15 Structure of Atoms and Nuclei

Multiple Choice Questions (1 Mark Each)

1. When an electron jumps from higher energy orbit to lower energy orbit, the difference in the energies in the two orbits is radiated as quantum (photon) of
   (A) $E = mc^2$
   (B) $E = h/\nu$
   (C) $E = \frac{hc}{\lambda}$
   (D) $E = \frac{\lambda}{hc}$

2. The radii of Bohr orbit are directly proportional to….
   (A) Principal quantum number
   (B) Square of principal quantum number
   (C) Cube of principal quantum number
   (D) Independent of principal quantum number

3. According to Bohr second postulate, the angular momentum of electron is the integral multiple of $\frac{h}{2\pi}$. The S.I unit of Planck constant h is same as
   (A) Linear momentum
   (B) angular momentum
   (C) Energy
   (D) Centripetal force

4. The ionization energy of Hydrogen atom in its ground state is
   (A) 3.4 e V
   (B) 10.2 eV
   (C) 13.6 eV
   (D) –13.6 eV

5. For hydrogen atom, the minimum excitation energy (of n = 2) is
   (A) 3.4 e V
   (B) 10.2 eV
   (C) 13.6 eV
   (D) –10.2 eV

Hint: $E_n = \frac{-E_1 Z^2}{n^2}$

For hydrogen, Z = 1,

∴ For n = 2

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

∴ Minimum excitation energy = $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

6. The dimensions of Rydberg's constant are

(A) $[M^0L^1T^0]$ **(B) $[M^0L^{-1}T^0]$**

(C) $[M^0L^1T^1]$ (D) $[M^0L^{-1}T^{-1}]$

7. In a Hydrogen, electron jumps from fourth orbit to second orbit. The wave number of the radiations emitted by electron is

(A) $\frac{R}{16}$ **(B) $\frac{3R}{16}$**

(C) $\frac{5R}{16}$ (D) $\frac{7R}{16}$

Hint: Wave number, $\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right] = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16}$

8. The speed of electron having de Broglie wavelength of 10^{-10} m is ($m_e = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J-s)

(A) 7.28×10^6 m/s (B) 4×10^6 m/s

(C) 8×10^5 m/s (D) 5.25×10^5 m/s

Hint: $\lambda = \frac{h}{mv}$

$\therefore \quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}} \approx 7.28 \times 10^6$ m/s

***[Note:** The question belongs to chapter 14, Dual Nature of Radiation and Matter.]*

9. The decay constant λ of a certain radioactive material is 0.2166 per day. The average life τ of the radioactive material is

(A) 5.332 days **(B) 4.617 days**

(C) 2.166 days (D) 1.083 days

Hint: Average life, $\tau = \frac{1}{\lambda} = \frac{1}{0.2166} = 4.617$ days

10. The ratio of areas of the circular orbit of an electron in the ground state to that of first excited state of an electron in hydrogen atom is…

(A) 16 : 1 (B) 4 :1

(C) 1 :4 **(D) 1 :16**

Hint: As, $r_n \propto n^2$

$$\frac{a_1}{a_2} = \frac{r_1^2}{r_2^2} = \left(\frac{n_1}{n_2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

Very Short Answer (VSA) (1 Mark Each)

1. What is the angular momentum of an electron in first exited state for hydrogen atom?

Ans: For 1st excited state, n = 2

$\therefore$ Angular momentum $= \frac{nh}{2\pi} = \frac{h}{\pi}$

2. If a_0 is the Bohr radius and n is the principal quantum number then, state the relation for the radius of n^{th} orbit of electron in terms of Bohr radius and principal quantum number.

Ans: The required relation is $r_n = a_0n^2$

3. In which region of electromagnetic spectrum for Hydrogen, does the Lyman series lies?

Ans: The Lyman series lies in ultra-violet (UV) region.

4. How much energy must be supplied to hydrogen atom, to free (remove) the electron in the ground state?

Ans: Energy that needs be supplied to hydrogen atom, to free the electron in the ground state is 13.6 eV

5. State the value of minimum excitation energy for Hydrogen atom.

Ans: Minimum excitation energy = $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

6. What is the energy of electron in hydrogen atom for $n = \infty$?

Ans: The energy of electron in hydrogen atom for $n = \infty$ is zero.

7. The radius of the smallest orbit of the electron (a_0) in hydrogen atom is 0.053 nm. What is the radius of the 4^{th} orbit of the electron in hydrogen atom?

Ans: Radius of the 4^{th} orbit of the electron in hydrogen atom,

$r_4 = a_0\, n^2 = 0.053 \times (4)^2 = 0.848$ nm

8. The half life of a certain radioactive species is 6.93×10^5 seconds. What is the decay constant?

Ans: Decay constant $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{6.93 \times 10^5} = 10^{-6}$ s

9. The linear momentum of the particle is 6.63 kg m/s. Calculate the de Broglie wavelength.

Ans: $\lambda = \frac{h}{p}$

$$= \frac{6.63 \times 10^{-34}}{6.63} = 10^{-34} \text{ m}$$

*[**Note:** The question belongs to chapter 14, Dual Nature of Radiation and Matter.]*

Short Answer I (SA1) (2 Marks Each)

1. Starting with $r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$, Show that the speed of electron in n^{th} orbit varies inversely to principal quantum number.

Ans: According to Bohr's second postulate,

$$m r_n v_n = \frac{nh}{2\pi}$$

$$\therefore \quad m^2 v_n^2 r_n^2 = \frac{n^2 h^2}{4\pi^2}$$

$$\therefore \quad v_n^2 = \frac{n^2 h^2}{4\pi^2 m^2 r_n^2}$$

Substituting, $r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$ in above relation,

$$v_n^2 = \frac{n^2 h^2}{4\pi^2 m^2} \times \left(\frac{\pi m Z e^2}{\varepsilon_0 h^2 n^2} \right)^2$$

$$= \frac{n^2 h^2}{4\pi^2 m^2} \times \frac{\pi^2 m^2 Z^2 e^4}{\varepsilon_0^2 h^4 n^4}$$

$$= \frac{Z^2 e^4}{4 \varepsilon_0^2 h^2 n^2}$$

$$\therefore \quad v_n^2 \propto \frac{1}{n^2}$$

$$\Rightarrow v_n \propto \frac{1}{n}$$

2. State Bohr second postulate for atomic model. Express it in its mathematical form.

Ans: Bohr's second postulate:

The radius of the orbit of an electron can only take certain fixed values such that the angular momentum of the electron in these orbits is an integral multiple of $\frac{h}{2\pi}$, h being the Planck's constant.

Mathematical form: $m_e r_n v_n = \frac{nh}{2\pi}$

where, m_e = mass of electron, r_n = radius of n^{th} Bohr's orbit, v_n = linear velocity of electron in n^{th} orbit, n = principal quantum number.

3. State any two limitations of Bohr's model for hydrogen atom.

Ans: Two limitations of Bohr's model for hydrogen atom:

i. Bohr's model for hydrogen atom could not explain the line spectra of atoms other than hydrogen. Even for hydrogen, more accurate study of the observed spectra showed multiple components in some lines which could not be explained on the basis of this model.

ii. It could not explain varying intensity of emission lines.

4. Using de Broglie's hypothesis, obtain the mathematical form of Bohr's second postulate.

Ans:

i. De Broglie suggested that instead of considering the orbiting electrons inside atoms as particles, they should be viewed as standing waves. Also, the length of the orbit of an electron should be an integral multiple of its wavelength.

ii. Now, the distance travelled by electron in one complete revolution in n^{th} orbit of radius r_n is $2\pi r_n$ and it should be integral multiple of wavelength.

∴ $2\pi r_n = n\lambda$ ....(1)

where, n = 1, 2, 3, 4….

iii. By de Broglie hypothesis,

$$\lambda = \frac{h}{p_n} = \frac{h}{m_e v_n}$$

iv. Substituting this value of 'λ' in equation (1),

momentum of electron, $p_n = \frac{nh}{2\pi r_n}$

∴ Angular momentum of electron $L_n = p_n r_n = \frac{nh}{2\pi}$.

Thus, mathematical form of Bohr's second postulate is obtained.

5. Show that half life period of radioactive material varies inversely to decay constant λ.

Ans: From law of radioactive decay,

$N = N_0 e^{-\lambda t}$

at $t = T_{1/2}$, $N = \frac{N_0}{2}$

∴ $\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$

∴ $\frac{1}{2} = e^{-\lambda T_{1/2}}$

∴ $e^{\lambda T_{1/2}} = 2$

∴ $\lambda T_{1/2} = \log_e 2 = 0.693$

∴ $T_{1/2} = \frac{0.693}{\lambda}$

$\Rightarrow T_{1/2} \propto \frac{1}{\lambda}$

6. Define (i) Excitation energy (ii) Ionization energy

Ans:

i. *The energy required to take an electron from the ground state to an excited state is called the* ***excitation energy*** *of the electron in that state.*

ii. *The* ***ionization energy*** *of an atom is the minimum amount of energy required to be given to an electron in the ground state of that atom to set the electron free.*

7. Calculate the longest wavelength in Paschen series.
(Given $R_H = 1.097 \times 10^7\ m^{-1}$)

Solution:

Given: $n = 3$, $m = 4$

To find: Longest wavelength in Paschen series (λ_L)

Formula: $\frac{1}{\lambda_L} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

Calculation: From formula,

$$\frac{1}{\lambda_L} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$\therefore \quad \frac{1}{\lambda_L} = R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$= R\left[\frac{16-9}{9\times16}\right] = \frac{1.097\times10^7\times7}{9\times16}$$

$$\therefore \quad \lambda_L = \frac{9\times16}{1.097\times7}\times10^{-7}$$

$$= \text{antilog}\ \{\log(9) + \log(16) - \log(1.097) - \log(7)\} \times 10^{-7}$$

$$= \text{antilog}\{0.9542 + 1.2041 - 0.0402 - 0.8451\} \times 10^{-7}$$

$$= \text{antilog}\{1.2730\} \times 10^{-7}$$

$$= 18.75 \times 10^{-7}\ \text{m}$$

$$\therefore \quad \lambda_L = \mathbf{18750\ Å}$$

Ans: The longest wavelength in Paschen series is **18750 Å.**

8. The angular momentum of electron in 3rd Bohr orbit of Hydrogen atom is 3.165×10^{-34} kg m^2/s. Calculate Planck's constant h.

Solution:

Given: $L_3 = 3.165 \times 10^{-34}$ kg m^2/s, n = 3

To find: Planck's constant (h)

Formula: $L_n = n\frac{h}{2\pi}$

Calculation: From formula,

$$h = \frac{2\pi L_n}{n}$$

$$= \frac{2 \times 3.142 \times 3.165 \times 10^{-34}}{3}$$

$$= 6.284 \times 1.055 \times 10^{-34}$$

$$= \text{antilog}\ \{\log(6.284) + \log(1.055)\} \times 10^{-34}$$

$$= \text{antilog}\ \{0.7982 + 0.0232\} \times 10^{-34}$$

$$= \text{antilog}\{0.8214\} \times 10^{-34}$$

$$= \mathbf{6.628 \times 10^{-34}\ Js}$$

Ans: Value of Planck's constant (h) is **6.628×10^{-34} Js.**

9. The half-life of a certain radioactive nucleus is 3.2 days. Calculate (i) decay constant (ii) average life of radioactive nucleus.

Solution:

Given: $T_{1/2} = 3.2$ days

To find:
i. decay constant (λ)
ii. average life (τ)

Formulae:
i. $T_{1/2} = \frac{0.693}{\lambda}$
ii. $\tau = \frac{1}{\lambda}$

Calculation: From formula (i),

$$3.2 = \frac{0.693}{\lambda}$$

$$\therefore \quad \lambda = \frac{0.693}{3.2}$$

$$\therefore \quad \lambda = \text{antilog}\{\log(0.693) - \log(3.2)\}$$

$$= \text{antilog}\{\bar{1}.8407 - 0.5051\}$$

$$= \text{antilog}\{\bar{1}.3356\}$$

$$= \mathbf{0.2166\ /day}$$

From formula (ii),

$$\tau = \frac{1}{0.2166}$$

Using reciprocal table,

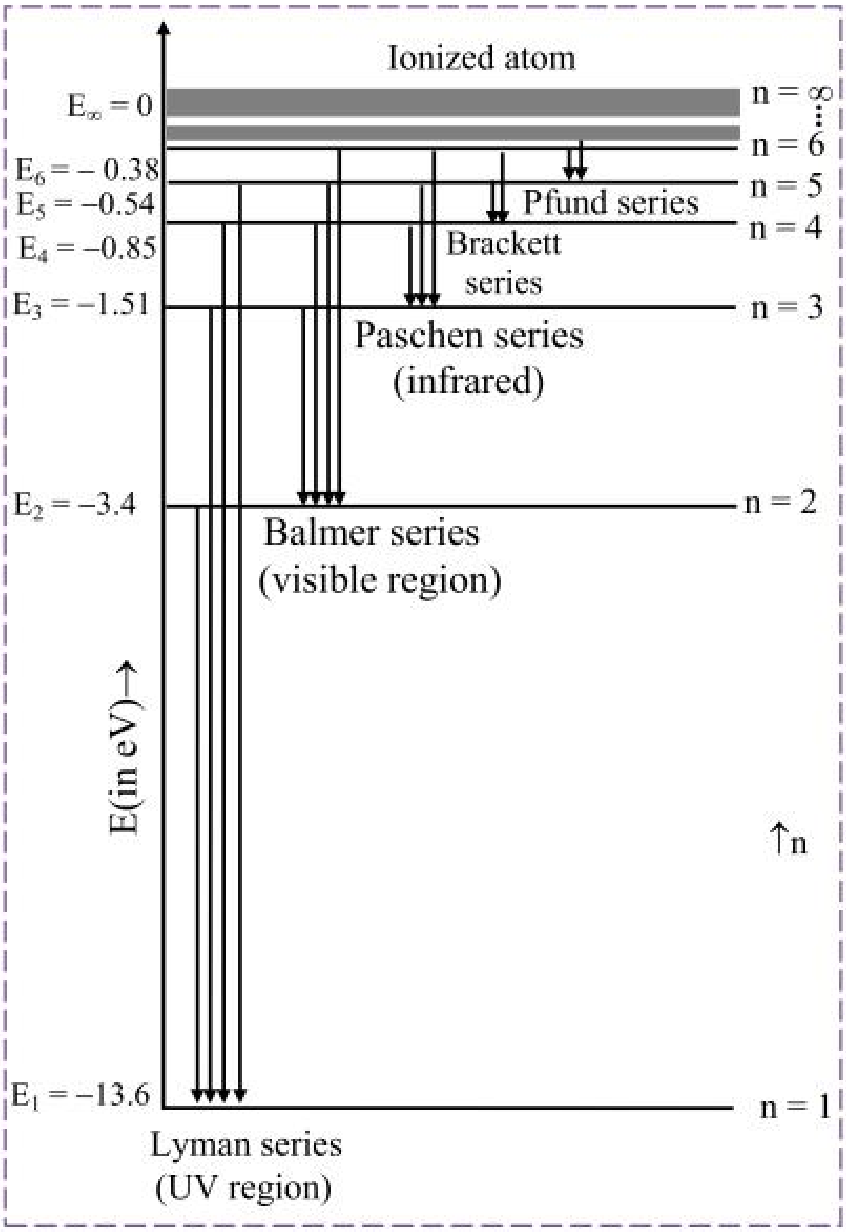
$\tau =$ **4.617 days**

Ans: i. The decay constant of reaction is **0.2166 /day**

ii. The mean life of the species is **4.617 days**.

10. Draw a neat labelled diagram showing energy levels and transition between them for hydrogen atom.

Ans: Energy levels and transition between them for hydrogen atom:



Short Answer II (SA2) (3 Marks Each)

1. Derive an expression for the radius of the n^{th} Bohr orbit for hydrogen atom.

Ans: Expression for radius of Bohr orbit in atom:

i. Let, m_e = mass of electron,

$-e$ = charge on electron,

r_n = radius of n^{th} Bohr's orbit,

$+e$ = charge on nucleus,

v_n = linear velocity of electron in n^{th} orbit,
Z = number of electrons in an atom,
n = principal quantum number.

ii. From Bohr's first postulate,
Coulomb's force F_e = Centripetal force F_{cp}

$$\therefore \quad \frac{Ze^2}{4\pi\varepsilon_0 r_n^2} = \frac{m_e v_n^2}{r_n}$$

$$\therefore \quad v_n^2 = \frac{Ze^2}{4\pi\varepsilon_0 r_n m_e} \qquad \ldots.(1)$$

iii. According to Bohr's second postulate,

$$m_e r_n v_n = \frac{nh}{2\pi}$$

$$\therefore \quad m_e^2 v_n^2 r_n^2 = \frac{n^2h^2}{4\pi^2}$$

$$\therefore \quad v_n^2 = \frac{n^2h^2}{4\pi^2 m_e^2 r_n^2} \qquad \ldots.(2)$$

iv. From equations (1) and (2),

$$\frac{n^2h^2}{4\pi^2 m_e^2 r_n^2} = \frac{Ze^2}{4\pi\varepsilon_0 r_n m_e}$$

$$\therefore \quad r_n = \frac{n^2h^2\varepsilon_0}{\pi m_e Ze^2}$$

$$\therefore \quad r_n = \left(\frac{\varepsilon_0 h^2}{\pi m_e Ze^2}\right)n^2 \qquad \ldots.(3)$$

This is the required expression for radius of n^{th} orbit.

2. Using the expression for energy of electron in the n^{th} orbit, show that $\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where symbols have their usual meaning.

Ans:

i. Let, E_m = Energy of electron in m^{th} higher orbit
E_n = Energy of electron in n^{th} lower orbit

ii. According to Bohr's third postulate,

$$E_m - E_n = h\nu$$

$$\therefore \quad \nu = \frac{E_m - E_n}{h} \qquad \ldots.(1)$$

iii. But $E_m = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2}$ ….(2)

$$E_n = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2} \quad \text{….(3)}$$

iv. From equations (1), (2) and (3),

$$\nu = \frac{-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2} - \left(-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2}\right)}{h}$$

$$\therefore \quad \nu = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[-\frac{1}{m^2} + \frac{1}{n^2}\right]$$

$$\therefore \quad \frac{c}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n^2} - \frac{1}{m^2}\right] \quad \text{….}[\because\ \nu = \frac{c}{\lambda}]$$

where, c = speed of electromagnetic radiation

$$\therefore \quad \frac{1}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

v. But, $\frac{m_e e^4}{8\varepsilon_0^2 h^3 c} = R_H$ = Rydberg's constant

$$= 1.097 \times 10^7 \text{ m}^{-1}$$

$$\therefore \quad \frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n^2} - \frac{1}{m^2}\right] \quad \text{….(4)}$$

For hydrogen Z = 1,

$$\therefore \quad \frac{1}{\lambda} = R_H \left[\frac{1}{n^2} - \frac{1}{m^2}\right] \quad \text{….(5)}$$

3. Show that for radioactive decay $N(t) = N_0\, e^{-\lambda t}$, where symbols have their usual meaning.

Ans: If 'N(t)' is the number of parent nuclei present at any instant 't', 'dN' is the number of nuclei disintegrated in short interval of time 'dt', then,

$dN \propto -N(t)\, dt$

$dN = -\lambda\, N(t)\, dt$

where, λ is known as decay constant or disintegration constant.

The negative sign indicates disintegration of atoms.

Integrating both sides of equation,

$$\int_{N_0}^{N(t)} \frac{dN}{N(t)} = \int_0^t -\lambda \, dt$$

where, N_0 is number of parent atoms at time t = 0.

$\therefore \quad \log_e \frac{N(t)}{N_0} = -\lambda t$

$\therefore \quad N(t) = N_0 \, e^{-\lambda t}$

This is the required relation.

4. Obtain an expression for half life time of radioactive material. Hence state the relation between average life and half life time of radioactive material.

Ans: Expression for half life period ($T_{1/2}$):

From law of radioactive decay, $N(t) = N_0 \, e^{-\lambda t}$

at $t = T_{1/2}$, $N(t) = \frac{N_0}{2}$

$\therefore \quad \frac{N_0}{2} = N_0 \, e^{-\lambda T(1/2)}$

$\therefore \quad e^{\lambda T(1/2)} = 2$

$\therefore \quad \lambda T_{1/2} = \log_e 2 = 0.693$

$\therefore \quad T_{1/2} = \frac{0.693}{\lambda}$

Relation between average life (τ) and half life time of radioactive material:

$T_{1/2} = \tau \, ln \, 2 = 0.693 \, \tau$

5. Calculate the wavelength for the first three lines in Paschen series. (Given $R_H = 1.097 \times 10^7 \, m^{-1}$)

Solution:

Given: $R_H = 1.097 \times 10^7 \, m^{-1}$,

For Paschen series, n = 3,

To find: Wavelength of first three lines of Paschen series

Formula: For Paschen series, $\frac{1}{\lambda} = R_H \left(\frac{1}{3^2} - \frac{1}{m^2} \right)$

Calculation: For first line of Paschen series,

From formula,

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= 1.097 \times 10^7 \times \left(\frac{7}{9 \times 16}\right)$$

$$= 0.05333 \times 10^7 \text{ m}^{-1}$$

Using reciprocal table,

$\lambda_1 = \mathbf{1.876 \times 10^{-6}\ m}$

For second line of Paschen series,

From formula,

$$\frac{1}{\lambda_2} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

$$= 1.097 \times 10^7 \times \left(\frac{16}{9 \times 25}\right)$$

$$= 0.075 \times 10^7 \text{ m}^{-1}$$

Using reciprocal table,

$\lambda_2 = \mathbf{1.282 \times 10^{-6}\ m}$

For third line of Paschen series,

From formula,

$$\frac{1}{\lambda_3} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{6^2}\right)$$

$$= 1.097 \times 10^7 \times \left(\frac{27}{9 \times 36}\right)$$

$$= 0.0914 \times 10^7 \text{ m}^{-1}$$

Using reciprocal table,

$\lambda_3 = \mathbf{1.094 \times 10^{-6}\ m}$

Ans: Wavelength of first three lines of Paschen series are $\mathbf{1.875 \times 10^{-6}\ m}$, $\mathbf{1.282 \times 10^{-6}\ m}$, and $\mathbf{1.094 \times 10^{-6}\ m}$, respectively.

6. Calculate the shortest wavelength in Paschen series if the longest wavelength in Balmar series is 6563 Å.

Solution:

Given: $(\lambda_B) = 6563\ \text{Å} = 6563 \times 10^{-10}\ \text{m}$

$= 6.563 \times 10^{-7}\ \text{m}$

To find: Shortest wavelength (λ_p)

Formula: $$\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

Calculation: For (λ_B), m = 3, n = 2

From formula,

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda_B} = \frac{5R}{36}$$

$$\therefore \quad \lambda_B = \frac{36}{5R} \quad \text{....(1)}$$

For Paschen series shortest wavelength (λ_p),

n = 3, m = ∞

$$\therefore \quad \frac{1}{\lambda_p} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right]$$

$$\therefore \quad \frac{1}{\lambda_p} = R\left[\frac{1}{9}\right]$$

$$\therefore \quad \frac{1}{\lambda_p} = \frac{R}{9}$$

$$\therefore \quad \lambda_p = \frac{9}{R} \quad \text{....(2)}$$

From equations (1) and (2),

$$\frac{\lambda_p}{\lambda_B} = \frac{9/R}{36/5R}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_B} = \frac{9}{R} \times \frac{5R}{36}$$

$$= \frac{5}{4}$$

$$\therefore \quad \lambda_p = \frac{5}{4} \times \lambda_B$$

$$= \frac{5}{4} \times 6563$$

$$\therefore \quad \lambda_p = \mathbf{8203.75\ Å}$$

Ans: The shortest wavelength in Paschen series is **8203.75 Å**.

7. **A radioactive substance decays to $(1/10)^{th}$ of its original value in 56 days. Calculate its decay constant.**

Ans: Here, $\frac{N(t)}{N} = \frac{1}{10}$ and t = 56 days

We have, $\frac{N(t)}{N_0} = e^{-\lambda t}$

$\therefore \quad \frac{1}{10} = e^{-\lambda t}$

$\therefore \quad e^{\lambda t} = 10$

$\therefore \quad \lambda t = \log_e 10$

$\therefore \quad \lambda = \frac{\log_e 10}{t}$

$= \frac{2.303 \times \log 10}{56}$

$= \frac{2.303}{56}$

= antilog {log(2.303) − log (56)}

= antilog {0.3623 − 1.7481}

= antilog $\{\bar{2}.6142\}$

= $\mathbf{4.113 \times 10^{-2}}$ **per day**

Ans: Decay constant of is $\mathbf{4.113 \times 10^{-2}}$ **per day**.

Long Answer (LA) (4 Marks Each)

1. **State the postulates of Bohr's atomic model. Hence show energy of electron varies inversely to the square of principal quantum number.**

Ans: Bohr's three postulates are:

i. *In a hydrogen atom, the electron revolves round the nucleus in a fixed circular orbit with constant speed.*

ii. *The radius of the orbit of an electron can only take certain fixed values such that the angular momentum of the electron in these orbits is an integral multiple of* $\frac{h}{2\pi}$, *h being the Planck's constant.*

iii. *An electron can make a transition from one of its orbits to another orbit having lower energy. In doing so, it emits a photon of energy equal to the difference in its energies in the two orbits.*

Expression for energy of electron in n^{th} orbit of Bohr's hydrogen atom:

i. **Kinetic energy:**

Let, m_e = mass of electron

r_n = radius of n^{th} orbit of Bohr's hydrogen atom

v_n = velocity of electron

$-e$ = charge of electron

$+e$ = charge on the nucleus

Z = number of electrons in an atom.

According to Bohr's first postulate,

$$\frac{m_e v_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0} \times \frac{Ze^2}{r_n^{\ 2}}$$

where, ε_0 is permittivity of free space.

$$\therefore \quad m_e v_n^2 = \frac{Z}{4\pi\varepsilon_0} \times \frac{e^2}{r_n} \qquad \text{....(1)}$$

The revolving electron in the circular orbit has linear speed and hence it possesses kinetic energy.

It is given by, $K.E = \frac{1}{2} m_e v_n^2$

$$\therefore \quad K.E = \frac{1}{2} \times \left(\frac{Z}{4\pi\varepsilon_0} \times \frac{e^2}{r_n} \right) \qquad \text{....[From equation (1)]}$$

$$\therefore \quad K.E = \frac{Ze^2}{8\pi\varepsilon_0 r_n} \qquad \text{....(2)}$$

ii. **Potential energy:**

Potential energy of electron is given by, P.E = V(−e)

where,

V = electric potential at any point due to charge on nucleus

− e = charge on electron.

In this case,

$$\therefore \quad P.E = \frac{1}{4\pi\varepsilon_0} \times \frac{e}{r_n} \times (-Ze)$$

$$\therefore \quad P.E = \frac{1}{4\pi\varepsilon_0} \times \frac{-Ze^2}{r_n}$$

$$\therefore \quad P.E = -\frac{Ze^2}{4\pi\varepsilon_0 r_n} \qquad \text{....(3)}$$

iii. **Total energy:**

The total energy of the electron in any orbit is its sum of P.E and K.E.

$\therefore$ T.E = K.E + P.E

$$= \left(\frac{Ze^2}{8\pi\varepsilon_0 r_n}\right) + \left(-\frac{Ze^2}{4\pi\varepsilon_0 r_n}\right) \quad \text{....[From equations (2) and (3)]}$$

$$\therefore \quad T.E = -\frac{Ze^2}{8\pi\varepsilon_0 r_n} \quad \text{....(4)}$$

iv. But, $r_n = \left(\frac{\varepsilon_0 h^2}{\pi m_e Ze^2}\right) \times n^2$

Substituting for r_n in equation (4),

$$\therefore \quad T.E = -\frac{1}{8\pi\varepsilon_0} \times \frac{Ze^2}{\left(\frac{\varepsilon_0 h^2}{\pi m_e Ze^2}\right)n^2}$$

$$= -\frac{1}{8\pi\varepsilon_0} \times \frac{Z^2 e^2 \pi m_e e^2}{\varepsilon_0 h^2 n^2}$$

$$\therefore \quad T.E = -\left(\frac{m_e Z^2 e^4}{8\varepsilon_0{}^2 h^2}\right) \times \frac{1}{n^2} \quad \text{....(5)}$$

$$\Rightarrow T.E. \propto \frac{1}{n^2}$$

2. Obtain an expression for wavenumber, when electron jumps from higher energy orbit to lower energy orbit. Hence show that the shortest wavelength for Balmar series is $4/R_H$.

Ans: Expression for wavenumber:

i. Let, E_m = Energy of electron in m^{th} higher orbit

E_n = Energy of electron in n^{th} lower orbit

ii. According to Bohr's third postulate,

$E_m - E_n = h\nu$

$$\therefore \quad \nu = \frac{E_m - E_n}{h} \quad \text{....(1)}$$

iii. But $E_m = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2}$ ....(2)

$$E_n = -\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2} \quad \text{....(3)}$$

iv. From equations (1), (2) and (3),

$$\nu = \frac{-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 m^2} - \left(-\frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^2 n^2}\right)}{h}$$

$$\therefore \quad \nu = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[-\frac{1}{m^2} + \frac{1}{n^2}\right]$$

$$\therefore \quad \frac{c}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n^2} - \frac{1}{m^2}\right] \qquad \ldots.[\because \ \nu = \frac{c}{\lambda}]$$

where, c = speed of electromagnetic radiation

$$\therefore \quad \frac{1}{\lambda} = \frac{Z^2 m_e e^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

v. But, $\frac{m_e e^4}{8\varepsilon_0^2 h^3 c} = R_H$ = Rydberg's constant

$$= 1.097 \times 10^7 \ m^{-1}$$

$$\therefore \quad \frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n^2} - \frac{1}{m^2}\right] \qquad \ldots.(4)$$

This is the required expression.

vi. For shortest wavelength for Balmer series:

n = 2 and m = ∞

$$\frac{1}{\lambda} = R_H \left[\frac{1}{2^2} - \frac{1}{\infty}\right]$$

$$= \frac{R_H}{4}$$

$$\therefore \quad \lambda = \frac{4}{R_H}$$

3. Obtain an expression for decay law of radioactivity. Hence show that the activity $A(t) = \lambda N_0 \ e^{-\lambda t}$.

Ans: Expression for decay law of radioactivity:

If 'N(t)' is the number of parent nuclei present at any instant 't', 'dN' is the number of nuclei disintegrated in short interval of time 'dt', then,

$dN \propto -N(t) \ dt$

$dN = -\lambda \ N(t) \ dt$

where, λ is known as decay constant or disintegration constant.

The negative sign indicates disintegration of atoms.

Integrating both sides of equation,

$$\int_{N_0}^{N(t)} \frac{dN}{N(t)} = \int_0^t -\lambda \, dt$$

where, N_0 is number of parent atoms at time t = 0.

$\therefore$ $\log_e \frac{N(t)}{N_0} = -\lambda t$

$\therefore$ $N(t) = N_0 \, e^{-\lambda t}$

Expression for activity:

i. The rate of decay, i.e., the number of decays per unit time $-\frac{dN(t)}{dt}$, is called as activity A(t).

ii. It is given as,

$$A(t) = -\frac{dN(t)}{dt}$$

$$= \lambda N(t)$$

$$= \lambda N_0 e^{-\lambda t}$$

At t = 0, the activity is $A_0 = \lambda N_0$.

$\therefore$ $A(t) = A_0 e^{-\lambda t}$.

4. Using the expression for the radius of orbit for Hydrogen atom, show that the linear speed varies inversely to principal quantum number n the angular speed varies inversely to the cube of principal quantum number n.

Ans: According to Bohr's second postulate,

$$mr_n v_n = \frac{nh}{2\pi}$$

$\therefore$ $m^2 v_n^2 r_n^2 = \frac{n^2 h^2}{4\pi^2}$

$\therefore$ $v_n^2 = \frac{n^2 h^2}{4\pi^2 m^2 r_n^2}$

Substituting, $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$ in above relation,

$$v_n^2 = \frac{n^2h^2}{4\pi^2 m^2} \times \left(\frac{\pi m Z e^2}{\varepsilon_0 h^2 n^2}\right)^2$$

$$= \frac{n^2h^2}{4\pi^2 m^2} \times \frac{\pi^2 m^2 Z^2 e^4}{\varepsilon_0^2 h^4 n^4}$$

$$= \frac{Z^2 e^4}{4\varepsilon_0^2 h^2 n^2}$$

$$\therefore \quad v_n^2 \propto \frac{1}{n^2}$$

$$\Rightarrow v_n \propto \frac{1}{n}$$

Expression for angular speed:

Since, $v_n = r_n\,\omega$ and $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m_e e^2}$

$$\therefore \quad \omega = \frac{v_n}{r_n} = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{1}{n} \bigg/ \frac{\varepsilon_0 h^2 n^2}{\pi m_e e^2}$$

$$\therefore \quad \omega = \frac{e^2}{2\varepsilon_0 hn} \times \frac{\pi m_e e^2}{\varepsilon_0 h^2 n^2} = \left(\frac{\pi m_e e^4}{2\varepsilon_0^2 h^3}\right)\frac{1}{n^3}$$

$$\Rightarrow \omega \propto \frac{1}{n^3}$$

Two mark questions:-

Ex. 1 : The required area A = $8\sqrt{2}$ sq. units

Ex.2: Required area = 1 Sq. units.

Ex.3: Required area = 3 sq.units.

Ex.4: Required area = 128/3 sq.units.

Ex.5: Required area = 9 Sq.units.

Ex.6: Required area = $(4\sqrt{2}/3)[3\sqrt{3}-1]$ Sq.units.

Ex.7: Required area = $(8/\sqrt{3})[4-\sqrt{2}]$ Sq.units.

Ex.8: Required area = $\pi/2$ sq.units.

Ex.9: Required area = $25\pi/4$ sq.units.

Ex.10: Required area = 5 sq.units.

Three marks questions:-

Ex.1: Required area = 4 sq.units.

Ex.2: Required area = 21/4 sq.units.

Ex.3: Required area = 128/3 sq.units.

Ex.4: Required area = $(16\sqrt{2}/3)(4-\sqrt{2})$ sq.units.

Ex.5: Required area =4 Sq.units

Ex.6: Required area = 48π sq.units.

Four marks questions:-

Ex.1 : Required area A = $(16/3)\ a^2$ sq. units.

Ex.2: Required area = 4 sq.units.

Ex.3: Required area = 2π sq.units.

Ex.4: Required area = 1 sq.units.

Ex.5: Required area = $(9\pi/8)$ sq.units.

Ex.6: Required area = $(\pi/4 - 1/2)$ sq.units.

Ex.7: Required area = 64/3 sq.units.