th}$ term and $\left(\frac{7+1}{2}+1\right)^{th}=5^{th}$ term

$$T_4 = T_{3+1} = {}^7C_3(3)^{7-3}\left(-\frac{x^3}{6}\right)^3 = (-1)^3\frac{7!}{3!4!}.3^4\cdot\frac{x^9}{6^3}$$
$$= -\frac{7\cdot 6\cdot 5.4!}{3\cdot 2.4!}\cdot 3^4\cdot\frac{1}{2^3\cdot 3^3}\cdot x^9 = -\frac{105}{8}x^9$$
$$T_5 = T_{4+1} = {}^7C_4(3)^{7-4}\left(-\frac{x^3}{6}\right)^4 = (-1)^4\frac{7!}{4!3!}(3)^3\cdot\frac{x^{12}}{6^4}$$
$$= \frac{7\cdot 6\cdot 5.4!}{4!.3\cdot 2}\cdot\frac{3^3}{2^4\cdot 3^4}\cdot x^{12} = \frac{35}{48}x^{12}$$

Thus, the middle terms in the expansion of $\left(3-\frac{x^3}{6}\right)^7$ are $-\frac{105}{8}x^9 \text{ and } \frac{35}{48}x^{12}$.

Q8 :

Find the middle terms in the expansions of $\left(\frac{x}{3}+9y\right)^{10}$

Answer :

It is known that in the expansion $(a+b)^n$, if n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{th}$ term.

Therefore, the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is $\left(\frac{10}{2}+1\right)^{th}=6^{th}$ term

$$T_6=T_{5+1}={}^{10}C_5\left(\frac{x}{3}\right)^{10-5}(9y)^5=\frac{10!}{5!5!}\cdot\frac{x^5}{3^5}\cdot 9^5\cdot y^5$$
$$=\frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5!}{5\cdot 4\cdot 3\cdot 2\cdot 5!}\cdot\frac{1}{3^5}\cdot 3^{10}\cdot x^5y^5 \qquad \left[9^5=\left(3^2\right)^5=3^{10}\right]$$
$$=252\times 3^5\cdot x^5\cdot y^5=61236x^5y^5$$

Thus, the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is 61236 x^5y^5.

Q9 :

In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

Answer :

It is known that $(r+1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by $T_{r+1}={}^nC_r a^{n-r}b^r$.

Assuming that a^m occurs in the $(r+1)^{th}$ term of the expansion $(1+a)^{m+n}$, we obtain

$$T_{r+1}={}^{m+n}C_r(1)^{m+n-r}(a)^r={}^{m+n}C_r a^r$$

Comparing the indices of a in a^m and in T_{r+1}, we obtain $r=$

m

Therefore, the coefficient of a^m is

$${}^{m+n}C_m=\frac{(m+n)!}{m!(m+n-m)!}=\frac{(m+n)!}{m!n!} \qquad \ldots(1)$$

Assuming that a^n occurs in the $(k+1)^{th}$ term of the expansion $(1+a)^{m+n}$, we obtain

$$T_{k+1}={}^{m+n}C_k(1)^{m+n-k}(a)^k={}^{m+n}C_k(a)^k$$

Comparing the indices of *a* in a^n and in T_{k+1}, we obtain $k =$

n

Therefore, the coefficient of a^n is

$$^{m+n}C_n = \frac{(m+n)!}{n!(m+n-n)!} = \frac{(m+n)!}{n!m!} \qquad ...(2)$$

Thus, from (1) and (2), it can be observed that the coefficients of a^m and a^n in the expansion of $(1 + a)^{m+n}$ are equal.

Q10 :

The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1:3:5. Find *n* and *r*.

Answer :

It is known that $(k + 1)^{th}$ term, (T_{k+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{k+1} = {}^nC_k a^{n-k} b^k$.

Therefore, (*r* â€“ 1)th term in the expansion of $(x+1)^n$ is $T_{r-1} = {}^nC_{r-2}(x)^{n-(r-2)}(1)^{(r-2)} = {}^nC_{r-2}x^{n-r+2}$

r^{th} term in the expansion of $(x+1)^n$ is $T_r = {}^nC_{r-1}(x)^{n-(r-1)}(1)^{(r-1)} = {}^nC_{r-1}x^{n-r+1}$

$(r+1)^{th}$ term in the expansion of $(x+1)^n$ is $T_{r+1} = {}^nC_r(x)^{n-r}(1)^r = {}^nC_r x^{n-r}$

Therefore, the coefficients of the (*r*â€“ 1)th, r^{th}, and $(r + 1)^{th}$ terms in the expansion of $(x+1)^n$ are ${}^nC_{r-2}$, ${}^nC_{r-1}$, and ${}^nC_r$ respectively. Since these coefficients are in the ratio 1:3:5, we obtain

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3} \text{ and } \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$$

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)(n-r+1)!}$$

$$= \frac{r-1}{n-r+2}$$

$$\therefore \frac{r-1}{n-r+2} = \frac{1}{3}$$
$$\Rightarrow 3r - 3 = n - r + 2$$
$$\Rightarrow n - 4r + 5 = 0 \qquad ...(1)$$
$$\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{n!}{(r-1)!(n-r+1)} \times \frac{r!(n-r)!}{n!} = \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)(n-r)!}$$
$$= \frac{r}{n-r+1}$$
$$\therefore \frac{r}{n-r+1} = \frac{3}{5}$$
$$\Rightarrow 5r = 3n - 3r + 3$$
$$\Rightarrow 3n - 8r + 3 = 0 \qquad ...(2)$$

Multiplying (1) by 3 and subtracting it from (2), we obtain

$4r$ â€“ 12 = 0 $\Rightarrow$

r= 3

Putting the value of $r$in (1), we obtain nâ€“

12 + 5 = 0

$\Rightarrow$ n= 7

Thus, n = 7 and r = 3

Q11 :

Prove that the coefficient of x^nin the expansion of $(1 + x)^{2n}$is twice the coefficient of x^nin the expansion of $(1 + x)^{2n-1}$.

Answer :

It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^{n}C_{r}a^{n-r}b^{r}$. Assuming that x^noccurs in the $(r+ 1)^{th}$term of the expansion of $(1 + x)^{2n}$, we obtain

$$T_{r+1} = {}^{2n}C_{r}(1)^{2n-r}(x)^{r} = {}^{2n}C_{r}(x)^{r}$$

Comparing the indices of $x$in $x^n$and in T_{r+1}, we obtain r= n

Therefore, the coefficient of x^nin the expansion of $(1 + x)^{2n}$is

$$^{2n}C_n = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!} = \frac{(2n)!}{(n!)^2} \qquad ...(1)$$

Assuming that x^noccurs in the $(k+1)^{th}$term of the expansion $(1 + x)^{2n\ \text{â€“}\ 1}$, we obtain

$$T_{k+1} = {}^{2n-1}C_k (1)^{2n-1-k} (x)^k = {}^{2n-1}C_k (x)^k$$

Comparing the indices of $x$in $x^n$and T_{k+1}, we obtain k=

n

Therefore, the coefficient of x^nin the expansion of $(1 + x)^{2n\ \text{â€“}1}$is

$$^{2n-1}C_n = \frac{(2n-1)!}{n!(2n-1-n)!} = \frac{(2n-1)!}{n!(n-1)!}$$

$$= \frac{2n.(2n-1)!}{2n.n!(n-1)!} = \frac{(2n)!}{2.n!n!} = \frac{1}{2}\left[\frac{(2n)!}{(n!)^2}\right] \qquad ...(2)$$

From (1) and (2), it is observed that

$$\frac{1}{2}\left({}^{2n}C_n\right) = {}^{2n-1}C_n$$

$$\Rightarrow {}^{2n}C_n = 2\left({}^{2n-1}C_n\right)$$

Therefore, the coefficient of x^nin the expansion of $(1 + x)^{2n}$is twice the coefficient of x^nin the expansion of $(1 + x)^{2n\text{â€“}1}$.

Hence, proved.

Q12 :

Find a positive value of *m*for which the coefficient of x^2in the expansion $(1 + x)^m$is 6.

Answer :

It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$. Assuming that x^2occurs in the $(r + 1)^{th}$term of the expansion $(1 +x)^m$, we obtain

$$T_{r+1} = {}^mC_r (1)^{m-r} (x)^r = {}^mC_r (x)^r$$

Comparing the indices of $x$in $x^2$and in T_{r+1}, we obtain r=

2

Therefore, the coefficient of $x^2$is ${}^mC_2$.

It is given that the coefficient of x^2in the expansion $(1 + x)^m$is 6.

$$\therefore {}^{m}C_2 = 6$$

$$\Rightarrow \frac{m!}{2!(m-2)!} = 6$$

$$\Rightarrow \frac{m(m-1)(m-2)!}{2\times(m-2)!} = 6$$

$$\Rightarrow m(m-1) = 12$$

$$\Rightarrow m^2 - m - 12 = 0$$

$$\Rightarrow m^2 - 4m + 3m - 12 = 0$$

$$\Rightarrow m(m-4) + 3(m-4) = 0$$

$$\Rightarrow (m-4)(m+3) = 0$$

$$\Rightarrow (m-4) = 0 \text{ or } (m+3) = 0$$

$$\Rightarrow m = 4 \text{ or } m = -3$$

Thus, the positive value of *m*, for which the coefficient of x^2in the expansion (1 + *x*)^{m}is 6, is 4.

Exercise Miscellaneous : Solutions of Questions on Page Number : **175**

Q1 :

Find *a*, *b*and *n* in the expansion of (*a*+ *b*)^{n}if the first three terms of the expansion are 729, 7290 and 30375, respectively.

Answer :

It is known that (r + 1)th term, (T_{r+1}), in the binomial expansion of (a + b)^n is given by $T_{r+1} = {}^{n}C_r a^{n-r} b^r$.

The first three terms of the expansion are given as 729, 7290, and 30375 respectively.

Therefore, we obtain

$$T_1 = {}^{n}C_0 a^{n-0} b^0 = a^n = 729 \qquad ...(1)$$

$$T_2 = {}^{n}C_1 a^{n-1} b^1 = na^{n-1}b = 7290 \qquad ...(2)$$

$$T_3 = {}^{n}C_2 a^{n-2} b^2 = \frac{n(n-1)}{2} a^{n-2} b^2 = 30375 \qquad ...(3)$$

Dividing (2) by (1), we obtain

$$\frac{na^{n-1}b}{a^n} = \frac{7290}{729}$$

$$\Rightarrow \frac{nb}{a} = 10 \qquad ...(4)$$

Dividing (3) by (2), we obtain

$$\frac{n(n-1)a^{n-2}b^2}{2na^{n-1}b} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1)b}{2a} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1)b}{a} = \frac{30375 \times 2}{7290} = \frac{25}{3}$$

$$\Rightarrow \frac{nb}{a} - \frac{b}{a} = \frac{25}{3}$$

$$\Rightarrow 10 - \frac{b}{a} = \frac{25}{3} \qquad [\text{Using } (4)]$$

$$\Rightarrow \frac{b}{a} = 10 - \frac{25}{3} = \frac{5}{3} \qquad ...(5)$$

From (4) and (5), we obtain

$$n \cdot \frac{5}{3} = 10$$

$$\Rightarrow n = 6$$

Substituting n = 6 in equation (1), we obtain

$a^6 = 729$

$$\Rightarrow a = \sqrt[6]{729} = 3$$

From (5), we obtain

$$\frac{b}{3} = \frac{5}{3} \Rightarrow b = 5$$

Thus, a = 3, b= 5, and n= 6.

Q2 :

Find *a* if the coefficients of x^2 and x^3 in the expansion of (3 + ax)^9 are equal.

Answer :

It is known that $(r + 1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$. Assuming that x^2 occurs in the $(r+ 1)^{th}$ term in the expansion of $(3 + ax)^9$, we obtain

$$T_{r+1} = {}^9C_r (3)^{9-r} (ax)^r = {}^9C_r (3)^{9-r} a^r x^r$$

Comparing the indices of x in x^2 and in T_{r+1}, we obtain r=

2

Thus, the coefficient of x^2 is

$$^9C_2(3)^{9-2}a^2 = \frac{9!}{2!7!}(3)^7 a^2 = 36(3)^7 a^2$$

Assuming that x^3 occurs in the $(k+1)^{th}$ term in the expansion of $(3 + ax)^9$, we obtain

$$T_{k+1} = {}^9C_k(3)^{9-k}(ax)^k = {}^9C_k(3)^{9-k}a^k x^k$$

Comparing the indices of x in x^3 and in T_{k+1}, we obtain

$k = 3$

Thus, the coefficient of x^3 is

$$^9C_3(3)^{9-3}a^3 = \frac{9!}{3!6!}(3)^6 a^3 = 84(3)^6 a^3$$

It is given that the coefficients of x^2 and x^3 are the same.

$$84(3)^6 a^3 = 36(3)^7 a^2$$
$$\Rightarrow 84a = 36 \times 3$$
$$\Rightarrow a = \frac{36 \times 3}{84} = \frac{104}{84}$$
$$\Rightarrow a = \frac{9}{7}$$

Thus, the required value of a is $\frac{9}{7}$.

Q3 :

Find the coefficient of x^5 in the product $(1 + 2x)^6(1 - x)^7$ using binomial theorem.

Answer :

Using Binomial Theorem, the expressions, $(1 + 2x)^6$ and (1 â€“ x)^7, can be expanded as

$$\begin{aligned}(1+2x)^6 &= {}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 \\ &\quad + {}^6C_5(2x)^5 + {}^6C_6(2x)^6 \\ &= 1 + 6(2x) + 15(2x)^2 + 20(2x)^3 + 15(2x)^4 + 6(2x)^5 + (2x)^6 \\ &= 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6\end{aligned}$$

$$
\begin{aligned}
(1-x)^7 &= {}^7C_0 - {}^7C_1(x) + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 \\
&\quad - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7 \\
&= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7
\end{aligned}
$$

$$
\begin{aligned}
&\therefore (1+2x)^6(1-x)^7 \\
&= \left(1+12x+60x^2+160x^3+240x^4+192x^5+64x^6\right)\left(1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7\right)
\end{aligned}
$$

The complete multiplication of the two brackets is not required to be carried out. Only those terms, which involve x^5, are required.

The terms containing $x^5$are

$$
\begin{aligned}
&1\left(-21x^5\right)+(12x)\left(35x^4\right)+\left(60x^2\right)\left(-35x^3\right)+\left(160x^3\right)\left(21x^2\right)+\left(240x^4\right)(-7x)+\left(192x^5\right)(1) \\
&= 171x^5
\end{aligned}
$$

Thus, the coefficient of x^5in the given product is 171.

Q4 :

If *a* and *b* are distinct integers, prove that *a* - *b* is a factor of a^n - b^n, whenever *n* is a positive integer. [Hint: write a^n = (*a* - *b* + *b*)^n and expand]

Answer :

In order to prove that (*a*â€“ *b*) is a factor of (a^nâ€“ b^n), it has to be proved that a^nâ€“

b^n= *k* (*a*â€“ *b*), where *k* is some natural number

It can be written that, *a*= *a* â€“ *b* + *b*

$$
\begin{aligned}
\therefore a^n &= (a-b+b)^n = \left[(a-b)+b\right]^n \\
&= {}^nC_0(a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + {}^nC_nb^n \\
&= (a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + b^n \\
\Rightarrow a^n - b^n &= (a-b)\left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right] \\
\Rightarrow a^n - b^n &= k(a-b) \\
\text{where, } k &= \left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right] \text{ is a natural number}
\end{aligned}
$$

Thisshows that (*a*â€“ *b*) is a factor of (a^nâ€“ b^n), where *n* is a positive integer.

Q5 :

Evaluate $\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$ **.**

Answer :

Firstly, the expression (a+ b)^{6}â€“ (aâ€“ b)^{6}is simplified by using Binomial Theorem. This can be done as

$$(a+b)^6 = {}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5a^1b^5 + {}^6C_6b^6$$
$$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$
$$(a-b)^6 = {}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5a^1b^5 + {}^6C_6b^6$$
$$= a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$
$$\therefore (a+b)^6 - (a-b)^6 = 2\left[6a^5b + 20a^3b^3 + 6ab^5\right]$$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6 = 2\left[6\left(\sqrt{3}\right)^5\left(\sqrt{2}\right) + 20\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3 + 6\left(\sqrt{3}\right)\left(\sqrt{2}\right)^5\right]$$
$$= 2\left[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6}\right]$$
$$= 2 \times 198\sqrt{6}$$
$$= 396\sqrt{6}$$

Q6 :

Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4 + \left(a^2-\sqrt{a^2-1}\right)^4$ **.**

Answer :

Firstly, the expression (x+ y)^4+ (x â€“ y)^{4}is simplified by using Binomial Theorem.

This can be done as

$$(x+y)^4 = {}^4C_0x^4 + {}^4C_1x^3y + {}^4C_2x^2y^2 + {}^4C_3xy^3 + {}^4C_4y^4$$
$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$
$$(x-y)^4 = {}^4C_0x^4 - {}^4C_1x^3y + {}^4C_2x^2y^2 - {}^4C_3xy^3 + {}^4C_4y^4$$
$$= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$$
$$\therefore (x+y)^4 + (x-y)^4 = 2\left(x^4 + 6x^2y^2 + y^4\right)$$

Putting $x = a^2$ and $y = \sqrt{a^2-1}$, we obtain

$$\left(a^2+\sqrt{a^2-1}\right)^4 + \left(a^2-\sqrt{a^2-1}\right)^4 = 2\left[\left(a^2\right)^4 + 6\left(a^2\right)^2\left(\sqrt{a^2-1}\right)^2 + \left(\sqrt{a^2-1}\right)^4\right]$$
$$= 2\left[a^8 + 6a^4\left(a^2-1\right) + \left(a^2-1\right)^2\right]$$
$$= 2\left[a^8 + 6a^6 - 6a^4 + a^4 - 2a^2 + 1\right]$$
$$= 2\left[a^8 + 6a^6 - 5a^4 - 2a^2 + 1\right]$$
$$= 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2$$

Q7 :

Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

Answer :

0.99 = 1 â€“ 0.01

$$\therefore (0.99)^5 = (1-0.01)^5$$
$$= {}^5C_0(1)^5 - {}^5C_1(1)^4(0.01) + {}^5C_2(1)^3(0.01)^2 \quad \text{(Approximately)}$$
$$= 1 - 5(0.01) + 10(0.01)^2$$
$$= 1 - 0.05 + 0.001$$
$$= 1.001 - 0.05$$
$$= 0.951$$

Thus, the value of $(0.99)^5$ is approximately 0.951.

Q8 :

Find *n*, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$

Answer :

In the expansion, $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + ... + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$,

Fifth term from the beginning $= {}^nC_4a^{n-4}b^4$

Fifth term from the end $= {}^nC_{n-4}a^4b^{n-4}$

Therefore, it is evident that in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$, the fifth term from the beginning is ${}^nC_4\left(\sqrt[4]{2}\right)^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^4$ and the fifth term from the end is ${}^nC_{n-4}\left(\sqrt[4]{2}\right)^4\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$.

$${}^nC_4\left(\sqrt[4]{2}\right)^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^4 = {}^nC_4\frac{\left(\sqrt[4]{2}\right)^n}{\left(\sqrt[4]{2}\right)^4}\cdot\frac{1}{3} = {}^nC_4\frac{\left(\sqrt[4]{2}\right)^n}{2}\cdot\frac{1}{3} = \frac{n!}{6.4!(n-4)!}\left(\sqrt[4]{2}\right)^n \qquad ...(1)$$

$${}^nC_{n-4}\left(\sqrt[4]{2}\right)^4\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4} = {}^nC_{n-4}.2\cdot\frac{\left(\sqrt[4]{3}\right)^4}{\left(\sqrt[4]{3}\right)^n} = {}^nC_{n-4}.2\cdot\frac{3}{\left(\sqrt[4]{3}\right)^n} = \frac{6n!}{(n-4)!4!}\cdot\frac{1}{\left(\sqrt[4]{3}\right)^n} \qquad ...(2)$$

It is given that the ratio of the fifth term from the beginning to the fifth term from the end is $\sqrt{6}:1$. Therefore, from (1) and (2), we obtain

$$\frac{n!}{6.4!(n-4)!}\left(\sqrt[4]{2}\right)^n : \frac{6n!}{(n-4)!4!}\cdot\frac{1}{\left(\sqrt[4]{3}\right)^n} = \sqrt{6}:1$$

$$\Rightarrow \frac{\left(\sqrt[4]{2}\right)^n}{6} : \frac{6}{\left(\sqrt[4]{3}\right)^n} = \sqrt{6}:1$$

$$\Rightarrow \frac{\left(\sqrt[4]{2}\right)^n}{6}\times\frac{\left(\sqrt[4]{3}\right)^n}{6} = \sqrt{6}$$

$$\Rightarrow \left(\sqrt[4]{6}\right)^n = 36\sqrt{6}$$

$$\Rightarrow 6^{\frac{n}{4}} = 6^{\frac{5}{2}}$$

$$\Rightarrow \frac{n}{4} = \frac{5}{2}$$

$$\Rightarrow n = 4\times\frac{5}{2} = 10$$

Thus, the value of *n* is 10.

Q9 :

Expand using Binomial Theorem $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4, x \neq 0$.

Answer :

Using Binomial Theorem, the given expression $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$ can be expanded as

$$\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$$

$$= {}^4C_0\left(1+\frac{x}{2}\right)^4 - {}^4C_1\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + {}^4C_2\left(1+\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2 - {}^4C_3\left(1+\frac{x}{2}\right)\left(\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{2}{x}\right)^4$$

$$= \left(1+\frac{x}{2}\right)^4 - 4\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + 6\left(1+x+\frac{x^2}{4}\right)\left(\frac{4}{x^2}\right) - 4\left(1+\frac{x}{2}\right)\left(\frac{8}{x^3}\right) + \frac{16}{x^4}$$

$$= \left(1+\frac{x}{2}\right)^4 - \frac{8}{x}\left(1+\frac{x}{2}\right)^3 + \frac{24}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} - \frac{16}{x^2} + \frac{16}{x^4}$$

$$= \left(1+\frac{x}{2}\right)^4 - \frac{8}{x}\left(1+\frac{x}{2}\right)^3 + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4} \qquad \ldots(1)$$

Again by using Binomial Theorem, we obtain

$$\left(1+\frac{x}{2}\right)^4 = {}^4C_0(1)^4 + {}^4C_1(1)^3\left(\frac{x}{2}\right) + {}^4C_2(1)^2\left(\frac{x}{2}\right)^2 + {}^4C_3(1)^1\left(\frac{x}{2}\right)^3 + {}^4C_4\left(\frac{x}{2}\right)^4$$

$$= 1 + 4\times\frac{x}{2} + 6\times\frac{x^2}{4} + 4\times\frac{x^3}{8} + \frac{x^4}{16}$$

$$= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} \qquad \ldots(2)$$

$$\left(1+\frac{x}{2}\right)^3 = {}^3C_0(1)^3 + {}^3C_1(1)^2\left(\frac{x}{2}\right) + {}^3C_2(1)\left(\frac{x}{2}\right)^2 + {}^3C_3\left(\frac{x}{2}\right)^3$$

$$= 1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8} \qquad \ldots(3)$$

From(1), (2), and (3), we obtain

$$\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$$

$$=1+2x+\frac{3x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-\frac{8}{x}\left(1+\frac{3x}{2}+\frac{3x^2}{4}+\frac{x^3}{8}\right)+\frac{8}{x^2}+\frac{24}{x}+6-\frac{32}{x^3}+\frac{16}{x^4}$$

$$=1+2x+\frac{3}{2}x^2+\frac{x^3}{2}+\frac{x^4}{16}-\frac{8}{x}-12-6x-x^2+\frac{8}{x^2}+\frac{24}{x}+6-\frac{32}{x^3}+\frac{16}{x^4}$$

$$=\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-5$$

Q10 :

Find the expansion of $\left(3x^2-2ax+3a^2\right)^3$ using binomial theorem.

Answer :

Using Binomial Theorem, the given expression $\left(3x^2-2ax+3a^2\right)^3$ can be expanded as

$$\left[\left(3x^2-2ax\right)+3a^2\right]^3$$

$$={}^3C_0\left(3x^2-2ax\right)^3+{}^3C_1\left(3x^2-2ax\right)^2\left(3a^2\right)+{}^3C_2\left(3x^2-2ax\right)\left(3a^2\right)^2+{}^3C_3\left(3a^2\right)^3$$

$$=\left(3x^2-2ax\right)^3+3\left(9x^4-12ax^3+4a^2x^2\right)\left(3a^2\right)+3\left(3x^2-2ax\right)\left(9a^4\right)+27a^6$$

$$=\left(3x^2-2ax\right)^3+81a^2x^4-108a^3x^3+36a^4x^2+81a^4x^2-54a^5x+27a^6$$

$$=\left(3x^2-2ax\right)^3+81a^2x^4-108a^3x^3+117a^4x^2-54a^5x+27a^6 \quad \ldots(1)$$

Again by using Binomial Theorem, we obtain

$$\left(3x^2-2ax\right)^3$$

$$={}^3C_0\left(3x^2\right)^3-{}^3C_1\left(3x^2\right)^2(2ax)+{}^3C_2\left(3x^2\right)(2ax)^2-{}^3C_3(2ax)^3$$

$$=27x^6-3\left(9x^4\right)(2ax)+3\left(3x^2\right)\left(4a^2x^2\right)-8a^3x^3$$

$$=27x^6-54ax^5+36a^2x^4-8a^3x^3 \quad \ldots(2)$$

From (1) and (2), we obtain

$$\left(3x^2-2ax+3a^2\right)^3$$

$$=27x^6-54ax^5+36a^2x^4-8a^3x^3+81a^2x^4-108a^3x^3+117a^4x^2-54a^5x+27a^6$$

$$=27x^6-54ax^5+117a^2x^4-116a^3x^3+117a^4x^2-54a^5x+27a^6$$