PRACTICE SET -5

1. The domain of definition of the function $y(x)$ given by the equation $2^x + 2^y = 2$ is:
   a. $0 < x \le 1$ b. $x \le x \le 1$
   c. $-\infty < x \le 0$ d. $-\infty < x < 1$.

2. cosh $(\alpha + i\beta) - \cosh(\alpha - i\beta)$ is equal to
   a. 2 sinh α sinh β b. 2 cosh α cosh β
   c. 2i sinh α sin β d. 2 cosh α cos β

3. Matrix $\begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$ is
   a. Orthogonal b. Idempotent
   c. Skew-symmetric d. Symmetric

4. For a sequence $< a_n >, a_1 = 2$ and $\frac{a_{n+1}}{a_n} = \frac{1}{3}$. Then $\sum_{r=1}^{20} a_r$ is
   a. $\frac{20}{2}[4 + 19 \times 3]$ b. $3\left(1 - \frac{1}{3^{20}}\right)$
   c. $2(1 - 3^{20})$ d. None of these

5. Coefficients of $x^r [0 \le r \le (n-1)]$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2)$ $+ (x+3)^{n-3}(x+2)^2 + \ldots + (x+2)^{n-1}$
   a. ${}^nC_r(3^r - 2^n)$ b. ${}^nC_r(3^{n-r} - 2^{n-r})$
   c. ${}^nC_r(3^r + 2^{n-r})$ d. None of these

6. $1 + \frac{2}{3!} + \frac{3}{5!} + \frac{4}{7!} + \ldots \infty =$
   a. e b. 2e
   c. e/2 d. e/3

7. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals
   a. $\frac{1}{4}$ b. $\frac{1}{7}$
   c. $\frac{1}{8}$ d. $\frac{1}{49}$

8. $\sin(\pi + \theta)\sin(\pi - \theta)\operatorname{cosec}^2\theta =$
   a. 1 b. -1
   c. sin θ d. $-$sin θ

9. $\sinh^2 x$ equals
   a. $\cosh 2x - 1$ b. $\cosh^2 x + 1$
   c. $\frac{1}{2}(\cosh 2x - 1)$ d. $\frac{1}{2}(\cosh 2x + 1)$

10. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point l meters just above A is β. The height of the tower is
    a. $l \tan\beta \cot\alpha$ b. $l \tan\alpha \cot\beta$
    c. $l \tan\alpha \tan\beta$ d. $l \cot\alpha \cot\beta$

11. If $y = (1+x)^x$, then $\frac{dy}{dx} =$
    a. $(1+x)^x\left[\frac{x}{1+x} + \log ex\right]$
    b. $\frac{x}{1+x} + \log(1+x)$
    c. $(1+x)^x\left[\frac{x}{1+x} + \log(1+x)\right]$
    d. None of these

12. $\frac{x}{1 + x\tan x}$ is maxima at
    a. $x = \sin x$ b. $x = \cos x$
    c. $x = \frac{\pi}{3}$ d. $x = \tan x$

13. $\int \frac{1}{x\sqrt{1+\log x}}\,dx =$
    a. $\frac{2}{3}(1+\log x)^{3/2} + c$ b. $(1+\log x)^{3/2} + c$
    c. $2\sqrt{1+\log x} + c$ d. $\sqrt{1+\log x} + c$

14. The area of the triangle formed by the tangent to the hyperbola $xy = a^2$ and coordinate axes is
    a. a^2 b. $2a^2$ c. $3a^2$ d. $4a^2$

15. The solution of $\frac{dy}{dx} = x\log x$ is
    a. $y = x^2 \log x - \frac{x^2}{2} + c$
    b. $y = \frac{x^2}{2}\log x - x^2 + c$
    c. $y = \frac{1}{2}x^2 + \frac{1}{2}x^2\log x + c$
    d. None of these

16. If the coordinates of one end of the diameter of the circle $x^2+y^2-8x-4y+c=0$ are (–3, 2), then the coordinates of other end are

a. (5, 3) b. (6, 2)
c. (1, –8) d. (11, 2)

17. Let (x,y) by any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0,0)$ to (x,y) in the ratio $1:3$. Then, the locus of P is

a. $x^2 = y$ b. $y^2 = 2x$
c. $y^2 = x$ d. $x^2 = 2y$

18. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=$

a. 1 b. 3
c. – 3/2 d. 3/2

19. The direction cosines of three lines passing through the origin are $l_1,m_1,n_1; l_2,m_2,n_2$ and l_3,m_3,n_3. The lines will be coplanar, if

a. $\begin{vmatrix} l_1 & n_1 & m_1 \\ l_2 & n_2 & m_2 \\ l_3 & n_3 & m_3 \end{vmatrix}=0$

b. $\begin{vmatrix} l_1 & m_2 & n_3 \\ l_2 & m_3 & n_1 \\ l_3 & m_1 & n_2 \end{vmatrix}=0$

c. $l_1l_2l_3+m_1m_2m_3+n_1n_2n_3=0$

d. None of these

20. $\sim(p\vee(\sim q))$ is equal to

a. $\sim p\vee q$ b. $(\sim p)\wedge q$
c. $\sim p\vee\sim p$ d. $\sim p\wedge\sim q$

21. A real root of the equation $\log_4\{\log_2(\sqrt{x+8}-\sqrt{x})\}=0$ is

a. 1 b. 2 c. 3 d. 4

22. The number of ways in which the following prizes be given to a class of 20 boys, first and second Mathematics, first and second Physics, first Chemistry and first English is.

a. $20^4\times19^2$ b. $20^3\times19^3$
c. $20^2\times19^4$ d. None of these

23. The value of k so that the function

$$f(x)=\begin{cases} k(2x-x^2), & \text{when } x<0 \\ \cos x, & \text{when } x\ge0 \end{cases}$$

is continuous at $x=0$, is

a. 1 b. 2
c. 4 d. None of these

24. The distance of point (–2, 3) from the line $x-y=5$ is

a. $5\sqrt{2}$ b. $2\sqrt{5}$
c. $3\sqrt{5}$ d. $5\sqrt{3}$

25. Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is atleast one more than the number of girls ahead of her, is

a. 1/2 b. 1/3
c. 2/3 d. $\frac{3}{4}$

26. Let $g(x)=\dfrac{(x-1)^n}{\log\cos^m(x-1)}$; $0<x<2$, m and n are integers, $m\neq0$, $n>0$, and let p be the left hand derivative of $|x-1|$ at $x=1$. If $\lim_{x\to1^+}g(x)=p$ then

a. $n=1, m=1$ b. $n=1, m=-1$
c. $n=2, m=2$ d. $n>2, m=n$

27. Let $g(x)=\log(f(x))$ where $f(x)$ is a twice differentiable positive function on $(0,\infty)$ such that $f(x+1)=x\,f(x)$. Then, for $N=1, 2, 3,...,$

a. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N-1)^2}\right\}$

b. $4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N-1)^2}\right\}$

c. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N+1)^2}\right\}$

d. $4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N+1)^2}\right\}$

28. Let $I=\int\frac{e^x}{e^{4x}+e^{2x}+1}dx$, $J=\int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. Then for an arbitrary constant C, the value of $J-I$ equals

a. $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$

b. $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

c. $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$

d. $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

29. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals:

a. $\pi-4$ b. $\frac{2\pi}{3}-4-4\sqrt{3}$

c. $4\sqrt{3}-4$ d. $4\sqrt{3}-4-\frac{\pi}{3}$

30. Let the population of rabbits surviving at a time t be government by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$. If $p(0)=100$, then $p(t)$ equals:

a. $400-300e^{t/2}$ b. $300-200e^{-t/2}$

c. $600-500e^{t/2}$ d. $400-300e^{-t/2}$

31. A straight line L through the point (3, –2) is inclined at an angle $60°$ to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, then the equation of L is

a. $y+\sqrt{3}x+2-3\sqrt{3}=0$

b. $y-\sqrt{3}x+2+3\sqrt{3}=0$

c. $\sqrt{3}y-x+3+2\sqrt{3}=0$

d. $\sqrt{3}y+x-3+2\sqrt{3}=0$

32. Let $y=y(x)$ be the solution of the differential equation

$\sin x\frac{dy}{dx}+y\cos x=4x, x\in(0,\pi)$. $y=\left(\frac{\pi}{2}\right)=0$, then

$y\left(\frac{\pi}{6}\right)$ is equal to:

a. $-\frac{4}{9}\pi^2$ b. $\frac{4}{9\sqrt{3}}\pi^2$

c. $\frac{-8}{9\sqrt{3}}\pi^2$ d. $-\frac{8}{9}\pi^2$

33. Let y = y(x) be the solution of the differential equation,

$(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$ such that $y(0)=0$. If

$\sqrt{a}y(1)=\frac{\pi}{32}$, then the value of 'a' is:

a. 1/2 b. 1/16

c. 1/4 d. 1

34. If $y=y(x)$ is the solution of the differential equation, $y=\frac{dy}{dx}+2y=x^2$ satisfying $y(1)=1$, then $y\left(\frac{1}{2}\right)$ is equal to:

a. $\frac{7}{64}$ b. $\frac{13}{16}$

c. $\frac{49}{16}$ d. $\frac{1}{4}$

35. Let $f:[0,1]\to R$ be such that $f(xy)=f(x).f(y)$ for all x,y,ε [0,1], and $f(0)\neq 0$. If $y=y(x)$ satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with $y(0)=1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

a. 4 b. 3

c. 5 d. 2

36. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals:

a. $\frac{1}{3}+e^6$ b. $\frac{1}{3}$

c. $-\frac{4}{3}$ d. $\frac{1}{3}+e^3$

37. Let f be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x}$, $(x>0)$ and $f(1)\neq 4$. Then $\lim_{x\to 0^+}xf\left(\frac{1}{x}\right)$:

a. Exists and equals 4 b. Does not exist

c. Exist and equals 0 d. Exists and equals $\frac{4}{7}$

38. The curve amongst the family of curves, represented by the differential equation, $(x^2-y^2)dx+2xy\,dy=0$ which passes through (1,1) is:

a. A circle with centre on the y-axis

b. A circle with centre on the x-axis

c. An ellipse with major axis along the y-axis

d. A hyperbola with transverse axis along the x-axis

39. If y(x) is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}, x>0$, where $y(1)=\frac{1}{2}e^{-2x}$, then:

a. y(x) is decreasing in (0,1)

b. y(x) is decreasing in $\left(\frac{1}{2},1\right)$

c. $y(\log_e 2)=\frac{\log_c 2}{4}$

d. $y(\log_e 2)=\log_c 4$

40. Match the statement of Column I with those in Column II:

Column I	Column II
(A) The minimum and maximum distance of a point (2, 6) from the ellipse $9x^2+8y^2-36x-16y-28=0$ are L and G, then	1. $L+G=10$
(B) The minimum and maximum distance of a point (1, 2) from the ellipse $4x^2+9y^2+8x-36y+4=0$ are L and G, then	2. $L+G=6$
(C) The minimum and maximum distance of a point $\left(\frac{9}{5},\frac{12}{5}\right)$ from the ellipse $4(3x+4y)^2+9(4x-3y)^2=9000$ are L and G, then	3. $G-L=6$
	4. $G-L=4$
	5. $L^G-G^L=6$

a. A→2,3,4; B→1,2; C→4,5
b. A→1,3; B→2,4,5; C→2,3
c. A→3,4,5; B→1,3; C→2,3
d. A→1,5; B→2,4,5; C→3,4

41. Match the statement of Column I with those in Column II:

Column I	Column II
(A) Direction circles of $x^2-2y^2=2$ and $x^2+2y^2=2$ are	1. $x^2+y^2=1$
(B) Direction circles of $3x^2+2y^2=6$ and $3x^2-2y^2=6$ are	2. $x^2+y^2=2$
(C) Direction circles of $5x^2-9y^2=45$ and $x^2+y^2=1$ are 0	3. $x^2+y^2=3$
	4. $x^2+y^2=4$
	5. $x^2+y^2=5$

a. A→1,3; B→5; C→2,4
b. A→2,3; B→4,5; C→3
c. A→1,2; B→3; C→2,4
d. A→4,5; B→1,3; C→3

42. If at every point x of an interval [a, b] the inequalities $g(x)\le f(x)\le h(x)$ are fulfilled, then $\int_a^b g(x)\,dx\le\int_a^b f(x)\,dx\le\int_a^b h(x)\,dx$, a < b. Match the entries from the following two columns:

p	Column II
(A) If $\mu<\int_0^1\frac{x^7dx}{\sqrt[3]{(1+x^6)}}<\lambda$, then	1. $[\lambda+\mu]=2$, where [.] denotes the greatest integer function.
(B) If $\mu<\int_0^1\frac{dx}{\sqrt{(1+x^6)}}<\lambda$, then	2. $[\lambda+\mu]=4$, where [.] denotes the greatest integer function.
(C) If $\mu<\int_0^1\frac{dx}{\sqrt{(4-x^2-x^3)}}<\lambda$, then	3. $[\lambda-\mu]=0$, where [.] denotes the greatest integer function.
	4. $[\lambda-\mu]=3$, where [.] denotes the greatest integer function.
	5. $[\lambda+\mu]=0$, where [.] denotes the greatest integer function.

a. A→3,4; B→1,3; C→2,5
b. A→3,5; B→1,3; C→2,4
c. A→1,3; B→3,5; C→2,4
d. A→3,5; B→1,2; C→3,4

43. Match the statement of Column I with those in Column II:

Column I	Column II
(A) For the line $4x+3y-6=0$ and $5x+12y+9=0$, acute angle bisector and obtuse angle bisectors represented by A and O respectively, then	1. $A:7x-9y-3=0$
(B) For the line $4x-3y-6=0$ and $5x-12y+9=0$, acute angle bisector and obtuse angle bisectors	2. $A:7x-9y+3=0$

represented by A and O respectively, then	
(C) For the straight line $4x-3y+6=0$ and $5x-12y-9=0$, acute angle bisector and obtuse angle bisectors represented by A and O respectively, then	3. $A: 7x+9y-3=0$
(D) 0	4. $O: 9x+7y-41=0$
	5. $O: 9x-7y-41=0$

a. A→3,5; B→1,2; C→2,4
b. A→3,5; B→1,4; C→2
c. A-1,4 B-3,2 C→2,4
d. A→1,4; B→2,3; C→3

44. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is:

a. $\frac{\pi}{3}$ b. $\frac{\pi}{6}$
c. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ d. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

45. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P[P_{ij}]$ be a $n\times n$ matrix with $p_{ij}=\omega^{i+j}$. Then when $P^2 \neq 0$, n =

a. 57 b. 55 c. 58 d. 56

46. If m is the A.M. of two distinct real numbers ℓ and $n(\ell, n>1)$ and G_1, G_2 and G_3 are three geometric means between ℓ and n, then $G_1^4+2G_2^4+G_3^4$ equals,

a. $4\ell^2 mn$ b. $4\ell m^2 n$
c. $4\ell mn^2$ d. $4\ell^2 m^2 n^2$

47. Let $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(s)

a. 1056 b. 1088
c. 1120 d. 1332

48. If $3^x=4^{x-1}$, then $x=$

a. $\frac{2\log_3 2}{2\log_3 2-1}$ b. $\frac{2}{2-\log_2 3}$
c. $\frac{1}{1-\log_4 3}$ d. $\frac{2\log_2 3}{2\log_2 3-1}$

49. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is:

a. 216 b. 192
c. 120 d. 72

50. Let y = y(x) be the solution of the differential equation, $x\frac{dy}{dx}+y=x\log_e x, (x>1)$. If $2y(2)=\log_e 4-1$, then $y(e)$ is equal to:

a. $\frac{e^2}{4}$ b. $\frac{e}{4}$ c. $-\frac{e}{2}$ d. $-\frac{e^2}{2}$

Answers and Solutions

1. (d) $2^y=2-2^x$

$\Rightarrow$ $y=\log_2(2-2^x)$ for domain $2-2^x \Rightarrow x<1$

2. (c) $\cosh(\alpha+i\beta)-\cosh(\alpha-i\beta)$

$=\cosh\alpha\cosh(i\beta)+\sinh\alpha\sinh(i\beta)$

$-\cosh\alpha\cosh(i\beta)+\sinh\alpha\sinh(i\beta)$

$=2\sinh\alpha\sinh i\beta=2i\sinh\alpha\sin\beta.$

3. (c) It is skew-symmetric.

4. (b) The sequence is a G.P. with common ratio $\frac{1}{3}$.

Now from $\frac{a(1-r^n)}{1-r}, \frac{2[1-(1/3)^{20}]}{1-(1/3)}$

$=3\left[1-\frac{1}{3^{20}}\right]$

5. (b) We have $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots+(x+2)^{n-1}$

$=\frac{(x+3)^n-(x+2)^n}{(x+3)-(x+2)}=(x+3)^n-(x+2)^n$

$\left(\because \frac{x^n-a^n}{x-a}=x^{n-1}+x^{n-2}a^1+x^{n-3}a^2+\ldots+a^{n-1}\right)$

Therefore coefficient of x^r in the given expression

$=$ Coefficient of x^r in $[(x+3)^n-(x+2)^n]$

$={}^nC_r 3^{n-r}-{}^nC_r 2^{n-r}={}^nC_r(3^{n-r}-2^{n-r})$

6. (c) $S=\frac{1}{1!}+\frac{2}{3!}+\frac{3}{5!}+\frac{4}{7!}+\ldots+\frac{n}{(2n-1)!}+\ldots$

Here $T_n=\frac{1}{2}\cdot\frac{2n}{(2n-1)!}=\frac{1}{2}\frac{(2n-1)+1}{(2n-1)!}$

$$= \frac{1}{2}\left\{\frac{1}{(2n-2)!} + \frac{1}{(2n-1)!}\right\}$$

$\Rightarrow$ $$S = \sum T_n = \frac{1}{2}\left\{\frac{e+e^{-1}}{2} + \frac{e-e^{-1}}{2}\right\} = \frac{e}{2}.$$

Trick: The sum of this series upto 4 terms is 1.359…and this is value of e/2 approximately.

7. (a) $7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, ...$

Therefore, for 7^r, $r \in N$ the number ends at unit place 7, 9, 3, 1, 7, …..

$\therefore$ $7^m + 7^n$ will be divisible by 5 if it end at 5 or 0.

But it cannot end at 5. Also for end at 0. For this m and n should be as follows

	m	n
1	$4r$	$4r-2$
2	$4r-1$	$4r-3$
3	$4r-2$	$4r$
4	$4r-3$	$4r-1$

For any given value of m, there will be 25 values of n.

Hence, the probability of the required event is

$$\frac{100\times 25}{100\times 100} = \frac{1}{4}$$

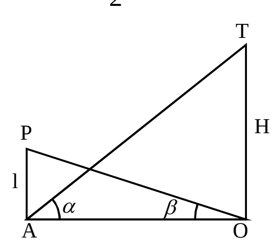
Note: Power of prime numbers have cyclic numbers in their unit place.

8. (b) $\sin(\pi+\theta)\sin(\pi-\theta)\,\mathrm{cosec}^2\theta$

$$= -\sin\theta\sin\theta\frac{1}{\sin^2\theta} = -1.$$

9. (c) $\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$

10. (b)



From figure, we can deduce $H = l\tan\alpha\cot\beta$.

11. (c) $y = (1+x)^x$

Taking log on both sides,

$\log y = x\log(1+x)$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \log(1+x) + x\frac{1}{(1+x)}$$

Thus $$\frac{dy}{dx} = (1+x)^x\left[\frac{x}{1+x} + \log(1+x)\right]$$

12. (b) If $\frac{x}{1+x\tan x}$ is maxima, then its reciprocal $\frac{1+x\tan x}{x}$ will be minima.

Let $y = \frac{1+x\tan x}{x} = \frac{1}{x} + \tan x$

$$\therefore \quad \frac{dy}{dx} = -\frac{1}{x^2} + \sec^2 x,$$

$$\frac{d^2y}{dx^2} = \frac{2}{x^3} + 2\sec x\sec x\tan x$$

On putting $\frac{dy}{dx} = 0$,

$$-\frac{1}{x^2} + \sec^2 x = 0$$

$$\Rightarrow \quad \sec^2 x = \frac{1}{x^2}$$

$$\Rightarrow \quad x^2 = \cos^2 x$$

$$\Rightarrow \quad x = \cos x$$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{2}{\cos^3 x} + 2\sec^2 x\tan x$$

$= 2\sec^2 x(\sec x + \tan x)$, which is positive.

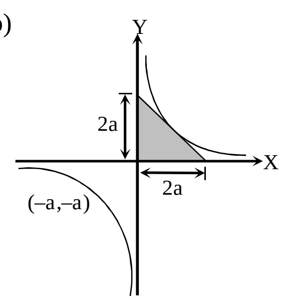
At $x = \cos x$, $\frac{1+x\tan x}{x}$ is minimum.

So $\frac{x}{1+x\tan x}$ will be maximum.

13. (c) Put $t = 1+\log x \Rightarrow dt = \frac{1}{x}dx$, then

$$\int \frac{dx}{x\sqrt{1+\log x}} = \int \frac{dt}{t^{1/2}} = 2t^{1/2} + c = 2(1+\log x)^{1/2} + c.$$

14. (b)



Given $xy = a^2$ or $y = \frac{a^2}{x}$ . . .(i)

There are two points on the curve $(a, a), (-a, -a)$

The equation of the line at (a, a) is,

$$y - a = \left(\frac{dy}{dx}\right)_{(a,a)} (x - a)$$

$$= \left(\frac{-a^2}{x^2}\right)_{(a,a)} (x - a)$$

$y - a = -(x - a)$ therefore, equation of the tangent at (a,a) is $x + y = 2a$. The interception of line $x + y = 2a$ with x-axis is $2a$ and with y-axis is $2a$.

$\therefore$ Required area $= \frac{1}{2} \times 2a \times 2a = 2a^2$.

15. (d) $\frac{dy}{dx} = x \log x$

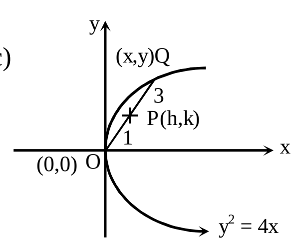
$\Rightarrow$ $dy = x \log x dx$

$\Rightarrow$ $\int dy = \int x \log x dx$

$\Rightarrow$ $y = \frac{x^2}{2} \log x - \frac{x^2}{4} + c.$

16. (d) Obviously the centre of the circle is (4, 2) which should be the middle point of the ends of diameter.
Hence the other end is (11, 2).

17. (c)



By section formula, $h = \frac{x+0}{4}, k = \frac{y+0}{4}$

$\therefore$ $x = 4h, y = 4k$

Substituting in $y^2 = 4x, (4k)^2 = 4(4h) \Rightarrow k^2 = h$

Or $y^2 = x$ is required locus.

18. (c) Squaring $(\vec{a} + \vec{b} + \vec{c}) = 0$,

we get $\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 0$

$\Rightarrow$ $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow$ $2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$

$\Rightarrow$ $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$

19. (a) Here, three given lines are coplanar if they have common perpendicular

Let d.c.'s of common perpendicular be l, m, n

$\Rightarrow$ $ll_1 + mm_1 + nn_1 = 0$ . . .(i)

$ll_2 + mm_2 + nn_2 = 0$ . . .(ii)

and $ll_3 + mm_3 + nn_3 = 0$ . . .(iii)

Solving (ii) and (iii), we get

$$\frac{l}{m_2 n_3 - n_2 m_3} = \frac{m}{n_2 l_3 - n_3 l_2} = \frac{n}{l_2 m_3 - l_3 m_2} = k$$

$\Rightarrow$ $l = k(m_2 n_3 - n_2 m_3), m = k(n_2 l_3 - n_3 l_2), n = k(l_2 m_3 - l_3 m_2)$

Substituting in (i), we get

$l_1(m_2 n_3 - n_2 m_3) + m_1(n_2 l_3 - n_3 l_2) + n_1(l_2 m_3 - l_3 m_2) = 0$

$$\Rightarrow \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$$

$$\Rightarrow -\begin{vmatrix} l_1 & n_1 & m_1 \\ l_2 & n_2 & m_2 \\ l_3 & n_3 & m_3 \end{vmatrix} = 0$$

20. (b) $\sim (p \vee (\sim q)) \equiv \sim p \wedge \sim (\sim q) \equiv (\sim p) \wedge q$.

21. (a) $\log_4 \left\{\log_2 (\sqrt{x+8} - \sqrt{x})\right\} = 0$

$\Rightarrow$ $4^0 = \log_2 \left(\sqrt{x+8} - \sqrt{x}\right)$

$\Rightarrow$ $2^1 = \sqrt{x+8} - \sqrt{x}$

$\Rightarrow$ $4 = x + 8 + x - 2\sqrt{x^2 + 8x}$

$\Rightarrow$ $2\sqrt{x^2 + 8x} = 2x + 4$

$\Rightarrow$ $x^2 + 8x = x^2 + 4 + 4x$

$\Rightarrow$ $4x = 4$

$\Rightarrow$ $x = 1.$

22. (a) Four first prizes can be given in 20^4 ways since first prize of Mathematics can be given in 20 ways, first prize of Physics also in 20 ways, similarly first prizes of Chemistry and English can be given in 20 ways each. (Note that a boy can stand first in all the four subjects).

Then two second prizes can be given in 19^2 ways since a boy cannot get both the first and second prizes. Hence the required number of ways

$= 20^4 \times 19^2$.

23. (d) $f(0-) = \lim_{x \to 0-} k(2x - x^2) = 0;$

$f(0+) = \lim_{x \to 0+} \cos x = 1$

$\therefore \quad f(0) = \cos x = 1$

Hence no value of k can make $f(0-) = 1$.

24. (a) Distance of point (–2, 3) from the line $x - y = 5$ is $\left|\frac{-2-3-5}{\sqrt{2}}\right| = \left|\frac{-10}{\sqrt{2}}\right| = 5\sqrt{2}.$

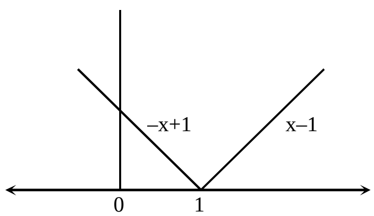
25. (a) Total number of ways to arrange 3 boys and 2 girls are 5!. According to given condition, following cases may arise.

B G G B B
G G B B B
G B G B B
G B B G B
B G B G B

So, number of favourable ways $= 5 \times 3! \times 2! = 60$

$\therefore$ Required probability $= \frac{60}{120} = \frac{1}{2}$

26. (c) From graph, $p = -1$



$\Rightarrow \lim_{x\to 1^+} g(x) = -1$

$\Rightarrow \lim_{h\to 0} g(1+h) = -1$

$\Rightarrow \lim_{h\to 0}\left(\frac{h^n}{\log\cos^m h}\right) = -1$

$\Rightarrow \lim_{h\to 0}\left(\frac{n\cdot h^{n-1}}{m\cdot(-\tan h)}\right) = -\left(\frac{n}{m}\right)\lim_{h\to 0}\left(\frac{h^{n-1}}{\tan h}\right) = -1$, which holds if $n = m = 2$.

27. (a) $g(x+1) = \log(f(x+1)) = \log x + \log(f(x)) = \log x + g(x)$

$\Rightarrow g(x+1) - g(x) = \log x$

$\Rightarrow g''(x+1) - g''(x) = -\frac{1}{x^2}$

$g''\left(1+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4$

$g''\left(2+\frac{1}{2}\right) - g''\left(1+\frac{1}{2}\right) = -\frac{4}{9}$ ....

$g''\left(N+\frac{1}{2}\right) - g''\left(N-\frac{1}{2}\right) = -\frac{4}{(2N-1)^2}$

Summing up all terms

Hence, $g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4\left(1+\frac{1}{9}+\ldots+\frac{1}{(2N-1)^2}\right).$

28. (c) $J - I = \int \frac{e^x(e^{2x}-1)}{e^{4x}+e^{2x}+1}dx = \int \frac{(Z^2-1)}{z^4+z^2+1}dz$

where $z = e^x$ $= \int \frac{\left(1-\frac{1}{z^2}\right)dz}{\left(z+\frac{1}{z}\right)^2-1} = \frac{1}{2}\ln\left(\frac{e^x+e^{-x}-1}{e^x+e^{-x}+1}\right) + c$

$\therefore \quad J - I = \frac{1}{2}\ln\left(\frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right) + c.$

29. (d) $I = \int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}$

$I = \int_0^{\pi}\left|1-2\sin\frac{x}{2}\right|dx$

$I = \int_0^{\frac{\pi}{3}}\left(1-2\sin\frac{x}{2}\right)dx + \int_{\frac{\pi}{3}}^{\pi}-\left(1-2\sin\frac{\pi}{2}\right)dx$

$1 = \left[x+4\cos\frac{x}{2}\right]_0^{\frac{\pi}{3}} + \left[-x-4\cos\frac{x}{2}\right]_{\frac{\pi}{3}}^{\pi}$

$1 = \frac{\pi}{3} + 4\left(\frac{\sqrt{3}}{2}-1\right) - \left(\pi - \frac{\pi}{3}\right) - 4\left(0 - \frac{\sqrt{3}}{2}\right)$

$I = \frac{\pi}{3} + 2\sqrt{3} - 4 - \frac{2\pi}{3} + 2\sqrt{3} - 4\sqrt{3}\,4 - -\frac{\pi}{3}$

30. (a) Rearranging the equation we get,

$\frac{dp(t)}{p(t)-400} = \frac{1}{2}dt$ . . .(i)

Integrating (1) on both sides we get

$p(t) = 400 + ke^{t/2}$,

where k is a constant of integration.

Using $p(0) = 100$, we get $k = -300$

$\therefore$ The relation is $P(t) = 400 - 300e^{1/2}$

31. (b) Inclination of line $\sqrt{3}x + y = 1$ is 150°

Inclination of line $L = 150° \pm 60° = 210°, 90°$

Slope of line

$L = \tan 210° = \tan 30° = \frac{1}{\sqrt{3}}$

Equation of = Line L

$\Rightarrow y + 2 = \frac{1}{\sqrt{3}}(x-3)$

$\Rightarrow \quad \sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

32. (d) $\sin x\frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$

$\frac{dy}{dx} + y\cot x = \frac{4x}{\sin x}$

$\therefore$ I.F $= e^{\int \cot x dx} = \sin x$

$\therefore$ Solution is given by

$y\sin x = \int \frac{4x}{\sin x} \cdot \sin x dx$

$y\sin x = 2x^2 + c$

When $x = \frac{\pi}{2}, y = 0 \Rightarrow c = -\frac{x^2}{2}$

$\therefore$ Equation is: $y\sin x = 2x^2 - \frac{\pi^2}{2}$

When $x = \frac{\pi}{6}$ then $y \cdot \frac{1}{2} = 2 \cdot \frac{\pi^2}{36} - \frac{\pi^2}{2}$

$\therefore \quad y = -\frac{8}{9}\pi^2$

33. (b) $\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore$ I.F. $= e^{\ell n(x^2+1)} = (x^2+1)$

So, general solution $y.(x^2+1) = \tan^{-1} x + c$

As y(0) = 0 $\Rightarrow$ c = 0

$\therefore \quad y(x) = \frac{\tan^{-1} x}{x^2+1}$ As, $\sqrt{a} \cdot y(1) = \frac{\pi}{32}$

$\Rightarrow \quad \sqrt{a} = \frac{1}{4} \Rightarrow a = \frac{1}{16}$

34. (c) $\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$

$\Rightarrow$ I.F. $= x^2$

$\therefore \quad yx^2 = \frac{x^4}{4} + \frac{3}{4}$ (As $y(1) = 1$)

$\therefore \quad y\left(x = \frac{1}{2}\right) = \frac{49}{16}$

35. (b) $f(xy) = f(x). f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow \quad f(x) = 1$

$\frac{dy}{dx} = f(x) = 1$

$\Rightarrow \quad y = x + c$

At, $x = 0, y = 1 \Rightarrow c = 1$

$y = x + 1$

$y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$

36. (a) $\frac{dy}{dx} + 3\sec^2 x. y = \sec^2 x$

I.F. $= e^{3\int \sec^2 x dx} = e^{3\tan x}$

or $\quad y.e^{3\tan x} = \int \sec^2 x.e^{3\tan x} dx$

or $\quad y.e^{3\tan x} = \frac{1}{3}e^{3\tan x} + C \qquad$ . . .(i)

Given

$y\left(\frac{\pi}{4}\right) = \frac{4}{3}$

$\therefore \quad \frac{4}{3}.e^3 = \frac{1}{3}e^3 + C$

$\therefore \quad C = e^3$

New put $x = -\frac{\pi}{4}$ in equation (i)

$\therefore \quad y.e^{-3} = \frac{1}{3}e^{-3} + e^3$

$\therefore \quad y = \frac{1}{3} + e^6$

$\therefore \quad y\left(-\frac{\pi}{4}\right) = \frac{1}{3} + e^6$

37. (a) $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x} (x > 0)$

given $f(1) \neq 4 \lim_{x\to 0^+} xf\left(\frac{1}{x}\right) = ?$ $\frac{dy}{dx} + \frac{3}{4}\frac{y}{x} = 7$ (This is LDE)

$IF = e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ln|x|} = x^{\frac{3}{4}}$ $y.x^{\frac{3}{4}} = \int 7. x^{\frac{3}{4x}} dx$

$y.x^{\frac{3}{4}} = 7 \cdot \frac{x^{7/4}}{7/4} + C$ $f(x) = 4x + C.x^{-3/4}$

$f\left(\frac{1}{x}\right) = \frac{4}{x} + C.x^{\frac{3}{4}}$

$\lim_{x\to 0^+} xf\left(\frac{1}{x}\right) = \lim_{x\to 0^+}\left(4 + C x^{\frac{7}{4}}\right) = 4$

38. (b) $(x^2 - y^2) dx + 2 xy dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $\quad y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

solving we get

$$\int \frac{2v}{v^2+1}dv = \int -\frac{dx}{x}$$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$y^2 + x^2 = 2x$

Hence, centre of circle lies on x-axis.

39. **(b)** $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$

I.F. $= e^{\int\left(\frac{2x+1}{x}\right)dx} = e^{\int\left(2+\frac{1}{x}\right)dx} = e^{2x+\ell nx} = e^2 x \cdot X$

So, $y(xe^{2x}) = \int e^{-2x} \cdot xe^{2x} + C$

$\Rightarrow \quad xye^{2x} = \int x\,dx + C$

$\Rightarrow \quad 2xye^{2x} = x^2 + 2C$

It passes through $\left(1, \frac{1}{2}e^{-2}\right)$ we get $C = 0$

$$y = \frac{xe^{-2x}}{2}$$

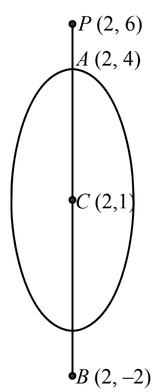
$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x+1)$

$\Rightarrow \quad f(x)$ is decreasing is $\left(\frac{1}{2}, 1\right)$

$$y(\log_e 2) = \frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$$

Hence, y(x) is decreasing in $\frac{1}{2}, 1$

40. **(b)** A→1,3; B→2,4,5; C→2,3



(A) Let $S = 9x^2 + 8y^2 - 36x - 16y - 28$

$\therefore$ Value of S at is (2, 6)

$S_1 = 9(2)^2 + 8(6)^2 - 36(2) - 16(6) - 28$

$= 36 + 288 - 72 - 96 - 28 = 128 > 0$

$\therefore$ Point (2, 6) are outside the ellipse.

The equation of the given ellipse be rewritten as

$9(x-2)^2 + 8(y-1)^2 = 72$

$$\Rightarrow \quad \frac{(x-2)^2}{8} + \frac{(y-1)^2}{9} = 1$$

Centre of ellipse is (2, 1) and axis parallel to y-axis

$\therefore$ Vertices are $x - 2 = 0$

and $y - 1 = \pm 3$

or (2, –2) and (2, 4)

$\therefore$ Minimum distance $L = PA = 2$ and maximum distance $G = PB = 8$

Then, $L + G = 10, G - L = 6$

(B) Let $S = 4x^2 + 9y^2 + 8x - 36y + 4$

$\therefore$ Value of S at (1, 2) is

$S_1 = 4(1)^2 + 9(2)^2 + 8(1) - 36(2) + 4$

$= 4 + 36 + 8 - 72 + 4 = -20 < 0$

$\therefore$ Point (1, 2) are outside the ellipse.

The equation of the given ellipse be rewritten as $4(x+1)^2 + 9(y-2)^2 = 36$

$$\Rightarrow \quad \frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$$

Centre of ellipse is (–1, 2) and axis parallel to x-axis

$\therefore$ Vertices are $x + 1 = \pm 3$

and $y - 2 = 0$ or (–4, 2) and (2, 2)

$\therefore$ Minimum distance $L = PA = 1$ and maximum distance

$G = PA' = AA' - PA = 6 - 1 = 5$

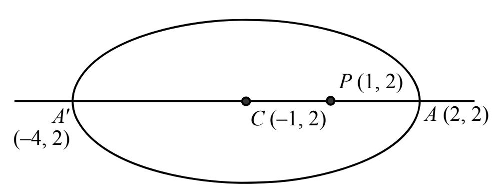
$\therefore \quad L + G = 6, G - L = 4, L^G + G^L = 6$

(C) Here $3x + 4y = 0$ and $4x - 3y = 0$ are mutually perpendicular lines,

then substituting $\frac{3x+4y}{\sqrt{3^2+4^2}} = X$

and $\frac{4x-3y}{\sqrt{(4)^2+(-3)^2}} = Y$

Then, the given equation can be written as $4X^2 + 9Y^2 = 36$



$\Rightarrow \quad \frac{X^2}{9}+\frac{Y^2}{4}=4$

$\therefore$ Vertices, $X=\pm 3, Y=0$

Or $\quad \frac{3x+4y}{5}\pm 3, \frac{4x-3y}{5}=0$

Or $\quad 3x+4y\pm 15, \ y=\frac{4x}{3}$

Vertices are $\left(\frac{9}{5},\frac{12}{5}\right)$ and $\left(-\frac{9}{5},-\frac{12}{5}\right)$

$\because$ Given point is a vertex.

$\therefore$ Minimum distance $L=0$ and maximum distance $G=$ Length of major axis

$=2\times 3=6$

Then $L+G=6, G-L=6$

41. (a) $A\rightarrow 1,3$; $B\rightarrow 5$; $C\rightarrow 2,4$

(A) $\because \ x^2-2y^2=2$

$\Rightarrow \quad \frac{x^2}{2}-\frac{y^2}{1}=1$

$\therefore$ Director circle is $x^2+y^2=2-1=1$

i.e., $x^2+y^2=1$ and $x^2+2y^2=2$

$\Rightarrow \quad \frac{x^2}{2}+\frac{y^2}{1}=1$

Director circle is $x^2+y^2=2+1=3$

i.e., $x^2+y^2=3$

(B) $\because \ 3x^2+2y^2=6$

$\Rightarrow \quad \frac{x^2}{2}+\frac{y^2}{3}=1$

$\therefore$ Director circle is $x^2+y^2=2+3=5$

i.e., $x^2+y^2=5$ and $3x^2-2y^2=6$

$\Rightarrow \quad \frac{x^2}{2}-\frac{y^2}{3}=1$

Director circle is $x^2+y^2=2-3=-1$ (not defined)

(C) $\quad 5x^2-9y^2=45$

$\Rightarrow \quad \frac{x^2}{9}-\frac{y^2}{5}=1$

$\therefore$ Director circle is $x^2+y^2=9-5=4$

i.e., $x^2+y^2=4$ and director circle of $x^2+y^2=1$ is $x^2+y^2=2$

Put $\quad x=\tan\theta \Rightarrow dx=\sec^2\theta\, d\theta$

$\therefore \quad I=2\int_0^{\pi/4} d\theta=\frac{\pi}{2}$

(B) Let $I=\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Put $\quad x=\sin\theta \Rightarrow dx=\cos\theta\, d\theta$

$\therefore \quad I=\int_0^{\pi/2} 1\, d\theta=\frac{\pi}{2}$

(C) $\int_2^3 \frac{dx}{1-x^2}=\frac{1}{2}\left[\log\left(\frac{1+x}{1-x}\right)\right]_2^3=\frac{1}{2}\left[\log\left(\frac{4}{-2}\right)-\log\left(\frac{3}{-1}\right)\right]$

$=\frac{1}{2}\left[\log\left(\frac{2}{3}\right)\right]$

(D) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}=[\sec^{-1}x]_1^2=\frac{\pi}{3}-0=\frac{\pi}{3}$

42. (b) $A\rightarrow 3,5$; $B\rightarrow 1,3$; $C\rightarrow 2,4$

(A) Since, $0<\frac{x^7}{\sqrt[3]{(1+x^8)}}<x^7 \ \forall\ 0<x<1$

Then, $\int_0^1 0\, dx<\int_0^1 \frac{x^7}{\sqrt[3]{(1+x^8)}}dx<\int_0^1 x^7 dx$

Hence, $\quad 0<\int_0^1 \frac{x^7\, dx}{\sqrt[3]{(1+x^8)}}<\frac{1}{8}$

$\therefore \quad \lambda=\frac{1}{8}, \mu=0 \quad [\lambda+\mu]=0, [\lambda-\mu]=0\in(3,5)$

(B) Since, $\sqrt{(1-x^2)}<\sqrt{(1+x^6)}<\sqrt{(1+x^2)} \ \forall\, x\in(0,1)$

$\Rightarrow \quad \frac{1}{\sqrt{(1-x^2)}}>\frac{1}{\sqrt{(1+x^6)}}>\frac{1}{\sqrt{(1+x^2)}} \ \forall\, x\in(0,1)$

$\Rightarrow \quad \int_0^1 \frac{dx}{\sqrt{(1+x^2)}}<\int_0^1 \frac{dx}{\sqrt{(1+x^6)}}<\int_0^1 \frac{dx}{\sqrt{(1-x^2)}}$

$\Rightarrow \quad \ln\{x+\sqrt{(1+x^2)}\}_0^1<\int_0^1 \frac{dx}{\sqrt{(1+x^6)}}<\{\sin^{-1}x\}_0^1$

$\Rightarrow \quad \ln 2<\int_0^1 \frac{dx}{\sqrt{(1+x^6)}}<\frac{\pi}{2}$

$\therefore \quad \lambda=\frac{\pi}{2}\approx 1.57, \mu=\ln 2\approx 0.693$

$[\lambda+\mu]=2, [\lambda-\mu]=0$ (1,3)

(C) Since, $4-x^2>4-x^2-x^3>4-2x^2 \ \forall\, x\in(0,1)$

$\Rightarrow \quad \sqrt{(4-x^2)}<\sqrt{(4-x^2-x^3)}<\sqrt{(4-2x^2)} \ \forall\, x\in(0,1)$

$$\Rightarrow \quad \frac{1}{\sqrt{(4-x^2)}} < \frac{1}{\sqrt{(4-x^2-x^3)}} < \frac{1}{\sqrt{(4-2x^2)}}\ x\ \forall \in (0,1)$$

$$\therefore \quad \int_0^1 \frac{dx}{\sqrt{(4-x^2)}} < \int_0^1 \frac{dx}{\sqrt{(4-x^2-x)}} < \int_0^1 \frac{dx}{\sqrt{(1-2x^2)}}$$

$$\Rightarrow \quad \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1 < \int_0^1 \frac{dx}{\sqrt{(4-x^2-x^3)}} < \frac{1}{\sqrt{2}}\left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1$$

$$\Rightarrow \quad \frac{\pi}{6} < \int_0^1 \frac{dx}{\sqrt{(4-x^2-x^3)}} < \frac{\pi}{4\sqrt{2}}$$

$$\therefore \quad \lambda = \frac{\pi}{4\sqrt{2}} \approx 4.43$$

and $\mu = \frac{\pi}{6} \approx 0.52$

$[\lambda+\mu]=4, [\lambda-\mu]=3 (2, 4)$

43. (b) A→3,5; B→1,4; C→2

(A) $\because\ 4x+3y-6=0$

$\Rightarrow\ -4x-3y+6=0$

and $5x+12y+9=0$

$\because\ (-4)(5)+(-3)(12)=-56<0$

$\therefore$ Bisectors are $\left(\frac{-4x-3y+6}{5}\right)=\pm\left(\frac{5x+12y+9}{13}\right)$

$\Rightarrow\ (-52x-39y+78)=\pm(25x+60y+45)$

$O: (-52x-39y+78)=+(25x+60y+45)$

or $27x-21y-123=0$

or $9x-7y-41=0$

$\therefore\ O: 9x-7y-41=0$ (5)

And $A: (-52x-39y+78)=-(25x+60y+45)$

or $77x+99y-33=0$

or $7x+9y-3=0$

$\therefore\ A: 7x+9y-3=0$ (3)

(B) $\because\ 4x-3y-6=0$

$\Rightarrow\ -4x+3y+6=0$

and $5x-12y+9=0$

$\because\ (-4)(5)+3(-12)=-56<0$

$\therefore$ Bisectors are $\left(\frac{-4x+3y+6}{5}\right)=\pm\left(\frac{5x-12y+9}{13}\right)$

$\Rightarrow\ (-52x+39y+78)=\pm(25x-60y+45)$

$O: (-52x+39y+78)=+(25x-60y+45)$

or $27x+21y-123=0$

or $9x+7y-41=0$

$\therefore\ O: 9x+7y-41=0$ (4)

And $A: (-52x+39y+78)=-(25x-60y+45)$

or $77x-99y-33=0$

or $7x-9y-3=0$

$\therefore\ A: 7x-9y-3=0$ (1)

(C) $\because\ 4x-3y+6=0$

and $5x-12y-9=0$

or $-5x+12y+9=0$

$\because\ (4)(-5)+(-3)(12)=-56<0$

$\therefore$ Bisectors are $\left(\frac{4x-3y+6}{5}\right)=\pm\left(\frac{-5x+12y+9}{13}\right)$

$\Rightarrow\ (52x-39y+78)=\pm(-25x+60y+45)$

$O: (52x-39y+78)=+(-25x+60y+45)$

or $27x+21y+123=0$

or $9x+7y+41=0$

$\therefore\ O: 9x+7y+41$

And $A: (52x-39y+78)=-(-25x+60y+45)$

or $77x-99y+33=0$

or $7x-9y+3=0$

$\therefore\ A: 7x-9y+3=0$ (2)

44. (d) $\frac{2+3i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta}$

$2-6\sin^2\theta=0$ (For purely imaginary)

$\sin^2\theta=\frac{1}{3}$

$\sin\theta=\frac{1}{\sqrt{3}},\ \theta=\sin^{-1}\frac{1}{\sqrt{3}}$

45. (b,c,d) $P=\begin{bmatrix}\omega^2 & \omega^3 & \omega^4 & \dots & \omega^{n+2}\\ \omega^3 & \omega^4 & \omega^5 & \dots & \omega^{n+3}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \omega^{n+2} & \omega^{n+3} & \dots & \dots & \omega^{2n+4}\end{bmatrix}$

$$P^2=\begin{bmatrix}\omega^4+\omega^6\dots & \omega^5+\omega^7+\omega^9 & \dots & \dots\\ \omega^5+\omega^7+\omega^9\dots & \dots & \dots & \dots\\ \vdots & \vdots & \vdots & \dots\\ \omega^{n+4}+\omega^{n+6}\dots & \dots & \dots & \omega^{2n+4}+\omega^{2n+6}\dots\end{bmatrix}$$

P^2 = Null matrix if n is a multiple of 3

46. (b) Given m is AM. between ℓ and n

$\Rightarrow$ $2m = \ell + n$

Given ℓ, G_1, G_2, G_3, n in G.P.

$$r = \left(\frac{n}{\ell}\right)^{1/4} \Rightarrow r^4 = \frac{n}{\ell}$$

$\therefore$ $G_1 = \ell r, G_2 = \ell r^2, G_3 = \ell r^3$

So, $G_1^4 + 2G_2^4 + G_3^4 = \ell^4 r^4[1+2r^4+r^8]$

$$= \ell^4 \cdot \left(\frac{n}{\ell}\right)\left[1+2\left(\frac{n}{\ell}\right)+\left(\frac{n}{\ell}\right)^2\right]$$

$$= n\ell^3\left[1+\frac{n}{\ell}\right]^2 = n\ell^3\frac{(n+\ell)^2}{\ell^2} = n\ell(2m)^2 = 4\ell m^2 n$$

47. (a,d) $S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}} k^2 \sum_{r=0}^{(n-1)}((4r+4)^2+(4r+3)^2-(4r+2)^2-(4r+1)^2)$

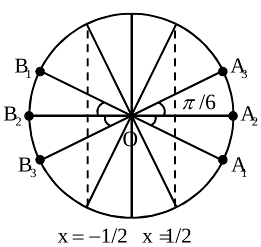
$$= \sum_{r=0}^{(n-1)}(2(8r+6)+2(8r+4))$$

$$= \sum_{r=0}^{(n-1)}(32r+20)$$

$$= 16(n-1)n + 20n$$

$$= 4n(4n+1) = \begin{cases} 1056 \text{ for } n=8 \\ 1332 \text{ for } n=9 \end{cases}$$

48. (a,b,c) $\log_2 3^x = (x-1)\log_2 4 = 2(x-1)$



$\Rightarrow$ $x\log_2 3 = 2x - 2$

$\Rightarrow$ $x = \dfrac{2}{2-\log_2 3}$

Rearranging, we get

$$x = \frac{-2}{2-\dfrac{1}{\log_3 2}} = \frac{2\log_3 2}{2\log_3 2 - 1}$$

Rearranging again,

$$x = \frac{\log_3 4}{\log_3 4 - 1} = \frac{\dfrac{1}{\log_4 3}}{\dfrac{1}{\log_4 3} - 1} = \frac{1}{1-\log_4 3}.$$

49. (b) Case-1 Any 5 – digit number > 6000 is all 5 -digits number

Total number > 6000 using 5-digits = 5! = 120

Case-2: Using 4-digits

Can be 6.7 or 8   4 ways   3 ways   2 ways

i.e., 3 ways Total number $= 3\times4\times3\times2 = 72$

Total ways $= 120 + 72 = 192$

50. (b) $\dfrac{dy}{dx} = \dfrac{y}{x} = \ell nx$

$e^{\int \frac{1}{x}dx} = x$

$xy = \int x\ell nx + C$

$\ell nx\dfrac{x^2}{2}\int\dfrac{1}{x}\cdot\dfrac{x^2}{2}$

$xy = \dfrac{x}{2}\ell nx - \dfrac{x^2}{4} + C$, for

$2y(2) = 2\ell n2 - 1$

$\Rightarrow$ C = 0

$y = \dfrac{x}{2}\ell nx - \dfrac{x}{4}$

$y(e) = \dfrac{e}{4}$