Sets

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. To check the understanding of sets, a Math teacher writes two sets A and B having finite numbers of elements. The sum of cardinal numbers of two finite sets A and B is 9. The ratio of a cardinal number of the power set of A is to a cardinal number of the power set of B is 8:1.

(A) The cardinal number of set A is:

(a) 2

(b) 3

(c) 6

(d) 8

(B) The cardinal number of set B is:

(a) 2

(b) 3

(c) 6

(d) 8

(C) The maximum value of n(A∪B) is:

(a) 3

(b) 6

(c) 8

(d) 9

(D) The minimum value of n (A∪B) is:

(a) 3

(b) 6
(c) 8
(d) 9

(E) If B⊂A, then n (A∩B) is:

(a) 3
(b) 6
(c) 8
(d) 6

Ans. (A) (c) 6

Explanation: Let the cardinal numbers of sets A and B be n(A) and n(B) respectively.

Given, n(A) + n(B) = 9 -(i)

Also, the cardinal number of the power set

of A = $2^n(4)$

And the cardinal number of the power set of

B = $2^n(B)$

Given, $\frac{2^{n(A)}}{2^{n(B)}} = \frac{8}{1}$

$\Rightarrow \quad 2^{n(A)-n(B)} = 2^3$

$\Rightarrow \quad n(A) - n(B) = 3 \qquad ...(ii)$

On adding (i) and (ii) , we get

$2n(A) = 12$

$\Rightarrow \quad n(A) = 6$

Thus, the cardinal number of set A is 6.

(B) (b) 3

Explanation: On subtracting (ii) from (i), we get

2n(B) = 6

n(B) = 3

Thus, the cardinal number of set B is 3.

(C) (d) 9

Explanation: We have,

n(A∪B) = n(A) + n(B) - n(A∪B)

The value of n(A∪B) will be maximum

when n(A∩B) will be minimum.

The minimum value of n(A∩B) = 0.
So, maximum value of
n(A∩B) = n(A) + n(B) = 6+3=9
(D) (b) 6
Explanation: We have,
n(A∪B) = n(A) + n(B) - n(A∪B)
The value of n(A∪B) will be minimum when
n(AB) will be maximum.
The maximum value of n(A∩B) = 3.
So, minimum value of
(E) (a) 3
n(A∪B) = n(A) + n(B) - n(A∩B)
=6+3-3=6
Explanation: Given BCA
➞ A∩B=B
➞ n(A∩B) = n(B)
⇒ n(A(∩B) = 3

2. In a library, 25 students are reading books on physics, chemistry, and mathematics. It was found that 15 students were reading mathematics, 12 reading physics and 11 reading chemistry, 5 students reading both mathematics and chemistry, 9 students reading both physics and mathematics, 4 students reading both physics and chemistry, and 3 students reading all three subjects.

(A) Find the number of students reading only Chemistry.
(B) Find the number of students reading only Mathematics.
(C) Find the number of students reading at least one of the subject and also find the number of students reading none of the subjects.

Ans. Let M denote a set of students who are reading mathematics, P denotes who is reading physics and C denotes who is reading chemistry. We have,

$n(U)=25$, $n(M) = 15$, $n(P) = 12$, $n(C)=11$

$n(M\cap C)=5$, $n(M\cap P) = 9$, $n(P\cap C)=4$

$n(M\cap P\cap C)=3$

(A) The number of students reading only chemistry

$= n(M'\cap P'\cap OQ)$

But, $n(M'\cap P'\cap C)= n((M\cap P)\ '\cap\ C)$

$= n(C)-n((M\cap P)\cap C$

[since, $n(A\cap B)= n(A) - n(A\cap B)$]

$= n(C)-n((M\cap C)\cup(P\cap C))$

$= n(C)-n(M\cap C)+n(P\cap C)-n(M\cap P\cap C))$

$=11-(5+ 4-3)=5$

(B) The number of students reading only Mathematics $n(M\cap P\cap C)$

But, $n(M\cap P\cap C) = n(M\cap n\ (P\cap C)')$

$= n(M)-n(M\cap(P\cap C))$

$= n(M)-n((M\cap P)\cup(M\cap C))$

$= n(M)-\ (n(M\cap P) +n(M\cap C)-n(M\cap P\cap C))$

$=15-(9+5-3)= 4$

(C) The number of students reading at least one of the subject = $n(\ M\cup P\cup C)$

$n(M\cup P\cup C)= n(M) + n(P) + n(C) - n(M\cap P)$

$-n(P\cap C)-n(M\cap C) + (M\cap P\cap C)$

$15+12+11-9-4-5+3$

$= 41-18=23$

The number of students reading none of the subjects

$= n(M'\cap P'\cap C') = n(M\cup P\cup C)$

But, $n(M\cup P\cup C)$

$= n(U)-((M\cup P\cup C)=25-23=2$