Pair of Linear Equations in Two Variables

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1. Half the perimeter of a rectangle garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Ans. Let length of rectangular garden = x metres
Let width of rectangular garden = y metres
According to given conditions, perimeter = 36 m
$\Rightarrow$ x + y = 36 ……(i)
And x = y+4
$\Rightarrow x - y = 4$ ……..(ii)
Adding eq. (i) and (ii),
2x = 40
$\Rightarrow$ x = 20 m
Subtracting eq. (ii) from eq. (i),
2y = 32
$\Rightarrow$ y = 16 m
Hence, length = 20 m and width = 16 m

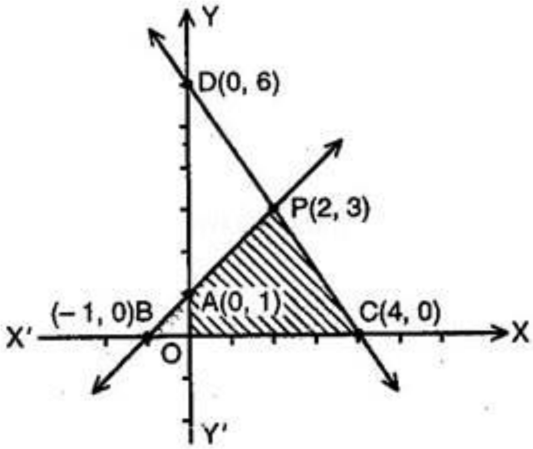
2. Draw the graphs of the equations x – y+1=0 and 3x+2y – 12=0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Ans. For equation x – y + 1 = 0, we have following points which lie on the line

x	0	-1
y	1	0

For equation 3x + 2y – 12 = 0, we have following points which lie on the line

x	4	0
y	0	6



We can see from the graphs that points of intersection of the lines with the x–axis are (–1, 0), (2, 3) and (4, 0).

3. The age of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Ans. Let the age of Ani and Biju be x years and y years respectively.

Age of Dharam = 2x years and Age of Cathy = $\frac{y}{2}$ years

According to question,

x – y = 3 … (1)

And $2x-\frac{y}{2}=30$

$\Rightarrow$ 4x – y = 60 … (2)

Subtracting (1) from (2), we obtain:

3x = 60 – 3 = 57

$\Rightarrow$ x = Age of Ani = 19 years

Age of Biju = 19 – 3= 16 years

Again, According to question, y – x= 3 … (3)

And $2x-\frac{y}{2}=30$

$\Rightarrow$ 4x – y = 60 … (4)

Adding (3) and (4), we obtain:

3x = 63

$\Rightarrow$ x = 21

Age of Ani = 21 years

Age of Biju = 21 + 3 = 24 years

4. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital?

Ans. Let the money with the first person and second person be Rs x and Rs y respectively.
According to the question,
$x + 100 = 2(y - 100)$
$\Rightarrow x + 100 = 2y - 200$
$\Rightarrow x - 2y = 300$ ... (1)
Again, $6(x - 10) = (y + 10)$
$\Rightarrow 6x - 60 = y + 10$
$\Rightarrow 6x - y = 70$ ... (2)
Multiplying equation (2) by 2, we obtain:
$12x - 2y = 140$ ... (3)
Subtracting equation (1) from equation (3), we obtain:
$11x = 140 + 300$
$\Rightarrow 11x = 440$
$\Rightarrow x = 40$
Putting the value of x in equation (1), we obtain:
$40 - 2y = -300$
$\Rightarrow 40 + 300 = 2y$
$\Rightarrow 2y = 340$
$\Rightarrow y = 170$
Thus, the two friends had Rs 40 and Rs 170 with them.

5. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.
Ans. Let the number of rows be x and number of students in a row be y.
Total number of students in the class = Number of rows x Number of students in a row = xy
According to the question,
Total number of students = $(x - 1)(y + 3)$
$\Rightarrow xy = (x - 1)(y + 3)$
$\Rightarrow xy = xy - y + 3x - 3$
$\Rightarrow 3x - y - 3 = 0$
$\Rightarrow 3x - y = 3$ ... (1)
Total number of students = $(x + 2)(y - 3)$
$\Rightarrow xy = xy + 2y - 3x - 6$
$\Rightarrow 3x - 2y = -6$ ... (2)
Subtracting equation (2) from (1), we obtain:
$y = 9$

Substituting the value of y in equation (1), we obtain:
$3x - 9 = 3$
$\Rightarrow 3x = 9 + 3 = 12$
$\Rightarrow x = 4$
Number of rows = x = 4
Number of students in a row = y = 9
Hence, Total number of students in a class = xy = 4 x 9 = 36

6. Find the values of α and β for which the following system of linear equations has infinite number of solutions, $2x + 3y = 7$, $2\alpha x + (\alpha+\beta) y = 28$.

Ans. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (Infinite solution)

$$\frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{-7}{-28}$$

$\Rightarrow \alpha = 4$, and $\beta = 8$

7. Find the condition for which the system of equations $\frac{x}{a} + \frac{y}{b} = c$ and $bx + ay = 4ab$ ($a, b \neq 0$) is inconsistent.

Ans. Inconsistent

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1/a}{b} = \frac{1/b}{a} \neq \frac{c}{4ab}$$

i.e. $\frac{1}{ab} = \frac{1}{ab} \neq \frac{c}{4ab}$

or $c \neq 4$

8. Find the value of 'α' so that the following linear equations have no solution

$(3\alpha+1)x + 3y - 2 = 0$, $(\alpha^2+1)x + (\alpha-2)y - 5 = 0$

Ans. No solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

i.e. $\dfrac{3\alpha + 1}{\alpha^2 + 1} = \dfrac{3}{\alpha - 2} \neq \dfrac{-2}{-5}$

$$3\alpha^2 - 6\alpha + \alpha - 2 = 3\alpha^2 + 3$$

$$-5\alpha = 5$$

or $\alpha = -1$

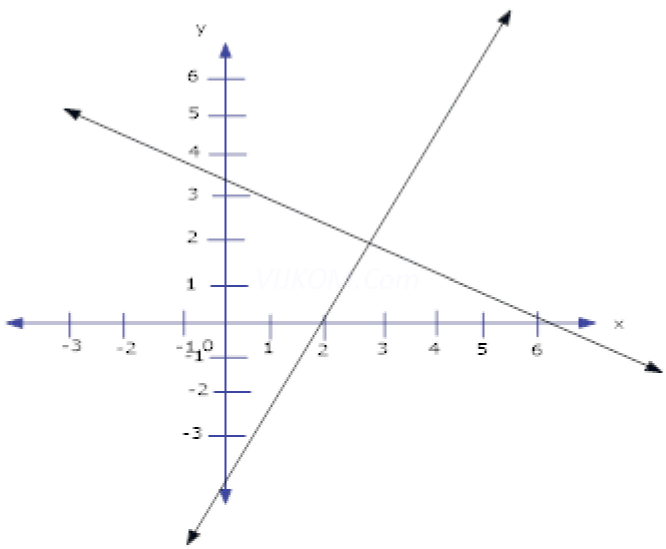
or $\dfrac{3}{\alpha - 2} \neq \dfrac{2}{5}$

$\Rightarrow \ \alpha \neq \dfrac{19}{2}$

9. Solve for x and y: ax + by = a – b and bx – ay = a + b

Ans. $ax + by = a - b \ ]\times a$

$bx - ay = a + b \ ]\times b$



$$a^2x + aby = a^2 - ab$$
$$bx - aby = ab + b^2$$
$$\overline{(a^2 + b^2)x = a^2 + b^2}$$

$$\Rightarrow \; x = 1$$

$$\therefore \; a + by = a - b$$

$$by = -b$$

$$y = -1$$

$$\therefore \; x = 1$$

$$y = -1$$

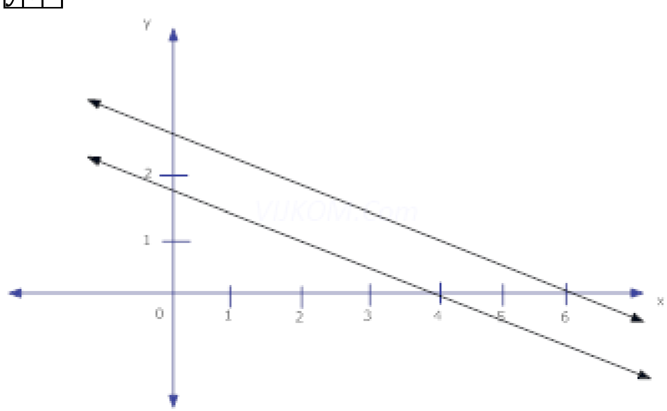
10. The path of a train A is given by the equation x + 2y – 4 = 0 and the path of another train B is given by the equation 2x + 4y – 12 = 0 represent this situation graphically.

Ans. $x + 2y - 4 = 0$

$2x + 4y - 12 = 0$

$when\, x = 4 - 2y$

x	4	2
y	0	1



$$when\; x = \frac{12 - 4y}{2}$$

x	6	2
y	0	2

11. For what value of 'α 'the system of linear equations α .x + 3y = α – 3, 12x + α y = α has no solution.

Ans. $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$i.e.\ \frac{\alpha}{12}=\frac{3}{\alpha}\neq\frac{\alpha-3}{\alpha}$

$If\ \frac{\alpha}{12}=\frac{3}{\alpha}\Rightarrow\alpha^2=36$

or $\alpha=\pm 6\ \rightarrow\ (i)$

$If\ \frac{3}{\alpha}\neq\frac{\alpha-3}{\alpha}$

$\Rightarrow\alpha^2-3\alpha\neq 3\alpha$

or $\alpha^2\neq 6\alpha$

or $\alpha=0$ and $\alpha=6\ \rightarrow\ (ii)$

$\therefore$ from eq (i) and (ii)

The value of α is 6.

12. Find the values of 'a' and 'b' for which the following system of linear equations has infinite number of solutions. 2x + 3y = 7, (a + b + 1) x +(a + 2b + 2) y = 4 (a + b) +1

Ans. If infinite number of number of solutions

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

or $\frac{2}{a+b+1}=\frac{3}{a+2b+2}=\frac{7}{4a+4b+1}$

If $\frac{2}{a+b+1}=\frac{3}{a+2b+2}$

$\Rightarrow\ a-b=1$

and if $\frac{3}{a+2b+2}=\frac{7}{4a+4b+1}$

$\Rightarrow\ 5a-2b=11$

on solving we get,

$a=3$ and $b=2$

13. Solve for 'x' and 'y' where x + y = a – b, ax – by = $a^2 + b^2$

Ans. $x + y = a - b$

and $ax - by = a^2 + b^2$

$$bx + by = ab - b^2$$

$$ax \underset{+}{-} by = a^2 + b^2$$

$$\cancel{(a+b)}x = a\cancel{(a+b)}$$

$x = a$

$x + y = a - b$

$a + y = a - b$

$y = -b$

14. The cost of two kg of apples and 1 kg of grapes on a day was found to be Rs. 160. After a month the cost of 4 kg apples and 2 kg grapes is Rs. 300. Represent the situation algebraically and graphically.

Ans. Let the cost of one Kg of apple is x and one Kg of grapes is y.

According to question,

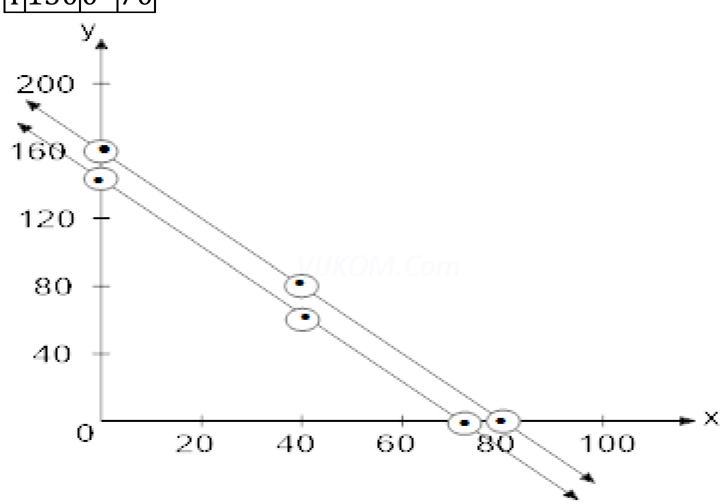
2x + y = 160 and 4x + 2y = 300

2x + y = 160

x	0	80	40
Y	160	0	80

4x + 2y = 300

x	0	75	40
Y	150	0	70



15. Find the value of 'k' for which the system of equation kx + 3y = k – 3 and 12x + ky = k will have no solution.

Ans. $kx+3y=k-3$

$12x+ky=k$

The system has no solution.

If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\frac{k}{12}=\frac{3}{k}\neq\frac{k-3}{k}$

$k^2=36$

$\Rightarrow k=\pm 6$ (i)

If $\frac{3}{k}\neq\frac{k-3}{k}$

$3k\neq k^2-3k$

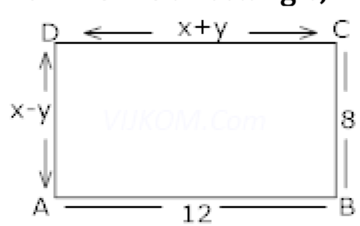
$k^2-6k\neq 0$

$k(k-6)\neq 0$

$k\neq 6\ (ii)$

$\therefore\ k=-6$

16. ABCD is a rectangle, find the values of x and y.



Ans. $x+y=12...(i)$

$x-y=8...(ii)$

on adding (i) and (ii),

$2x=20$

$\Rightarrow x=10$

$\therefore\ 10+y=12$

$\Rightarrow y=2$

17. **Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variable such that the geometrical representation of the pair so formed is**
(a) intersecting lines
(b) parallel lines
(c) overlapping

Ans. 2x + 3y – 8 = 0 another linear equation representing.

(i) Intersecting lines is $x+3y=8$

(ii) Parallel lines is $4x+6y=3$

(iii) Overlapping lines is $6x+9y=24$

18. **Find the value of 'k' for which the system of equation has infinitely many solutions 2x + (k – 2) y = k and 6x + (2k – 1) y = 2k + 5**

Ans. for infinitely many solution

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

i.e. $\frac{2}{6}=\frac{k-2}{2k-1}=\frac{k}{2k+5}$

if $\frac{1}{3}=\frac{k-2}{2k-1}$

$2k-1=3k-6$

$k=5$

or if $\frac{k}{2k+5}=\frac{1}{3}$

or $3k=2k+5$

$k=5$

if $\frac{k-2}{2k-1}=\frac{k}{2k+5}$

$\Rightarrow 2k^2+5k-4k-10$

$=2k^2-k$

19. **Find the relation between a, b, c and d for which the equations ax + by = c and cx + dy = a have a unique solution.**

Ans. $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ i.e. $\frac{a}{c}\neq\frac{b}{d}$

or $ad\neq bc$

20. Solve for ‘x’ and ‘y’:

(a – b) x + (a + b) y = a² – b² – 2ab

(a + b) (x + y) = a² + b²

Ans.

$$(a-b)x+(a+b)y=a^2-b^2-2ab$$

$$\underline{(a+b)x+(a+b)y=a^2+b^2}$$

$$-2bx \qquad\qquad = -2b(b+a)$$

$$x=a+b$$

$$\therefore\ (a-b)(a+b)+(a+b)y=a^2-b^2-2ab$$

$$a^2-b^2+(a+b)y$$

$$=a^2-b^2-2ab$$

$$y=\frac{-2ab}{a+b}$$