CHAPTER

4 DETERMINANTS

Syllabus

Determinant of a square matrix (up to 3 × 3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Chapter Analysis

The analysis given here gives you an analytical picture of this chapter and will help you to identify the concepts of the chapter that are to be focussed more from exam point of view.

TOPIC	2016		2017		2018
	Delhi	OD	Delhi	OD	Delhi/OD
Expansion	1. Q. (1 Mark)	–	–	–	–
Area of Triangle/Equation of line	–	–	–	–	–
Property	1. Q. (6 Marks)	1. Q. (1 Mark) 1. Q. (6 Marks)	1. Q. (1 Mark)	1. Q. (1 Mark) 1. Q. (2 Marks) 1. Q. (4 Marks)	1. Q. (4 Marks)
Solution of equations	1. Q. (4 Marks) 1. Q. (6 Marks)	1. Q. (4 Marks)	1. Q. (6 Marks)	1. Q. (6 Marks)	1. Q. (6 Marks)

TOPIC-1
Determinants, Minors & Co-factors

Revision Notes

Determinants, Minors & Co-factors

(a) Determinant : A unique number (real or complex) can be associated to every square matrix $A = [a_{ij}]$ of order m. This number is called the determinant of the square matrix A, where $a_{ij} = (i, j)^{\text{th}}$ element of A.

For instance, if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then, determinant of matrix A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = det (A) and its value is given by $ad - bc$.

(b) Minors : Minors of an element a_{ij} of a determinant *(or a determinant corresponding to matrix A)* is the determinant obtained by deleting its i^{th} row and j^{th} column in which a_{ij} lies. Minor of a_{ij} is denoted by M_{ij}. Hence, we can get 9 minors corresponding to the 9 elements of a third order (*i.e.*, 3 × 3) determinant.

(c) Co-factors : Cofactor of an element a_{ij}, denoted by A_{ij}, is defined by $A_{ij} = (-1)^{(i+j)} M_{ij}$, where M_{ij} is minor of a_{ij}. Sometimes C_{ij} is used in place of A_{ij} to denote the co-factor of element a_{ij}.

1. ADJOINT OF A SQUARE MATRIX :

Let $A = [a_{ij}]$ be a square matrix. Also, assume $B = [A_{ij}]$, where A_{ij} is the cofactor of the elements a_{ij} in matrix A. Then the transpose B^T of matrix B is called the **adjoint of matrix** A and it is denoted by "*adj* (*A*)".

To find adjoint of a 2 × 2 matrix : Follow this, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or $adj\,A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

For example, consider a square matrix of order 3 as $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 0 & 5 \end{bmatrix}$, then in order to find the adjoint matrix A, we find a matrix B (formed by the co-factors of elements of matrix A as mentioned above in the definition)

i.e., $B = \begin{bmatrix} 15 & -2 & -6 \\ -10 & -1 & 4 \\ -1 & 2 & -1 \end{bmatrix}$. Hence, $adj\,A = B^T = \begin{bmatrix} 15 & -10 & -1 \\ -2 & -1 & 2 \\ -6 & 4 & -1 \end{bmatrix}$

2. SINGULAR MATRIX AND NON-SINGULAR MATRIX :

(a) Singular matrix : A square matrix A is said to be singular if $|A| = 0$ *i.e.*, its determinant is zero.

e.g.
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 12 \\ 1 & 1 & 3 \end{bmatrix}$$
$$= 1(15-12) - 2(12-12) + 3(4-5) = 3 - 0 - 3 = 0$$

∴ A is singular matrix.

$$B = \begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix} = 12 - 12 = 0$$

∴ B is singular matrix.

(b) Non-singular matrix : A square matrix A is said to be non-singular if $|A| \neq 0$.

e.g.
$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$
$$= 0\,(0-1) - 1(0-1) + 1(1-0)$$
$$= 0 + 1 + 1 = 2 \neq 0$$

∴ A is non-singular matrix.

- A square matrix A is **invertible** if and only if A is **non-singular**.

3. ALGORITHM TO FIND A^{-1} BY DETERMINANT METHOD :

STEP 1 : Find $|A|$.

STEP 2 : If $|A| = 0$, then, write "A is a singular matrix and hence not invertible". Else write "A is a non-singular matrix and hence invertible".

STEP 3 : Calculate the cofactors of elements of matrix A.

STEP 4 : Write the matrix of cofactors of elements of A and then obtain its transpose to get *adj.A* (*i.e.*, adjoint A).

STEP 5 : Find the inverse of A by using the relation $A^{-1} = \dfrac{1}{|A|}(adj\,A)$.

4. PROPERTIES ASSOCIATED WITH VARIOUS OPERATIONS OF MATRICES AND THE DETERMINANTS :

(a) $AB = I = BA$

(b) $AA^{-1} = I$ or $A^{-1}\,I = A^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$

(d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(e) $(A^{-1})^{-1} = A$

(f) $(A^T)^{-1} = (A^{-1})^T$

(g) $A(adjA) = (adj\,A)A = |A|\,I$

(h) $adj(AB) = adj(B)\,adj\,(A)$

(i) $adj(A^T) = (adj\,A)^T$

(j) $(adj\,A)^{-1} = (adj\,A^{-1})$

(k) $|adj\,A| = |A|^{n-1}$, if $|A| \neq 0$, where n is of the order of A.

(l) $|AB| = |A|\,|B|$

(m) $|A\,adj\,A| = |A|^n$, where n is of the order of A.

(n) $|A^{-1}| = \dfrac{1}{|A|}$, if matrix A is invertible.

(o) $|A| = |A^T|$

- $|kA| = k^n\,|A|$, where n is of the order of square matrix A and k is any scalar.
- If A is a non-singular matrix (*i.e.*, when $|A| \neq 0$) of order n, then $|adj\,A| = |A|^{n-1}$.
- If A is a non-singular matrix of order n, then $adj\,(adj\,A) = |A|^{n-2}\,A$.

5. PROPERTIES OF DETERMINANTS :

(a) If any two rows or columns of a determinant are proportional or identical, then its value is equal to zero.

e.g., $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$ [As R_1 and R_3 are the same]

(b) The value of a determinant remains unchanged if its rows and columns are interchanged.

e.g., $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Here, rows and columns have been interchanged, but there is no effect on the value of determinant.

(c) If each element of a row or a column of a determinant is multiplied by a constant k, then the value of new determinant is k times the value of the original determinant.

e.g., $$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ or } \Delta_1 = k\Delta$$

(d) If any two rows or columns are interchanged, then the determinant retains its absolute value, but its sign is changed.

e.g., $$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} \text{ or } \Delta_1 = -\Delta \qquad [\text{Here } R_1 \leftrightarrow R_3]$$

(e) If every element of some column or row is the sum of two terms, then the determinant is equal to the sum of two determinants; one containing only the first term in place of each sum, the other only the second term. The remaining elements of both determinants are the same as given in the original determinant.

e.g., $$\Delta = \begin{vmatrix} a_1+\alpha & b_1 & c_1 \\ a_2+\beta & b_2 & c_2 \\ a_3+\gamma & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha & b_1 & c_1 \\ \beta & b_2 & c_2 \\ \gamma & b_3 & c_3 \end{vmatrix}$$

6. AREA OF TRIANGLE :

Area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ sq. units}$$

- Since area is a positive quantity, we take absolute value of the determinant.
- If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then $\Delta = 0$.
- The equation of a line passing through the points (x_1, y_1) and (x_2, y_2) can be obtained by the expression given here :

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Objective Type Questions

(1 mark each)

Q. 1. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$**, then the value of** x **is**

(a) 3 **(b)** ±3
(c) ±6 **(d)** 6

[NCERT Exemp.]

Ans. Correct option : (c)

Explanation **:** Given that

$$\therefore \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix},$$

$$\Rightarrow 2x^2 - 40 = 18 + 14$$
$$\Rightarrow 2x^2 = 32 + 40$$
$$\Rightarrow x^2 = \frac{72}{2} = 36$$
$$\therefore x = \pm 6$$

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Q. 2. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$ **is**

(a) $a^3 + b^3 + c^3$ **(b)** $3bc$
(c) $a^3 + b^3 + c^3 - 3abc$ **(d)** None of these

[NCERT Exemp.]

Ans. Correct option : (d)

Explanation : We have

$$\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix} = \begin{vmatrix} a+c & b+c+a & a \\ b+c & c+a+b & b \\ c+b & a+b+c & c \end{vmatrix}$$

$[\because C_1 \to C_1 + C_2 \text{ and } C_2 \to C_2 + C_3]$

$$= (a+b+c)\begin{vmatrix} a+c & 1 & a \\ b+c & 1 & b \\ c+b & 1 & c \end{vmatrix}$$

[Taking $(a+b+c)$ common from C_2]

$[\because R_2 \to R_2 - R_3 \text{ and } R_1 \to R_1 - R_3]$

$$= (a+b+c)\begin{vmatrix} a-b & 0 & a-c \\ 0 & 0 & b-c \\ c+b & 1 & c \end{vmatrix}$$

[Expanding along R_2]

$$= (a+b+c)(b-c)(b-a)$$

Q. 3. The area of a triangle with vertices (–3, 0), (3, 0) and (0, *k*) is 9 sq. units. Then, the value of *k* will be

(a) 9 **(b) 3**
(c) –9 **(d) 6**

[NCERT Exemp.]

Ans. Correct option : (b)

Explanation **:** We know that, area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \quad \Delta = \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$$

[Expanding along R_1]

$$9 = \frac{1}{2}\left[-3(-k) - 0 + 1(3k)\right]$$

$$\Rightarrow 18 = 3k + 3k = 6k$$

$$\therefore \quad k = \frac{18}{6} = 3$$

Q. 4. The determinant $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$ **is equal to**

(a) ***abc*(*b* – *c*)(*c* – *a*)(*a* – *b*)**
(b) (*b* – *c*)(*c* – *a*)(*a* – *b*)
(c) (*a* + *b* + *c*)(*b* – *c*)(*c* – *a*)(*a* – *b*)
(d) None of these

[NCERT Exemp.]

Ans. Correct option : (d)

Explanation **:** We have

$$\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$$

$$= (b-a)^2\begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

[On taking $(b-a)$ common from C_1 and C_3 each]

$$= (b-a)^2\begin{vmatrix} b-c & b-c & b \\ a-b & a-b & a \\ c-a & c-a & a \end{vmatrix}$$

$[\because C_1 \to C_1 - C_3]$

$$= 0$$

[Since, two columns C_1 and C_2 are identical, so the value of determinant is zero.]

Q. 5. If $A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$**. Then A^{-1} exist if**

(a) $\lambda = 2$ **(b)** $\lambda \neq 2$
(c) $\lambda \neq -2$ **(d) None of these**

[NCERT Exemp.]

Ans. Correct option : (d)

Explanation **:** Given that,

$$A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$$

Expanding along R_1,

$$|A| = 2(6-5) - \lambda(-5) - 3(-2)$$
$$= 2 + 5\lambda + 6$$

We know that A^{-1} exists, if A is non-singular matrix,
i.e., $|A| \neq 0$

$$\therefore 2 + 5\lambda + 6 \neq 0$$
$$\Rightarrow \quad 5\lambda \neq -8$$
$$\therefore \quad \lambda \neq \frac{-8}{5}$$

So, A^{-1} exists if and only if $\lambda \neq \frac{-8}{5}$.

Q. 6. If *A* and *B* are invertible matrices, then which of the following is not correct?

(a) $adj.\ A = |A|.A^{-1}$ **(b)** $\det(A^{-1}) = [\det(A)]^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$ **(d)** $(A+B)^{-1} = B^{-1} + A^{-1}$

[NCERT Exemp.]

Ans. Correct option : (d)

Explanation **:** Since, *A* and *B* are invertible matrices, so, we can say that

$$(AB)^{-1} = B^{-1}A^{-1} \quad \text{...(i)}$$

Also, $$A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\Rightarrow \quad adj\ A = A^{-1}.|A| \quad \text{...(ii)}$$

Also, $$\det(A)^{-1} = [\det(A)]^{-1}$$

$$\Rightarrow \quad \det(A)^{-1} = \frac{1}{[\det(A)]}$$

$$\Rightarrow \det(A).\det(A)^{-1} = 1 \quad \text{...(iii)}$$

From equation (iii), we conclude that it is true.

Again, $$(A+B)^{-1} = \frac{1}{|(A+B)|}adj\ (A+B)$$

$$\Rightarrow \quad (A+B)^{-1} \neq B^{-1} + A^{-1} \quad \text{...(iv)}$$

Q. 7. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ **then** x **is equal to**

(a) 6 **(b)** ±6

(c) −6 **(d)** 0

[NCERT Ex.]

Ans. Correct option : (b)

Explanation : Given that,

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$\Rightarrow x^2 - 36 = 36 - 36$$

$$\Rightarrow \quad x^2 - 36 = 0$$

$$\Rightarrow \quad x^2 = 36$$

$$\Rightarrow \quad x = \pm 6$$

Q. 8. Let A **be a non-singular square matrix of order 3 × 3. Then** $|adj\, A|$ **is equal to**

(a) $|A|$ **(b)** $|A|^2$

(c) $|A|^3$ **(d)** $3|A|$

[NCERT Ex.]

Ans. Correct option : (b)

Explanation : We know that,

$$(adj\, A)A = |A|\, I = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$\Rightarrow \quad |(adj\, A)A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\Rightarrow |adj\, A|\,|A| = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |A|^3\, (I)$$

$$\therefore \quad |adj\, A| = |A|^2$$

Q. 9. If A **is an invertible matrix of order 2, then det(**A^{-1}**) is equal to**

(a) det(A) **(b)** $\dfrac{1}{\det(A)}$

(c) 1 **(d)** 0

[NCERT Ex.]

Ans. Correct option : (b)

Explanation : Given that A is an invertible matrix,

A^{-1} exists and $A^{-1} = \dfrac{1}{|A|} adj.A.$

As matrix A is order of 2, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then, $|A| = ad - bc$ and $adj.A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now,

$$A^{-1} = \frac{1}{|A|} adj.A = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\therefore \quad |A^{-1}| = \begin{vmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{vmatrix}$$

$$= \frac{1}{|A|^2} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$= \frac{1}{|A|^2}(ad - bc)$$

$$= \frac{1}{|A|^2} . |A|$$

$$= \frac{1}{|A|}$$

$$\therefore \det(A^{-1}) = \frac{1}{\det(A)}$$

Very Short Answer Type Questions

(1 mark each)

Q. 1. Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

R&U **[Delhi Set I, II, III 2016]**

Sol. Let the maximum value be

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

Expanding, we get

$$\Delta = \sin\theta \,.\, \cos\theta$$

$$= \frac{2\sin\theta.\cos\theta}{2} = \frac{1}{2}\sin 2\theta \qquad ½$$

$$\therefore \text{Maximum value} = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$= \frac{1}{2} \qquad [\text{as} -1 \le \sin 2\theta \le 1] \qquad ½$$

Q. 2. If $x \in N$ **and** $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$**, then find the value of** x**.**

R&U **[O.D. Set I, II, III 2016]**

Sol. Given $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$

or $2x(x + 3) - (-2)(-3x) = 8$

or $2x^2 + 6x - 6x = 8$

or $x^2 = \dfrac{8}{2}$ ½

or $x^2 = 4$

or $x = \pm 2$ ½

Since $x \in N$, so, $x = 2$

Commonly Made Error

- Sometimes Students attempt this question using long calculations instead of using simple fundamental concepts.

Answering Tips

- Illustrate properties of determinants in detail.

Q. 3. In the interval $\frac{\pi}{2} < x < \pi$, find the value of x for which that matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular.

R&U **[O.D. Set I, II, III Comptt. 2015]**

Sol. Let $A = \begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$

For singular matrix

$|A| = 0$

$\therefore \quad 4\sin^2 x - 3 = 0$ ½

or $\sin x = \pm\frac{\sqrt{3}}{2}$

or $x = \frac{2\pi}{3}$

as $\frac{\pi}{2} < x < \pi$ ½

[CBSE Marking Scheme 2015]

Q. 4. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$

R&U **[Delhi Set I Comptt. 2014]**

Sol. $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = 1$ 1

[CBSE Marking Scheme 2014]

Detailed Answer :

$\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = p^2 - (p^2 - 1)$

$= p^2 - p^2 + 1$ ½

$= 1$ ½

Q. 5. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.

R&U **[NCERT Exemplar, Delhi Set I, 2014]**

Sol. $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

or $x = \pm 6$ 1

[CBSE Marking Scheme 2014]

Alternative Method :

$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$2x^2 - 40 = 18 + 14$

or $2x^2 = 32 + 40$ ½

or $2x^2 = 72$

or $x^2 = 36$

or $x = \pm 6$ ½

Q. 6. If A is a 3 × 3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k.

A **[Foreign Set I, II, III, 2014]**

Sol. $k = 27$ 1

[CBSE Marking Scheme 2014]

Detailed Answer :

Since $|kA| = k^n|A|$, where n is the order of matrix. ½

$\therefore \quad |3A| = 3^3|A|$

$= 27|A|$

or $k = 27$ ½

Q. 7. If A and B are invertible matrices of order 3, $|A| = 2$ and $|(AB)^{-1}| = -\frac{1}{6}$. Find $|B|$. A **[S.Q.P. 2018]**

Sol. $\frac{1}{|AB|} = -\frac{1}{6}$

$\Rightarrow \quad \frac{1}{|A||B|} = -\frac{1}{6}$

$\Rightarrow \quad |B| = -3.$ 1

[CBSE Marking Scheme 2018]

Q. 8. If $A = \begin{pmatrix} 4 & 6 \\ 7 & 5 \end{pmatrix}$, then what is the value of $A.(adj\ A)$? R&U **[S.Q.P. 2014]**

Sol. $A.(adj\ A) = \begin{pmatrix} -22 & 0 \\ 0 & -22 \end{pmatrix}$ 1

[CBSE Marking Scheme 2014]

Detailed Answer :

$A = \begin{pmatrix} 4 & 6 \\ 7 & 5 \end{pmatrix}$

$adj\ A = \begin{pmatrix} 5 & -6 \\ -7 & 4 \end{pmatrix}$

$A\cdot(adj\ A) = \begin{pmatrix} 4 & 6 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 5 & -6 \\ -7 & 4 \end{pmatrix}$ ½

$= \begin{pmatrix} 20-42 & -24+24 \\ 35-35 & -42+20 \end{pmatrix} = \begin{pmatrix} -22 & 0 \\ 0 & -2 \end{pmatrix}$ ½

Q. 9. If $\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = -\frac{1}{2}$ where α and β are acute angles, then write the value of $\alpha + \beta$.

R&U **[S.Q.P. 2014]**

Sol. $(\alpha + \beta) = \frac{\pi}{3}$ 1

[CBSE Marking Scheme 2014]

Alternative Method :

$\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = -\frac{1}{2}$

or $\sin\alpha\sin\beta - \cos\alpha\cos\beta = -\frac{1}{2}$

or $-\cos(\alpha + \beta) = -\frac{1}{2}$

or $\cos(\alpha + \beta) = \cos\frac{\pi}{3}$ ½

or $\cos(\alpha + \beta) = \cos\frac{\pi}{3}$

or $(\alpha + \beta) = \frac{\pi}{3}$ ½

Q. 10. Evaluate x if : $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$.

R&U **[Delhi Set I, II, III Comptt. 2016]**

Sol. $2 - 20 = 2x^2 - 24$ **1**

or $x = \pm\sqrt{3}$

[CBSE Marking Scheme 2016]

Q. 11. Given $A = \begin{pmatrix} 4 & 2 & 5 \\ 2 & 0 & 3 \\ -1 & 1 & 0 \end{pmatrix}$, **write the value of det. ($2AA^{-1}$).** A **[Outside Dec. Set I, II, III Comptt. 2016]**

Sol. $|2AA^{-1}| = (2)^3$ $[\because AA^{-1} = I]$

$= 8$ **1**

[CBSE Marking Scheme 2016]

Q. 12. If A is square matrix of order 3 such that $|adj\ A| = 64$, find $|A|$. A **[Delhi Set I Comptt. 2013]**

Sol. $|A| = 8$ **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

$|adj\ A| = |A|^{n-1}$, where n is the order of the matrix ½

$|A|^2 = 64$

Or, $|A| = 8$ ½

Q. 13. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$**, write the value of x.**

A **[Delhi Set I Comptt. 2013]**

Sol. $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$ Or $x = 1$ **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

$$\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$$

or, $2x(x + 1) - (x + 3)\ [2(x + 1)] = 3 - 15$ ½

or, $2x^2 + 2x - 2x^2 - 8x - 6 = -12$

or, $-6x - 6 = -12$

or, $-6x = -12 + 6$

or, $x = 1$ ½

Answering Tips

- Give extensive practice in different types of questions based on properties of determinants.

Q. 14. For what value of x, the given matrix $A = \begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix}$ **is a singular matrix ?**

R&U **[O.D. Set I Comptt. 2013]**

Sol. $x = 1$ **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

$$A = \begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix}$$

Since A is a singular matrix. *i.e.,* $|A| = 0$

$$\begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix} = 0$$ ½

or, $4(3 - 2x) - 2(x + 1) = 0$

or, $12 - 8x - 2x - 2 = 0$

or, $-10x + 10 = 0$

or, $x = 1$ ½

Q. 15. If A is an invertible square matrix of order 3 and $|A| = 5$, then find the value of $|adj\ A|$.

A **[O.D. Set I Comptt. 2013]**

Sol. $|adj\ A| = 25$ **1**

[CBSE Marking Scheme 2013]

Alternative Method :

$|adj\ A| = |A|^{n-1}$, where n is the order of the matrix ½

$\therefore$ $|adj\ A| = 5^2$ Or $|adj\ A| = 25$ ½

Q.16. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$**, then write the value of x.**

R&U **[Delhi Set I 2013]**

Sol. $x = 2$ **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

$$\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$$

or, $(x + 1)(x + 2) - (x - 3)(x - 1) = 12 + 1$ ½

or, $x^2 + 3x + 2 - x^2 + 4x - 3 = 13$

or, $7x - 1 = 13$

or, $7x = 14$

or, $x = 2$ ½

Q. 17. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$**, then write the value of $a_{32}A_{32}$.** A **[O.D. Set I, 2013]**

Sol. $a_{32}A_{32} = 110$ **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

$$a_{32}A_{32} = -5\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$$

$= -5(8 - 30)$ ½

$= -5\ (-22) = 110$ ½

AI **Q. 18. If A is square matrix of order 2 and $|adj\ A| = 9$, find $|A|$.**

A **[Outside Delhi Set I, II, III Comptt., 2016]**

Sol. $|\text{adj } A| = |A|^{2-1}$

or, $|A| = 9$ **1**

[CBSE Marking Scheme 2016]

Q. 19. If A is a square matrix and $|A| = 2$, then write the value of $|AA'|$, where A' is the transpose of matrix A. A **[Foreign Set I, II, III, 2013]**

Sol. $|AA'| = 4$ **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

$$|AA'| = |A||A'|$$
$$= 2 \times 2 \qquad [\because |A| = |A'|] \ ½$$
$$= 4 \qquad ½$$

Q. 20. If $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$, then write A^{-1}.

R&U **[Foreign Set I, II, III, 2013]**

Sol. $A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

$$A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\,A) \qquad ½$$

$$|A| = \begin{vmatrix} 3 & 10 \\ 2 & 7 \end{vmatrix} = 21 - 20 = 1$$

$$adj\,A = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} \text{ or } A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

½

Q. 21. If A is a square matrix of order 3 such that $|adj\,A| = 225$, find $|A'|$. A **[S.Q.P. 2012]**

Sol. $|A'| = \pm 15$ **1**

[CBSE Marking Scheme 2012]

Detailed Answer :

$|adj\,A| = |A|^{n-1}$, where n is the order of the matrix. ½

$|A|^2 = 15^2$ Or $|A| = \pm 15$ Or $|A'| = \pm 15$ ½

Q. 22. Write the inverse of the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

A **[S.Q.P. 2012]**

Sol. $A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ **1**

[CBSE Marking Scheme 2012]

Detailed Answer :

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\,A)$$

$$|A| = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \qquad ½$$

$$= \cos^2\theta + \sin^2\theta = 1$$

$$adj\,A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

or $$A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \qquad ½$$

Q. 23. The value of the determinant of a matrix A of order 3×3 is 4. Find the value of $|5A|$.

A **[Delhi Set I Comptt. 2012]**

Sol. $|5A| = 500$ **1**

[CBSE Marking Scheme 2012]

Detailed Answer :

$$|kA| = k^n|A| \qquad ½$$
$$\Rightarrow \quad |5A| = 5^3(4) = 125(4) = 500 \qquad ½$$

Q. 24. Is A is square matrix of order 3 and $|2A| = k|A|$, then find the value of k.

A **[S.Q.P. 2016-17]**

Sol. $k = 2^3 = 8$ **1**

[CBSE Marking Scheme 2016]

Q. 25. If the determinant of matrix A of order 3×3 is of value 4, write the value of $|3A|$.

A **[O.D. Set I Comptt. 2012]**

Sol. $|3A| = 108$ **1**

[CBSE Marking Scheme 2012]

Detailed Answer :

$$|kA| = k^n|A|$$
or $$|3A| = 3^3(4) \qquad ½$$
$$= 27\,(4)$$
$$= 108 \qquad ½$$

Q. 26. Write the value of the determinant :

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

R&U **[Foreign Set I, 2012]**

Sol. $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$ **1**

[CBSE Marking Scheme 2012]

Detailed Answer :

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} \qquad \left(R_1 \to \frac{1}{6}R_1\right) \ ½$$

$$= 0 \qquad (\because R_1 \text{ and } R_3 \text{ are identical}) \ ½$$

Q. 27. For what value of x, the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix ? R&U **[O.D. Set I Comptt. 2012]**

Sol. $x = \dfrac{13}{15}$ **1**

[CBSE Marking Scheme 2012]

Detailed Answer :

Since, A is a singular matrix

$$\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0$$

or $$8(1 + x) - 7(3 - x) = 0 \qquad ½$$
or $$8 + 8x - 21 + 7x = 0$$
or $$15x - 13 = 0 \qquad ½$$
or $$x = \frac{13}{15}$$

Q. 28. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}. R&U **[Delhi Set I, 2012]**

Sol. $a_{23} = 7$ **1**

[CBSE Marking Scheme 2012]

Detailed Answer :

Given $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$ ½

$a_{23} = 10 - 3 = 7$ ½

Q. 29. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$ R&U **[O.D. Set I Comptt. 2014]**

Sol. $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$ **1**

[CBSE Marking Scheme 2014]

Alternative Method :

$\Delta = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

$C_1 : C_1 + 9C_2$

$\begin{vmatrix} 65 & 7 & 65 \\ 75 & 8 & 75 \\ 86 & 9 & 86 \end{vmatrix} = 0$ **1**

Since two columns are same, hence the determinant value will be zero

Q. 30. If $A = [a_{ij}]$ is is a matrix of order 2 × 2, such that $|A| = -15$, and C_{ij} represents the cofactor of a_{ij}, then find $a_{21}c_{21} + a_{22}c_{22}$ R **[S.Q.P. 2017–18]**

Sol. $a_{21}c_{21} + a_{22}c_{22} = |A| = -15$ **1**

[CBSE Marking Scheme 2017-18]

AI **Q. 31. If for any 2 × 2 square matrix A, A(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.**

R **[OD Set I 2017]**

Sol. $|A| = 8$ **[CBSE Marking Scheme 2017]**

OR

1) $A(\text{adj} A) = |A| I_n$

$A(\text{adj} A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$|A| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = 8$

[Topper's Answer, 2017]

Q. 32. If A is a square matrix such that |A| = 5, write the value of $|AA^T|$. R&U **[OD Set II 2016]**

Sol

$|A| = 5$

$|AA^T| = |A||A^T|$ (As A is a square matrix)

$= |A||A|$ (As $|A| = |A^T|$)

$= |A|^2 = 25$

[Topper's Answer, 2016]

Q. 33. If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -4 \\ 3 & -2 \end{pmatrix}$, find $|AB|$. R&U **[OD Set II 2016]**

Sol. $|AB| = |A||B|$ (Provided A & B are square matrices)

$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 1 \times (-1) - 2 \times 3 = -7$

$|B| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} = 1 \times (-2) - 3 \times (-4) = 10$

$\therefore |AB| = -7 \times 10 = -70$

[Topper's Answer, 2016]

Short Answer Type Questions

(2 marks each)

Q. 1. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$.

Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

R&U **[S.Q.P. 2017-18]**

Sol. $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} = \frac{1}{9-10}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ **1 + ½**

$\therefore P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$ **½**

[CBSE Marking Scheme 2017-18]

Q. 2. If $A = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & 0 \\ 3 & 3 & 2 \end{vmatrix}$, **find** $M_{12} \times M_{21} + C_{21} \times C_{12}$ **when M_{ij} is called minor and C_{ij} is called co-factors of A.**

R&U

Sol. We have, $A = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & 0 \\ 3 & 3 & 2 \end{vmatrix}$ **1**

$M_{12} = \begin{vmatrix} 4 & 0 \\ 3 & 2 \end{vmatrix} = 8 - 0 = 8$

$M_{21} = \begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = 6 + 3 = 9$

$C_{21} = -\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = -(6+3) = -9$

$C_{12} = -\begin{vmatrix} 4 & 0 \\ 3 & 2 \end{vmatrix} = -(8-0) = -8$

$M_{12} \times M_{21} + C_{21} \times C_{12}$

$8 \times (9) + (-9)(-8) = 72 + 72 = 144$ **1**

Q. 3. If $A = \begin{bmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, **then show that** $|2A| = 8|A|$.

R&U

Sol. We have, $|A| = \begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

$|A| = 6(4-0) - 0(0-0) + 1(0-0)$

$= 24$ **1**

$|2A| = \begin{vmatrix} 12 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 8 \end{vmatrix}$

$= 12(16-0) - 0(0-0) + 2(0-0)$

$= 192$ **1**

$|2A| = 8 \times 24 = 8|A|$ **Hence proved.**

Q. 4. If A is a skew-symmetric matrix of order 3, then prove that det $A = 0$. R&U **[OD Set I 2017]**

Sol. Any skew symmetric matrix of order 3 is

$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

or $\Rightarrow |A| = -a(bc) + a(bc) = 0$ **1**

Since A is a skew-symmetric matrix $\therefore A^T = -A$

$\therefore |A^T| = |-A| = (-1)^3 . |A|$ **½**

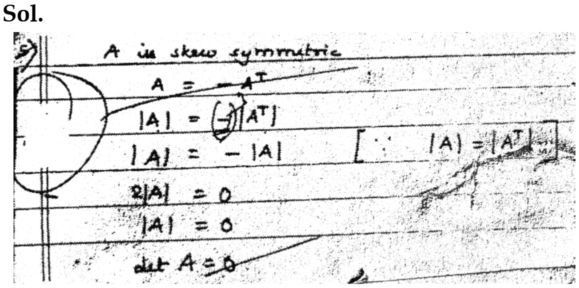
or $|A^T| = -|A|$

or $2|A| = 0$ or $|A| = 0$. **½**

[CBSE Marking Scheme 2017]

OR

Sol.


A is skew symmetric
A = −A^T
|A| = (−1)|A^T|
|A| = − |A| [∵ |A| = |A^T|]
2|A| = 0
|A| = 0
det A = 0

[Topper's Answer, 2017]

Q. 5. If A, B are square matrices of the same order, then prove that $adj\ (AB) = (adj\ B)\ (adj\ A)$.

Sol. We know that

$(AB)\ adj(AB) = |AB| = adj(AB)(AB)$ ...(i)

$(AB)(adj\ B\ adj\ A)$

$= A{\cdot}B\ adj\ B{\cdot}adj\ A = A(B\ adj\ B)\ adj\ A$

$= A(|B|I)\ adj\ A$ $[\because B\ adj\ B = |B|I]$

$= |B|(A{\cdot}\ adj\ A)$

$= |B||A|I$ $[\because A\ adj\ A = |A|I]$

$= |A||B|I$ **1**

$= |AB|I$ ...(ii)

From (i) and (ii), we get

$AB(adj\ AB) = AB(adj\ B{\cdot}adj\ A)$

On multiplying both sides by $(AB)^{-1}$, we get

$(AB)^{-1}[(AB)\ adj\ AB]$

$= (AB)^{-1}\ [(AB)\ adj\ B{\cdot}adj\ A]$

or $adj\ AB = adj\ B{\cdot}adjA$ **1**

Q. 6. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then find the value of $|2AB|$.

R&U **[Foreign 2017]**

Sol. $|2AB| = 2^3 \times |A| \times |B|$ **1**

$= 8 \times (-1) \times 3 = -24$ **1**

[CBSE Marking Scheme 2017]

Long Answer Type Questions-I

(4 marks each)

Q. 1. Using properties of determinants, prove that :

$$\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\(a+2)(a+3) & a+3 & 1\\(a+3)(a+4) & a+4 & 1\end{vmatrix} = -2$$

R&U **[Delhi Set I, II, III Comptt. 2015]**

Sol. $$\text{LHS} = \begin{vmatrix}(a+1)(a+2) & a+2 & 1\\(a+2)(a+3) & a+3 & 1\\(a+3)(a+4) & a+4 & 1\end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ **1**

$$= \begin{vmatrix}(a+1)(a+2) & (a+2) & 1\\2(a+2) & 1 & 0\\(a+3)(a+4) & (a+4) & 1\end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$

$$= \begin{vmatrix}(a+1)(a+2) & (a+2) & 1\\2(a+2) & 1 & 0\\4a+10 & 2 & 0\end{vmatrix}$$ **1**

Expanding along C_3

$= 4a + 8 - 4a - 10$ **1½**

$= -2 = \text{RHS}$ **½**

[CBSE Marking Scheme 2015]

Q. 2. If $A = \begin{bmatrix}2 & 3\\1 & -4\end{bmatrix}$, $B = \begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}$, **verify that**

$$(AB)^{-1} = B^{-1}A^{-1}$$

A **[NCERT] [O.D. Set I, II, III Comptt. 2015]**

Sol. Given $A = \begin{bmatrix}2 & 3\\1 & -4\end{bmatrix}$

and $B = \begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}$

then $AB = \begin{bmatrix}2 & 3\\1 & -4\end{bmatrix}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix} = \begin{bmatrix}-1 & 5\\5 & -14\end{bmatrix}$ **1**

Taking L.H.S. $= (AB)^{-1} = \dfrac{adj(AB)}{|AB|}$

Here $adj(AB) = \begin{bmatrix}-14 & -5\\-5 & -1\end{bmatrix}$ and

$|AB| = 14 - 25 = -11$

$\therefore \quad (AB)^{-1} = -\dfrac{1}{11}\begin{bmatrix}-14 & -5\\-5 & -1\end{bmatrix} = \dfrac{1}{11}\begin{bmatrix}14 & 5\\5 & 1\end{bmatrix}$ **1**

$adj\,B = \begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$

$|B| = 3 - 2 = 1$

$adj\,A = \begin{bmatrix}-4 & -3\\-1 & 2\end{bmatrix}$

$|A| = -8 - 3 = -11$

$B^{-1} = \dfrac{1}{|B|}(adj\,B) = \begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$

$A^{-1} = -\dfrac{1}{11}\begin{bmatrix}-4 & -3\\-1 & 2\end{bmatrix}$

Taking R.H.S. $= B^{-1}A^{-1}$

$= \begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}^{-1}\begin{bmatrix}2 & 3\\1 & -4\end{bmatrix}^{-1}$

$= 1\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix} \times -\dfrac{1}{11}\begin{bmatrix}-4 & -3\\-1 & 2\end{bmatrix}$

$= \dfrac{1}{11}\begin{bmatrix}14 & 5\\5 & 1\end{bmatrix}$ **1+1**

$\therefore$ L.H.S. = R.H.S.

[CBSE Marking Scheme 2015]

AI **Q. 3. Using properties of determinants, solve for x :**

$$\begin{vmatrix}a+x & a-x & a-x\\a-x & a+x & a-x\\a-x & a-x & a+x\end{vmatrix} = 0$$

R&U **[OD Comptt. 2015] [OD 2011] [S.Q.P. 2015-16]**

Sol. $$\begin{vmatrix}a+x & a-x & a-x\\a-x & a+x & a-x\\a-x & a-x & a+x\end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix}3a-x & 3a-x & 3a-x\\a-x & a+x & a-x\\a-x & a-x & a+x\end{vmatrix} = 0$$ **1**

Applying $C_2 \to C_2 - C_1$

$$\begin{vmatrix}3a-x & 0 & 3a-x\\a-x & 2x & a-x\\a-x & 0 & a+x\end{vmatrix} = 0$$ **1**

Applying $C_3 \to C_3 - C_1$

$$\begin{vmatrix}3a-x & 0 & 0\\a-x & 2x & 0\\a-x & 0 & 2x\end{vmatrix} = 0$$ **1**

Expanding along C_3

$4x^2(3a - x) = 0$

$\therefore \quad x = 0, 3a$ **1**

[CBSE Marking Scheme 2015]

Q. 4. Using properties of determinants, prove that :

$$\begin{vmatrix}1 & 1+p & 1+p+q\\3 & 4+3p & 2+4p+3q\\4 & 7+4p & 2+7p+4q\end{vmatrix} = 1$$

R&U **[OD 2009] [S.Q.P. 2015-16]**

Sol. Taking $$\text{L.H.S.} = \begin{vmatrix}1 & 1+p & 1+p+q\\3 & 4+3p & 2+4p+3q\\4 & 7+4p & 2+7p+4q\end{vmatrix}$$

Applying $R_2 \to R_2 - 3R_1$

$$= \begin{vmatrix}1 & 1+p & 1+p+q\\0 & 1 & -1+p\\4 & 7+4p & 2+7p+4q\end{vmatrix}$$ **1**

Applying $R_3 \to R_3 - 4R_1$

$$= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & -1+p \\ 0 & 3 & -2+3p \end{vmatrix} \quad 1$$

Applying $R_3 \to R_3 - 3R_2$

$$= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & -1+p \\ 0 & 0 & 1 \end{vmatrix} \quad 1$$

Expanding along R_3 $\begin{vmatrix} 1 & 1+P \\ 0 & 1 \end{vmatrix}$

$= 1 = \text{R.H.S.}$ 1

[CBSE Marking Scheme 2015]

Q. 5. Without expanding the determinant at any stage; prove that :

$$\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = 0.$$

R&U **[S.Q.P. 2015-16]**

Sol. Let $A = \begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix}$

Interchanging rows and columns

$$A = \begin{vmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} \quad 1½$$

Multiplying R_1, R_2 and R_3 by (– 1)

$$A = (-1)(-1)(-1)\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix}$$

1½

or $A = (-1)A$ or $2A = 0$ ½

or $A = 0$ ½

Hence proved

[CBSE Marking Scheme 2015]

Commonly Made Error

- Mostly candidates expand the determinant to solve it, though it is clearly determinants.

Answering Tips

- Give extensive practice in different types of question based on properties a of determinants.

Q. 6. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$**, find** $(A')^{-1}$**.**

A **[Delhi, 2015]**

Sol. Getting $A' = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ 1

then $|A'| = 1(-9) - 2(-5)$

$= -9 + 10 = 1 \neq 0$ ½

Hence A' is invertible.

Cofactors of A' are :

$C_{11} = -9, C_{21} = -8, C_{31} = -2$

$C_{12} = 8, \quad C_{22} = 7, \quad C_{32} = 2$

$C_{13} = -5, C_{23} = -4, C_{33} = -1$ 1

$$\text{adj } A' = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\therefore \quad (A')^{-1} = \frac{adj\,A'}{|A'|} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix} \quad 1½$$

[CBSE Marking Scheme 2015]

AI **Q. 7. Using properties of determinants, prove the following :**

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(bc+ca+ab).$$

R&U **[NCERT] [Delhi Comptt. 2011] [OD Comptt. 2013] [Delhi Set I Comptt. 2014]**

OR

Using properties of determinants, prove that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

[All India 2013C, Delhi 2011C]

Sol. $\Delta = \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$

Using $R_1 \to a\,R_1$, $R_2 \to b\,R_2$, $R_3 \to c\,R_3$

$$\Delta = \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} \quad ½$$

Taking abc common from C_3

$$\Delta = \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \quad ½$$

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} a^2 & a^3 & 1 \\ b^2-a^2 & b^3-a^3 & 0 \\ c^2-a^2 & c^3-a^3 & 0 \end{vmatrix} \quad 1½$$

Taking common $(b-a)$ from R_2, $(c-a)$ from R_3

$$\Delta = (b-a)(c-a)\begin{vmatrix} a^2 & a^3 & 1 \\ b+a & b^2+ab+a^2 & 0 \\ c+a & c^2+ac+a^2 & 0 \end{vmatrix} \quad ½$$

Expanding along C_3, we get

$$\Delta = (b-a)(c-a)\begin{vmatrix} b+a & b^2+ab+a^2 \\ c+a & c^2+ac+a^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$$\Delta = (b-a)(c-a)\begin{vmatrix} b+a & b^2+ab+a^2 \\ c-b & c^2-b^2+ac-ab \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} b+a & b^2+ab+a^2 \\ -(b-c) & -(b-c)(a+b+c) \end{vmatrix}$$

$$= (b-a)(c-a)(b-c)\begin{vmatrix} b+a & b^2+ab+a^2 \\ -1 & -(a+b+c) \end{vmatrix}$$

$$= -(b-a)(c-a)(b-c)\begin{vmatrix} a+b & a^2+b^2+ab \\ 1 & a+b+c \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)\{a^2+ab+ac+ab+b^2 + bc - a^2 - b^2 - ab\}$$

$= (a-b)(b-c)(c-a)(ab + bc + ac)$ **1**

[CBSE Marking Scheme 2014]

Q. 8. Using properties of determinants, prove that following :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

R&U [NCERT] [O.D. Set I Comptt. 2014]
[O.D. Set I, 2012]

Sol. $$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Using $R_1 \to R_1 - R_2 - R_3$

$$= \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ **1**

$R_2 \to cR_2$, $R_3 \to bR_3$ and taking common (-2) from R_1

$$= -\frac{2}{bc}\begin{vmatrix} 0 & c & b \\ bc & c^2+ac & bc \\ bc & bc & ab+b^2 \end{vmatrix}$$

Taking common bc from C_1, c from C_2 & b from C_3

$$= -2bc\begin{vmatrix} 0 & 1 & 1 \\ 1 & c+a & c \\ 1 & b & a+b \end{vmatrix}$$ **1**

$C_2 \to C_2 - C_3$

$$= -2bc\begin{vmatrix} 0 & 0 & 1 \\ 1 & a & c \\ 1 & -a & a+b \end{vmatrix}$$ **1**

Expanding along R_1, we get

$\Delta = -2bc(-2a)$

$\Delta = 4abc$ **1**

[CBSE Marking Scheme 2014]

Q. 9. Using properties of determinants, prove that

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z).$$

R&U [NCERT Exemplar] [Foreign Set I, 2014]

Sol. $$\Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Using $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$$

Taking common $(a + x + y + z)$ from C_1

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$ **1**

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix}$$ **2**

Expanding along C_1,

$\Delta = (a + x + y + z)(a^2 - 0)$

Hence, $\Delta = a^2(a + x + y + z)$ **1**

[CBSE Marking Scheme 2014]

Q. 10. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$

R&U [CBSE Delhi/Outside Delhi Set I, II, III 2018]

Sol. $$\text{LHS} = \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$$ **1**

$$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$$ **1**

(Using $C_3 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$)

$= 1 \times (9yz) + 3x(3z + 9yz + 3y)$

(Expanding along R_1) **1**

$= 9(3xyz + xy + yz + zx) = \text{RHS}$ **1**

[CBSE Marking Scheme 2018]

Commonly Made Error

- Students should know the properties of determinants. Before applying the properties of determinants, students directly expand the determinant, which is wrong.

Q.11. Using properties of determinants, prove that

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda - x)^2.$$

R&U [NCERT] [Delhi 2011, 2009]
[Foreign Set II, 2014]

Sol. $$\Delta = \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

Using $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 5x+\lambda & 2x & 2x \\ 5x+\lambda & x+\lambda & 2x \\ 5x+\lambda & 2x & x+\lambda \end{vmatrix}$$

Taking $(5x + \lambda)$ common from C_1

$$= (5x+\lambda)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+\lambda & 2x \\ 1 & 2x & x+\lambda \end{vmatrix} \quad \textbf{1}$$

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (5x+\lambda)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x+\lambda & 0 \\ 0 & 0 & -x+\lambda \end{vmatrix} \quad \textbf{2}$$

Expanding along C_1, we get

$\Delta = (5x + \lambda)\{(\lambda - x)^2 - 0\}$

$\Delta = (5x + \lambda)(\lambda - x)^2$ **1**

[CBSE Marking Scheme 2014]

Q.12. Prove the following :

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

R&U **[NCERT] [OD 2015] [Foreign Set III, 2014]**

Sol. $$\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking a, b and c common from C_1, C_2 and C_3 respectively

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} \quad \textbf{1}$$

Using $C_1 \to C_1 + C_2 - C_3$

$$= abc\begin{vmatrix} 0 & c & a+c \\ 2b & b & a \\ 2b & b+c & c \end{vmatrix} \quad \textbf{1}$$

Using $R_2 \to R_2 - R_3$

$$= abc\begin{vmatrix} 0 & c & a+c \\ 0 & -c & a-c \\ 2b & b+c & c \end{vmatrix} \quad \textbf{1}$$

Expanding along C_1,

$\Delta = abc[0 - 0 + 2b(ca - c^2 + ca + c^2)]$

or $\Delta = abc(4abc)$ or $\Delta = 4a^2b^2c^2$ **1**

[CBSE Marking Scheme 2014]

Q. 13. Using properties of determinants, prove that :

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3.$$

R&U **[NCERT] [Delhi Set I, 2014]**

OR

Prove, using properties of determinants

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$

[NCERT] [Foreign 2011]

Sol. $$\text{LHS} = \begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

Using $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} \quad \textbf{1}$$

Using $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} (x+y+z) & 0 & 0 \\ 2z & 0 & -(x+y+z) \\ x-y-z & (x+y+z) & (x+y+z) \end{vmatrix} \quad \textbf{2}$$

Expanding with respect to R_1,

$= (x + y + z)\{0(x + y + z) + (x + y + z)^2\}$ **1**

$= (x + y + z)^3$

[CBSE Marking Scheme 2014]

AI **Q. 14. Using properties of determinants, prove that :**

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

R&U **[NCERT] [Delhi Set II, 2014]**
[Delhi Set I Comptt. 2012]
[Foreign 2011, OD Comptt. 2009]

Sol. $$\Delta = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

Using $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix} \quad \textbf{1}$$

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & c+a+b \end{vmatrix} \quad \textbf{2}$$

Expanding along C_1,

$\Delta = 2(a + b + c)\{(a + b + c)^2 - 0\}$

$\Delta = 2(a + b + c)^3$ **1**

[CBSE Marking Scheme 2014]

Q. 15. Using properties of determinants, prove that :

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2.$$

R&U **[Delhi Set III, 2014], [NCERT]**

OR

Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$$

[OD Comptt. 2011] [Foreign 2009] [Foreign Set I, 2013]

Sol. $$\Delta = \begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix}$$

Using $R_1 \to xR_1, R_2 \to yR_2, R_3 \to zR_3$

$$= \frac{1}{xyz}\begin{vmatrix} x(x^2+1) & x^2y & x^2z \\ xy^2 & y(y^2+1) & y^2z \\ xz^2 & yz^2 & z(z^2+1) \end{vmatrix} \quad 1$$

Taking common x from C_1, y from C_2 & z from C_3

$$= \frac{xyz}{xyz}\begin{vmatrix} x^2+1 & x^2 & x^2 \\ y^2 & y^2+1 & y^2 \\ z^2 & z^2 & z^2+1 \end{vmatrix} \quad ½$$

Using $R_1 \to R_1 + R_2 + R_2$

$$= \begin{vmatrix} 1+x^2+y^2+z^2 & 1+x^2+y^2+z^2 & 1+x^2+y^2+z^2 \\ y^2 & y^2+1 & y^2 \\ z^2 & z^2 & z^2+1 \end{vmatrix} \quad 1$$

Using $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= \begin{vmatrix} (1+x^2+y^2+z^2) & 0 & 0 \\ y^2 & 1 & 0 \\ z^2 & 0 & 1 \end{vmatrix} \quad 1$$

Expanding along C_1

$$\Delta = (1 + x^2 + y^2 + z^2) \quad ½$$

[CBSE Marking Scheme 2014]

OR

Part is Same
Put $a = x, b = y$ and $c = z$
Rest same
Solution as above.

Q. 16. Using properties of determinants, prove that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$$

R&U **[O.D. Set I, 2014]**

[OD 2009] [O.D. Set I Comptt. 2013]

Sol. Operating $R_2 \to R_2 - 4R_1$ and $R_3 \to R_3 - 8R_1$, we get

$$\text{RHS} = \begin{vmatrix} x+y & x & x \\ x & 0 & -2x \\ 2x & 0 & -5x \end{vmatrix} \quad 2$$

Expanding along C_2, we get

$$= -x(-5x^2 + 4x^2) = x^3 \quad 1 + 1$$

[CBSE Marking Scheme 2014]

AI **Q. 17. Using properties of determinants, prove that :**

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

R&U **[NCERT] [OD Comptt. 2010][O.D. Set II, 2014] [Delhi Set I, 2012] [O.D. Set I Comptt. 2013]**

OR

Using properties of determinants, prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

[NCERT] [Delhi 2012]

Sol. Operating $C_1 \to C_1 - (C_2 + C_3)$, we get

$$\text{LHS} = \begin{vmatrix} -2a & c+a & a+b \\ -2p & r+p & p+q \\ -2x & z+x & x+y \end{vmatrix} \quad 1½$$

Taking (– 2) Common from C_1

$$= -2\begin{vmatrix} a & c+a & a+b \\ p & r+p & p+q \\ x & z+x & x+y \end{vmatrix} \quad ½$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\text{LHS} = -2\begin{vmatrix} a & c & b \\ p & r & q \\ x & z & y \end{vmatrix} \quad 1½$$

$$C_2 \leftrightarrow C_3 = +2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = \text{RHS} \quad ½$$

[CBSE Marking Scheme 2014]

Q. 18. Using properties of determinants, prove that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab.$$

R&U **[NCERT] [OD 2009] [Foreign 2016] [O.D. Set III 2014, 2012]**

Sol. $R_1 \to \frac{1}{a}R_1, R_2 \to \frac{1}{b}R_2, R_3 \to \frac{1}{c}R_3$

$$\therefore \quad \text{LHS} = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} \quad 1$$

$R_1 \to R_1 + R_2 + R_3$

$\Rightarrow$ LHS

$$= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} \quad 1$$

Taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from R_1

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} \quad ½$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix} \quad 1$$

Expand along R_1

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(1)$$

$= abc + bc + ca + ab = \text{RHS}$ ½

[CBSE Marking Scheme 2014]

Commonly Made Error

- Most of the candidates make errors while applying the properties of determinants in the correct order.

Answering Tips

- Elucidate all properties of determinants and their applications.

Q. 19. **If $a + b + c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then using properties of determinants, prove that $a = b = c$.**

R&U **[OD Comptt. 2017]**

[CBSE OD 2015][NCERT Exemplar] [S.Q.P. 2013]

Sol. $$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Using $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$ ½

Taking $a + b + c$ common from C_1

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$ ½

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$ 1

Expanding along C_1

$\Delta = (a + b + c)[(c - b)(b - c) - (a - c)(a - b)]$

or $\Delta = (a + b + c)[ab + bc + ca - a^2 - b^2 - c^2]$ ½

Multiplying and dividing by 2

$$\Delta = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$ ½

As $\Delta = 0$ and $(a + b + c) \neq 0$

thus $[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$ ½

$\therefore a - b = 0, b - c = 0, c - a = 0$ *i.e.*, $a = b = c$ ½

[CBSE Marking Scheme 2013]

Q. 20. **Using properties of determinants, prove that :**

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c).$$

R&U **[Delhi Set I Comptt. 2013]**

[Delhi Comptt. 2009]

OR

Using properties of determinants, prove that :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c).$$

[NCERT] [Delhi Comptt. 2011] [Delhi Set II, 2012]

Sol. $$\Delta = \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$$

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3-a^3 \\ 0 & c-a & c^3-a^3 \end{vmatrix}$$ 1

Taking $(b - a)$ and $(c - a)$ common from R_2 and R_3 respectively

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+a^2+ab \\ 0 & 1 & c^2+a^2+ac \end{vmatrix}$$ 1

Using $R_3 \to R_3 - R_2$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+a^2+ab \\ 0 & 0 & c^2-b^2+ac-ab \end{vmatrix}$$ ½

Taking $(c - b)$ from R_3

$$= (b-a)(c-a)(c-b)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+a^2+ab \\ 0 & 0 & a+b+c \end{vmatrix}$$ ½

Expanding with respect to C_1

$\Delta = (a - b)(b - c)(c - a)(a + b + c)$ 1

[CBSE Marking Scheme 2013]

Q. 21. **Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$ using determinants and find the value of k if $D(k, 0)$ is a point such that area of ΔABD is 3 square units.**

A **[NCERT] [O.D. Set I Comptt. 2013]**

Sol. Using determinants, the line joining $A(1, 3)$ and $B(0, 0)$ is given by $\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$ 1

Expanding along R_3, we get $1(3x - y) = 0$

or $y = 3x$ 1

Now, $D(k, 0)$ is a point such that area $\Delta ABD = 3$ sq. units

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = |3|$$ 1

Expanding along R_2

$(0 - 3k) = \pm 6$ ½

or $k = \pm 2$ ½

[CBSE Marking Scheme 2013]

Q. 22. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2.$$

R&U [NCERT] [Delhi Set I 2013]

OR

Using properties of determinants, prove that

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1 - a^3)^2.$$

[NCERT] [Foreign 2015, 2009]

Sol. $\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

Using $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix} \quad ½$$

Taking $(1 + x + x^2)$ common from C_1

$$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix} \quad ½$$

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x-x^2 \\ 0 & x^2-x & 1-x^2 \end{vmatrix} \quad 1$$

Taking $(1 - x)$ common from R_2 and R_3

$$= (1+x+x^2)(1-x)^2\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x \\ 0 & -x & 1+x \end{vmatrix} \quad 1$$

Expanding with respect to C_1

$\Delta = (1 + x + x^2)(1 - x)^2 (1 + x + x^2)$

$= \{(1 - x)(1 + x + x^2)\}^2$

$\Delta = (1 - x^3)^2$ 1

[CBSE Marking Scheme 2013]

Note : OR part is same.

Put $x = a$ & solve by same method

AI **Q. 23. Using properties of determinants, prove that following :**

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x + y).$$

R&U **[NCERT Exemplar] [Delhi 2017] [OD 2017] [O.D. Set I, 2013]**

Sol. $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$= 3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix} \quad 1$$

$R_1 \to R_1 - R_2$, $R_3 \to R_3 - R_2$

$$= 3(x+y)\begin{vmatrix} 0 & y & y \\ 1 & x & x+y \\ 0 & 2y & -y \end{vmatrix} \quad 1 + 1$$

$$= -3(x+y)(-y^2 - 2y^2) = 9y^2(x+y) \quad 1$$

[CBSE Marking Scheme 2017]

Q. 24. Using properties of determinants, prove the following :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1 + a^2 + b^2)^3.$$

R&U **[NCERT] [Delhi 2009] [Foreign Set III, 2013]**

Sol. $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

Using $C_1 \to C_1 - bC_3$, $C_2 \to C_2 + aC_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix} \quad 1$$

Taking $(1 + a^2 + b^2)$ common from C_1 and C_2

$$= (1+a^2+b^2)^2\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix} \quad ½$$

Using $R_3 \to R_3 + aR_2 - bR_1$

$$= (1+a^2+b^2)^2\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix} \quad 1$$

Taking $(1 + a^2 + b^2)$ common from R_3

$$= (1+a^2+b^2)^3\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & 0 & 1 \end{vmatrix} \quad ½$$

Expanding with respect to R_3

$\Delta = (1 + a^2 + b^2)^3(1)$

or $\Delta = (1 + a^2 + b^2)^3$ 1

[CBSE Marking Scheme 2013]

Commonly Made Error

- Sometimes candidates fail to take common factor $(1 + a^2 + b^2)^3$ and could not simplify the determinant.

Q. 25. Using properties of determinants, prove that following :

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

R&U **[Delhi Comptt. 2010]**
[Delhi Set II Comptt. 2012]

Sol. $$\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$

Using $R_1 \to R_1 + R_3$

$$= \begin{vmatrix} \alpha+\beta+\gamma & \gamma+\alpha+\beta & \alpha+\beta+\gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} \quad \frac{1}{2}$$

Taking $\alpha+\beta+\gamma$ common from R_1

$$= \alpha+\beta+\gamma \begin{vmatrix} 1 & 1 & 1 \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} \quad \frac{1}{2}$$

Using $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= (\alpha+\beta+\gamma) \begin{vmatrix} 1 & 0 & 0 \\ \alpha^2 & \beta^2-\alpha^2 & \gamma^2-\alpha^2 \\ \beta+\gamma & \alpha-\beta & \alpha-\gamma \end{vmatrix} \quad 1$$

Taking $(\beta - \alpha)$ and $(\gamma - \alpha)$ common from C_2 and C_3

$$= (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha) \begin{vmatrix} 1 & 0 & 0 \\ \alpha^2 & \beta+\alpha & \gamma+\alpha \\ \beta+\gamma & -1 & -1 \end{vmatrix} \quad 1$$

Expanding with respect to R_1

$\Delta = (\alpha + \beta + \gamma)(\beta - \alpha)(\gamma - \alpha)(- \beta - \alpha + \gamma + \alpha)$

$\Delta = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha + \beta + \gamma)$ **1**

[CBSE Marking Scheme 2012]

AI **Q. 26. Using properties of determinants, prove the following :**

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3.$$

R&U **[NCERT Exemplar] [SQP Dec. 2016-17]**
[O.D. Set I Comptt. 2012]

Sol. $$\Delta = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

Using $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} \quad \frac{1}{2}$$

Taking $(a + b + c)$ common from R_1

$$= (a+b+c) \begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} \quad \frac{1}{2}$$

Using $C_1 \to C_1 - 2C_3$

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ c+a-2b & b-c & b \\ a+b-2c & c-a & c \end{vmatrix} \quad 1$$

Using $C_1 \to C_1 + C_2$

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & b \\ b-c & c-a & c \end{vmatrix} \quad 1$$

Expanding with respect to R_1

$\Delta = (a + b + c)[(c - a)(a - b) - (b - c)^2]$

or $\Delta = (a + b + c)(- a^2 - b^2 - c^2 + ab + bc + ca)$ ½

or $\Delta = (3abc - a^3 - b^3 - c^3)$ ½

[CBSE Marking Scheme 2012]

Q. 27. Using properties of determinants, prove the following :

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$$

R&U **[NCERT Exemplar] [Delhi 2009]**
[Delhi Set III Comptt. 2012]

Sol. $$\Delta = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

Using $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix} \quad \frac{1}{2}$$

Taking $(a + b + c)$ common from C_1

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix} \quad \frac{1}{2}$$

Using $R_3 \to R_3 - 2R_1$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix} \quad 1$$

Using $R_3 \to R_3 + R_2$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & a-b & b-c \end{vmatrix} \quad 1$$

Expanding with respect to C_1

$\Delta = (a + b + c)[(b - c)^2 - (c - a)(a - b)]$

$\Delta = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ ½

$\Delta = (a^3 + b^3 + c^3 - 3abc)$ ½

[CBSE Marking Scheme 2012]

Q. 28. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

R&U **[O.D. Set II Comptt. 2012]**

Sol. Let $\Delta = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$

Using $C_2 \to C_2 - 2C_1 - 2C_3$

$$= \begin{vmatrix} a^2 & -a^2-b^2-c^2 & bc \\ b^2 & -a^2-b^2-c^2 & ca \\ c^2 & -a^2-b^2-c^2 & ab \end{vmatrix} \qquad 1$$

Taking $(a^2 + b^2 + c^2)$ common from C_2

$$= (a^2+b^2+c^2)\begin{vmatrix} a^2 & -1 & bc \\ b^2 & -1 & ca \\ c^2 & -1 & ab \end{vmatrix} \qquad 1$$

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (a^2+b^2+c^2)\begin{vmatrix} a^2 & -1 & bc \\ b^2-a^2 & 0 & c(a-b) \\ c^2-a^2 & 0 & b(a-c) \end{vmatrix} \qquad ½$$

Taking $(a - b)$ and $(c - a)$ common from R_2 and R_3 and multiplying C_2 by (-1)
Using $R_2 \to R_2 + R_3$

$$= (a-b)(c-a)(a^2+b^2+c^2)\begin{vmatrix} a^2 & 1 & bc \\ -a-b & 0 & c \\ c+a & 0 & -b \end{vmatrix} \qquad ½$$

$R_2 \to R_2 + R_3$

$$= (a-b)(c-a)(a^2+b^2+c^2)\begin{vmatrix} a^2 & 1 & bc \\ c-b & 0 & c-b \\ c+a & 0 & -b \end{vmatrix} \qquad ½$$

$$(a-b)(c-a)(c-b)(a^2+b^2+c^2)\begin{vmatrix} a^2 & 1 & bc \\ 1 & 0 & 1 \\ c+a & 0 & -b \end{vmatrix}$$

Expanding along C_2

$(a - b)(c - a)(c - b)(a^2 + b^2 + c^2)$

$[+1(-b-c-a)]$

$$= (a - b)(b - c)(c - a)(a^2 + b^2 + c^2)(a + b + c)]$$

[CBSE Marking Scheme 2012]

Q. 29. If $A + B + C = \pi$. then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

R&U **[SQP Dec. 2016-17]**

Sol. As $A + B + C = \pi$

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} \qquad 1$$

$$= \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix} \qquad ...(i)$$

$$= 0 \times \begin{vmatrix} 0 & \tan A \\ -\tan A & 0 \end{vmatrix} - \sin B \times \begin{vmatrix} -\sin B & \tan A \\ -\cos & 0 \end{vmatrix} \qquad 1$$

$$+ \cos C \times \begin{vmatrix} -\sin B & 0 \\ -\cos C & -\tan A \end{vmatrix}$$

$= 0 - \sin B \tan A \cos C + \cos C \sin B \tan A$ 1

$= 0$ ...(i) 1

[CBSE Marking Scheme 2016]

Q. 30. Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$. Find the other roots. R&U **[Delhi Set I, 2012]** **[NCERT Exemplar]**

Sol. LHS $= \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ ...(i)

(Expanding along R_1) **1**

or $x\begin{vmatrix} x & 2 \\ 6 & x \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 7 & x \end{vmatrix} + 7\begin{vmatrix} 2 & x \\ 7 & 6 \end{vmatrix} = 0$ **1**

or $x(x^2-12) - 3(2x-14) + 7(12-7x) = 0$

or $x^3 - 12x - 6x + 42 + 84 - 49x = 0$

$x^3 - 67x + 126 = 0$ ...(ii)

$x = -9$ is a root of (ii).

$\therefore$ $x + 9$ is a factor of $x^3 - 67x + 126$. **1**

Dividing $x^3 - 67x + 126$ by $x + 9$, the quotient is $x^2 - 9x + 14$.

$\therefore$ (ii) $(x + 9)(x^2 - 9x + 14) = 0$

Solving $x^2 - 9x + 14 = 0$, we get

$$x = \frac{9 \pm \sqrt{81-56}}{2}$$

$$= \frac{9 \pm 5}{2} = 2, 7.$$

$\therefore$ The other roots are 2 and 7. **1**

[CBSE Marking Scheme 2012]

AI **Q. 31. Using properties of determinants, prove that :**

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

R&U **[SQP 2018-19]**
[O.D. Marking Scheme 2012]

Sol. LHS $= \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}$ $C_1 \to aC_1$

$$= \frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ca & b & c-a \\ a^2-ab & b+a & c \end{vmatrix} \qquad 1$$

$C_1 \to C_1 + bC_2 + cC_3$

$$= \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2-ab & b+a & c \end{vmatrix}$$

Taking $(a_2 + b_2 + c_2)$ common from C_1

$$= \frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

$[R_2 \to R_2 - R_1]$
$[R_3 \to R_3 - R_1]$

$$= \frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix}$$

½+½

Expanding with respect to C_1

$$= \frac{(a^2+b^2+c^2)}{a}(-bc+a^2+ac+ba+bc)$$

$= (a^2 + b^2 + c^2)(a + b + c) = \text{RHS}$ **1**

[CBSE Marking Scheme 2018-19]

Q. 32. Using the properties of determinants, show that :

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$$

R&U **[NCERT] [Foreign Set I, 2012]**

Sol. $$\Delta = \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

Using $C_2 \to C_2 - C_1$

$$= \begin{vmatrix} a & b & a+b+c \\ 2a & a+2b & 4a+3b+2c \\ 3a & 3a+3b & 10a+6b+3c \end{vmatrix}$$ **1**

Using $C_3 \to C_3 - C_1 - C_2$

$$= \begin{vmatrix} a & b & c \\ 2a & a+2b & a+b+2c \\ 3a & 3a+3b & 4a+3b+3c \end{vmatrix}$$ **1**

Using $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$= \begin{vmatrix} a & b & c \\ 0 & a & a+b \\ 0 & 3a & 4a+3b \end{vmatrix}$$ **1**

Expanding with respect to C_1

$\Delta = a[a(4a + 3b) - 3a(a + b)]$

$= a[4a^2 + 3ab - 3a^2 - 3ab]$ ½

$= a^3$ ½

[CBSE Marking Scheme 2012]

Q. 33. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$

then find the value of $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}$.

R&U **[SQP 2017-18]**

Sol. Let $\Delta_1 = \begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix} = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$. **2**

Where C_{ij} = the cofactor of a_{ij} and a_{ij} = the (i, j)th element of determinant Δ

We know that $\begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix} = \Delta^2$ **1**

$\therefore \Delta_1 = \Delta^2 = (-4)^2 = 16$ **1**

[CBSE Marking Scheme 2017-18]

Q. 34. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

R&U **[OD Set I 2017] [NCERT Exemplar]**

Sol.

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+2a-2a-1 & 2a+1 & 1 \\ 2a+1-a-2 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix} \quad [C_1 \to C_1 - C_2]$$

$$= \begin{vmatrix} a^2-1 & 2a+1 & 1 \\ a-1 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= (a-1)\begin{vmatrix} a+1 & 2a+1 & 1 \\ 1 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$ [Taking $(a-1)$ common out of C_1]

$$= (a-1)\begin{vmatrix} a+1 & 2a-2 & 0 \\ 1 & a-1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$$ $\left[\begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{matrix}\right]$

$$= (a-1)\begin{vmatrix} a+1 & 2(a-1) & 0 \\ 1 & (a-1) & 0 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= (a-1)^2\begin{vmatrix} a+1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$$ [Taking $(a-1)$ common out of C_2]

$$= (a-1)^2\begin{vmatrix} a+1 & 2 \\ 1 & 1 \end{vmatrix}$$

$= (a-1)^2 (a+1-2)$ [On expanding the determinant]

$= (a-1)^2 (a-1)$

$= (a-1)^3$

Proved.

[Topper's Answer, 2017]

Q. 35. Using properties of determinants show that

$$\begin{vmatrix} 1 & 1 & 1+x \\ 1 & 1+y & 1 \\ 1+z & 1 & 1 \end{vmatrix} = xyz + yz + zx + xy.$$

R&U **[Foreign 2017]**

Sol. Since a negative sign is missing in the question, so the equality can not be proved. So, 4 marks may be given for genuine attempt.

[CBSE Marking Scheme 2017]

Detailed Answer :

From L.H.S. $\begin{vmatrix} 1 & 1 & 1+x \\ 1 & 1+y & 1 \\ 1+z & 1 & 1 \end{vmatrix}$

Apply $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_5$

$$= \begin{vmatrix} 0 & -x & 1+x \\ -y & y & 1 \\ z & 0 & 1 \end{vmatrix}$$

Now, expanding above

$$= 0\begin{vmatrix} y & 1 \\ 0 & 1 \end{vmatrix} - (-x)\begin{vmatrix} -y & 1 \\ z & 1 \end{vmatrix} + (1+x)\begin{vmatrix} -y & y \\ z & 0 \end{vmatrix}$$

$= 0 + x(-y-z) + (1+x)(0-zy)$

$= -xy - xz - zy - xyz$

$= -(xy + yz + zx + xyz)$

$=$ RHS

Q. 36. If $\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, **then using properties of determinants. find the value of** $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}$, **where** $x, y, z \neq 0$. R&U **[Delhi Comptt. 2017]**

Sol. Taking x, y, z common from C_1, C_2, C_3 respectively. we get

$$xyz\begin{vmatrix} \frac{a}{x} & \frac{b}{y}-1 & \frac{c}{z}-1 \\ \frac{a-1}{x} & \frac{b}{y} & \frac{c}{z}-1 \\ \frac{a}{x}-1 & \frac{b}{y}-1 & \frac{c}{z} \end{vmatrix} = 0$$ **1**

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} \frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2 & \frac{b}{y}-1 & \frac{c}{z}-1 \\ \frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2 & \frac{b}{y} & \frac{c}{z}-1 \\ \frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2 & \frac{b}{y}-1 & \frac{c}{z} \end{vmatrix} = 0$$ **1**

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix}1 & \frac{b}{y-1} & \frac{c}{z-1}\\ 1 & \frac{b}{y} & \frac{c}{z-1}\\ 1 & \frac{b}{y-1} & \frac{c}{z}\end{vmatrix}=0$$ ½

$R_2 \to R_2 - R_1 \cdot R_3 \to R_3 - R_1$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix}1 & \frac{b}{y-1} & \frac{c}{z-1}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=0$$ 1

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\cdot 1=0 \text{ or } \frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$ ½

[CBSE Marking Scheme 2017]

Answering Tips

- Properties of determinants and their applications.

Q. 37. Let $A = \begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}$, then verify the following :

$A(adj\ A) = (adj\ A)\ A = |A|I$, where I is the identity matrix of order 2. A **[S.Q.P. 2015-16]**

Sol. $A = \begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}$

then $adj\ A = \begin{bmatrix}4 & -3\\ -1 & -2\end{bmatrix}^T = \begin{bmatrix}4 & -1\\ -3 & -2\end{bmatrix}$ 1

Taking $A\ (adj\ A) = \begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}4 & -1\\ -3 & -2\end{bmatrix}$

$= \begin{bmatrix}-11 & 0\\ 0 & -11\end{bmatrix} = -11\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ 1

Taking $(adj\ A)\cdot A = \begin{bmatrix}4 & -1\\ -3 & -2\end{bmatrix}\begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}$

$= \begin{bmatrix}-11 & 0\\ 0 & -11\end{bmatrix} = -11\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ 1

Getting $|A| = \begin{vmatrix}-2 & 1\\ 3 & 4\end{vmatrix} = -11$

$\therefore$ $|A|I = -11\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}$ 1

$A\cdot(adj\ A) = (adj\ A)\cdot A = |A|I$

Hence proved.

[CBSE Marking Scheme 2015]

Q. 38. If $f(x)=\begin{vmatrix}a & -1 & 0\\ ax & a & -1\\ ax^2 & ax & a\end{vmatrix}$, using properties of determinants, find the value of $f(2x) - f(x)$. A **[Delhi 2015]**

Sol. Given, $f(x)=\begin{vmatrix}a & -1 & 0\\ ax & a & -1\\ ax^2 & ax & a\end{vmatrix}$

On taking a common from C_1, we get

$$f(x)=a\begin{vmatrix}1 & -1 & 0\\ x & a & -1\\ x^2 & ax & a\end{vmatrix}$$ 1

On applying $C_1 \to C_1 + C_2$, we get

$$f(x)=a\begin{vmatrix}0 & -1 & 0\\ x+a & a & -1\\ x^2+ax & ax & a\end{vmatrix}$$ 1

Now, on expanding along R_1, we get

$f(x) = a\ [1\ \{a\ (x + a) + 1\ (x^2 + ax)\}]$

$= a\ (ax + a^2 + x^2 + ax)$

$= a\ (x^2 + 2ax + a^2)$

$= a\ (x + a)^2$ 1

$\therefore$ $f(2x) = a(2x + a^2)$

[put $x = 2x$]

Now, $f(2x) - f(x) = a(2x + a)^2 - a(x + a)^2$

$= a[(2x + a)^2 - (x + a)^2]$

$= a[(2x + a + x + a)\ (2x + a - x - a)]$

[$\because$ $(a + b)(a - b) = a^2 - b^2$]

$= a[(3x + 2a)\ (x)]$

$= x(3x + 2a)a = ax\ (3x + 2a)$ 1

[CBSE Marking Scheme 2015]

Q. 39. Show that $\Delta = \Delta_1$, where

$$\Delta=\begin{vmatrix}Ax & x^2 & 1\\ By & y^2 & 1\\ Cz & z^2 & 1\end{vmatrix},\ \Delta_1=\begin{vmatrix}A & B & C\\ x & y & z\\ zy & zx & xy\end{vmatrix}.$$

A **[NCERT Exemplar]**
[All India 2014C]

Sol. Given, $\Delta=\begin{vmatrix}Ax & x^2 & 1\\ By & y^2 & 1\\ Cz & z^2 & 1\end{vmatrix}$

On taking x, y and z common from R_1, R_2 and R_3 respectively, we get

$$\Delta = xyz\begin{vmatrix}A & x & 1/x\\ B & y & 1/y\\ C & z & 1/z\end{vmatrix}$$ 1

Now, on applying $C_3 \to xyzC_3$, we get

$$\Delta = \frac{xyz}{xyz}\begin{vmatrix}A & x & yz\\ B & y & zx\\ C & z & xy\end{vmatrix} = \begin{vmatrix}A & x & yz\\ B & y & zx\\ C & z & xy\end{vmatrix}$$ 1

On interchanging rows and columns we get

$$\Delta_1=\begin{vmatrix}A & B & C\\ x & y & z\\ zy & zx & xy\end{vmatrix}$$ [$\because$ $|A'| = |A|$] 1½

or $\Delta = \Delta_1$ ½

Q. 40. Using properties of determinants, prove that

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

R&U [NCERT] [All India 2013]

Sol. To prove $\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$

$= 3(x+y+z)(xy+yz+zx)$

$$\text{LHS} = \begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$ **1**

On taking $(x + y + z)$ common from C_1, we get

$$\text{LHS} = (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 1 & 3y & x-y \\ 1 & y-z & 3z \end{vmatrix}$$ **1**

On applying $R_1 \to R_2 - R_1, R_3, \to R_3 - R_1$, we get

$$\text{LHS} = (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 0 & 2y+x & x-y \\ 0 & x-z & 2z+x \end{vmatrix}$$ **1**

Now, on expanding along C_1, we get

$\text{LHS} = (x+y+z)\cdot 1\cdot\{(2y+x)(2z+x) - (x-y)(x-z)\}$

$= (x+y+z)\{4yz + 2xz + 2xy + x^2 - x^2 + xy + zx - yz\}$

$= (x+y+z)\cdot(3xy + 3yz + 3zx)$

$= 3(x+y+z)\cdot(xy+yz+zx) = \text{RHS}$ **1**

Hence proved.

Q. 41. Using properties of determinants, prove that

$$\begin{vmatrix} 5a & -2a+b & -2a+c \\ -2b+a & 5b & -2b+c \\ -2c+a & -2c+b & 5c \end{vmatrix} = 12(a+b+c)(ab+bc+ca)$$

R&U [Comptt. 2018 Set I, II, III]

Sol. $C_1 \to C_1 + C_2 + C_3$ gives LHS as

$$\begin{vmatrix} a+b+c & -2a+b & -2a+c \\ a+b+c & 5b & -2b+c \\ a+b+c & -2c+b & 5c \end{vmatrix}$$ **1**

Taking common $(a+b+c)$ from C_1,

$$= (a+b+c)\begin{vmatrix} 1 & -2a+b & -2a+c \\ 1 & 5b & -2b+c \\ 1 & -2c+b & 5c \end{vmatrix}$$ ½

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ gives

$$= (a+b+c)\begin{vmatrix} 1 & -2a+b & -2a-2b \\ 0 & 2a+4b & 2a-2b \\ 0 & 2a-2c & 4c+2a \end{vmatrix}$$ **1**

On expanding along C_1, we get

$$= (a+b+c)\begin{vmatrix} 2a+4b & 2a-2b \\ 2a-2c & 4c+2a \end{vmatrix}$$ ½

$$= 4(a+b+c)\begin{vmatrix} a+2b & a-b \\ a-c & 2c+a \end{vmatrix}$$

$= 12(a+b+c)(ab+bc+ac)$ ½+½

[CBSE Marking Scheme 2018]

Q. 42. Using properties of determinants, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$

R&U [Delhi 2011; All India 2011C]

Sol. To prove $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

$$\text{LHS} = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

On taking a, b and c common from R_1, R_2 and R_3 respectively, we get

$$\text{LHS} = abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$ **1**

Again, on taking a, b and c common from C_1, C_2 and C_3 respectively, we get

$$\text{LHS} = a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$ **1**

On applying $C_1 \to C_1 + C_2$, we get

$$\text{LHS} = a^2b^2c^2\begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$ **1**

$= a^2b^2c^2\,[2(1+1)]$ [expanding along C_1]

$= 4a^2b^2c^2 = \text{RHS}$ **Hence proved. 1**

Q. 43. Using properties of determinants, prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

R&U [Delhi 2011, 2010C]

Sol. To prove

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

$$\text{LHS} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$ ½

[taking x, y and z common from C_1, C_2 and C_3, respectively]

On applying $C_1 \to C_1 - C_2$ and then $C_2 \to C_2 - C_3$, we get

$$\text{LHS} = xyz\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix}$$ **1½**

On expanding along R_1, we get

$$\text{LHS} = xyz\begin{vmatrix} x-y & y-z \\ x^2-y^2 & y^2-z^2 \end{vmatrix}$$ **1**

On taking $(x-y)$ common from C_1 and $(y-z)$ from C_2, we get

$$\text{LHS} = xyz(x-y)(y-z)\begin{vmatrix} 1 & 1 \\ x+y & y+z \end{vmatrix}$$

$= xyz\,(x-y)(y-z)[(y+z-(x+y))]$

$= xyz\,(x-y)(y-z)(z-x)$ **1**

$=$ RHS **Hence proved.**

Q. 44. Using properties of determinants, solve the following for x

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$$ R&U **[All India 2011]**

Sol. Given,

$$\Rightarrow \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

On applying $R_1 \to R_1 - R_2$ and then $R_2 \to R_2 - R_3$, we get

$$\Rightarrow \begin{vmatrix} 2 & 6 & 12 \\ 4 & 18 & 48 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$ **1½**

On taking 2 common from R_1 and R_2, we get

$$\Rightarrow 2\times 2\begin{vmatrix} 1 & 3 & 6 \\ 2 & 9 & 24 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$ **½**

On applying $R_2 \to R_2 - 2R_1$, we get

$$\Rightarrow 4\begin{vmatrix} 1 & 3 & 6 \\ 0 & 3 & 12 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$ **1**

On expanding along C_1, we get

$$4\begin{vmatrix} 1 & 3 & 6 \\ 0 & 3 & 12 \\ x-8 & 2x-27 & 3x-6x \end{vmatrix} = 0$$

$C_3 \to C_3 - 4C_2$

$$4\begin{vmatrix} 1 & 3 & -6 \\ 0 & 3 & 0 \\ x-8 & 2x-27 & -5x+44 \end{vmatrix} = 0$$

on expanding along R_2

$$4\times 3\begin{vmatrix} 1 & -6 \\ x-8 & -5x+44 \end{vmatrix} = 0$$

$12(-5x+44+6x-48) = 0$

$12\,(x-4) = 0$

$x-4 = 0$

$x = 4.$ **1**

Q. 45. Using properties of determinants, solve the following for x.

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

R&U **[NCERT] [All India 2011]**

Sol. Given, $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix} = 0$$ **1**

On taking $(3x+a)$ common from C_1, we get

$$(3x+a)\begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix} = 0$$ **½**

Now, on applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$(3x+a)\begin{vmatrix} 1 & x & x \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = 0$$ **1½**

On expanding along C_1, we get

$(3x+a)(1\cdot a\cdot a) = 0$

$\Rightarrow \quad a^2(3x+a) = 0$

$\therefore \quad x = -\dfrac{a}{3}$ **1**

Q. 46. Find the adjoint of the matrix

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$ **and hence show that**

$A(adj\,A) = |A|I_3$. R&U **[All India 2015]**

Sol. Given, $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Let A_{ij} be the cofactor of an element a_{ij} of $|A|$.
Then, cofactor of elements of $|A|$ are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = (1-4) = -3$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2+4) = -6$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = (-4-2) = -6$$ **½**

$$A_{21} = (-1)^{2+1}\begin{vmatrix}-2 & -2\\-2 & 1\end{vmatrix} = -(-2-4) = 6$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix}-1 & -2\\2 & 1\end{vmatrix} = (-1+4) = 3$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix}-1 & -2\\2 & -2\end{vmatrix} = -(4+2) = -6 \quad ½$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix}-2 & -2\\1 & -2\end{vmatrix} = (2+4) = 6$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix}-1 & -2\\2 & -2\end{vmatrix} = (-2+4) = -6$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix}-1 & -2\\2 & 1\end{vmatrix} = (-1+4) = 3 \quad ½$$

Clearly, the adjoint of the matrix A is given by

$$adj\,A = \begin{bmatrix}A_{11} & A_{21} & A_{31}\\A_{12} & A_{22} & A_{32}\\A_{13} & A_{23} & A_{33}\end{bmatrix} = \begin{bmatrix}-3 & 6 & 6\\-6 & 3 & -6\\-6 & -6 & 3\end{bmatrix}$$

½

Now, $$|A| = \begin{vmatrix}-1 & -2 & -2\\2 & 1 & -2\\2 & -2 & 1\end{vmatrix}$$

$$= -1(1-4) + 2(2+4) - 2(-4-2)$$
$$= -1(-3) + 2(6) - 2(-6)$$
$$= 3 + 12 + 12 = 27 \quad 1$$

and $$A.(adj\,A) = \begin{bmatrix}-1 & -2 & -2\\2 & 1 & -2\\2 & -2 & 1\end{bmatrix}\begin{bmatrix}-3 & 6 & 6\\-6 & 3 & -6\\-6 & -6 & 3\end{bmatrix}$$

$$= \begin{bmatrix}3+12+12 & -6-6+12 & -6+12-6\\-6-6+12 & 12+3+12 & 12-6-6\\-6+12-6 & 12-6-6 & 12+12+3\end{bmatrix}$$

$$= \begin{bmatrix}27 & 0 & 0\\0 & 27 & 0\\0 & 0 & 27\end{bmatrix} = 27\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$= 27I_3 = |A|I_3$ **Hence proved. 1**

Q. 47. If a, b, c are real numbers, then prove that

$$\begin{vmatrix}a & b & c\\b & c & a\\c & a & b\end{vmatrix} = -(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega),$$

where ω is a complex number and cube root of unity. AE

Sol. $$\Delta = \begin{vmatrix}a & b & c\\b & c & a\\c & a & b\end{vmatrix}$$

$$= \begin{vmatrix}a+b+c & b & c\\b+c+a & c & a\\c+a+b & a & b\end{vmatrix}$$

[Applying $C_1 \to C_1 + C_2 + C_3$]

$$= (a+b+c)\begin{vmatrix}1 & b & c\\1 & c & a\\1 & a & b\end{vmatrix} \quad 1$$

[Taking out $(a + b + c)$ from C_1]

$$= (a+b+c)\begin{vmatrix}1 & b & c\\0 & c-b & a-c\\0 & a-b & b-c\end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$$= (a+b+c)\begin{vmatrix}c-b & a-c\\a-b & b-c\end{vmatrix} \quad 1$$

[Expanding along C_1]

$= (a + b + c)\{-(b - c)^2 - (a - c)(a - b)\}$

LHS $= -(a + b + c)\{(a^2 + b^2 + c^2 - ab - bc - ca)$

Also RHS $= -(a + b + c)(a + bw + cw^2)(a + bw^2 + cw)$

$= -(a + b + c)(a^2 + abw^2 + acw + abw$ **1**

$+ b^2w^3 + bcw^2 + acw^2 + bcw^4 + c^2w^3)$

$= -(a + b + c)[(a^2 + b^2 + c^2 + ab(w^2 + w)$

$+ bc(w^2 + w^4) + ca(w + w^2)]$

$= -(a + b + c)[(a^2 + b^2 + c^2 - ab - bc - ca)$

$=$ LHS **1**

Q. 48. Without expanding, evaluate the determinant :

$$\begin{vmatrix}(a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1\\(a^y+a^{-y})^2 & (a^y-a^{-y})^2 & 1\\(a^z+a^{-z})^2 & (a^z-a^{-z})^2 & 1\end{vmatrix},$$

where $a > 0$ and $x, y, \in R$. C

Sol. Let Δ be the given determinant.

Applying $C_1 \to C_1 - C_2$, we get

$$\Delta = \begin{vmatrix}4 & (a^x-a^{-x}) & 1\\4 & (a^y-a^{-y})^2 & 1\\4 & (a^z-a^{-z})^2 & 1\end{vmatrix} \quad 2$$

[Using $(a + b)^2 - (a - b)^2 = 4ab$]

Taking out 4 from C_1, we get

$$\Delta = 4\begin{vmatrix}1 & (a^x-a^{-x})^2 & 1\\1 & (a^y-a^{-y})^2 & 1\\1 & (a^z-a^{-z})^2 & 1\end{vmatrix} \quad 2$$

⇒ $\Delta = 4 \times 0 = 0$[∵ C_1 and C_3 are identical]

Q. 49. Let $f(t) = \begin{vmatrix}\cos t & t & 1\\2\sin t & t & 2t\\\sin t & t & t\end{vmatrix}$**, then find** $\lim_{t\to 0}\frac{f(t)}{t^2}$.

AE **[NCERT Exemplar]**

Sol. Given,

$$f(t) = \begin{vmatrix}\cos t & t & 1\\2\sin t & t & 2t\\\sin t & t & t\end{vmatrix} = \begin{vmatrix}\cos t & t & 1\\0 & -t & 0\\\sin t & t & t\end{vmatrix}$$

[Applying $R_2 \to R_2 - 2R_3$]

$$= t\begin{vmatrix}\cos t & 1 & 1\\0 & -1 & 0\\\sin t & 1 & t\end{vmatrix} \quad 2$$

Expanding an along R_2, we get

$t[(-1)(t\cos t - \sin t)]$

$= -t^2\cos t + t\sin t$

$$\therefore \lim_{t\to 0}\frac{f(t)}{t^2} = \lim_{t\to 0}\frac{-t^2\cos t + t\sin t}{t^2}$$

$$= \lim_{t\to 0}\left(\frac{-t^2\cos t}{t^2} + \frac{t\sin t}{t^2}\right)$$

$$= \lim_{t\to 0}\left(-\cos t + \frac{\sin t}{t}\right) = -1 + \lim_{t\to 0}\frac{\sin t}{t}$$

$= -1 + 1 = 0$ **2**

Long Answer Type Questions-II

(6 marks each)

AI Q. 1. Using properties of determinants, prove that :

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

R&U **[OD Set I 2016]**

Sol.

TP: $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

Proof : $C_1 \to C_1 - 2C_3$

$\Delta = \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ac \\ a^2+b^2 & c^2 & ab \end{vmatrix}$

$C_1 \to C_1 + C_2$

Now taking $a^2+b^2+c^2$ common from C_1

$\Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ac \\ 1 & c^2 & ab \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2-a^2 & ac-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$

Taking $(a-b)$ common from R_2 & $(c-a)$ common from C_3

$\Delta = (a^2+b^2+c^2)(a-b)(c-a)\begin{vmatrix} 1 & a^2 & bc \\ 0 & -(a+b) & c \\ 0 & a+c & -b \end{vmatrix}$

Expanding along C_1

$\Delta = (a^2+b^2+c^2)(a-b)(c-a)\,(-(a+b)\times b + (a+c)c)$

$= (a^2+b^2+c^2)(a-b)(c-a)\,(b^2 - c^2 + bc - ba)$

$= (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$ H.P

As $(b-c)(a+b+c) = ab + b^2 + bc - ac - bc - c^2$

$b^2 - c^2 + ab - ac$

[Topper's Answer, 2016]

Q. 2. Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ **is divisible by $(x+y+z)$, and hence find the quotient.**

R&U **[Delhi Set I, II, III, 2016]**

Sol. $= \begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ **1**

$$= \begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ (x-y)(x+y+z) & (y-z)(x+y+z) & (z-x)(x+y+z) \\ (x-z)(x+y+z) & (y-x)(x+y+z) & (z-y)(x+y+z) \end{vmatrix}$$

Taking $(x+y+z)$ common from R_2 and R_3

$$= (x+y+z)^2 \begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ x-y & y-z & z-x \\ x-z & y-x & z-y \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ **1**

$= (x+y+z)^2$

$$\begin{vmatrix} (xy+yz+zx)-(x^2+y^2+z^2) & zx-y^2 & xy-z^2 \\ 0 & y-z & z-x \\ 0 & y-x & z-y \end{vmatrix}$$

Expanding along C_1

$= (x+y+z)^2[xy+yz+zx-(x^2+y^2+z^2)]$ **1½**

$[(y-z)(z-y)-(z-x)(y-x)]$

$= (x+y+z)^2[xy+yz+zx-(x^2+y^2+z^2)]$ **1½**

$[yz-y^2-z^2+yz-yz+xz+xy-x^2]$

$= (x+y+z)^2[xy+yz+zx-(x^2+y^2+z^2)]^2$

Hence, it is divisble by $(x+y+z)$ and the quotient is $(x+y+z)[xy+yz+zx-(x^2+y^2+z^2)]^2$. **1**

AI **Q. 3. Using properties of determinants, prove that :**

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3.$$

R&U **[Outside Delhi Comptt. Set I, II, III 2016]**

Sol. LHS $= \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix}$

Apply $(R_1 \to xR_1, R_2 \to yR_2, R_3 \to zR_3)$ **1**

$$= \frac{1}{xyz}\begin{vmatrix} x(y+z)^2 & x^2y & zx^2 \\ xy^2 & y(x+z)^2 & y^2z \\ xz^2 & yz^2 & z(x+y)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$$ **1**

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= \begin{vmatrix} (y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2 \\ y^2 & (x+z)^2-y^2 & 0 \\ z^2 & 0 & (x+y)^2-z^2 \end{vmatrix}$$ **1**

Taking $(x+y+z)$ common from C_2 and C_3.

$$= (x+y+z)^2 \begin{vmatrix} (y+z)^2 & x-y-z & x-y-z \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

Apply, $R_1 \to R_1 - R_2 - R_3$ **½**

$$= (x+y+z)^2 \begin{vmatrix} 2yz & -2z & -2y \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

Apply, $C_2 \to yC_2, C_3 \to zC_3$ **½**

$$= \frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & -2yz & -2yz \\ y^2 & xy+zy-y^2 & 0 \\ z^2 & 0 & zx+yz-z^2 \end{vmatrix}$$

Apply, $C_2 \to C_2 + C_1$ & $C_3 \to C_3 + C_1$ **1**

$$= \frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & 0 & 0 \\ y^2 & xy+zy & y^2 \\ z^2 & z^2 & zx+yz \end{vmatrix}$$

$$= \frac{(x+y+z)^2}{yz}\cdot\left|(2yz)\cdot(x^2yz+xy^2z+xyz^2+y^2z^2-y^2z^2)\right|$$

$= 2xyz(x+y+z)^3 =$ RHS **1**

[CBSE Marking Scheme 2016]

Commonly Made Error

- Sometimes candidates make errors while applying the properties of determinant in the correct order.

Q. 4. Using properties of determinants, prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ac)^3.$$

R&U **[Delhi Set I, II, III Comptt. 2016]**

Sol. $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3,$

LHS $= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$ **1½**

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$ **½**

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ac)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$ **1**

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= (ab+bc+ac)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & 0 \\ ac+bc & 0 & -(ab+bc+ca) \end{vmatrix}$$

1 + 1

Expanding along R_1

$= (ab+bc+ca)^3$. **1**

[CBSE Marking Scheme 2016]

Q. 5. $A(adj\ A) = (adj\ A)A = |A|\ I$ for matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$. R&U **[S.Q.P. Dec. 2016-17]**

Sol. Here, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$

$= 1(0 + 0) + 1(9 + 2) + 2(0 – 0)$
$= 11$ **1**

$\Rightarrow \quad |A|I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ ...(i) ½

$adj\ A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$ **2**

Now, $A(adj\ A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ ...(ii) **1**

and $(adj\ A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$ **1**

$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ ...(iii) ½

From (i), (ii) and (iii)
Thus, it is verified that $A(adj\ A) = (adj\ A)A = |A|I$
[CBSE Marking Scheme 2016]

Q. 6. Find the value of x, y and z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$. R&U **[SQP Dec. 2016-17]**

Sol. The relation $A' = A^{-1}$ gives $A'A = A^{-1}A = I$ **1**

Thus, $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 1½

$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **2**

$\Rightarrow \quad 2x^2 = 1;\ 6y^2 = 1 \text{ and } 3z^2 = 1$

$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}};\ y = \pm\frac{1}{\sqrt{6}};$

$z = \pm\frac{1}{\sqrt{3}};$ 1½

[CBSE Marking Scheme 2016]

Q. 7. Using properties of determinants, prove that

$$\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3.$$

R&U **[SQP Dec. 2016-17]**

Sol. LHS $= \frac{1}{abc}\begin{vmatrix} (a+b)^2 & c^2 & c^2 \\ a^2 & (b+c)^2 & a^2 \\ b^2 & b^2 & (c+a)^2 \end{vmatrix}$

$C_1 \to C_1 - C_3,\ C_2 \to C_2 - C_3$ ½

$= \frac{1}{abc}\begin{vmatrix} (a+b+c)(a+b-c) & 0 & c^2 \\ 0 & (b+c+a)(b+c-a) & a^2 \\ (b+c+a)(b-c-a) & (b+c+a)(b-c-a) & (c+a)^2 \end{vmatrix}$

1

$= \frac{(a+b+c)^2}{abc}\begin{vmatrix} (a+b-c) & 0 & c^2 \\ 0 & (b+c-a) & a^2 \\ (b-c-a) & (b-c-a) & (c+a)^2 \end{vmatrix}$ ½

$R_3 \to R_3 - R_1 - R_2$

$= \frac{(a+b+c)^2}{abc}\begin{vmatrix} (a+b-c) & 0 & c^2 \\ 0 & (b+c-a) & a^2 \\ (-2a) & (-2c) & 2ca \end{vmatrix}$

taking common $2ac$ from R_3

$= \frac{(2ac)(a+b+c)^2}{ba^2c^2}\begin{vmatrix} ac+bc-c^2 & 0 & c^2 \\ 0 & ab+ca-a^2 & a^2 \\ -1 & -1 & 1 \end{vmatrix}$ ½

$(C_1 \to C_1 + C_3,\ C_2 \to C_2 + C_3)$

$= \frac{2ac(a+b+c)^2}{ba^2c^2}\begin{vmatrix} (ac+bc) & c^2 & c^2 \\ a^2 & (ba+ca) & a^2 \\ 0 & 0 & 2ca \end{vmatrix}$ 1

Taking c from R_1 & a from R_2

$= \frac{2ac(a+b+c)^2}{(ac)b}\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ 0 & 0 & 1 \end{vmatrix}$ 1

Expanding along R_3

$= \frac{2ac(a+b+c)^2}{abc}\left[ab+ac+b^2+bc-ac\right]$

$= \frac{2abc(a+b+c)^3}{abc}$

$= 2(a+b+c)^3$ ½

[CBSE Marking Scheme 2016]

Q. 8. If $p \neq 0$, $q \neq 0$ and $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, then using properties of determinants, prove that at least one of the following statements is true :

(a) p, q, r are in G.P.,

(b) α is a root of the equation $px^2 + 2qx + r = 0$.

R&U [SQP 2016-17]

Sol. Given equation, $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$ **1**

Given equation $\Rightarrow \dfrac{1}{pq}\begin{vmatrix} pq & q^2 & pq\alpha+q^2 \\ pq & pr & pq\alpha+pr \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$

$(R_1 \to R_1 - R_2)$ **1**

$\Rightarrow \dfrac{1}{pq}\begin{vmatrix} 0 & q^2-pr & q^2-pr \\ pq & pr & pq\alpha+pr \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$ **1**

On Taking Common $q^2 - pr$ from R_1

$\Rightarrow \dfrac{q^2-pr}{pq}\begin{vmatrix} 0 & 1 & 1 \\ pq & pr & pq\alpha+pr \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$ **1**

On Taking Common P from R_2

$\Rightarrow \dfrac{q^2-pr}{pq}p\begin{vmatrix} 0 & 1 & 1 \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$ **1**

$(C_2 \to C_2 - C_3)$

$\Rightarrow \dfrac{q^2-pr}{pq}p\begin{vmatrix} 0 & 0 & 1 \\ q & -q\alpha & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$ **1**

Expanding along R_1

$\Rightarrow \dfrac{q^2-pr}{q}(q^2\alpha + rq + pq\alpha^2 + q^2\alpha) = 0$

$\Rightarrow \quad (q^2 - pr)(2q\alpha + r + p\alpha^2) = 0$

$\Rightarrow \quad q^2 - pr = 0$

(*i.e.*, p, q, r are in GP) or $2q\alpha + r + p\alpha^2 = 0$

(*i.e.*, α is root or the equation $(2qx + r + px^2 = 0)$)

[CBSE Marking Scheme 2016]

Q. 9. If a, b, c are positive and unequal, show that the following determinant is negative.

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

R&U [All India 2010] [NCERT]

Sol. Given, $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

On applying $R_1 \to R_2 + R_3$, we get

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$ **1**

On taking common $(a + b + c)$ from R_1, we get

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$ **½**

On applying $C_2 \to C_2 - C_1$ and $C_3 - C_1$, we get

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ b & c-b & a-b \\ c & a-c & b-c \end{vmatrix}$$ **1**

On expanding along R_1, we get

$\Delta = (a + b + c)\,[-(b-c)^2 - (a-b)(a-c)]$

$= (a + b + c)\,[-(b^2 + c^2 - 2bc) - (a^2 - ac - ab + bc)]$

$= (a + b + c)\,[-b^2 - c^2 - 2bc + a^2 + ac + ab - bc]$

$= (a + b + c)\,(ab + bc + ca - a^2 - b^2 - c^2)$

$= -(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca)$ **1**

$= -\dfrac{1}{2}(a + b + c)\,(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$ **1**

[multiplying and divide by 2]

$= \dfrac{-1}{2}(a + b + c)\,\{(a-b)^2 + (b-c)^2 + (c-a)^2\} < 0$ **1**

$\therefore \Delta < 0$ $\quad [\because a, b, c > 0$ and $a \neq b \neq c]$

Hence proved. ½

[CBSE marking Scheme 2010]

Q. 10. Using properties of determinants, prove that following

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1 + pxyz)(x-y)(y-z)(z-x)$$

R&U [NCERT] [All India 2010]

Sol. To prove $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}$

$= (1 + pxyz)(x-y)(y-z)(z-x)$

$$\text{LHS} = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix}$$ **1**

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$ **1**

[Taking common p from C_3, x from R_1, y from R_2 and z from R_3]

On interchanging C_1 and C_3 is Ist determinant, we get

$$\text{LHS} = \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

On interchanging C_2 and C_3 in Ist determinant, we get ½

$$\text{LHS} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad ½$$

$$\Rightarrow (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

1

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = (1 + pxyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & (y-x)(y+x) \\ 0 & z-x & (z-x)(z+x) \end{vmatrix}$$

$$= (y-x)(z-x)(1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$$

[Taking common $y - x$ from R_2 and $z - x$ from R_3]

$$= (y-x)(z-x)(1+pxyz)(z+x-y-x)$$

1

$$= (1+pxyz)(x-y)(y-z)(z-x) \quad \textbf{1}$$

Hence proved.

Q. 11. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **find *adj A* and verify that *A* (*adj A*) = (*adj A*) *A* = |*A*| I_3.**

R&U [Foreign 2015]

Sol. We have, $A = \Delta \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Clearly, the cofactors of elements of $|A|$ are given by

$A_{11} = \cos\alpha$; $A_{12} = -\sin\alpha$; $A_{13} = 0$;
$A_{21} = \sin\alpha$; $A_{22} = \cos\alpha$; $A_{23} = 0$
$A_{31} = 0$; $A_{32} = 0$ and $A_{33} = 1$ **2**

$$\therefore \quad adj(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}'$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{1}$$

Now, A $(adj\ A)$

$$= \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\alpha+\sin^2\alpha & 0 & 0 \\ 0 & \sin^2\alpha+\cos^2\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ...(i)\ ½$$

$$(adj\ A)\cdot(A) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\alpha+\sin^2\alpha & 0 & 0 \\ 0 & \sin^2\alpha+\cos^2\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ½$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ...(ii)\ ½$$

and $$|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 1\cdot(\cos^2\alpha + \sin^2\alpha) = 1 \quad ...(iii)\ ½$$

(expanding along R_3)

From Eqs. (i), (ii) and (iii), we get

$$A\ (adj\ A) = (adj\ A)\cdot A = |A|\ I_3 \quad \textbf{1}$$

TOPIC-2
Solutions of System of Linear Equations

Revision Notes

SOLVING SYSTEM OF EQUATIONS BY MATRIX METHOD [INVERSE MATRIX METHOD

(a) Consistent and Inconsistent system : A system of equations is consistent if it has one or more solutions otherwise it is said to be an inconsistent system. In other words an inconsistent system of equations has no solution.

(b) Homogeneous and Non-homogeneous system : A system of equations $AX = B$ is said to be a homogeneous system if $B = 0$. Otherwise it is called a non-homogeneous system of equations.

$$a_1x + b_1y + c_1z = d_1,$$
$$a_2x + b_2y + c_2z = d_2,$$
$$a_3x + b_3y + c_3z = d_3$$

STEP 1 : Assume $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

STEP 2 : Find $|A|$. Now there may be following situations :

(i) $|A| \neq 0 \Rightarrow A^{-1}$ exists. It implies that the given system of equations is consistent and therefore, the system has **unique solution**. In that case, write

$$AX = B$$

$$\Rightarrow \quad X = A^{-1}B \quad \left[\text{where } A^{-1} = \frac{1}{|A|}(adj\, A)\right]$$

Then by using the definition of equality of matrices, we can get the values of x, y and z.

(ii) $|A| = 0 \Rightarrow A^{-1}$ does not exist. It implies that the given system of equations may be consistent or inconsistent. In order to check proceed as follow :

$\Rightarrow$ Find $(adj\ A)\ B$. Now, we may have either $(adj\ A)\ B \neq 0$ or $(adj\ A)\ B = 0$.

- If $(adj{\cdot}A)B = 0$, then the given system may be consistent or inconsistent.

  To check, put $z = k$ in the given equations and proceed in the same manner in the new two variables system of equations assuming $d_i - c_ik$, $1 \leq i \leq 3$ as constant.
- And if $(adj\ A)\ B \neq 0$, then the given system is inconsistent with no solutions.

Very Short Answer Type Questions (1 mark each)

Q. 1. For what values of k, the system of linear equations

$$x + y + z = 2$$
$$2x + y - z = 3$$
$$3x + 2y + kz = 4$$

has a unique solution ? A **[Outside Delhi 2016]**

Sol. For a unique solution

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \qquad 1$$

$$k + 2 + 2k + 3 + 4 - 3 \neq 0$$

$$\Rightarrow \quad k \neq -2$$

[CBSE Marking Scheme 2016]

Short Answer Type Questions (2 mark each)

Q. 1. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$. A **[Delhi 2018, Set I, II, III]**

Sol. $|A| = 2,$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \qquad 1$$

$$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix},$$

$$\text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

LHS = RHS 1

[CBSE Marking Scheme 2018]

Q. 2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, be such that $A^{-1} = kA$, then find the value of k. A **[Comptt. 2018, Set I, II, III]**

Sol. Finding $A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$ 1

$$\Rightarrow \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2k & 3k \\ 5k & -2k \end{bmatrix} \qquad ½$$

$$\Rightarrow \quad k = \frac{1}{19} \qquad ½$$

[CBSE Marking Scheme 2018]

Long Answer Type Questions-I (4 marks each)

Q. 1. If $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ then using A^{-1}, solve the following system of equations : $x - 2y = -1$, $2x + y = 2$. R&U **[S.Q.P. 2016-17]**

Sol.

$$|A| = 5$$

$$adj\ A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \quad 1$$

Given system of equations is $AX = B$, where

where $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$

$$B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad 1$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1+4 \\ 2+2 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \end{bmatrix} \quad 1$$

$$x = \frac{3}{5} \text{ and } y = \frac{4}{5} \quad 1$$

[CBSE Marking Scheme 2016]

Q. 2. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Then compute AB.

Hence, solve the following system of equations : $2x + y = 4$, $3x + 2y = 1$. A **[S.Q.P. 2015-16]**

Sol.

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

and $B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$

then $AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I \quad 1$$

$\Rightarrow$ $A\left(\frac{1}{2}B\right) = I$

On multiplying by A^{-1}

$$A^{-1} = \frac{1}{2}B$$

$$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \quad 1$$

The given system of equations are equivalent to $A'X = C$,

where $X = \begin{bmatrix} x \\ y \end{bmatrix}$ 1

and $C = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, A' = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$$X = (A')^{-1}C = (A^{-1})'C$$

$\Rightarrow$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \end{bmatrix}$

$\therefore$ $x = 7$

and $y = -10$ 1

[CBSE Marking Scheme 2015]

Long Answer Type Questions-II (6 marks each)

Q. 1. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, Find A^{-1}.

Hence, solve the system of equations :

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$2x - 3y - z = 5$$

A **[SQP 2018-19]**

Sol.

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$$

$$|A| = 3(-2) - 1(3) + 2(-4)$$
$$= -6 - 3 - 8 = -17 \neq 0 \quad 1$$

$\therefore$ A^{-1} exists. ½

$$\text{Cofactor matrix of A} = \begin{bmatrix} -2 & -3 & -4 \\ 1 & -7 & 2 \\ -7 & 15 & 3 \end{bmatrix} \quad 2$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}A$$

$$= \frac{1}{-17}\begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix} \quad ½$$

Now for given system of equations.

$$\begin{bmatrix}3 & 3 & 2\\1 & 2 & 0\\2 & -3 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\4\\5\end{bmatrix}$$

$$(A^t)X = B$$

$\Rightarrow$ $X = (A^t)^{-1}B$

$\Rightarrow$ $X = (A^{-1})^tB$ $[\because (A^t)^{-1} = (A^{-1})^t]$ ½

$$X = \frac{1}{-17}\begin{bmatrix}-2 & -3 & -4\\1 & -7 & 2\\-7 & 15 & 3\end{bmatrix}\begin{bmatrix}1\\4\\5\end{bmatrix}$$

$$X = \frac{1}{-17}\begin{bmatrix}-34\\-17\\68\end{bmatrix} = \begin{bmatrix}2\\1\\-4\end{bmatrix}$$

$x = 2, y = 1, z = -4$ 1½

[CBSE Marking Scheme 2018]

Q. 2. If $A = \begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}$, Find A^{-1}. Use it to solve the system of equations

Hence, solve the system of equations :

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

A [Delhi OD 2018 Set I, II, III]

Sol. $|A| = -1 \neq 0 \therefore A^{-1}$ exists

Co-factors of A are : 1

$A_{11} = 0$; $A_{12} = 2$; $A_{13} = 1$
$A_{21} = -1$; $A_{22} = -9$; $A_{23} = -5$
$A_{31} = 2$; $A_{32} = 23$; $A_{23} = 13$

} 1 m for 4 correct cofactors 2

$$\text{adj}(A) = \begin{bmatrix}0 & -1 & 2\\2 & -9 & 23\\1 & -5 & 13\end{bmatrix}$$

$\Rightarrow$ $A^{-1} = \frac{1}{|A|}.adj(A)$

$$= \begin{bmatrix}0 & 1 & -2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix}$$ ½

For, $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B = \begin{bmatrix}11\\-5\\-3\end{bmatrix}$, the system of equation is $A \cdot X = B$ ½

$$\therefore X = A^{-1}\cdot B = \begin{bmatrix}0 & 1 & -2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$ 1

$\therefore x = 1, y = 2, z = 3$ 1

[CBSE Marking Scheme 2018]

Q. 3. If $A = \begin{bmatrix}1 & 2 & -3\\2 & 3 & 2\\3 & -3 & -4\end{bmatrix}$, find A^{-1}. Hence solve the following system of equations :

$x + 2y - 3z = -4, 2x + 3y + 2z = 2, 3x - 3y - 4z = 11.$

R&U [S.Q.P. 2012] [Delhi Set III Comptt. 2012]

Sol. $A = \begin{bmatrix}1 & 2 & -3\\2 & 3 & 2\\3 & -3 & -4\end{bmatrix}$

Co factors are :

$C_{11} = -6$, $C_{21} = 17$, $C_{31} = 13$
$C_{12} = 14$, $C_{22} = 5$, $C_{32} = -8$
$C_{13} = -15$, $C_{23} = 9$, $C_{33} = -1$ 2

$|A| = -6 + 28 + 45 = 67$

$$\therefore \quad A^{-1} = \frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\14 & 5 & -8\\-15 & 9 & -1\end{bmatrix}$$ 1

Now the equations are :

$$x + 2y - 3z = -4,$$
$$2x + 3y + 2z = 2$$
and $3x - 3y - 4z = 11$

They may be written as

$$\begin{bmatrix}1 & 2 & -3\\2 & 3 & 2\\3 & -3 & -4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-4\\2\\11\end{bmatrix}$$ ½

where $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B = \begin{bmatrix}-4\\2\\11\end{bmatrix}$

$\Rightarrow$ $AX = B$

$\Rightarrow$ $X = A^{-1}B$

$\Rightarrow$ $X = \frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\14 & 5 & -8\\-15 & 9 & -1\end{bmatrix}\begin{bmatrix}-4\\2\\11\end{bmatrix}$ 1

$$= \frac{1}{67}\begin{bmatrix}201\\-134\\67\end{bmatrix} = \begin{bmatrix}3\\-2\\1\end{bmatrix}$$ ½

$\Rightarrow$ $x = 3, y = -2$ and $z = 1$ ½

[CBSE Marking Scheme 2012]

Q. 4. If $A = \begin{bmatrix}1 & 2 & 1\\-1 & 1 & 1\\1 & -3 & 1\end{bmatrix}$, find A^{-1}. Hence solve the system of equations :

$x + 2y + z = 4, -x + y + z = 0, x - 3y + z = 4.$

R&U [Delhi Set I Comptt. 2012]

Sol. Given $A = \begin{bmatrix}1 & 2 & 1\\-1 & 1 & 1\\1 & -3 & 1\end{bmatrix}$

$|A| = 1(1 + 3) - 2(-1 - 1) + 1(3 - 1)$
$= 4 + 4 + 2 = 10$ 1

Co-factor Matrix is :

$$\begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}$$ 1

$\therefore$ *adj A* = transpose of above matrix

$$= \begin{bmatrix}4 & -5 & 1\\2 & 0 & -2\\2 & 5 & 3\end{bmatrix}$$ 1

$$\therefore \quad A^{-1} = \frac{adj(A)}{|A|}$$

$$= \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} 2/5 & -1/2 & 1/10 \\ 1/5 & 0 & -1/5 \\ 1/5 & 1/2 & 3/10 \end{bmatrix}$$ **1**

Given set of equations are :

$$x + 2y + z = 4$$
$$-x + y + z = 0$$
$$x - 3y + z = 4$$ ½

$$\Rightarrow \quad \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow \quad A \cdot X = B$$

Multiplying both sides by A^{-1}, we get

$$A^{-1}AX = A^{-1}B$$

$$\Rightarrow \quad X = A^{-1}B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2/5 & -1/2 & 1/10 \\ 1/5 & 0 & -1/5 \\ 1/5 & 1/2 & 3/10 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$ ½

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{8}{5} + \frac{2}{5} \\ \frac{4}{5} - \frac{4}{5} \\ \frac{4}{5} + \frac{6}{5} \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore \quad x = 2;\ y = 0,\ z = 2$$ **1**

[CBSE Marking Scheme 2012]

Q. 5. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$**, and use it to solve the system of equations :**

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.$ **R&U [Delhi Set I, II, III-2017, Delhi Set II Comptt. 2012]**

Sol. Getting $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ ...(i) **1½**

Given equations can be written as $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$ **1**

or $AX = B$

From (i) $A^{-1} = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ **1**

$$\therefore \quad X = A^{-1}B = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$$ **1**

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$ **1**

or $x = 3, y = -2, z = -1$ **[CBSE Marking Scheme 2017]** ½

OR

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-4)1 + 4(1) + 4(2) & -4(-1) + 4(-2) + 4(1) & -4(1) + 4(-2) + 4(3) \\ -7(1) + 1(1) + 3(2) & -7(-1) + 1(-2) + 3(1) & -7(1) + 1(-2) + 3(+3) \\ 5(1) - 3(1) - 1(2) & 5(-1) - 3(-2) - 1(1) & 5(1) - 3(-2) - 1(3) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} A = 8I$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -4(4) + 4(9) + 4(1) \\ -7(4) + 1(9) + 3(1) \\ 5(4) - 3(9) - 1(1) \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -16 + 36 + 4 \\ -28 + 9 + 3 \\ 20 - 27 - 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

Hence $x = 3$, $y = -2$, $z = -1$

[Topper's Answer 2017]

Q. 6. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ **are square matrices, find** $A \cdot B$ **and hence solve the system of equations :**

$x - y = 3,\ 2x + 3y + 4z = 17$ **and** $y + 2z = 7.$

R&U [O.D. Set I Comptt. 2012] [NCERT Exemplar]

Sol. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ ½

$$\therefore AB = \begin{bmatrix} 2+4 & 2-2 & -4+4 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ -4+4 & 2-2 & -4+10 \end{bmatrix}$$ ½

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$ **1**

$\therefore \quad AB = 6I$

On multiplying by A^{-1}

$$A^{-1}AB = 6A^{-1}I$$

$\Rightarrow \quad IB = 6A^{-1}I \quad (\because A^{-1}A = I)$

$\Rightarrow \quad B = 6A^{-1} \quad (\because IX = X)$

$\Rightarrow \quad A^{-1} = \frac{1}{6}B$

Given equations are :

$$x - y = 3$$
$$2x + 3y + 4z = 17$$ ½
$$y + 2z = 7$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$ ½

$AX = C$, where $C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ ½

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ ½

$\Rightarrow \quad A^{-1}AX = A^{-1}C$

$\Rightarrow \quad X = A^{-1}C$ **1**

$$\Rightarrow X = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$ ½

$\therefore \quad x = 2, y = -1$ and $z = 4$ ½

[CBSE Marking Scheme 2012]

Q. 7. If $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$, **find** A^{-1}. **Hence solve the following system of equations :**

$x + 2y + 5z = 10, x - y - z = -2, 2x + 3y - z = -11.$

R&U **[O.D. Set III Comptt. 2012]**

Sol. $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

$a_{11} = (1 + 3) = 4$

$a_{12} = -(-1 + 2) = -1$

$a_{13} = (3 + 2) = 5$ ½

$a_{21} = -(-2 - 15) = 17$

$a_{22} = (-1 - 10) = -11$

$a_{23} = -(3 - 4) = 1$ ½

$a_{31} = (-2 + 5) = 3$

$a_{32} = -(-1 - 5) = 6$

$a_{33} = (-1 - 2) = -3$ ½

$|A| = 1 \times 4 + 2 \times (-1) + 5 \times (5)$

$= 4 - 2 + 25 = 27$ ½

$$\Rightarrow \quad A^{-1} = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4/27 & 17/27 & 1/9 \\ -1/27 & -11/27 & 2/9 \\ 5/27 & 1/27 & -1/9 \end{bmatrix}$$ **1**

Given set of equations can be written as

$x + 2y + 5z = 10,$

$x - y - z = -2$

and $2x + 3y - z = -11$

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$ **1**

$\Rightarrow \quad AX = B,$

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$

On multiplying by A^{-1}

$X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4/27 & 17/27 & 1/9 \\ -1/27 & -11/27 & 2/9 \\ 5/27 & 1/27 & -1/9 \end{bmatrix}\begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

1

$$= \begin{bmatrix} \frac{40}{27} - \frac{34}{27} - \frac{33}{27} \\ -\frac{10}{27} + \frac{22}{27} - \frac{66}{27} \\ \frac{50}{27} - \frac{2}{27} + \frac{33}{27} \end{bmatrix} = \begin{bmatrix} -\frac{27}{27} \\ -\frac{54}{27} \\ \frac{81}{27} \end{bmatrix}$$ ½

$$= \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

$\Rightarrow \quad x = -1, y = -2$ and $z = 3$ ½

[CBSE Marking Scheme 2012]

Q. 8. Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, **compute** $(AB)^{-1}$.

R&U **[Comptt. 2018 Set I, II, III]**

Sol. $|A| = 5(-1) + 4(1) = -1$ **1**

$C_{11} = -1 \quad C_{21} = 8 \quad C_{31} = -12$

$C_{12} = 0 \quad C_{22} = 1 \quad C_{32} = -2$ **2**

$C_{13} = 1 \quad C_{23} = -10 \quad C_{33} = 15$

$$A^{-1} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$ **1**

$(AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$ **1**

$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$ **1**

[CBSE Marking Scheme 2018]

Q. 9. Using matrices, solve the following system of linear equations :

$x - y + 2z = 7, 3x + 4y - 5z = -5, 2x - y + 3z = 12.$

R&U **[Delhi Set I, II, III, 2012]**

Sol. Given equations can be written as

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} \text{ or } AX = B$$ **1**

$|A| = 1(7) + 1(19) + 2(-11)$

$= 4 \neq 0$

$\therefore$ therefore A^{-1} exists.

Co-factors **1**

$$\left.\begin{matrix} a_{11} = 7, & a_{12} = -19 & a_{13} = -11 \\ a_{21} = 1, & a_{22} = -1 & a_{23} = -1 \\ a_{31} = -3 & a_{32} = 11 & a_{33} = 7 \end{matrix}\right\}$$

{1 mark for any four correct cofactors} **2**

$$\Rightarrow \quad A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$ ½

$$\therefore \quad X = A^{-1}B$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} \quad \mathbf{1}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad ½$$

$$\therefore \quad x = 2, y = 1 \text{ and } z = 3$$

[CBSE Marking Scheme 2012]

Q.10. Using matrices, solve the following system of linear equations :
$x - y + z = 4, 2x + y - 3z = 0, x + y + z = 2.$

R&U **[Foreign Set I, 2012]**

Sol. Given equations are

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
and $$x + y + z = 2$$

We can write this system of equations as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \quad ½$$

Let $AX = B,$

where $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix},$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ ½

$$\therefore \quad |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1(1 + 3) - (-1)(2 + 3) + 1(2 - 1)$$
$$= 4 + 5 + 1$$
$$= 10 \quad \mathbf{1}$$

Now, $X = A^{-1}B$

For A^{-1}, we have cofactors matrix as

$$P = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} \quad \mathbf{1½}$$

$$\therefore \quad adj\ A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \quad \mathbf{1½}$$

Thus, $X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$= \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} \quad ½$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

The required solution is ½

$\therefore x = 2, y = -1$ and $z = 1.$

[CBSE Marking Scheme 2012]

Q. 11. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$, find $(AB)^{-1}$.

R&U **[Foreign Set I, 2012]**

Sol. $$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= 1(3 - 0) - 2(-1 - 0) - 2(2 - 0)$$
$$= 3 + 2 - 4 = 1 \neq 0 \quad ½$$

i.e., B is invertible matrix.

$\Rightarrow B^{-1}$ exists. ½

Now $C_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = 3 - 0 = 3$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2 \quad ½$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -(2-4) = 2$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = -(-2-0) = 2 \quad ½$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} = 0 + 6 = 6$$

$$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = -(0-2) = 2$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = (3+2) = 5 \quad ½$$

$$\therefore \quad adj\ B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \quad ½$$

$$\Rightarrow \quad B^{-1} = \frac{1}{|B|}(adj\ B)$$

$$= \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \quad \mathbf{1}$$

Now $(AB)^{-1} = B^{-1}\cdot A^{-1}$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \quad 1$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad 1$$

[CBSE Marking Scheme 2012]

Q. 12. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}. Hence using A^{-1} solve the system of equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.

U&A **[NCERT] [O.D. Set-II, III 2017]**

Sol.

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

or $|A| = 2(0) + 3(-2) + 5(1) = -1$ 1

$A_{11} = 0, A_{12} = 2, A_{13} = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$ 2

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

or $$A^{-1} = -1\begin{pmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{pmatrix}$$

$$= -1\begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}$$

½

Given equations can be written as

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \text{ or } AX = B$$

or $X = A^{-1}B$

or $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

or $x = 1, y = 2, z = 3$. 1½

[CBSE Marking Scheme 2017]

Q. 13. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$

U&A **[Delhi Set-I 2017]**

Sol. $$AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 1$$

$AB = I$

or $$A^{-1} = B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \quad 1$$

Given equations in matrix form are :

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} \quad 1$$

$AX = C$ ½

or $X = (A)^{-1} C = (A^{-1}) C$ 1

or $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

or $x = 0, y = 5, z = 3$ ½

[CBSE Marking Scheme 2017]

Q. 14. If $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{pmatrix}$ find A^{-1} and hence solve the system of equations $2x + y - 3z = 13$, $3x + 2y + z = 4$, $x + 2y - z = 8$.

U&A **[Delhi Set III 2017]**

Sol. Try yourself like Q.13 long Answer Type Q. II.

Q. 15. In $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$ find A^{-1}. Using A^{-1} solve the system of equations $\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 2$; $\frac{4}{x}+\frac{6}{y}+\frac{5}{z} = 5$; $\frac{6}{x}+\frac{9}{y}+\frac{20}{z} = 4$

U&A **[Delhi Set-II 2017]**

Sol. Here $|A| = 1200$

Co-factors are

$C_{11} = 75, C_{21} = 150, C_{31} = 75$

$C_{12} = 110, C_{22} = -100, C_{32} = 30$ 2

$C_{13} = 72, C_{23} = 0, C_{33} = -24$

$$A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad ½$$

Given equation in matrix from is :

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad 1½$$

or $AX = B$

or $X = A^{-1}B$

or $\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix}$ **1½**

or $x = 2, y = -3, z = 5$

[CBSE Marking Scheme 2017] ½

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