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QUADRATIC EQUATION

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1. QUADRATIC EXPRESSION

The general form of a quadratic expression in x is,

$f(x) = ax^2 + bx + c$, where a, b, c $\in$ R & $a \neq 0$.

and general form of a quadratic equation in x is,

$ax^2 + bx + c = 0$, where a, b, c $\in$ R & $a \neq 0$.

2. ROOTS OF QUADRATIC EQUATION

(a) The solution of the quadratic equation,

$ax^2 + bx + c = 0$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression $D = b^2 - 4ac$ is called the discriminant of the quadratic equation.

(b) If α & β are the roots of the quadratic equation

$ax^2 + bx + c = 0$, then ;

(i) $\alpha + \beta = -b/a$ (ii) $\alpha\beta = c/a$

(iii) $|\alpha - \beta| = \dfrac{\sqrt{D}}{|a|}$.

(c) A quadratic equation whose roots are α & β is $(x - \alpha)(x - \beta) = 0$ i.e.

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e.

x^2 – (sum of roots) x + product of roots = 0.

Note...

$$y = (ax^2 + bx + c) \equiv a(x - \alpha)(x - \beta)$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a}$$

3. NATURE OF ROOTS

(a) Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ R & $a \neq 0$ then;

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal).

(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal).

(iii) $D < 0 \Leftrightarrow$ roots are imaginary.

(iv) If p + i q is one root of a quadratic equation, then the other must be the conjugate p – i q & vice versa. $\left(p,\ q \in R\ \&\ i = \sqrt{-1}\right)$.

(b) Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ Q & $a \neq 0$ then;

(i) If D > 0 & is a perfect square, then roots are rational & unequal.

(ii) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational & $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ & vice versa.

Note...

Remember that a quadratic equation cannot have three different roots & if it has, it becomes an identity.

4. GRAPH OF QUADRATIC EXPRESSION

Consider the quadratic expression, $y = ax^2 + bx + c$, $a \neq 0$ & $a, b, c \in R$ then ;

(i) The graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upwards & if $a < 0$ then the shape of the parabola is concave downwards.

(ii) $y > 0 \ \forall \ x \in R$, only if $a > 0 \ \& \ D < 0$

(iii) $y < 0 \ \forall \ x \in R$, only if $a < 0 \ \& \ D < 0$

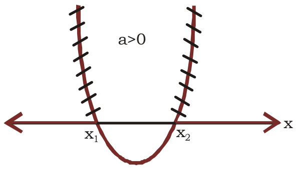
5. SOLUTION OF QUADRATIC INEQUALITIES

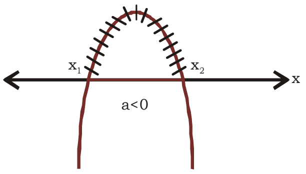
$ax^2 + bx + c > 0 \ (a \neq 0)$.

(i) If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two different roots $(x_1 < x_2)$.

Then $a > 0 \quad \Rightarrow \quad x \in (-\infty, x_1) \cup (x_2, \infty)$

$a < 0 \quad \Rightarrow \quad x \in (x_1, x_2)$



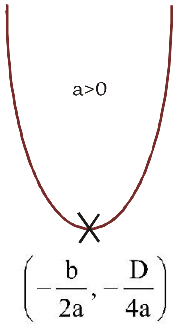


(ii) Inequalities of the form $\dfrac{P(x)}{Q(x)} \gtrless 0$ can be quickly solved using the method of intervals (wavy curve).

6. MAX. & MIN. VALUE OF QUADRATIC EXPRESSION

Maximum & Minimum Value of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as :

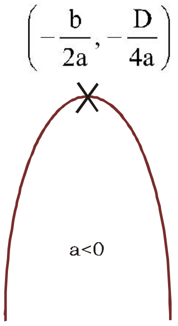
For a > 0, we have :



$$y \in \left[\frac{4ac - b^2}{4a}, \infty\right)$$

$$y_{min} = \frac{-D}{4a} \text{ at } x = \frac{-b}{2a}, \text{ and } y_{max} \to \infty$$

For a < 0, we have :



$$y \in \left(-\infty, \frac{4ac - b^2}{4a}\right]$$

$$y_{max} = \frac{-D}{4a} \text{ at } x = \frac{-b}{2a}, \text{ and } y_{min} \to -\infty$$

7. THEORY OF EQUATIONS

If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are the roots of the n^{th} degree polynomial equation :

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0$$

where $a_0, a_1, \ldots\ldots\; a_n$ are all real & $a_0 \neq 0$,

Then,

$$\sum \alpha_1 = -\frac{a_1}{a_0};$$

$$\sum \alpha_1\, \alpha_2 = +\frac{a_2}{a_0};$$

$$\sum \alpha_1\, \alpha_2\, \alpha_3 = -\frac{a_3}{a_0};$$

…………

$$\alpha_1\, \alpha_2\, \alpha_3 \ldots\ldots \alpha_n = (-1)^n\, \frac{a_n}{a_0}$$

8. LOCATION OF ROOTS

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'k' are :

$D \geq 0$ & $f(k) > 0$ & $(-b/2a) > k$.

(ii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'k' (in other words the number 'k' lies between the roots of $f(x) = 0$ is:

$af(k) < 0$.

(iii) Conditions for exactly one root of $f(x) = 0$ to lie in the interval (k_1, k_2) i.e. $k_1 < x < k_2$ are :

$D > 0$ & $f(k_1) . f(k_2) < 0$.

(iv) Conditions that both roots of $f(x) = 0$ to be confined between the numbers k_1 & k_2 are $(k_1 < k_2)$:

$D \geq 0$ & $f(k_1) > 0$ & $f(k_2) > 0$ & $k_1 < (-b/2a) < k_2$.

Note...

Remainder Theorem : If $f(x)$ is a polynomial, then $f(h)$ is the remainder when $f(x)$ is divided by $x - h$.

Factor theorem : If $x = h$ is a root of equation $f(x) = 0$, then $x-h$ is a factor of $f(x)$ and conversely.

9. MAX. & MIN. VALUES OF RATIONAL EXPRESSION

Here we shall find the values attained by a rational expresion of the form $\frac{a_1x^2 + b_1x + c_1}{a_2x^2 + b_2x + c_2}$ for real values of x.

Example No. 4 will make the method clear.

10. COMMON ROOTS

(a) Only One Common Root

Let α be the common root of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$, such that $a, a' \neq 0$ and $a\,b' \neq a'b$.

Then, the condition for one common root is :

$$(ca' - c'a)^2 = (ab' - a'b)\,(bc' - b'c).$$

(b) Two Common Roots

Let α, β be the two common roots of

$ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$,

such that $a, a' \neq 0$.

Then, the condition for two common roots is :

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

11. RESOLUTION INTO TWO LINEAR FACTORS

The condition that a quadratic function

$f(x, y) = ax^2 + 2\,hxy + by^2 + 2\,gx + 2\,fy + c$

may be resolved into two linear factors is that ;

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

OR $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

12. FORMATION OF A POLYNOMIAL EQUATION

If $\alpha_1, \alpha_2, \alpha_3, \ldots.., \alpha_n$ are the roots of the nth degree polynomial equation, then the equation is

$x^n - S_1x^{n-1} + S_2x^{n-2} + S_3x^{n-3} + \ldots\ldots + (-1)^n S_n = 0$

where S_k denotes the sum of the products of roots taken k at a time.

Particular Cases

(a) **Quadratic Equation** if α, β be the roots the quadratic equation, then the equation is :

$x^2 - S_1x + S_2 = 0$ *i.e.* $x^2 - (\alpha + \beta)\,x + \alpha\beta = 0$

(b) **Cubic Equation** if α, β, γ be the roots the cubic equation, then the equation is :

$x^3 - S_1x^2 + S_2x - S_3 = 0$ *i.e.*

$x^3 - (\alpha + \beta + \gamma)\,x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\,x - \alpha\beta\gamma = 0$

(i) If α is a root of equation $f(x) = 0$, the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$. In other words, $(x - \alpha)$ is a factor of $f(x)$ and conversely.

(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If there be any two real numbers 'a' & 'b' such that f (a) & f (b) are of opposite signs, then $f(x) = 0$ must have atleast one real root between 'a' and 'b'.

(iv) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

13. TRANSFORMATION OF EQUATIONS

(i) To obtain an equation whose roots are reciprocals of the roots of a given equation, it is obtained by replacing x by 1/x in the given equation

(ii) Transformation of an equation to another equation whose roots are negative of the roots of a given equation–replace x by – x.

(iii) Transformation of an equation to another equation whose roots are square of the roots of a given equation–replace x by $\sqrt{x}$.

(iv) Transformation of an equation to another equation whose roots are cubes of the roots of a given equation–replace x by $x^{1/3}$.

SOLVED EXAMPLES

Example – 1

If the remainder on dividing $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence find the zeros of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

Sol. Let $p(x) = x^3 + 2x^2 + kx + 3$.

We are given that when p (x) is divided by the linear polynomial $x - 3$, the remainder is 21.

⇒ $p(3) = 21$ (Remainder Theorem)

⇒ $3^3 + 2 \times 3^2 + k \times 3 + 3 = 21$

⇒ $27 + 18 + 3k + 3 = 21$

⇒ $3k = 21 - 27 - 18 - 3$

⇒ $3k = -27$

⇒ $k = -9$

Hence, $p(x) = x^3 + 2x^2 - 9x + 3$.

To find the quotient obtained on dividing p(x) by x–3, we perform the following division :

$$x^2 + 5x + 6$$
$$x - 3\overline{\big)\, x^3 + 2x^2 - 9x + 3}$$
$$x^3 - 3x^2$$
$$- \quad +$$
$$5x^2 - 9x + 3$$
$$5x^2 - 15x$$
$$- \quad +$$
$$6x + 3$$
$$6x - 18$$
$$- \quad +$$
$$21$$

Hence, $p(x) = (x^2 + 5x + 6)(x - 3) + 21$

(Divisor × Quotient + Remainder)

⇒ $x^3 + 2x^2 - 9x + 3 - 21 = (x^2 + 5x + 6)(x - 3)$

⇒ $x^3 + 2x^2 - 9x - 18 = (x^2 + 3x + 2x + 6)(x - 3)$

⇒ $x^3 + 2x^2 - 9x - 18 = (x + 3)(x + 2)(x - 3)$

Hence, the zeros of $x^3 + 2x^2 - 5x - 18$ are given by $x + 3 = 0, x + 2 = 0, x - 3 = 0$

⇒ $x = -3, -2, 3$

∴ The zeros of $x^3 + 2x^2 - 9x - 18$ are $-3, -2, 3$.

Example – 2

Find all the zeros of the polynomial $x^4 + x^3 - 9x^2 - 3x + 18$ if it is given that two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

Sol. Given polynomial $f(x) = x^4 + x^3 - 9x^2 - 3x + 18$ has two of its zeros $-\sqrt{3}$ and $\sqrt{3}$.

⇒ $(x + \sqrt{3})(x - \sqrt{3})$ is a factor of f (x),

i.e., $x^2 - 3$ is a factor of f (x).

Now, we apply the division algorithm to the given polynomial with $x^2 - 3$.

$$x^2 + x - 6$$
$$x^2 - 3\overline{\big)\, x^4 + x^3 - 9x^2 - 3x + 18}$$
$$x^4 \qquad - 3x^2$$
$$- \qquad +$$
$$x^3 - 6x^2 - 3x + 18$$
$$x^3 \qquad - 3x$$
$$- \qquad +$$
$$-6x^2 + 18$$
$$-6x^2 + 18$$
$$+ \qquad -$$
$$0 = \text{Remainder}$$

Thus, $x^4 + x^3 - 9x^2 - 3x + 18$

$= (x^2 - 3)(x^2 + x - 6)$

$= (x^2 - 3) \times \{x^2 + 3x - 2x - 6\}$

$= (x^2 - 3) \times \{x(x + 3) - 2(x + 3)\}$

$= (x^2 - 3) \times (x + 3)(x - 2)$

Putting $x + 3 = 0$ and $x - 2 = 0$

we get $x = -3$ and $x = 2$, i.e., –3 and 2 are the other two zeros of the given polynomial.

Hence $-\sqrt{3}, \sqrt{3}, -3$, 2 are the four zeros of the given polynomial.

Example – 3

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Sol. By division algorithm

$$x^4 - 6x^3 + 16x^2 - 25x + 10 = (x^2 - 2x + k)\, q(x) + (x + a)$$

where q (x) is the quotient.

As the degree on L.H.S. is 4; therefore, q (x) must be of degree 2.

Let $q(x) = lx^2 + mx + n,\ l \neq 0.$

Then $x^4 - 6x^3 + 16x^2 - 25x + 10 = (x^2 - 2x + k)(lx^2 + mx + n) + x + a$

$\Rightarrow\ x^4 - 6x^3 + 16x^2 - 25x + 10 = lx^4 + (m - 2l)x^3 + (n - 2m + kl)x^2 + (mk - 2n + 1)x + nk + a$

Equating coefficients of like powers of x on the two sides,

we obtain

$l = 1$ ... (1)

$m - 2l = -6$ ... (2)

$n - 2m + kl = 16$ ... (3)

$mk - 2n + l = -25$ ... (4)

and $nk + a = 10$ ... (5)

From (2), $m = -6 + 2l = -6 + 2 \times 1 = -4$ and

then from (3), $n = 16 + 2m - kl = 16 + 2 \times (-4) - k \times 1$

$\Rightarrow\ n = 8 - k$ ... (6)

From (4) and (6), we get

$(-4)k - 2(8 - k) + 1 = -25$

$\Rightarrow\ -4k - 16 + 2k = -25$

$\Rightarrow\ -2k = -25 + 16 - 1$

$\Rightarrow\ -2k = -10 \Rightarrow k = 5$

Substituting this value of k in (6), we have

$n = 8 - 5 = 3$ and then from (5),

we get

$a = 10 - nk = 10 - 3 \times 5 = -5.$

Example – 4

Solve the inequality, $\dfrac{3x^2 - 7x + 8}{x^2 + 1} \leq 2$

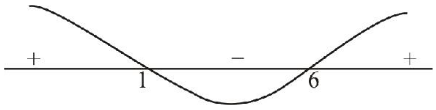
Sol. Domain : $x \in R$

Given inequality is equivalent to

$$\frac{3x^2 - 7x + 8}{x^2 + 1} - 2 \leq 0$$

$$\Rightarrow \quad \frac{3x^2 - 7x + 8 - 2x^2 - 2}{x^2 + 1} \leq 0$$

$$\Rightarrow \quad \frac{3x^2 - 7x + 6}{x^2 + 1} \leq 0 \ \Rightarrow \frac{(x-1)(x-6)}{x^2 + 1} \leq 0$$



$\Rightarrow\ x \in [1, 6]$

Example – 5

Solve the equation $25x^2 - 30x + 11 = 0$ by using the general expression for the roots of a quadratic equation.

Sol. Comparing the given equation with the general form of a quadratic equation $ax^2 + bx + c = 0$, we get

$a = 25, b = -30$ and $c = 11.$

Substituting these values in

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha = \frac{30 + \sqrt{900 - 1100}}{50} \text{ and } \beta = \frac{30 - \sqrt{900 - 1100}}{50}$$

$$\Rightarrow \quad \alpha = \frac{30 + \sqrt{-200}}{50} \text{ and } \beta = \frac{30 - \sqrt{-200}}{50}$$

$$\Rightarrow \quad \alpha = \frac{30 + 10i\sqrt{2}}{50} \text{ and } \beta = \frac{30 - 10i\sqrt{2}}{50}$$

$$\Rightarrow \quad \alpha = \frac{3}{5} + \frac{\sqrt{2}}{5}i \text{ and } \beta = \frac{3}{5} - \frac{\sqrt{2}}{5}i$$

Hence, the roots of the given equation are $\dfrac{3}{5} \pm \dfrac{\sqrt{2}}{5}i$

Example – 6

Solve the following quadratic equation by factorization method :

$x^2 - 5ix - 6 = 0$

Sol. The given equation is

$x^2 - 5ix - 6 = 0$

$\Rightarrow \quad x^2 - 5ix + 6i^2 = 0$

$\Rightarrow \quad x^2 - 3ix - 2ix + 6i^2 = 0$

$\Rightarrow \quad x(x - 3i) - 2i(x - 3i) = 0$

$\Rightarrow \quad (x - 3i)(x - 2i) = 0$

$\Rightarrow \quad x - 3i = 0, x - 2i = 0$

$\Rightarrow \quad x = 3i, x = 2i$

Hence, the roots of the given equation are 3i and 2i.

Example – 7

Solve the equation

(i) $15.2^{x+1} + 15.2^{2-x} = 135.$

(ii) $3^{x-4} + 5^{x-4} = 34$

(iii) $5^x \sqrt[x]{8^{x-1}} = 500$

Sol. (i) The equation rewrite in the form

$$30.2^x + \frac{60}{2^x} = 135$$

Let $t = 2^x$

then $30t^2 - 135t + 60 = 0$

$6t^2 - 27t + 12 = 0$

$\Rightarrow \quad 6t^2 - 24t - 3t + 12 = 0$

$\Rightarrow \quad (t - 4)(6t - 3) = 0$

then $t_1 = 4$ and $t_2 = \frac{1}{2}$

thus given equation is equivalent to

$2^x = 4$ and $2^x = \frac{1}{2}$

then $x = 2$ and $x = -1$

Hence roots of the original equation are $x_1 = 2$ and $x_2 = -1$

Note...

An equation of the form

$a^{f(x)} + b^{f(x)} = c$

where $a, b, c \in R$

and a, b, c satisfies the condition $a^2 + b^2 = c$

then solution of the equation is $f(x) = 2$ and no other solution of this equation.

(ii) Here, $3^2 + 5^2 = 34$, then given equation has a solution $x - 4 = 2$

$\therefore \quad x = 6$ is a root of the original equation

Note...

An equation of the form $\{f(x)\}^{g(x)}$ is equivalent to the equation

$\{f(x)\}^{g(x)} = 10^{g(x)\log f(x)}$ where $f(x) > 0$

(iii) We have $5^x \sqrt[x]{8^{x-1}} = 500$

$\Rightarrow \quad 5^x \sqrt[x]{8^{x-1}} = 5^3 . 2^2$

$\Rightarrow \quad 5^x.8^{\left(\frac{x-1}{x}\right)} = 5^3.2^2$

$\Rightarrow \quad 5^x.2^{\frac{3x-3}{x}} = 5^3.2^2$

$\Rightarrow \quad 5^{x-3}.2^{\left(\frac{x-3}{x}\right)} = 1$

$\Rightarrow \quad (5.2^{1/x})^{(x-3)} = 1$

is equivalent to the equation

$$10^{(x-3)\log\left(5.2^{1/x}\right)} = 1$$

$\Rightarrow \quad (x - 3)\log(5.2^{1/x}) = 0$

Thus original equation is equivalent to the collection of equations

$x - 3 = 0, \log(5.2^{1/x}) = 0$

$\therefore \quad x = 3, 5.2^{1/x} = 1$

$\Rightarrow \quad 2^{1/x} = (1/5)$

$\therefore \quad x = -\log_5 2$

Hence roots of the original equation are

$x = 3$ and $x = -\log_5 2$

Example – 8

Form an equation whose roots are cubes of the roots of equation $ax^3 + bx^2 + cx + d = 0$

Sol. Replacing x by $x^{1/3}$ in the given equation, we get

$a(x^{1/3})^3 + b(x^{1/3})^2 + c(x^{1/3}) + d = 0$

$\Rightarrow \quad ax + d = -(bx^{2/3} + cx^{1/3}) \qquad \text{....... (i)}$

$\Rightarrow \quad (ax + d)^3 = -(bx^{2/3} + cx^{1/3})^3$

$\Rightarrow \quad a^3x^3 + 3a^2dx^2 + 3ad^2x + d^3$

$= -[b^3x^2 + c^3x + 3bcx(bx^{2/3} + cx^{1/3})]$

$\Rightarrow \quad a^3x^3 + 3a^2dx^2 + 3ad^2x + d^3$

$[-b^3x^2 - c^3x + 3bcx(ax + d)]$ [From Eq. (i)]

$\Rightarrow \quad a^3x^3 + x^2(3a^2d - 3abc + b^3)$

$+ x(3ad^2 - 3bcd + c^3) + d^3 = 0$

This is the requied equation.

Example – 9

If α, β, γ be the roots of the equation

$x(1 + x^2) + x^2(6 + x) + 2 = 0,$

then the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is

(a) –3

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) None of these

Sol. $2x^3 + 6x^2 + x + 2 = 0$ has roots α, β, γ.

So, $2x^3 + x^2 + 6x + 2 = 0$ has roots $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$

(writing coefficients in revers order, since roots are reciprocal)

Hence, Sum of the roots $= -\frac{(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$

$\therefore \quad \alpha^{-1} + \beta^{-1} + \gamma^{-1} = -\frac{1}{2}$

Hence, (c) is the correct answer.

Example – 10

If P (x) = $ax^2 + bx + c$ and Q (x) = $-ax^2 + bx + c$, where $ac \neq 0$, show that the equation

$P(x) \cdot Q(x) = 0$ has at least two real roots.

Sol. Roots of the equation P (x) Q (x) = 0

i.e., $(ax^2 + bx + c)(-ax^2 + bx + c) = 0$ will be roots of the equations

$ax^2 + bx + c = 0 \qquad \text{....... (i)}$

and $-ax^2 + bx + c = 0 \qquad \text{........ (ii)}$

If D_1 and D_2 be discriminants of (i) and (ii) then

$D_1 = b^2 - 4ac$ and $D_2 = b^2 + 4ac$

Now $D_1 + D_2 = 2b^2 \geq 0$

(since, b may be zero)

i.e., $D_1 + D_2 \geq 0$

Hence, at least one of D_1 and $D_2 \geq 0$

i.e., at least one of the equations (i) and (ii) has real roots and therefore, equation P (x) Q (x) = 0 has at least two real roots.

Alternative Sol.

Since, $ac \neq 0$

$\therefore \quad ac < 0$ or $ac > 0$

Case I :

If $ac < 0 \Rightarrow -ac > 0$

then $D_1 = b^2 - 4ac > 0$

Case II :

If $ac > 0$

then $D_2 = b^2 + 4ac > 0$

So, at least one of D_1 and $D_2 > 0$.

Hence, at least one of the equations (i) and (ii) has real roots.

Hence, equation $P(x) \cdot Q(x) = 0$ has at least two real roots.

Example – 11

If $x^2 - ax + b = 0$ and $x^2 - px + q = 0$ have a root in common and the second equation has equal roots.

show that $b + q = \frac{ap}{2}$.

Sol. Given equations are $x^2 - ax + b = 0$ ...(i)

and $x^2 - px + q = 0$ ...(ii)

Let α be the common root. Then roots of Eq. (ii) will be α and α. Let β be the other root of Eq. (i). Thus roots of Eq. (i) are α, β and those of Eq. (ii). are α, α

$\alpha + \beta = a$ ...(iii)

$\alpha\beta = b$ ...(iv)

$2\alpha = p$ ...(v)

$\alpha^2 = q$ ...(vi)

LHS $= b + q = \alpha\beta + \alpha^2 = \alpha(\alpha + \beta)$ ...(vii)

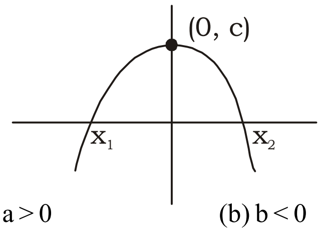
and RHS $= \frac{ap}{2} = \frac{(\alpha+\beta)2\alpha}{2} = \alpha(\alpha+\beta)$ ...(viii)

From Eqs. (vii) and (viii), LHS = RHS

Example – 12

The diagram shows the graph of

$y = ax^2 + bx + c$, Then,



(a) $a > 0$ (b) $b < 0$

(c) $c > 0$ (d) $b^2 - 4ac = 0$

Sol. As it is clear from the figure that it is a parabola opens downwards i.e. $a < 0$.

⇒ It is $y = ax^2 + bx + c$ i.e. degree two polynomial

Now, if $ax^2 + bx + c = 0$

⇒ it has two roots x_1 and x_2 as it cuts the axis at two distinct point x_1 and x_2.

Now from the figure it is also clear that $x_1 + x_2 < 0$

(i.e. sum of roots are negative)

⇒ $\frac{-b}{a} < 0 \Rightarrow \frac{b}{a} > 0$

⇒ $b < 0$ ($\because$ $a < 0$) (b) is correct.

As the graph of $y = f(x)$ cuts the + y-axis at (0, c)

where $c > 0$ ⇒ (c) is correct.

Example – 13

Find all roots of the equation $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$ if one root is $2 + \sqrt{3}$.

Sol. All coefficients are real, irrational roots will occur in conjugate pairs.

Hence another roots is $2 - \sqrt{3}$.

∴ Product of these roots $= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$

$= (x - 2)^2 - 3$

$= x^2 - 4x + 1$

Dividing $x^4 + 2x^3 - 16x^2 - 22x + 7$ by $x^2 - 4x + 1$ then the other quadratic factor is $x^2 + 6x + 7$

then the given equation reduce in the form

$(x^2 - 4x + 1)(x^2 + 6x + 7) = 0$

∴ $x^2 + 6x + 7 = 0$

then $x = \frac{-6 \pm \sqrt{36 - 28}}{2}$

$= -3 \pm \sqrt{2}$

Hence roots $2 \pm \sqrt{3}, -3 \pm \sqrt{2}$

Example – 14

If x is real, then prove that the values of $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ lies between $\frac{1}{7}$ and 7.

Sol. Let $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$. Then,

$x^2(y - 1) + 3x(y + 1) + 4(y - 1) = 0$

∴ for $x \in$ real, $D \geq 0$

⇒ $9(y + 1)^2 - 16(y - 1)^2 \geq 0$

⇒ $-7y^2 + 50y - 7 \geq 0$

⇒ $(7y - 1)(y - 7) \leq 0$

Hence, the given expression lies between $\frac{1}{7}$ and 7.

Example – 15

a, b, c $\in$ R, a $\neq$ 0 and the quadratic equation $ax^2 + bx + c = 0$ has no real roots, then,

(a) $a + b + c > 0$ (b) $a(a + b + c) > 0$

(c) $b(a + b + c) > 0$ (d) $c(a + b + c) > 0$

Sol. Let $f(x) = ax^2 + bx + c$. It is given that $f(x) = 0$ has no real roots. So, either $f(x) > 0$ for all $x \in R$ or $f(x) < 0$ for all $x \in R$ i.e. f(x) has same sign for all values of x.

$\therefore$ $f(0)\, f(1) > 0$

$\Rightarrow$ $c(a + b + c) > 0$

Also, $af(1) > 0$

$\Rightarrow$ $a(a + b + c) > 0$.

Example – 16

Find the values of a for which the inequality $(x - 3a)(x - a - 3) < 0$ is satisfied for all x such that $1 \leq x \leq 3$.

Sol. $(x - 3a)(x - a - 3) < 0$

Case I :

Let $3a < a + 3 \quad \Rightarrow a < 3/2$ ...(i)

Solution set of given inequality is $x \in (3a, a + 3)$

Now for given inequality to be true for all $x \in [1, 3]$, set [1, 3] should be the subset of (3a, a + 3) i.e. 1 and 3 lie inside 3a and a + 3 on number line.

So we can take, $3a < 1$ and $a + 3 > 3$ ...(ii)

Combining (i) and (ii), we get :

$\Rightarrow$ $a \in (0, 1/3)$

Case II :

Let $3a > a + 3 \quad \Rightarrow a > 3/2$ ...(iii)

Solution set of given inequality is

$x \in (a + 3, 3a)$

As in Case I, [1, 3] should be the subset of

(a + 3, 3a)

i.e., $a + 3 < 1$ and $3a > 3$ ...(iv)

Combining (iii) and (iv), we get :

$a \in \phi$ i.e. No solution ...(v)

Combining both cases, we get : $a \in (0, 1/3)$

Alternate Sol.

Let $f(x) = (x - 3a)(x - a - 3)$

for given equality to be true for all values of $x \in [1, 3]$, 1 and 3 should lie between the roots of $f(x) = 0$.

$\Rightarrow$ $f(1) < 0$ and $f(3) < 0$

Consider $f(1) < 0$:

$\Rightarrow$ $(1 - 3a)(1 - a - 3) < 0$

$\Rightarrow$ $(3a - 1)(a + 2) < 0$

$\Rightarrow$ $a \in (-2, 1/3)$ ...(i)

Consider $f(3) < 0$:

$\Rightarrow$ $(3 - 3a)(3 - a - 3) < 0$

$\Rightarrow$ $(a - 1)(a) < 0$

$\Rightarrow$ $a \in (0, 1)$ ...(ii)

Combining (i) and (ii), we get :

$a \in (0, 1/3)$

Example – 17

If $ax^2 - bx + 5 = 0$ does not have two distinct real roots, then find the minimum value of 5a + b.

Sol. Let $f(x) = ax^2 - bx + 5$

Since, f (x) = 0 does not have two distinct real roots, we have either

$$f(x) \geq 0 \ \forall x \in R \text{ or } f(x) \leq 0 \ \forall x \in R$$

But $f(0) = 5 > 0$, so $f(x) \geq 0 \ \forall x \in R$

In particular $f(-5) \geq 0 \Rightarrow 5a + b \geq -1$

Hence, the least value of 5a + b is –1.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Basics, Sum & Product of Roots

1. Roots of the equation $x^2 + x(2 - p^2) - 2p^2 = 0$ are

(a) $-p^2$ and -2 (b) p^2 and -2

(c) $-p^2$ and 2 (d) p^2 and 2

2. If $\sqrt{x+1} - \sqrt{x-1} = 1,$ then x is equal to

(a) 2/3 (b) 3/5

(c) 5/4 (d) 4/5

3. If $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{10}{3}$ and $x + y = 10$, then the value of xy will be

(a) 36 (b) 24

(c) 16 (d) 9

4. $\frac{8x^2+16x-51}{(2x-3)(x+4)} > 3$ if x is such that

(a) $x < -4$ (b) $-3 < x < 3/2$

(c) $x > 5/2$ (d) all these true

5. The sum of all real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$, is

(a) 0 (b) 8

(c) 4 (d) none of these

6. The equation $x^2 - 3|x| + 2 = 0$ has

(a) no real roots (b) one real root

(c) two real roots (d) four real roots

7. The sum of the real roots of the equation $x^2 + |x| - 6 = 0$ is

(a) 4 (b) 0

(c) -1 (d) none of these

8. The product of the real roots of the equation $|2x + 3|^2 - 3|2x + 3| + 2 = 0$, is

(a) 5/4 (b) 5/2

(c) 5 (d) 2

9. The roots of the equation $|x^2 - x - 6| = x + 2$ are

(a) $-2, 1, 4$ (b) $0, 2, 4$

(c) $0, 1, 4$ (d) $-2, 2, 4$

10. If $\sqrt{9x^2+6x+1} < (2-x),$ then

(a) $x \in \left(-\frac{3}{2}, \frac{1}{4}\right)$ (b) $x \in \left(-\frac{3}{2}, \frac{1}{4}\right]$

(c) $x \in \left[-\frac{3}{2}, \frac{1}{4}\right)$ (d) $x < \frac{1}{4}$

11. If α and β are the roots of $ax^2 + bx + c = 0$ then the value of $(a\alpha + b)^{-2} + (a\beta + b)^{-2}$ is equal to

(a) $\frac{b^2 - 2ac}{a^2c^2}$ (b) $\frac{c^2 - 2ab}{a^2b^2}$

(c) $\frac{a^2 - 2bc}{b^2c^2}$ (d) None

12. If α and β are the roots of $4x^2 + 3x + 7 = 0$, then the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is

(a) $-\frac{27}{64}$ (b) $\frac{63}{16}$

(c) $\frac{225}{343}$ (d) None of these

13. If α and β are the roots of the equation $x^2 + px + p^2 + q = 0,$ then the value of $\alpha^2 + \alpha\beta + \beta^2 + q =$

(a) 0 (b) 1

(c) q (d) 2q

14. If α, β are the roots of the equation $8x^2 - 3x + 27 = 0,$ then the value of $\left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3}$ is

(a) 1/4 (b) 1/3

(c) 7/2 (d) 4

15. If p, q are the roots of the equation $x^2 + px + q = 0$ where both p and q are non-zero, then (p, q) =

(a) (1, 2) (b) (1, –2)

(c) (–1, 2) (d) (–1, –2)

16. The product of the roots of the equation $mx^2 + 6x + (2m - 1) = 0$ is –1. Then m is equal to

(a) 1 (b) 1/3

(c) –1 (d) –1/3

17. If one root of the equation $ax^2 + bx + c = 0, a \neq 0$, is reciprocal of the other, then

(a) b = c (b) a = c

(c) a = 0 (d) b = 0

18. If the sum of the roots of the equation $ax^2 + 2x + 3a = 0$ is equal to their product, then value of a is

(a) $-\frac{2}{3}$ (b) –3

(c) 4 (d) $-\frac{1}{2}$

19. If the product of the roots of the quadratic equation $mx^2 - 2x + (2m - 1) = 0$ is 3 then the value of m

(a) 1 (b) 2

(c) –1 (d) 3

20. If the roots of $px^2 + qx + 2 = 0$ are reciprocals of each other, then

(a) p = 0 (b) p = –2

(c) p = ± 2 (d) p = 2

21. If the equation $(k - 2)x^2 - (k - 4)x - 2 = 0$ has difference of roots as 3 then the value of k is

(a) 1, 3 (b) 3, 3/2

(c) 2, 3/2 (d) 3/2, 1

22. If sin θ and cos θ are the roots of the equation $lx^2 + mx + n = 0$, then

(a) $l^2 - m^2 + 2ln = 0$ (b) $l^2 + m^2 + 2ln = 0$

(c) $l^2 - m^2 - 2ln = 0$ (d) $l^2 + m^2 - 2ln = 0$

23. The roots of the equation $x^2 + px + q = 0$ are tan 22° and tan 23° then

(a) p + q = 1 (b) p + q = –1

(c) p – q = 1 (d) p – q = –1

24. If α, β are the roots of the equation $x^2 - p(x + 1) - c = 0$, then $(\alpha + 1)(\beta + 1) =$

(a) c (b) c–1

(c) 1–c (d) none of these

25. If α and β are the roots of $x^2 - p(x + 1) - c = 0$, then the value of $\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c}$ is

(a) 2 (b) 1

(c) –1 (d) 0

26. If one root of $x^2 - x - k = 0$ be square of the other, then k is equal to

(a) $2 \pm \sqrt{3}$ (b) $3 \pm \sqrt{2}$

(c) $2 \pm \sqrt{5}$ (d) $5 \pm \sqrt{2}$

27. The number of values of a for which

$(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$

is an identity in x, is

(a) 0 (b) 2

(c) 1 (d) 3

28. If $p(x + 1)^2 + q(x^2 - 3x - 2) + x + 1 = 0$ be an identity in x, then p, q are

(a) 2, –2 (b) 1, –1

(c) 0, 0 (d) none

29. If the difference between the roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and a ≠ b, then

(a) a + b + 4 = 0 (b) a + b –4 = 0

(c) a – b –4 = 0 (d) a – b + 4 = 0

30. If roots of the equation $x^2 + ax + 25 = 0$ are in the ratio of 2 : 3 then the value of a is

(a) $\frac{\pm 5}{\sqrt{6}}$ (b) $\frac{\pm 25}{\sqrt{6}}$

(c) $\frac{\pm 5}{6}$ (d) none of these

31. If the roots of the equations $x^2 + 3x + 2 = 0$ & $x^2 - x + \lambda = 0$ are in the same ratio then the value of λ is given by

(a) 2/7 (b) 2/9

(c) 9/2 (d) 7/2

32. If α, β are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is

(a) $x^2 - 11x + 30 = 0$
(b) $(x - 3)^2 - 5(x - 3) + 6 = 0$
(c) Both (a) and (b)
(d) None of these

33. If α, β are roots of $Ax^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$, then p is equal to

(a) $(B^2 - 2AC)/A^2$
(b) $(2AC - B^2)/A^2$
(c) $(B^2 - 4AC)/A^2$
(d) $(4AC - B^2)A^2$

34. If α, β are roots of the equation

$ax^2 + 3x + 2 = 0\ (a < 0)$, then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is greater than

(a) 0
(b) 1
(c) 2
(d) none of these

35. In a quadratic equation with leading coefficient 1, a student reads the coefficient 16 of x wrongly as 19 and obtain the roots as –15 and –4. The correct roots are

(a) 6, 10
(b) –6, –10
(c) –7, –9
(d) none of these

Cubic, Biquadratic, Nature of Roots

36. If α, β and γ are the roots of the cubic equation $(x - 1)(x^2 + x + 3) = 0$, then the value of $\alpha + \beta + \gamma$ is equal to

(a) –1
(b) 0
(c) 2
(d) 3

37. If one root of equation $x^2 + ax + 12 = 0$ is 4 while the equation $x^2 + ax + b = 0$ has equal roots, then the value of b is

(a) $\frac{4}{49}$
(b) $\frac{49}{4}$
(c) $\frac{7}{4}$
(d) $\frac{4}{7}$

38. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(a) $-\frac{15}{4}$
(b) $\frac{15}{4}$
(c) $\frac{9}{4}$
(d) 4

39. The value of m for which the equation

$x^3 - mx^2 + 3x - 2 = 0$ has two roots equal in magnitude but opposite in sign, is

(a) 1/2
(b) 2/3
(c) 3/4
(d) 4/5

40. If α, β, γ are the roots of the equation

$x^3 + px^2 + qx + r = 0$, then

$(1 - \alpha^2)(1 - \beta^2)(1 - \gamma^2)$ is equal to

(a) $(1 + q)^2 - (p + r)^2$
(b) $(1 + q)^2 + (p + r)^2$
(c) $(1 - q)^2 + (p - r)^2$
(d) none of these

41. If $(x^2 - 3x + 2)$ is a factor of $x^4 - px^2 + q = 0$, then the values of p and q are

(a) –5, 4
(b) 5, 4
(c) 5, –4
(d) –5, –4

42. The least integral value of k which makes the roots of the equation $x^2 + 5x + k = 0$ imaginary is

(a) 4
(b) 5
(c) 6
(d) 7

43. The roots of the quadratic equation $7x^2 - 9x + 2 = 0$ are

(a) Rational and different
(b) Rational and equal
(c) Irrational and different
(d) Imaginary and different

44. The roots of the equation $x^2 - 2\sqrt{2}x + 1 = 0$ are

(a) Real and different
(b) Imaginary and different
(c) Real and equal
(d) Rational and different

45. If a, b, c, $\in$ Q and $b + c \neq 0$ then the roots of the equation $(b + c)x^2 - (a + b + c)x + a = 0$ are

(a) Real
(b) Real and Rational
(c) Non real and different
(d) Real and equal

46. If l, m, n are real, $l \neq m$, then the roots of the equation $(l - m)x^2 - 5(l + m)x - 2(l - m) = 0$ are

(a) real and equal
(b) Non real
(c) real and unequal
(d) none of these

47. If a,b,c are distinct real numbers then the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ has

(a) equal roots
(b) irrational roots
(c) rational roots
(d) none of these

48. If a,b,c are distinct rational numbers then roots of equation $(b + c - 2a)x^2 + (c + a - 2b)x + (a + b - 2c) = 0$ are

(a) rational
(b) irrational
(c) non-real
(d) equal

49. If a,b,c are distinct rational numbers and $a+b+c=0$, then the roots of the equation

$(c^2-ab)\,x^2-2\,(a^2-bc)\,x+(b^2-ac)=0$ are

(a) imaginary (b) real and equal

(c) real and unequal (d) none of these

50. If a,b,c are distinct rational numbers and $a+b+c=0$, then the roots of the equation

$(b+c-a)\,x^2+(c+a-b)\,x+(a+b-c)=0$ are

(a) imaginary (b) real and equal

(c) real and unequal (d) none of these

51. If $a\in Z$ and the equation $(x-a)(x-10)+1=0$ has integral roots, then the values of a are

(a) 8, 10 (b) 10, 12

(c) 12, 8 (d) none

52. The quadratic equation with rational coefficients whose one root is $2+\sqrt{3}$ is

(a) $x^2-4x+1=0$ (b) $x^2+4x+1=0$

(c) $x^2+4x-1=0$ (d) $x^2+2x+1=0$

53. The quadratic equation with real coefficients whose one root is $2-i\sqrt{3}$ is

(a) $x^2-4x+7=0$ (b) $x^2+4x-7=0$

(c) $x^2-4x-7=0$ (d) none of these

54. The equation of the smallest degree with real coefficients having 1 + i as one of the roots is

(a) $x^2+x+1=0$ (b) $x^2-2x+2=0$

(c) $x^2+2x+2=0$ (d) $x^2+2x-2=0$

55. If α, β are the roots of a quadratic equation $x^2-3x+5=0$ then the equation whose roots are $(\alpha^2-3\alpha+7)$ and $(\beta^2-3\beta+7)$ is

(a) $x^2+4x+1=0$ (b) $x^2-4x+4=0$

(c) $x^2-4x-1=0$ (d) $x^2+2x+3=0$

56. If the roots of $a_1x^2+b_1x+c_1=0$ are α_1, β_1 and those of $a_2x^2+b_2x+c_2=0$ are $\alpha_2\beta_2$ such that $\alpha_1\alpha_2=\beta_1\beta_2=1$, then

(a) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (b) $\frac{a_1}{c_2}=\frac{b_1}{b_2}=\frac{c_1}{a_2}$

(c) $a_1a_2=b_1b_2=c_1c_2$ (d) none of these

Common Root

57. If the equations $x^2+2x+3\lambda=0$ and $2x^2+3x+5\lambda=0$ have a non-zero common root, then $\lambda=$

(a) 1 (b) –1

(c) 3 (d) None

58. The value of a so that the equations

$(2a-5)\,x^2-4x-15=0$ and

$(3a-8)\,x^2-5x-21=0$ have a common root, is

(a) 4, 8 (b) 3, 6

(c) 1, 2 (d) None

59. If a,b,c $\in$ R, the equation $ax^2+bx+c=0\,(a,c\neq 0)$ and $x^2+2x+3=0$ have a common root, then a : b : c =

(a) 1 : 2 : 3 (b) 1 : 3 : 4

(c) 2 : 4 : 5 (d) None

60. If the equations $k(6x^2+3)+rx+2x^2-1=0$ and $6k\,(2x^2+1)+px+4x^2-2=0$ have both roots common, then the value of $(2r-p)$ is

(a) 0 (b) 1/2

(c) 1 (d) None of these

Range of Rational Expression

61. If x is real, then $\frac{x^2-2x+4}{x^2+2x+4}$ takes values in the interval

(a) $\left[\frac{1}{3},3\right]$ (b) $\left(\frac{1}{3},3\right)$

(c) $(3, 3)$ (d) $\left(-\frac{1}{3},3\right)$

62. If x is real then the value of the expression $\frac{x^2+14x+9}{x^2+2x+3}$ lies between

(a) –3 and 3 (b) –4 and 5

(c) –4 and 4 (d) –5 and 4

Graph of Quadratic Expression

63. If a, b $\in$ R & the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is

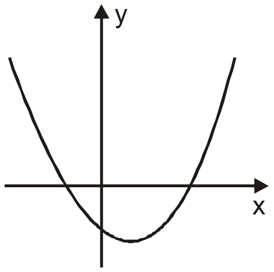
(a) positive

(b) negative

(c) zero

(d) depends on the sign of b

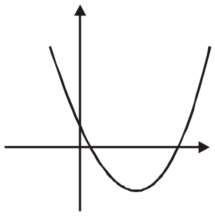
64. If a, b, c $\in$ R, the graph of the quadratic polynomial; $y = ax^2 + bx + c$ is as shown in the figure. Then

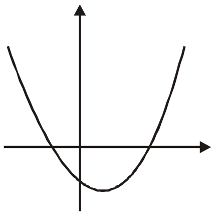


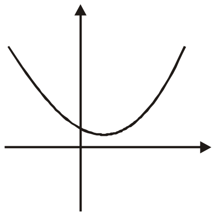
(a) $b^2 - 4ac > 0$ (b) $b < 0$

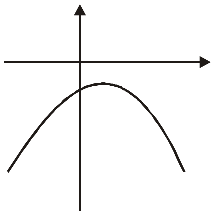
(c) $a > 0$ (d) $c < 0$

65. If a, b, c $\in$ R, Which of the following graph represents, $f(x) = ax^2 + bx + c$ when $a > 0$, $b < 0$ and $c < 0$?

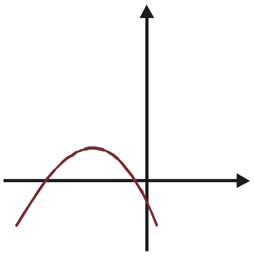
(a)


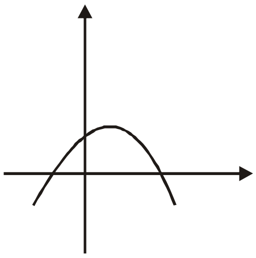
(b)


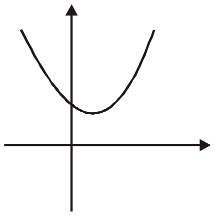
(c)


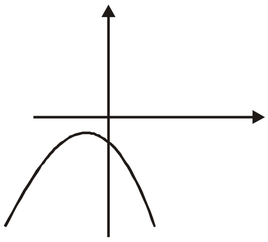
(d)


66. If a, b, c $\in$ R, for which of the following graphs of $y = ax^2 + bx + c$, the product a b c is negative.

(a)


(b)


(c)


(d)


67. The integer k for which the inequality

$x^2 - 2(4k-1)x + 15k^2 - 2k - 7 > 0$ is valid for any x, is

(a) 2 (b) 3

(c) 4 (d) none of these

68. If a $\in$ R and $x^2 + 2ax + 10 - 3a > 0$ for all x $\in$ R, then

(a) $-5 < a < 2$ (b) $a < -5$

(c) $a > 5$ (d) $2 < a < 5$

69. The real values of 'a' for which $(a^2 - 1)x^2 + 2(a-1)x + 2$ is positive for any x, are

(a) $a \geq 1$ (b) $a \leq 1$

(c) $a > -3$ (d) $a < -3$ or $a \geq 1$

70. The real value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assumes the least value, is

(a) 0 (b) 1

(c) 2 (d) 3

71. If a, b, c $\in$ R. Both roots of the equation

$(x-b)(x-c) + (x-c)(x-a) + (x-a)(x-b) = 0$ are

(a) positive (b) negative

(c) real (d) imaginary

72. If $a < b < c < d$, then roots of

$(x-a)(x-c)+2(x-b)(x-d)=0$ are

(a) real and equal (b) real and unequal

(c) imaginary (d) rational

Location of Roots

73. The value of k for which the equation

$3x^2+2x(k^2+1)+k^2-3k+2=0$

has roots of opposite signs, lies in the interval

(a) $(-\infty, 0)$ (b) $(-\infty, -1)$

(c) $(1, 2)$ (d) $(3/2, 2)$

74. The value of a for which the equation

$2x^2-2(2a+1)x+a(a-1)=0$ has roots, α and β such that $\alpha < a < \beta$ is

(a) $a \geq 0$ (b) $a < 0$

(c) $-3 < a < 0$ (d) None of these

75. The value of λ for which

$2x^2-2(2\lambda+1)x+\lambda(\lambda+1)=0$ may have one root less than λ and other root greater than λ are given by

(a) $1 > \lambda > 0$ (b) $-1 < \lambda < 0$

(c) $\lambda \geq 0$ (d) $\lambda > 0$ or $\lambda < -1$

76. The value of ‘a’ for which the equation

$x^2-2(a-1)x+(2a+1)=0$ has both roots positive is

(a) $a > 0$ (b) $0 < a < 4$

(c) $a \geq 4$ (d) None of these

77. If the equation $x^2+2(a+1)x+9a-5=0$ has only negative roots, then

(a) $a \in (-\infty, 6)$ (b) $a \in \left(\frac{5}{9}, 1\right] \cup (6, \infty)$

(c) $a \in (0, 6)$ (d) $a \geq 0$

78. The value of k for which both the roots of the equation $4x^2-20kx+(25k^2+15k-66)=0$ are less then 2, lies in

(a) $(4/5, 2)$ (b) $(2, 0)$

(c) $(-1, -4/5)$ (d) $(-\infty, -1)$

79. If the roots of $x^2+x+a=0$ exceed a, then

(a) $2 < a < 3$ (b) $a > 3$

(c) $-3 < a < 3$ (d) $a < -2$

80. The range of values of m for which the equation

$(m-5)x^2+2(m-10)x+m+10=0$ has real roots of the same sign, is given by

(a) $m > 10$ (b) $-5 < m < 5$

(c) $m < -10, 5 < m \leq 6$ (d) None of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. If $\alpha \neq \beta$ but $a^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation whose roots are α/β and β/α is **(2002)**

(a) $3x^2 - 25x + 3 = 0$ (b) $x^2 + 5x - 3 = 0$

(c) $x^2 - 5x + 3 = 0$ (d) $3x^2 - 19x + 3 = 0$

2. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then **(2002)**

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$

(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

3. Product of real roots of the equation

$$x^2 + |x| + 9 = 0$$ **(2002)**

(a) is always positive (b) is always negative

(c) does not exist (d) none of the above

4. If p and q are the roots of the equation $x^2 + px + q = 0$, then **(2002)**

(a) $p = 1, q = -2$ (b) $p = 0, q = 1$

(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

5. If a, b, c are distinct real numbers and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is **(2002)**

(a) less than 1 (b) equal to 1

(c) greater than 1 (d) any real no

6. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is **(2003)**

(a) $-2/3$ (b) $1/3$

(c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

7. The number of real solution of the equations $x^2 - 3|x| + 2 = 0$ is **(2003)**

(a) 4 (b) 1

(c) 3 (d) 2

8. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation **(2004)**

(a) $x^2 + 18x - 16 = 0$ (b) $x^2 - 18x + 16 = 0$

(c) $x^2 + 18x + 16 = 0$ (d) $x^2 - 18x - 16 = 0$.

9. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are **(2004)**

(a) 0, –1 (b) –1, 1

(c) 0, 1 (d) –1, 2

10. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is **(2004)**

(a) 3 (b) 12

(c) 49/4 (d) 4

11. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value, is **(2005)**

(a) 0 (b) 1

(c) 2 (d) 3

12. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals **(2005)**

(a) 3 (b) –2

(c) 1 (d) 2

13. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **(2005)**

(a) $(6, \infty)$ (b) $(5, 6]$

(c) $[4, 5]$ (d) $(-\infty, 4)$

14. If the roots of the quadratic equations $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively, then the value of $2 + q - p$ is **(2006)**

(a) 2 (b) 3

(c) 0 (d) 1

15. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval **(2006)**

(a) $-2 < m < 0$ (b) $m > 3$

(c) $-1 < m < 3$ (d) $1 < m < 4$

16. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **(2007)**

(a) $(3, \infty)$ (b) $(-\infty, -3)$

(c) $(-3, 3)$ (d) $(-3, \infty)$

17. The quadratic equations

$$x^2 - 6x + a = 0$$

and $$x^2 - cx + 6 = 0$$

have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **(2008)**

(a) 2 (b) 1

(c) 4 (d) 3

18. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is **(2009)**

(a) greater than 4ab (b) less than 4ab

(c) greater than –4ab (d) less than –4ab

19. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009}$ is equal to **(2010)**

(a) –2 (b) –1

(c) 1 (d) 2

20. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are **(2011)**

(a) –4, –3 (b) 6, 1

(c) 4, 3 (d) –6, –1

21. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has **(2012)**

(a) infinite number of real roots

(b) no real roots

(c) exactly one real root

(d) exactly four real roots

22. Let α and β be the roots of equation $px^2 + qx + r = 0, p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is : **(2014)**

(a) $\frac{2\sqrt{13}}{9}$ (b) $\frac{\sqrt{61}}{9}$

(c) $\frac{2\sqrt{17}}{9}$ (d) $\frac{\sqrt{34}}{9}$

23. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: **(2014)**

(a) $(-\infty, -2) \cup (2, \infty)$ (b) $(-1, 0) \cup (0, 1)$

(c) (1, 2) (d) (–2, –1)

24. If equations ax2 + bx + c = 0, (a, b $\in$ R, a $\neq$ 0) and $2x^2 + 3x + 4 = 0$ have a common root then a : b : c equals: **(2014/Online Set–1)**

(a) 1 : 2 : 3 (b) 2 : 3 : 4

(c) 4 : 3 : 2 (d) 3 : 2 : 1

25. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0$ ($a \neq 0$, a, b $\in$ R), then the equation, $x(x + b^3) + (a^3 - 3abx) = 0$ has roots: **(2014/Online Set–1)**

(a) $\alpha^{3/2}$ and $\beta^{3/2}$ (b) $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$

(c) $\sqrt{\alpha\beta}$ and $\alpha\beta$ (d) $\alpha^{-3/2}$ and $\beta^{-3/2}$

26. If α and β are roots of the equation, $x^2 - 4\sqrt{2}\,kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$ then $\alpha^3 + \beta^3$ is equal to: **(2014/Online Set–2)**

(a) $248\sqrt{2}$ (b) $280\sqrt{2}$

(c) $-32\sqrt{2}$ (d) $-280\sqrt{2}$

27. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is **(2014/Online Set–3)**

(a) 2 (b) -2

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

28. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has: **(2014/Online Set–4)**

(a) no solution

(b) exactly one solution

(c) exactly two solutions

(d) exactly four solutions

29. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to: **(2015)**

(a) 3 (b) –3
(c) 6 (d) –6

30. If 2 + 3i is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation **(2015/Online Set–1)**

(a) exists and is equal to 1

(b) exists and is equal to $-\frac{1}{2}$

(c) exists and is equal to $\frac{1}{2}$

(d) does not exist

31. A value of b for which the equations
$x2 + bx - 1 = 0, x2 + x + b = 0$, have one root in common is **(2015/Online Set–2)**

(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$
(c) $i\sqrt{5}$ (d) $\sqrt{2}$

32. The sum of all real values of x satisfying the equation
$\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is : **(2016)**

(a) –4 (b) 6
(c) 5 (d) 3

33. If $b \in C$ and the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from –1, then |b| is equal to : **(2016/Online Set–1)**

(a) $\sqrt{2}$ (b) 2
(c) 3 (d) $\sqrt{3}$

34. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to : **(2016/Online Set–2)**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$
(c) 2 (d) $2\sqrt{2}$

35. If, for a positive integer n, the quadratic equation,
$x(x+1) + (x+1)(x+2) + \ldots + (x+\overline{n-1})(x+n) = 10n$
has two consecutive integral solutions, then n is equal to: **(2017)**

(a) 12 (b) 9
(c) 10 (d) 11

36. Let p(x) be a quadratic polynomial such that p(0) = 1. If p(x) leaves remainder 4 when divided by x“ 1 and it leaves remainder 6 when divided by x + 1; then : **(2017/Online Set–1)**

(a) p(2) = 11 (b) p(2) = 19
(c) p(–2) = 19 (d) p (–2) = 11

37. The number of real values of λ for which the system of linear equations

$$2x + 4y - \lambda z = 0$$
$$4x + \lambda y + 2z = 0$$
$$\lambda x + 2y + 2z = 0$$

has infinitely many solutions, is **(2017/Online Set–1)**

(a) 0 (b) 1
(c) 2 (d) 3

38. The sum of all the real values of x satisfying the equation
$2^{(x-1)(x^2+5x-50)} = 1$ is : **(2017/Online Set–2)**

(a) 16 (b) 14
(c) –4 (d) –5

39. If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2-\lambda)x + (10-\lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is : **(2018/Online Set–1)**

(a) $4\sqrt{2}$ (b) $2\sqrt{5}$
(c) $2\sqrt{7}$ (d) 20

40. If f(x) is a quadratic expression such that f(1) + f(2) = 0, and -1 is a root of f(x) = 0, then the other root of f(x) = 0 is : **(2018/Online Set–2)**

(a) $-\frac{5}{8}$ (b) $-\frac{8}{5}$
(c) $\frac{5}{8}$ (d) $\frac{8}{5}$

41. Let p, q and r be real numbers $(p \neq q, r \neq 0)$, such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to : **(2018/Online Set–3)**

(a) $\frac{p^2+q^2}{2}$ (b) $p^2 + q^2$
(c) $2(p^2 + q^2)$ (d) $p^2 + q^2 + r^2$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. The number of real values of the triplet (a, b, c) for which $a\cos 2x + b\sin^2 x + c = 0$ is satisfied by all real x, is

(a) 0 (b) 2

(c) 3 (d) infinite

2. $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0, -1 < x < 0$, then the value of $\sin 2\alpha$ is

(a) 24/25 (b) –12/25

(c) –24/25 (d) 20/25

3. Set of all values of x satisfying the inequality $\sqrt{x^2 - 7x + 6} > x + 2$ is

(a) $x \in \left(-\infty, \frac{2}{11}\right)$ (b) $x \in \left(\frac{2}{11}, \infty\right)$

(c) $x \in (-\infty, 1] \cup [6, \infty)$ (d) $x \in [6, \infty)$

4. If the product of the roots of the quadratic equation $mx^2 - 2x + (2m - 1) = 0$ is 3 then the value of m

(a) 1 (b) 2

(c) –1 (d) 3

5. If $\frac{(x+1)}{(2x-1)(3x+1)} = \frac{A}{(2x-1)} + \frac{B}{(3x+1)}$, then $16A + 9B$ is equal to

(a) 4 (b) 5

(c) 6 (d) 8

6. If a, b, c $\in$ R. For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of roots is zero, then the sum of roots if $b + c \neq 0$ is

(a) 0 (b) $\frac{2ab}{b+c}$

(c) $\frac{2bc}{b+c}$ (d) $\frac{-2bc}{b+c}$

7. $\frac{8x^2+16x-51}{(2x-3)(x+4)} > 3$ if x is such that

(a) $x < -4$ (b) $-3 < x < 3/2$

(c) $x > 5/2$ (d) all these true

8. Let α, β be the roots of $ax^2 + bx + c = 0$ and γ, δ be the roots of $px^2 + qx + r = 0$; and D_1, D_2 the respective Discriminants of these equations. If $\alpha, \beta, \gamma, \delta$, are in A.P., then $D_1 : D_2$

(a) $\frac{a^2}{p^2}$ (b) $\frac{a^2}{b^2}$

(c) $\frac{b^2}{q^2}$ (d) $\frac{c^2}{r^2}$

9. If $0 \le x \le \pi$, then the solution of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is given by x equal to

(a) $\frac{\pi}{6}, \frac{\pi}{3}$ (b) $\frac{\pi}{3}, \frac{\pi}{2}$

(c) $\frac{\pi}{6}, \frac{\pi}{2}$ (d) none of these

10. Two real numbers α and β are such that $\alpha + \beta = 3$ and $|\alpha - \beta| = 4$, then α and β are the roots of the quadratic equation

(a) $4x^2 - 12x - 7 = 0$ (b) $4x^2 - 12x + 7 = 0$

(c) $4x^2 - 12x + 25 = 0$ (d) none of these

11. If $(x + 1)^2$ is greater then $5x - 1$ and less than $7x - 3$ then the integral value of x is equal to

(a) 1 (b) 2

(c) 3 (d) 4

12. The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is

(a) 0, 2 (b) 0, –2

(c) 2, –2 (d) none of these

13. If α, β and γ are the roots of the equation, $x^3 - x - 1 = 0$ then, $\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}$ has the value equal to

(a) zero (b) –1

(c) –7 (d) 1

14. If the quadratic equations, $3x^2 + ax + 1 = 0$ and $2x^2 + bx + 1 = 0$ have a common root, then the value of the expression $5ab - 2a^2 - 3b^2$ is

(a) 0 (b) 1

(c) –1 (d) none

15. If a, b, p, q are non-zero real numbers, the two equations, $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have

(a) no common root

(b) one common root if $2a^2 + b^2 = p^2 + q^2$

(c) two common roots if $3pq = 2ab$

(d) two common roots if $3qb = 2ap$

16. If $\frac{6x^2-5x-3}{x^2-2x+6} \le 4$, the least and the highest values of $4x^2$ are

(a) 0 and 81 (b) 9 and 81

(c) 36 and 81 (d) none of these

17. If p & q are roots of the equation $x^2 - 2x + A = 0$ and r & s be roots of the equation $x^2 - 18x + B = 0$ if $p < q < r < s$ be in A.P., then A and B are respectively

(a) – 3, 77 (b) 3, 77

(c) 3, – 77 (d) none of these

18. If α, β are roots of the equation $ax^2 + 3x + 2 = 0$ $(a < 0)$, then $\alpha^2/\beta + \beta^2/\alpha$ is greater than

(a) 0 (b) 1

(c) 2 (d) none of these

19. If α, β be the roots $x^2 + px - q = 0$ and γ, δ be the roots of $x^2 + px + r = 0$ then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} =$

(a) 1 (b) q

(c) r (d) q + r

20. If α, β be roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$ then $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$

(a) $p^2 + q^2$ (b) $p^2 - q^2$

(c) $q^2 - p^2$ (d) none of these

21. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then $a\ S_{n+1} + c\ S_{n-1} =$

(a) $b\ S_n$ (b) b^2S_n

(c) $2bS_n$ (d) $-bS_n$

22. If the roots of equation $x^2 + bx + ac = 0$ are α, β and roots of the equation $x^2 + ax + bc = 0$ are α, γ then the value of α, β, γ respectively

(a) a, b, c (b) b, c, a

(c) c, a, b (d) none of these

23. If the roots of the equation, $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equation, $x^3 - Ax^2 + Bx - C = 0$ where A, B, C, P and Q are constants then the value of A + B + C =

(a) 18 (b) 19

(c) 20 (d) none

24. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the equation $ax^2 - bx(x - 1) + c(x - 1)^2 = 0$ has roots

(a) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ (b) $\alpha - 1, \beta - 1$

(c) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ (d) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$

25. $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is $(a, b, c, d \in R)$

(a) –d/a (b) d/a

(c) (b – a)/a (d) (a – b)/a

26. If $a, b \in R$, $a \ne b$. The roots of the quadratic equation, $x^2 - 2(a + b)x + 2(a^2 + b^2) = 0$ are

(a) Rational and different (b) Rational and equal

(c) Irrational and different (d) Imaginary and different

27. If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ $(b \ne c)$ have a common root, then $a + 4b + 4c$ is equal to

(a) –2 (b) –1

(c) 0 (d) 1

28. If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \ne 0$, then, the common factor is

(a) (x – 3) (b) (x – 6)

(c) (x – 8) (d) none of these

29. Let a > 0, b > 0 and c > 0. Then both the roots of the equation $ax^2 + bx + c = 0$

(a) are real and negative (b) have negative real parts

(c) are rational numbers (d) none of these

30. If r and s are positive, then roots of the equation $x^2 - rx - s = 0$ are

(a) imaginary

(b) real and both positive

(c) real and of opposite signs

(d) real and both negative

31. If $a, b, c \in R$ and roots of the equation $ax^2 + 2bx + c = 0$ are real and different, then roots of the equation

$(a^2 + 2b^2 - ac)\,x^2 + 2b\,(a + c)\,x + (2b^2 + c^2 - ac) = 0$ are

(a) real and equal (b) real and unequal

(c) imaginary (d) none of these

32. If $p,q,r,s,\in$ R and α,β are roots of the equation $x^2 + px + q = 0$ and α^4 and β^4 are roots of $x^2 - rx + s = 0$, then the roots of $x^2 - 4qx + 2q^2 - r = 0$ are

(a) both real (b) both positive

(c) both negative (d) none of these

33. If the roots of the quadratic equation $x^2 - 4x - \log_3 a = 0$ are real, then the least value of a is

(a) 81 (b) 1/81

(c) 1/64 (d) none of these

34. If α, β, γ, δ are the roots of the equation, $x^4 - Kx^3 + Kx^2 + Lx + M = 0$ where K, L and M are real numbers then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is

(a) 0 (b) –1

(c) 1 (d) 2

35. The value of 'a' for which the sum of the squares of the roots of the equation $x^2 - (a - 2)\,x - a - 1 = 0$ assume the least value is

(a) 0 (b) 1

(c) 2 (d) 3

36. If the two roots of the equation, $x^3 - px^2 + qx - r = 0$ are equal in magnitude but opposite in sign then

(a) pr = q (b) qr = p

(c) pq = r (d) none

37. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then the ordered pair (x_1, x_2) is

(a) (–5, –7) (b) (1, –1)

(c) (–1, 1) (d) (5, 7)

38. If $(2x^2 - 3x + 1)\,(2x^2 + 5x + 1) = 9x^2$, then equation has

(a) four real roots

(b) two real and two imaginary roots

(c) all imaginary

(d) none of the above

39. If $(x + 2)\,(x + 3)\,(x + 8)\,(x + 12) = 4x^2$, then equation has

(a) no real roots (b) all real roots

(c) can't be discussed (d) none of these

40. If 'x' is real and satisfying the inequality, $|x| < \frac{a}{x}\,(a \in R)$, then

(a) $x \in \left(0,\ \sqrt{a}\right)$ for $a > 0$

(b) $x \in \left(-\sqrt{a},\ 0\right)$ for $a < 0$

(c) $x \in \left(-\sqrt{-a},\ 0\right)$ for $a < 0$

(d) $x \in \left(-\sqrt{a},\ \sqrt{a}\right)$ for $a > 0$

41. The set of real 'x' satisfying, $||x - 1| - 1| \leq 1$ is

(a) [0, 2] (b) [–1, 3]

(c) [–1, 1] (d) [1, 3]

42. If one root of the equation $4x^2 + 2x - 1 = 0$ is α, then other root is

(a) 2α (b) $4\alpha^3 - 3\alpha$

(c) $4\alpha^3 + 3\alpha$ (d) none of these

43. If α, β are the roots of $x^2 - p\,(x + 1) - c = 0$ then

$\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c}$ is equal to

(a) 0 (b) 1

(c) 2 (d) none of these

44. If the equation $(k-2)x^2-(k-4)x-2=0$ has difference of roots as 3 then the value of k is

(a) 1, 3
(b) 3, 3/2
(c) 2, 3/2
(d) 3/2, 1

45. If the equation $\sin^4 x-(k+2)\sin^2 x-(k+3)=0$ has a solution then k must lie in the interval

(a) $(-4,-2)$
(b) $[-3,2)$
(c) $(-4,-3)$
(d) $[-3,-2]$

46. The equation, $\pi^x=-2x^2+6x-9$ has

(a) no solution
(b) one solution
(c) two solutions
(d) infinite solutions

47. If both roots of the quadratic equation $x^2+x+p=0$ exceed p where $p \in R$ then p must lie in the interval

(a) $(-\infty, 1)$
(b) $(-\infty,-2)$
(c) $(-\infty,-2)\cup(0, 1/4)$
(d) $(-2, 1)$

48. If both roots of the quadratic equation $(2-x)(x+1)=p$ are distinct and positive then p must lie in the interval

(a) $p>2$
(b) $2<p<9/4$
(c) $p<-2$
(d) $-\infty<p<\infty$

49. The quadratic equation, $ax^2+bx+c=0$ will always have imaginary root if

(a) $a<-1, 0<c<1, b>0$
(b) $a<-1, -1<c<0, 0<b<1$
(c) $a<-1, c<0, b>1$
(d) $a<-1, c<-1, 1<b<2$

50. If $b>a$ and the equation $(x-a)(x-b)+1=0$ has real roots then

(a) both roots in (a, b)
(b) both roots in $(-\infty, a)$
(c) both roots in (b, ∞)
(d) one root in $(-\infty, a)$ and other in (b, ∞)

51. If α, β are the roots of the equation, $x^2-2mx+m^2-1=0$ then the range of values of m for which $\alpha, \beta \in (-2, 4)$ is

(a) $(-1, 3)$
(b) $(1, 3)$
(c) $(\infty, -1)\cup(3, \infty)$
(d) none

52. If α, β are the roots of the quadratic equation, $x^2-2p(x-4)-15=0$ then the set of values of p for which one root is less than 1 & the other root is greater than 2 is

(a) $(7/3, \infty)$
(b) $(-\infty, 7/3)$
(c) $x \in R$
(d) none

53. If $b<0$, then the roots x_1 and x_2 of the equation $2x^2+6x+b=0$, satisfy the condition $\left(\frac{x_1}{x_2}\right)+\left(\frac{x_2}{x_1}\right)<k$ where k is equal to

(a) – 3
(b) – 5
(c) – 6
(d) – 2

54. Consider $y=\frac{2x}{1+x^2}$, then the range of expression, y^2+y-2 is

(a) $[-1, 1]$
(b) $[0, 1]$
(c) $[-9/4, 0]$
(d) $[-9/4, 1]$

55. The least value of expression, $x^2+2xy+2y^2+4y+7$ is

(a) –1
(b) 1
(c) 3
(d) 7

56. If the graph of $|y|=f(x)$, where $f(x)=ax^2+bx+c$, $b, c \in R$, $a \neq 0$, has the maximum vertical height 4, then

(a) $a>0$
(b) $a<0$
(c) (b^2-4ac) is negative
(d) Nothing can be said

57. Set of all possible real values of a such that the inequality $(x-(a-1))(x-(a^2+2))<0$ holds for all $x \in (-1, 3)$ is

(a) $(1, \infty)$
(b) $(-\infty, -1]$
(c) $(-\infty, -1)$
(d) $(0, 1)$

58. If $a(p+q)^2+2bpq+c=0$ and $a(p+r)^2+2bpr+c=0$, $(a \neq 0)$ then

(a) $qr=p^2+\frac{c}{a}$
(b) $qr=p^2$
(c) $qr=-p^2$
(d) None of these

59. If $p(x)=ax^2+bx$ and $q(x)=lx^2+mx+n$ with $p(1)=q(1)$; $p(2)-q(2)=1$ and $p(3)-q(3)=4$, then $p(4)-q(4)$ is

(a) 0
(b) 5
(c) 6
(d) 9

60. If $x \in R$, then the maximum value of

$y=2(a-x)\left(x+\sqrt{x^2+b^2}\right)$ is

(a) a^2+b^2
(b) a^2-b^2
(c) a^2+2b^2
(d) none of these

61. If a, b, c $\in$ R, a > 0 and c $\neq 0$ Let α and β be the real and distinct roots of the equation $ax^2 + bx + c = |c|$ and p, q be the real and distinct roots of the equation $ax^2 + bx + c = 0$. Then

(a) p and q lie between α and β

(b) p and q do not lie between α and β

(c) Only p lies between α and β

(d) Only q lies between α and β

62. Let $f(x) = ax^2 + bx + c$; a, b, c $\in$ R. If $f(x)$ takes real values for real values of x and non-real values for non-real values of x, then a satisifes.

(a) $a > 0$ (b) $a = 0$

(c) $a < 0$ (d) $a \in R$

63. The value of a for which the equations $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root is

(a) –2 (b) –1

(c) 1 (d) 2

64. The equation $x^{\left[(\log_3 x)^2 - (9/2)\log_3 x + 5\right]} = 3\sqrt{3}$ has

(a) at least one real solution

(b) exactly three real solutions

(c) exactly one irrational solution

(d) non real roots

65. For a > 0, the roots of the equation

$\log_{ax} a + \log_x a^2 + \log_{a^2 x} a^3 = 0$, are given by

(a) $a^{-4/3}$ (b) $a^{-3/4}$

(c) $a^{-1/2}$ (d) a^{-1}

66. The roots of the equation, $(x^2 + 1)^2 = x(3x^2 + 4x + 3)$, are given by

(a) $2 - \sqrt{3}$ (b) $\left(-1 + i\sqrt{3}\right)/2,\ i = \sqrt{-1}$

(c) $2 + \sqrt{3}$ (d) $\left(-1 - i\sqrt{3}\right)/2,\ i = \sqrt{-1}$

67. If $0 < a < b < c$, and the roots α, β of the equation $ax^2 + bx + c = 0$ are non real complex roots, then

(a) $|\alpha| = |\beta|$ (b) $|\alpha| > 1$

(c) $|\beta| < 1$ (d) none of these

68. Let a, b, c $\in$ R. If $ax^2 + bx + c = 0$ has two real roots A and B where $A < -1$ and $B > 1$, then

(a) $1 + \left|\frac{b}{a}\right| + \frac{c}{a} < 0$ (b) $1 - \left|\frac{b}{a}\right| + \frac{c}{a} < 0$

(c) $|c| < |a|$ (d) $|c| < |a| - |b|$

69. $5^x + \left(2\sqrt{3}\right)^{2x} - 169 \le 0$ is true in the interval.

(a) $(-\infty, 2)$ (b) $(0, 2)$

(c) $(2, \infty)$ (d) $(0, 4)$

70. If a < b < c < d, then for any positive λ, the quadratic equation $(x - a)(x - c) + \lambda(x - b)(x - d) = 0$ has

(a) non-real roots

(b) one real root between a and c

(c) one real root between b and d

(d) irrational roots

71. Equation $\frac{\pi^e}{x-e} + \frac{e^\pi}{x-\pi} + \frac{\pi^\pi + e^e}{x-\pi-e} = 0$ has

(a) one real root in (e,π) and other in $(\pi - e, e)$

(b) one real root in (e,π) and other in $(\pi, \pi + e)$

(c) two real roots in $(\pi - e, \pi + e)$

(d) No real root

72. If a < 0, then root of the equation $x^2 - 2a|x - a| - 3a^2 = 0$ is

(a) $a\left(-1 - \sqrt{6}\right)$ (b) $a\left(1 - \sqrt{2}\right)$

(c) $a\left(-1 + \sqrt{6}\right)$ (d) $a\left(1 + \sqrt{2}\right)$

73. If a, b, c $\in$ R and α is a real root of the equation $ax^2 + bx + c = 0$, and β is the real root of the equation $-ax^2 + bx + c = 0$, then the equation $\frac{a}{2}x^2 + bx + c = 0$ has

(a) real roots

(b) none- real roots

(c) has a root lying between α and β

(d) None of these

Assertion Reason

(A) If both ASSERTION and REASON are true and reason is the correct explanation of the assertion.

(B) If both ASSERTION and REASON are true but reason is not the correct explanation of the assertion.

(C) If ASSERTION is true but REASON is false.

(D) If both ASSERTION and REASON are false.

(E) If ASSERTION is false but REASON is true.

74. **Assertion** : If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.

Reason : If a, b, c are odd integer then the roots of the equation $4abc\ x^2 + (b^2 - 4ac)\ x - b = 0$ are real and distinct.

(a) A (b) B
(c) C (d) D
(e) E

75. **Assertion** : If one roots is $\sqrt{5} - \sqrt{2}$ is then the equation of lowest degree with rational coefficient is $x^4 - 14x^2 + 9 = 0$.

Reason : For a polynomial equation with rational co-efficient irrational roots occurs in pairs.

(a) A (b) B
(c) C (d) D
(e) E

76. **Assertion** : The set of all real numbers a such that $a^2 + 2a$, $2a + 3$ and $a^2 + 3a + 8$ are the sides of a traiangle is $(5, \infty)$.

Reason : Since in a triangle sum of two sides is greater than the other and also sides are always positive.

(a) A (b) B
(c) C (d) D
(e) E

77. **Assertion** : The number of roots of the equation $\sin(2^x)\cos(2^x) = \frac{1}{4}(2^x + 2^{-x})$ is 2.

Reason : AM ≥ GM.

(a) A (b) B
(c) C (d) D
(e) E

78. **Assertion** : If $a > b > c$ and $a^3 + b^3 + c^3 = 3abc$, then the equation $ax^2 + bx + c = 0$ has one positive and one negative real roots.

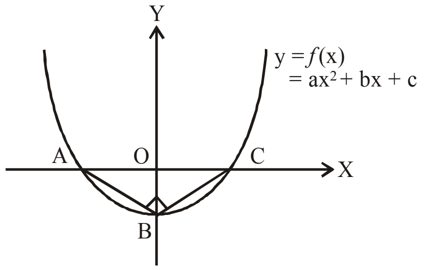
Reason : If roots of opposite nature, then product of roots < 0 and |sums of roots| ≥ 0.

(a) A (b) B (c) C (d) D (e) E

Using the following passage, solve Q.79 to Q.81

Passage –1

In the given figure vertices of Δ ABC lie on $y = f(x) = ax^2 + bx + c$. The Δ ABC is right angled isosceles triangle whose hypotenuse AC = $4\sqrt{2}$ units, then



79. y = f(x) is given by

(a) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$ (b) $y = \frac{x^2}{2} - 2$

(c) $y = x^2 - 8$ (d) $y = x^2 - 2\sqrt{2}$

80. Minimum value of y = f(x) is

(a) $2\sqrt{2}$ (b) $-2\sqrt{2}$

(c) 2 (d) –2

81. Number of integral value of k for which $\frac{k}{2}$ lies between the roots of f(x) = 0, is

(a) 9 (b) 10
(c) 11 (d) 12

Using the following passage, solve Q.82 to Q.84

Passage –2

If roots of the equation $x^4 - 12x^3 + bx^2 + cx + 81 = 0$ are positive then

82. Value of b is

(a) – 54 (b) 54

(c) 27 (d) –27

83. Value of c is

(a) 108 (b) –108

(c) 54 (d) – 54

84. Root of equation $2bx + c = 0$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) –1

Match the column

85. The value of k for which the equation $x^3 - 3x + k = 0$ has

Column–I	**Column–II**
(A) three distinct real roots	(P) $\|k\| > 2$
(B) two equal roots	(q) $k = -2, 2$
(C) exactly one real root	(R) $\|k\| < 2$
(D) three equal roots	(S) no value of k

86.

Column–I	**Column–II**
(A) Number of real solution of $\|x + 1\| = e^x$ is	(P) 2
(B) The number of non-negative real roots of $2^x - x - 1 = 0$ equal to	(Q) 3
(C) If p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$, then minimum value of $p^2 + q^2$ is equal to	(R) 6
(D) If α and β are the roots of $2x^2 + 7x + c = 0$ & $\|\alpha^2 - \beta^2\| = \frac{7}{4}$, then c is equal to	(S) 5

Subjective

87. When x^{100} is divided by $x^2 - 3x + 2$, the remainder is $(2^{k+1} - 1)x - 2(2^k - 1)$ where *k* is a numerical quantity, then *k* must be.

88. If roots x_1 and x_2 of $x^2 + 1 = \frac{x}{a}$ satisfy

$$\left|x_1^2 - x_2^2\right| > \frac{1}{a}, \text{ then } a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{\sqrt{k}}\right)$$

the numerical quantity k must be equal to

89. The integral part of positive value of *a* for which, the least value of $4x^2 - 4ax + a^2 - 2a + 2$ on [0, 2] is 3, is

90. If *x, y, z* are unequal and positive and if $x + y + z = 1$, the expression $\frac{(1+x)(1+y)(1+z)}{(1-x)(1-y)(1-z)}$ is greater than

(The best possible number)

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Single Answer Question

1. Let a > 0, b > 0 and c > 0. Then, both the roots of the equation $ax^2 + bx + c = 0$ **(1979)**

(a) are real and negative

(b) have negative real parts

(c) have positive real parts

(d) None of the above

2. Both the roots of the equation

$(x–b)(x–c) + (x–a)(x–c) + (x–a)(x–b) = 0$

are always **(1980)**

(a) positive (b) negative

(c) real (d) None of these

3. The least value of the expression $2\log_{10} x – \log_x (0.01)$, for $x > 1$, is **(1980)**

(a) 10 (b) 2

(c) –0.01 (d) None of these

4. The number of real solutions of the equation $|x|^2 – 3|x| + 2 = 0$ is **(1982)**

(a) 4 (b) 1

(c) 3 (d) 2

5. If $x_1, x_2, \ldots, x_n$ are any real numbers and n is any positive integer, then **(1982)**

(a) $n\sum_{i=1}^{n} x_i^2 < \left(\sum_{i=1}^{n} x_i\right)^2$ (b) $\sum_{i=1}^{n} x_i^2 \ge \left(\sum_{i=1}^{n} x_i\right)^2$

(c) $\sum_{i=1}^{n} x_i^2 \ge n\left(\sum_{i=1}^{n} x_i\right)^2$ (d) None of these

6. The largest interval for which $x^{12} – x^9 + x^4 – x + 1 > 0$ is **(1982)**

(a) $– 4 < x \le 0$ (b) $0 < x < 1$

(c) $– 100 < x < 100$ (d) $–\infty < x < \infty$

7. If a + b + c = 0, then the quadratic equation $3ax^2 + 2bx + c = 0$ has **(1983)**

(a) at least one root in (0, 1)

(b) one root in (2, 3) and the other in (–2, –1)

(c) imaginary roots

(d) None of the above

8. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has **(1984)**

(a) no root (b) one root

(c) two equal roots (d) infinitely many roots

9. If $\log_{0.3}(x–1) < \log_{0.09}(x–1)$, then x lies in the interval **(1985)**

(a) $(2, \infty)$ (b) 1, 2)

(c) (–2, –1) (d) None of these

10. If a, b and c are distinct positive numbers, then the expression $(b + c – a)(c + a – b)(a + b – c) – abc$ is **(1986)**

(a) positive (b) negative

(c) non-positive (d) non-negative

11. If α and β are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 – rx + s = 0$, then the equation $x^2 – 4qx + 2q^2 – r = 0$ has always, if α and β are real numbers. **(1989)**

(a) two real roots

(b) two positive roots

(c) two negative roots

(d) one positive and one negative root

12. The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has **(1989)**

(a) at least one real solution

(b) exactly three real solutions

(c) exactly one irrational solution

(d) complex roots

13. Let a,b,c be real numbers, $a \ne 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is the root of $a^2x^2 – bx – c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies **(1989)**

(a) $\gamma = \frac{\alpha+\beta}{2}$ (b) $\gamma = \alpha + \frac{\beta}{2}$

(c) $\gamma = \alpha$ (d) $\alpha < \gamma < \beta$

14. Let f (x) be a quadratic expression which is positive for all real values of x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x **(1990)**

(a) $g(x) < 0$ (b) $g(x) > 0$

(c) $g(x) = 0$ (d) $g(x) \ge 0$

15. The number $\log_2 7$ is **(1990)**

(a) an integer (b) a rational number

(c) an irrational number (d) a prime number

16. Let α, β be the roots of the equation

$$(x - a)(x - b) = c, c \neq 0$$

Then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are **(1992)**

(a) a, c (b) b, c

(c) a, b (d) a + c, b + c

17. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has **(1997)**

(a) no solution

(b) one solution

(c) two solutions

(d) more than two solutions

18. In a triangle PQR, $\angle R = \frac{\pi}{2}$, if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$, then **(1999)**

(a) a + b = c (b) b + c = a

(c) a + c = b (d) b = c

19. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then **(1999)**

(a) $a < 2$ (b) $2 \leq a \leq 3$

(c) $3 < a \leq 4$ (d) $a > 4$

20. If α and β $(\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then **(2000)**

(a) $0 < \alpha < \beta$ (b) $\alpha < 0 < \beta < |\alpha|$

(c) $\alpha < \beta < 0$ (d) $\alpha < 0\ |\alpha| < \beta$

21. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has **(2000)**

(a) both roots in (a, b)

(b) both roots in $(-\infty, a)$

(c) both roots in $(b, +\infty)$

(d) one root in $(-\infty, a)$ and the other in $(b, +\infty)$

22. For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the root is square of the other, then p is equal to **(2000)**

(a) 1/3 (b) 1

(c) 3 (d) 2/3

23. The number of solutions of $\log_4(x-1) = \log_2(x - 3)$ is **(2001)**

(a) 3 (b) 1

(c) 2 (d) 0

24. The number of values of k for which the system of equations

$$(k + 1)x + 8y = 4k$$
$$kx + (k + 3)y = 3k - 1$$

has infinitely many solution, is **(2002)**

(a) 0 (b) 1

(c) 2 (d) infinite

25. The set of all real numbers x for which

$x^2 - |x + 2| + x > 0$ is **(2002)**

(a) $(-\infty, -2) \cup (2, \infty)$ (b) $\left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

(c) $(-\infty, -1) \cup (1, \infty)$ (d) $\left(\sqrt{2}, \infty\right)$

26. For all 'x', $x^2 + 2ax + (10 - 3a) > 0$, then the interval in which 'a' lies is **(2004)**

(a) $a < -5$ (b) $-5 < a < 2$

(c) $a > 5$ (d) $2 < a < 5$

27. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is **(2004)**

(a) $p^3 - (3p - 1)q + q^2 = 0$ (b) $p^3 - q(3p + 1) + q^2 = 0$

(c) $p^3 + q(3p - 1) + q^2 = 0$ (d) $p^3 + q(3p + 1) + q^2 = 0$

28. If a, b, c are the sides of a triangle ABC such that $x^2 - 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ has real roots, then **(2006)**

(a) $\lambda < \frac{4}{3}$ (b) $\lambda > \frac{5}{3}$

(c) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$ (d) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$

29. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is **(2007)**

(a) 2/9 (p – q) (2q – p) (b) 2/9 (q – p) (2p – q)

(c) 2/9 (q – 2p) (2q – p) (d) 2/9 (2p – q) (2q – p)

30. Let p and q be the real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **(2010)**

(a) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(b) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(c) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(d) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

31. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **(2011)**

(a) 1 (b) 2

(c) 3 (d) 4

32. A value of b for which the equations $x^2 + bx - 1 = 0$, $x^2 + x + b = 0$ have one root in common is **(2011)**

(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$

(c) $i\sqrt{5}$ (d) $\sqrt{2}$

33. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots.

Then the equation

$$p(p(x)) = 0$$

has **(2014)**

(a) only purely imaginary roots

(b) all real roots

(c) two real and two purely imaginary roots

(d) neither real nor purely imaginary roots

34. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **(2015)**

(a) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (b) $\left(-\frac{1}{5}, 0\right)$

(c) $\left(0, \frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

35. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals. **(2016)**

(a) $2(\sec\theta - \tan\theta)$ (b) $2\sec\theta$

(c) $-2\tan\theta$ (d) 0

Assertion & Reason

(A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(C) If ASSERTION is true, REASON is false

(D) If ASSERTION is false, REASON is true

36. Let a, b, c, p, q be the real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$. **(2008)**

Assertion : $(p^2 - q)(b^2 - ac) \geq 0$

Reason : $b \notin pa$ or $c \notin qa$.

(a) (b) (c) (d)

Passage Q. 37–39

If a continuous f defined on the real line R, assumes positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative, then the equation $f(x) = 0$ has a root in R.

Consider $f(x) = ke^x - x$ for all real x where k is real constant. **(2007)**

37. The line $y = x$ meets $y = ke^x$ for $k \leq 0$ at

(a) no point (b) one point

(c) two points (d) more than two points

38. The positive value of k for which $ke^x - x = 0$ has only one root is

(a) $\frac{1}{e}$ (b) 1

(c) e (d) $\log_e 2$

39. For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots, is

(a) $\left(0, \frac{1}{e}\right)$ (b) $\left(\frac{1}{e}, 1\right)$

(c) $\left(\frac{1}{e}, \infty\right)$ (d) $(0, 1)$

Passage Q. 40 to 42

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$ **(2010)**

40. The real numbers s lies in the interval

(a) $\left(-\frac{1}{4}, 0\right)$ (b) $\left(-11, -\frac{3}{4}\right)$

(c) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (d) $\left(0, \frac{1}{4}\right)$

41. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(a) $\left(\frac{3}{4}, 3\right)$ (b) $\left(\frac{21}{64}, \frac{11}{16}\right)$

(c) (9, 10) (d) $\left(0, \frac{21}{64}\right)$

42. The function f′ (x) is

(a) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(b) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(c) increasing in (–t, t)

(d) decreasing in (–t, t)

Passage Q. 43 and 44

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For n = 0, 1, 2,……., let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b. **(2017)**

43. If $a_4 = 28$, then p + 2q =

(a) 12 (b) 21

(c) 14 (d) 7

44. $a_{12} =$

(a) $a_{11} + 2a_{10}$ (b) $a_{11} + a_{10}$

(c) $a_{11} - a_{10}$ (d) $2a_{11} + a_{10}$

Fill in the Blanks

45. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) = (…….). **(1982)**

46. If the products of the roots of the equation $x^2 – 3kx + 2e^{2 \log k} - 1 = 0$ is 7, then the roots are real for k = … . **(1984)**

47. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq b)$ have a common root, then the numerical value of a + b is … . **(1986)**

48. The solution of the equation $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$ is …. **(1986)**

49. If α, β, γ are the cube roots of p, p < 0, then for any x, y and z, then $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha} = \ldots$ **(1990)**

50. The sum of all the real roots of the equation $|x–2|^2 + |x–2| - 2 = 0$ is … . **(1997)**

True/False

51. If x – r is a factor of the polynomial

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$, repeated m times $(1 < m \leq n)$, then r is a root of $f'(x) = 0$ repeated m times. **(1983)**

52. The equation $2x^2 + 3x + 1 = 0$ has an irrational root. **(1983)**

53. If a < b < c < d, then the roots of the equation (x–a) (x–c) + 2 (x–b) (x–d) = 0 are real and distinct. **(1984)**

54. If P (x) = $ax^2 + bx + c$ and Q(x) = $–ax^2 + bx + c$, where $ac \neq 0$, then P (x) Q (x) has at least two real roots. **(1985)**

Subjective Questions

55. Show that the square of $\frac{\sqrt{26 - 15\sqrt{3}}}{5\sqrt{2} - \sqrt{(38 + 5\sqrt{3})}}$ is a rational number. **(1978)**

56. If α and β are the roots of the equation $x^2 + px + 1 = 0$; γ, δ are the roots of $x^2 + qx + 1 = 0$, then $q^2 - p^2 = (\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$ **(1978)**

57. Solve $2 \log_x a + \log_{ax} a + 3 \log_b a = 0$,

where a > 0, $b = a^2 x$ **(1978)**

58. If α and β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + rx + s = 0$, then evaluate $(\alpha - \gamma)(\beta - \gamma)(\alpha - \delta)(\beta - \delta)$ in terms of p, q, r and s. **(1979)**

59. Show that for any triangle with sides a,b,c; $3(ab + bc + ca) \leq (a + b + c)^2 \leq 4(ab + bc + ca)$ **(1979)**

60. Find the integral solutions of the following systems of inequalities

(a) $5x - 1 < (x + 1)^2 < 7x - 3$

(b) $\frac{x}{2x+1} \geq \frac{1}{4}, \frac{6x}{4x-1} < \frac{1}{2}$ **(1979)**

61. For what values of m, does the system of equations

3x + my = m

and 2x – 5y = 20

has solution satisfying the conditions x > 0, y > 0 ? **(1980)**

62. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the nth power of the other, then show that

$(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$ **(1983)**

63. Find all real values of x which satisfy $x^2 - 3x + 2 > 0$ and $x^2 - 2x - 4 \leq 0$. **(1983)**

64. If a > 0, b > 0 and c > 0 prove that

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$$ **(1984)**

65. Solve for $x\ (5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$ **(1985)**

66. For $a \leq 0$, determine all real roots of the equation

$x^2 - 2a\,|x-a| - 3a^2 = 0$ **(1986)**

67. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$ **(1987)**

68. Find the set of all x for which

$$\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$$ **(1987)**

69. Solve the x the following equation

$\log_{(2x+3)}(6x^2 + 23x + 21)$

$= 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$ **(1987)**

70. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then show that

$$1+\frac{c}{a}+\left|\frac{b}{a}\right|<0$$ **(1995)**

71. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the length of the sides of the quadrilateral, prove that $2 \leq a^2 + b^2 + c^2 + d^2 \leq 4$ **(1997)**

72. Find the set of all solutions of the equation

$2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$ **(1997)**

73. Let $f(x) = Ax^2 + Bx + C$ where, A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers 2A, A + B and C are all integers. Conversely, prove that if the numbers 2A, A + B and C are all integers, then $f(x)$ is an integer whenever x is an integer. **(1998)**

74. If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that

$$\frac{b^2-4ac}{a^2}=\frac{B^2-4AC}{A^2}$$ **(2000)**

75. Let $-1 \leq p < 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval [1/2, 1] and identify it. **(2001)**

76. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **(2001)**

77. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$, then find the values of a for which equation has unequal real roots for all values of b. **(2003)**

78. If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta$, $\beta + \delta$, are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constants δ, then prove that $\dfrac{b^2-4ac}{a^2}=\dfrac{B^2-4AC}{A^2}$ **(2004)**

79. If $x^2 - 10ax - 11b = 0$ have roots c & d, $x^2 - 10cx - 11d = 0$ have roots a and b. $(a \neq c)$ Find $a + b + c + d$. **(2006)**

Integer Answer Type Questions

80. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is.... **(2009)**

81. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations $3x - y - z = 0, -3x + z = 0, -3x + 2y + z = 0$. Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is...

(2009)

82. Let ω be the complex number $\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$. Then the number of distinct complex number z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$$ is equal to..... **(2010)**

83. Let *a*, *b*, *c* be three non-zero real numbers such that the equation $\sqrt{3}a\cos x + 2b\sin x = c, x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \dfrac{\pi}{3}$. . Then, the value of $\dfrac{b}{a}$ is _________. **(2018)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (b) 2. (c) 3. (d) 4. (d) 5. (c) 6. (d) 7. (b) 8. (b) 9. (d) 10. (a)
11. (a) 12. (c) 13. (a) 14. (a) 15. (b) 16. (b) 17. (b) 18. (a) 19. (c) 20. (d)
21. (b) 22. (a) 23. (d) 24. (c) 25. (b) 26. (c) 27. (c) 28. (d) 29. (a) 30. (b)
31. (b) 32. (c) 33. (b) 34. (d) 35. (b) 36. (b) 37. (b) 38. (a) 39. (b) 40. (a)
41. (b) 42. (d) 43. (a) 44. (a) 45. (a,b) 46. (c) 47. (c) 48. (a) 49. (b) 50. (b,c)
51. (c) 52. (a) 53. (a) 54. (b) 55. (b) 56. (b) 57. (b) 58. (a) 59. (a) 60. (a)
61. (a) 62. (d) 63. (a) 64. (a,b,c,d) 65. (b) 66. (a,b,c,d) 67. (b) 68. (a) 69. (d) 70. (b)
71. (c) 72. (b) 73. (c) 74. (d) 75. (d) 76. (c) 77. (b) 78. (d) 79. (d) 80. (c)

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (d) 2. (a) 3. (c) 4. (a) 5. (a) 6. (d) 7. (a) 8. (b) 9. (a) 10. (c)
11. (b) 12. (c) 13. (d) 14. (b) 15. (c) 16. (c) 17. (a) 18. (c) 19. (c) 20. (b)
21. (b) 22. (a) 23. (b) 24. (b) 25. (a) 26. (b) 27. (c) 28. (a) 29. (a) 30. (c)
31. (c) 32. (d) 33. (d) 34. (a) 35. (d) 36. (c) 37. (b) 38. (c) 39. (b) 40. (d)
41. (b)

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (d) 2. (a,c) 3. (a) 4. (c) 5. (c) 6. (d) 7. (d) 8. (a) 9. (a) 10. (a)
11. (c) 12. (b) 13. (c) 14. (b) 15. (a) 16. (a) 17. (a) 18. (d) 19. (a) 20. (c)
21. (d) 22. (c) 23. (a) 24. (c) 25. (a,d) 26. (d) 27. (c) 28. (c) 29. (b) 30. (c)
31. (c) 32. (d) 33. (b) 34. (b) 35. (b) 36. (c) 37. (a) 38. (a) 39. (b) 40. (a,c)
41. (b) 42. (b) 43. (b) 44. (b) 45. (d) 46. (a) 47. (b) 48. (b) 49. (d) 50. (a)
51. (a) 52. (b) 53. (d) 54. (c) 55. (c) 56. (b) 57. (b) 58. (a) 59. (d) 60. (a)
61. (a) 62. (b) 63. (a) 64. (a,b,c) 65. (a,c) 66. (a,b,c,d) 67. (a,b) 68. (a,b) 69. (a,b) 70. (b,c)
71. (b,c) 72. (b,c) 73. (a,c) 74. (b) 75. (a) 76. (a) 77. (e) 78. (a) 79. (a) 80. (b)
81. (c) 82. (b) 83. (b) 84. (c) 85. A–R; B–Q; C–P; D–S 86. A–Q; B–P; C–S; D–R
87. 0099 88. 0005 89. 0008 90. 0008

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (b) 2. (c) 3. (d) 4. (a) 5. (d) 6. (d) 7. (a) 8. (a) 9. (a) 10. (b)
11. (a,d) 12. (a,b,c) 13. (d) 14. (b) 15. (c) 16. (c) 17. (a) 18. (a) 19. (a) 20. (b)
21. (d) 22. (c) 23. (b) 24. (b) 25. (b) 26. (b) 27. (a) 28. (a) 29. (d) 30. (b)
31. (c) 32. (b) 33. (d) 34. (a, d) 35. (c) 36. (b) 37. (b) 38. (a) 39. (a) 40. (c)
41. (a) 42. (b) 43. (a) 44. (b) 45. (–4, 7) 46. k = 2 47. –1 48. x = 4 49. w^2 50. 4
51. False 52. False 53. True 54. True 57. $x = a^{-1/2}$ or $a^{-4/3}$ 58. $(q - s)^2 - rqp - rsp + sp^2 + qr^2$
60. (a) x = 3 (b) x = ϕ 61. $m \in \left(-\infty, -\frac{15}{2}\right) \cup (30, \infty)$ 63. $x \in [1-\sqrt{5}, 1) \cup [1+\sqrt{5}, 2)$ 65. $x = \pm 2, \pm\sqrt{2}$
66. $x = \{a\,(1-\sqrt{2}), a\,(\sqrt{6}-1)\}$ 67. $x = -4, (-1-\sqrt{3})$ 68. $x \in (-2,-1) \cup \left(-\frac{2}{3}, -\frac{1}{2}\right)$
69. $x = -\frac{1}{4}$ 72. $y \in \{-1\} \in [1, \infty)$ 75. $x = \cos\left(\frac{1}{3}\cos^{-1} p\right)$ 76. $x = \alpha^2\beta, \alpha\beta^2$
77. a > 1 79. 1210 80. k = 2 81. 7 82. 1 83. (0.5)