CBSE Board
Class XII Mathematics
Sample Paper 3

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. **All** the questions are **compulsory**.
2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is **not** permitted.

Part A

Q1 – Q20 are multiple choice type questions. Select the correct option.

1. If A and B are square matrices of order 2, then det (A + B) = 0 is possible only when

A. det (A) = 0 or det (B) = 0
B. det (A) + det (B) = 0
C. det (A) = 0 and det (B) = 0
D. A + B = 0

2. $\text{If } a \cdot b = a \cdot c \text{ and } a \times b = a \times c, a \neq 0, \text{then}$

A. $b = c$
B. $b = 0$
C. $b + c = 0$
D. $b - c = 0$

3. If one ball is drawn at random from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, then the probability that 2 white and 1 black balls will be drawn is

A. $\frac{13}{32}$
B. $\frac{1}{4}$
C. $\frac{1}{32}$

D. $\frac{3}{16}$

4. The point which does not lie in the half plane $3x + y \leq 6$ is

A. (1, 2)
B. (2, 1)
C. (1, 3)
D. (−3, 1)

5. If $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\} = \alpha$, then $x^2 =$

A. $\cos 2\alpha$
B. $\cos \alpha$
C. $\sin 2\alpha$
D. $\sin \alpha$

6. The set of points on which the function $f(x) = |x - 3| \cos x$ is differentiable is

A. R
B. R – {0}
C. Z
D. R – {3}

7. If $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adjA is

A. $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$
B. $\begin{bmatrix} -d & c \\ b & -a \end{bmatrix}$
C. $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
D. $\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

8. The equation of the normal to the curve $y = \sin x$ at (0, 0) is

A. $x + y = 0$
B. $x - y = 0$
C. $y = 0$
D. $x = 0$

9. The unit vector perpendicular to the plane passing through the points $P\left(\hat{i} - \hat{j} + 2\hat{k}\right), Q\left(2\hat{i} - \hat{k}\right)$ and $R\left(2\hat{j} + \hat{k}\right)$ is

A. $8\hat{i} + 4\hat{j} + 4k$

B. $\frac{1}{\sqrt{6}}\left(2\hat{i} + \hat{j} + k\right)$

C. $\frac{1}{2\sqrt{6}}\left(-5\hat{i} + 2\hat{j} - k\right)$

D. $\frac{1}{\sqrt{6}}\left(-5\hat{i} + 2\hat{j} - k\right)$

10. $\int_0^{\pi} \frac{1}{1 + \sin x}\, dx$ equal

A. 0

B. ∞

C. –2

D. 2

11. $\int \frac{1}{1 + e^x} dx$ equals

A. $-\log\left|e^{-x}\right| + c$

B. $\log\left|1 + e^{-x}\right| + c$

C. $-\log\left|1 + e^{-x}\right| + c$

D. $\log\left|e^{-x}\right| + c$

12. Let $*$ be a binary operation defined by $a * b = 3a + 4b - 2$. Find $2 * 7$.

A. 14

B. 27

C. 36

D. 32

13. If A = {a, b, c} and B = {1, 2, 3} and a function f: A → B is given by f = {(a, 2), (b, 3), (c, 1)}. Then f is:

A. One to one

B. Onto

C. Bijective

D. Real

14. Differentiate $e^{x^2 + \tan x}$ with respect to x.

A. $e^{x^2 + \tan x}\left(2x + \sec^2 x\right)$

B. $2x + \sec^2 x$

C. $e^{\tan x}\left(2x + \sec^2 x\right)$

D. $e^{x^2}\left(2x + \sec^2 x\right)$

15. Find the angle between the curves $y^2 = x$ and $x^2 = y$ at (1, 1).

A. $\tan^{-1}\left(\frac{5}{4}\right)$

B. $\tan^{-1}\left(\frac{3}{4}\right)$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

16. Find the equation of a line through (–2, 1, 3) and parallel to $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

A. $\frac{x+3}{-2} = \frac{y-4}{1} = \frac{z+8}{3}$

B. $\frac{x-3}{-2} = \frac{y-5}{1} = \frac{z-6}{3}$

C. $\frac{x+2}{3} = \frac{y-1}{5} = \frac{z-3}{6}$

D. $\frac{x+2}{-2} = \frac{y-1}{1} = \frac{z-3}{3}$

17. Area of the region bounded by the circle $x^2 + y^2 = 4$ in the 1st quadrant is equal to

A. $\frac{\pi}{2}$ units

B. $\left(2 + \frac{\pi}{2}\right)$ units

C. 2π units

D. π units

18. Find solution of the differential equation $\frac{dy}{dx} = \left(1 + x^2\right)\left(1 + y^2\right)$

A. $\tan^{-1} y = 1 + \frac{x^2}{2} + C$

B. $\tan^{-1} y = x + \frac{x^3}{3} + C$

C. $\tan^{-1} x = 1 + \frac{y^2}{2} + C$

D. $\tan^{-1} x = y + \frac{y^3}{3} + C$

19. Find the principal value of $\cot^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{5\pi}{4}$

D. $\frac{3\pi}{2}$

20. A line makes angles $30^\circ, 60^\circ$ and 90° with the positive directions of the x, y and z axes respectively. Find its direction cosines.

A. $1, \frac{\sqrt{3}}{2}$ and $\frac{1}{2}$

B. $0, \sqrt{3}$ and $\frac{1}{\sqrt{3}}$

C. $1, \sqrt{3}$ and $\frac{1}{\sqrt{3}}$

D. $0, \frac{1}{2}$ and $\frac{\sqrt{3}}{2}$

Part B

21. Differentiate $\frac{x^3\sqrt{5+x}}{(7-3x)^5\sqrt[3]{8+5x}}$ w.r.t x.

OR

If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2y'' + xy' + y = 0$.

22. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that y = 1 when x = 0.

23. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

OR

Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)\, dx$

24. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine: (i) k (ii) P(X < 3) (iii) P(X > 5) (iv) P(1 ≤ X < 3)

25. Solve the following for x:

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

OR

Solve the Equation:

$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, (x>0)$

26. Find the equation of a plane that is parallel to the x-axis and passes through the line common to two intersecting planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) - 1 = 0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k}) = -4$.

27. Find the Equation of the plane passing through the point (2, 5, –8), perpendicular to the plane 2x – 3y + 4z + 1 = 0, and 4x + y – 2z + 6 = 0.

28. If $f: R \to A, A = \{x : x \in R, -1 < x < 1\}, f(x) = \frac{x}{1+|x|}, x \in R$. Show that the function f is a bijective function

29. If x, y and z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then show that xyz = –1.

30. Evaluate: $\int \frac{x^2}{x^4 + x^2 - 2} dx$

31. For what value of k is the following function continuous at x = 1

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 4k, & x = 1 \end{cases}$$

Part C

32. Two trainee carpenters A and B earn Rs. 150 and Rs. 200 per day respectively. A can make 6 frames and 4 stools per day while B can make 10 frames and 4 stools per day. How many days shall each work, if it is desired to produce atleast 60 frames and 32 stools at a minimum labour cost? Solve the problem graphically.

OR

A brick manufacturer has two depots, A and B, with stocks of 30,000 and 20,000 bricks respectively. He receives orders from three builders P, Q and R for 15,000, 20,000 and 15,000 bricks respectively. The cost in rupees for transporting 1000 bricks to the builders from the depots are given below:

From \ To	P	Q	R
A	40	20	30
B	20	60	40

How should the manufacturer fulfil the orders so as to keep the cost of transportation minimum?

33. Calculate the area

(i) between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the x-axis between x = 0 to x = a

(ii)

Triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where OA = a and OB = b.

Find the area enclosed between the chord AB and the arc AB of the ellipse

(iii) Find the ratio of the two areas found.

34. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ is minimum.

35. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses it or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ be the probability that he copies it.

Assuming that a student, who copies the answer, will be correct with the probability ¾, what is the probability that student knows the answer, given that he answered it correctly?

Arjun does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts of unfair means? How would an act like the above hamper his character development in the coming years?

OR

Two bags A and B contain 3 red and 4 black balls, and 4 red and 5 black balls respectively. From bag A, one ball is transferred to bag B and then a ball is drawn from bag B. The ball is found to be red in colour. Find the probability that

a. The transferred ball is black?
b. The transferred ball is red?

36. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{x}{2}\right) \\ \tan\left(\frac{x}{2}\right) & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$.

37. Find the equation plane passing through points (1, 2, 3), (0, –1, 0) and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$.

OR

Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, –1, 2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$