CHAPTER 15

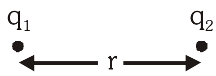
ELECTROSTATICS

ELECTRIC CHARGE

Charge of a material body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative. S.I. unit is coulomb. Charge is quantized, conserved, and additive.

COULOMB'S LAW

Force between two charges $\vec{F} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2} \hat{r}$



If medium is present then multiply $\in_0$ with $\in_r$ where $\in_r$ =relative permittivity

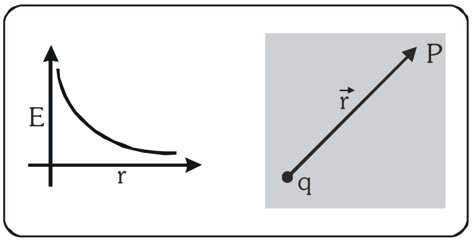
NOTE : The Law is applicable only for static and point charges. Moving charges may result in magnetic interaction. And if charges are spread on bodies then induction may change the charge distribution.

ELECTRIC FIELD OR ELECTRIC INTENSITY OR ELECTRIC FIELD STRENGTH (Vector Quantity)

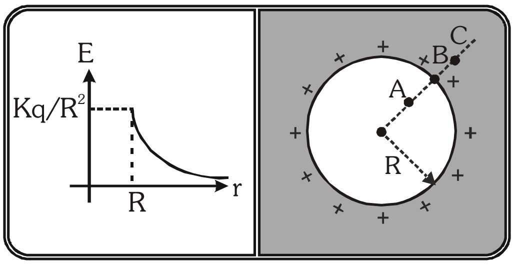
It is the net force on unit positive charge due to all other charges. $\vec{E} = \frac{\vec{F}}{q}$ unit is N/C or V/m.

ELECTRIC FIELD DUE TO SPECIAL CHARGE DISTRIBUTION

(a) Point charge $E = \frac{kq}{r^2}$



(b) Charged conducting sphere :

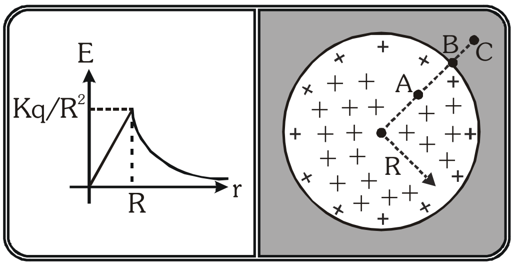


(i) $E_C = \frac{kq}{r^2}$; r > R for point out side the sphere

(ii) $E_B = \frac{kq}{R^2} = \frac{\sigma}{\in_0}$; r=R for point at the surface of the sphere

(iii) $E_A = 0$; r < R for point inside the sphere

(c) Uniformly charged non conducting sphere:

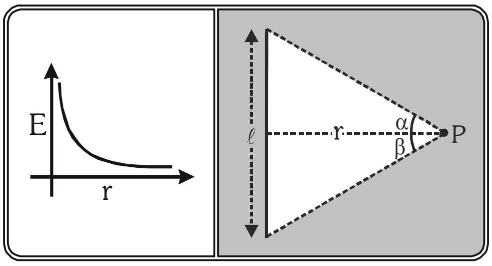


(i) $E_C = \frac{kq}{r^2}$; r > R

(ii) $E_B = \frac{kq}{R^2}$; r = R

(iii) $E_A = \frac{kqr}{R^3}$; r < R

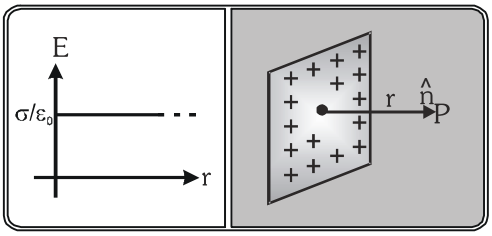
(d) Linear charge distribution of length ' ℓ '



$$E_P = \frac{\lambda \sin\left(\frac{\alpha+\beta}{2}\right)}{2\pi \in_0 r} = \frac{2k\lambda}{r} \sin\left(\frac{\alpha+\beta}{2}\right)$$

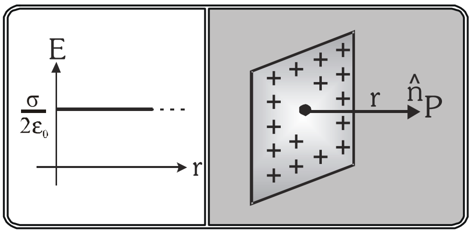
For infinite line of charge : $\vec{E}_P = \frac{2k\lambda}{r} \hat{r}$

(e) Infinite charged conducting plate



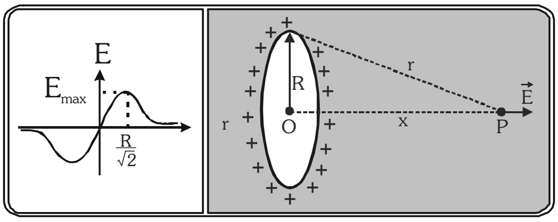
$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$$

(f) Infinite sheet of charge (or charged non conducting plate)



$$\vec{E} = \frac{\sigma}{2\,\epsilon_0}\hat{n}$$

(g) Charged circular ring at an axial point :

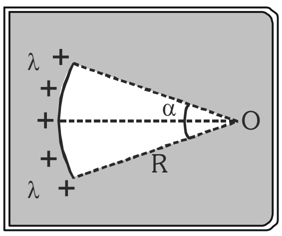


$$E_P = \frac{kQx}{\left(R^2 + x^2\right)^{3/2}}$$

Field will be maximum at $x = \pm\frac{R}{\sqrt{2}}$

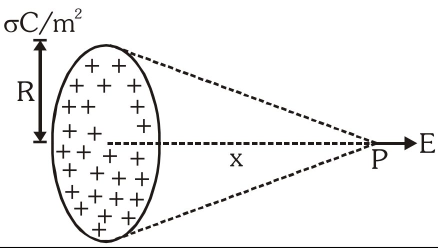
At centre of ring x = 0 so $E_0 = 0$

(h) Segment of ring :



$$E_0 = \frac{2k\lambda}{R}\sin\left(\frac{\alpha}{2}\right)$$

(i) Due to charged disk



$$E_P = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

Null point for two charges :

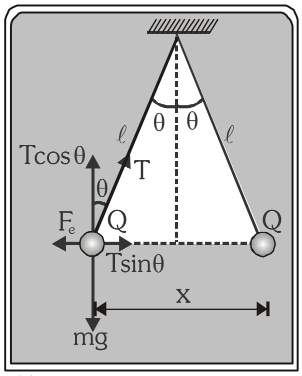
$$\underset{Q_1}{\bullet}\xleftrightarrow{\quad r \quad}\underset{Q_2}{\bullet}$$

If $|Q_1| > |Q_2| \Rightarrow$ Null point near Q_2 (Smaller charge)

$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} \pm \sqrt{Q_2}}r \text{ (distance of null point from } Q_1)$$

(+) for like charges; (–) for unlike charges

❒ **Equilibrium of suspended point charge ball system**

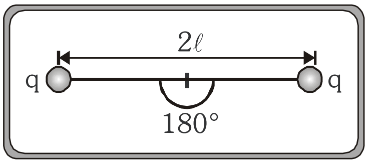


For equilibrium position $T\cos\theta = mg$ &

$$T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

- If whole set up is taken into an artificial satellite

$(g_{eff} \simeq 0)$ $T = F_e = \frac{kq^2}{4\ell^2}$



ELECTRIC FLUX

$$\phi = \int \vec{E}.d\vec{A}$$

(i) For uniform electric field; $\phi = \vec{E}.\vec{A} = EA\cos$ where θ = angle between $\vec{E}$ & area vector $(\vec{A})$. Flux is contributed only due to the component of electric field which is perpendicular to the plane.

(ii) If $\vec{E}$ is not uniform throughout the area A, then $\phi = \int \vec{E}.d\vec{A}$

(iii) $d\vec{A}$ represent area vector normal to the surface and pointing outwards from a closed surface.

Gauss's Law

$\oint \vec{E}.d\vec{s} = \frac{\sum q}{\epsilon_0}$ (Applicable only to closed surface)

$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\varepsilon_0}}$$

where q_{en}= net charge enclosed by the closed surface .

ϕ does not depend on the

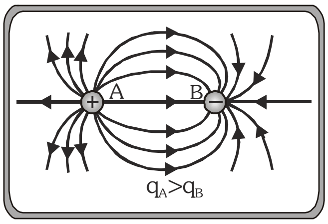
(i) Shape and size of the closed surface

(ii) The charges located outside the closed surface.

(iii) Electric field depends on charges both inside and outside the surface.

- **Electric field intensity at a point near a charged conductor** : $E = \frac{\sigma}{\epsilon_0}$
- **Electrostatics pressure on a charged conductor** : $P = \frac{\sigma^2}{2 \epsilon_0}$
- **Energy density in electric field:** $u_E = \frac{1}{2} \epsilon_0 E^2$

ELECTRIC FIELD LINES



Electric lines of electrostatic field have following properties

(i) Imaginary lines

(ii) Never intersect each other

(iii) Electrostatic field lines never forms closed loops

(iv) Field lines ends or starts normally at the surface of a conductor.

(v) If there is no electric field there will be no field lines.

(vi) Number of electric field lines per unit area normal to the area at a point represents magnitude of intensity,crowded lines represent strong field while distant lines weak field.

(vii) Tangent to the line of force at a point in an electric field gives the direction of intensity of electric field.

POTENTIAL DIFFERENCE

The potential difference between two points A and B is work done by external agent against electric field in taking a unit positive charge from B to A without acceleration (or keeping Kinetic Energy constant or $K_i = K_f$)) $V_A - V_B = \frac{(W_{BA})_{ext}}{q}$.

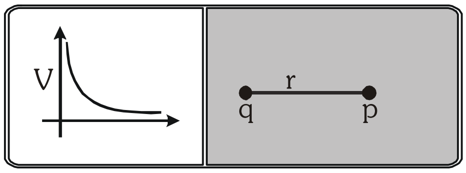
Electric potential

It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point without gaining any kinetic energy

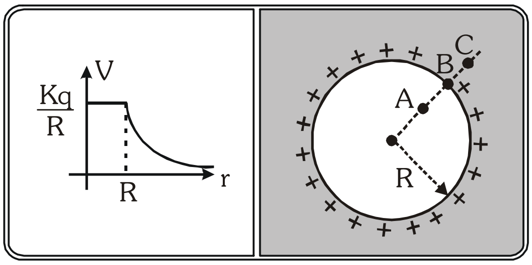
$$V_P = \left[\frac{(W_{\infty-P})_{ext}}{q}\right]$$

Potential Due to Special Charge Distribution

(i) Point charge $V = \frac{kq}{r}$



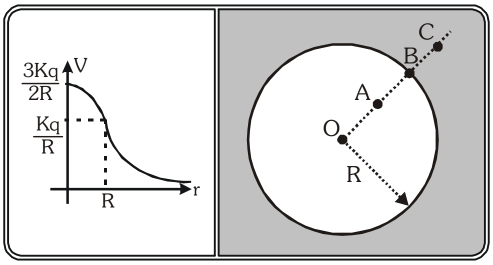
(ii) Charged conducting sphere



(a) $V_C = \frac{kq}{r}$; $r > R$ (b) $V_B = \frac{kq}{R}$; $r = R$

(c) $V_A = \frac{kq}{R}$; $r < R$

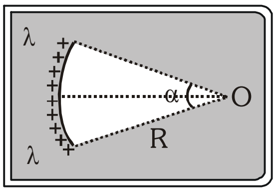
(iii) Uniformly charged non conducting sphere



(a) $V_C = \frac{kq}{r}$; $r > R$ (b) $V_B = \frac{kq}{R}$; $r = R$

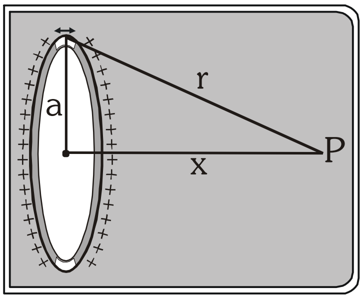
(c) $V_A = \frac{kq\left[3R^2 - r^2\right]}{2R^3}$; $r < R$

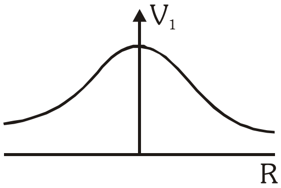
(iv) Segment of ring



$$V_0 = \frac{k\lambda.\alpha}{R} = \frac{kQ}{R} \text{ when } Q = \lambda\alpha R$$

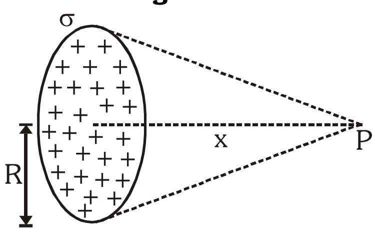
(v) Electric Potential at a Distant Point Along The Axis of a Charged Ring





$$V = \frac{q}{4\pi\varepsilon_0(a^2+x^2)^{1/2}} = \frac{kq}{r}$$

(vi) due to charged disk :-



$$V_P = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R^2} - x\right)$$

Relation between $\vec{E}$ & V

$$\vec{E} = -\text{grad } V = -\vec{\nabla}V, \quad E = -\frac{\partial V}{\partial r};$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}, \quad V = \int -\vec{E}.\vec{dr}$$

Electric potential energy of two charges

It is the amount of work required to bring the two point charges to a particular separation from ∞ without change in KE.

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

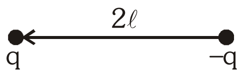
Equipotential Surface and Equipotential Region

In an electric field the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field line meet at right angles. The region where E = 0, Potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region like conducting bodies.

Electric dipole

If equal and opposite point charges are placed at very small separation then system is known as dipole.

Electric dipole moment = $q2\ell$



1. It is a vector quantity
2. Direction is from –ve to +ve charge

Electric dipole in uniform electric field

1. Torque $\vec{\tau} = \vec{p}\times\vec{E}$ or $\tau = pE\sin\theta$
2. Work done in rotation to dipole from θ_1 to θ_2 angle in external electric field.
   $$W = pE(\cos\theta_1 - \cos\theta_2)$$
3. Potential energy = $-pE\cos\theta$

Note :___________________________________

- In uniform field, force on a dipole = 0, torque may or may not be zero.
- In general in non-uniform field, force on a dipole ≠ 0 and torque may or may not be zero
- In non uniform $\vec{E}, F_e = \vec{p}.\frac{\overline{dE}}{dr}$

Electric field and potential due to dipole

	Electric field	Potential
1. at axial	$\frac{2kp}{r^3}$	$\frac{kp}{r^2}$
2. at equitorial	$\frac{kp}{r^3}$	0
3. at general position	$\frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$	$\frac{kp\cos\theta}{r^2}$

KEY POINTS

- Electric field in the bulk of the conductor (volume) is zero while it is perpendicular to the surface in electrostatics.
- Excess charge resides on the free surface of conductor in electrostatic condition.
- Potential throughout the volume of the conductor is same in electrostatics.
- Charge density at convex sharp points on a conductor is greater. Lesser is radius of curvature at a convex part, greater is the charge density.
- Potential difference between two points in an electric field does not depend on the path between them.
- Potential at a point due to positive charge is positive & due to negative charge is negative.
- When $\vec{P}$ is parallel to $\vec{E}$ then the dipole is in stable equilibrium
- When $\vec{P}$ is antiparallel to $\vec{E}$ then the dipole is in unstable equilibrium
- Self potential energy of a charged conducting spherical shell = $\frac{KQ^2}{2R}$.
- A spherically symmetric charge {i.e ρ depends only on r} behaves as if its charge is concentrated at its centre (for outside points).
- **Dielectric strength of material :** The minimum electric field required to ionize the medium or the maximum electric field which the medium can bear without break down.
- The particles such as photon or neutrino which have no (rest) mass can never has a charge because charge cannot exist without mass.
- Electric charge is invariant because value of electric charge does not depend on frame of reference.
- A charged particle is free to move in an electric field. It may or may not move along an electric line of force because initial conditions affect the motion of charged particle.
- Electrostatic experiments do not work well in humid days because water is a good conductor of electricity.
- A metallic shield in form of a hollow conducting shell may be built to block an electric field because in a hollow conducting shell, the electric field is zero at every point.